MATH 5490 — HPC-xTC

Home Work #2 **Group 5** Jorge Flores, Xiukun Hu, Grigorii Sarnitskii

1 Step 2

1.1 Results

The code is capable with square blocks (i.e. with `blk_rows` and `blk_cols` to be the same) and multiple blocks in A and B. The multiplication result will be written to disk in file(s) `C.*.*`, and can be printed on screen.

The result is examed by Matlab for 100×100, 1000×1000 and 5000×5000 block multiplication. The absolute difference between two results is less than $1e-11$ for all tests.

Total time spent for 5000×5000 matrices multiplication for this code is $1.37e3$ seconds and computing time is $1.36e3$ seconds, while for Matlab the computing time is only 4.28 seconds.

1.2 Requirement

Unzip the folder and every file needed is inside. Use `make run` command to run it.

1.3 Compute Bound

For MacBook Pro with 2.7GHz Intel Core i5, 8G DDR3 memory, it gets compute bound when the block is over 50×50.

For Lenovo with 2.4GHz Intel Core i7, running in a 64bit Red Hat virtual machine, it gets compute bound when the block is over 50×50.

2 Step 3

2.1 Algorithm

The pseudocode is as follow:

```
OMP PARALLEL
    OMP single
        ablock[tog] ← READ(A_0_0)                                   ▷ tog initialized as 0
        bblock[tog] ← READ(B_0_0)
    ENDOMP single                                                   ▷ Implicit barrier here
    while i < rows of blocks in C do
        OMP single nowait
            ablock[1 − tog] ←READ(next A block)
            bblock[1 − tog] ←READ(next B block)
        ENDOMP single nowait
        OMP for nowait dynamic
            for every continuous WIDTH (macro) elements in ablock do
                for each column of B do
                    temp+ = WIDTH elements of ablock[tog]∗WIDTH rows of bblock[tog]
                    OMP atomic update
                        cblock[ctog] corresponding element + = temp
                    ENDOMP atomic update
                end for
            end for
        ENDOMP for nowait dynamic
        if blk_cols cannot be divided by WIDTH then
            OMP for nowait dynamic
                for each row of the remainder columns in ablock[tog] do
                    update cblock[ctog]
                end for
            ENDOMP for nowait dynamic
        end if
      OMP BARRIER
        if k == columns of blocks in A then
            OMP single nowait
                WRITE cblock[ctog]
                fill cblock[ctog] with zeros
            ENDOMP single nowait
            ctog = 1 − ctog
        end if
        tog = 1 − tog
        update i, j, k to point to next block in A and B;
    end while
ENDOMP PARALLEL
```

2.2 Results

The code works for square blocks (i.e. with `blk_rows` and `blk_cols` being equal) and multiple blocks in A and B. The multiplication result will be written to disk in file(s) of the form `C.*.*`.

The result was verified to be correct using MATLAB for 1000×1000 block size and 10×10 blocks matrix multiplication.

We performed a one dimensional analysis on the performance of our code using different block sizes and block matrix sizes. These tests where performed on an Intel(R) Xeon(R) CPU E5-4650 @2.70GHz with 8 cores and 2 threads per core (for a total of 16 threads). The results are summarized in the table below, were the total time represents the time it took our code to multiply the matrices and write the resulting blocks of C to disk.

Block/Matrix Size Analysis		
Block Size	Block Matrix Size	Total Time (seconds)
100x100	1x1	1.130199×10^{-2}
100x100	2x2	5.166101×10^{-2}
100x100	5x5	6.825109×10^{-1}
1000x1000	1x1	2.701371×10^{-1}
1000x1000	2x2	1.635549
1000x1000	5x5	22.65355

It is obvious that the total time is $O(n^3)$.

We also performed a one dimensional analysis, this time focusing on the performance of our code using different thread numbers. These tests were performed on an Intel(R) Xeon(R) CPU E5-4650 @2.70GHz with 8 cores and 2 threads per core (for a total of 16 threads). The results are summarized in the table below.

Thread Number Analysis			
Number of Threads	Block Block Size	Block Matrix Size	Total Time (seconds)
4	1000x1000	5x5	43.33175
8	1000x1000	5x5	22.61952
12	1000x1000	5x5	20.78736
16	1000x1000	5x5	23.37497

We note that increasing the thread number from 4 threads to 8 threads, the total time nearly halved. However, changing from 8 to 12 and 16 threads, we see only a small change in the total time. This might be due to the fact that the system we used only has 8 physical cores.

2.3 Requirement

Unzip the folder called hw2.zip, every file needed to run the code is inside. Use `make matrix` to first generate and write to disk the matrices (block entries for each matrix) that will be used. (Notice that the block should be square.) Then use `make run` to multiply the matrices in parallel and write the resulting matrix (in blocks) to the disk. Alternatively, use `make runserial` to multiply the matrices in serial.

2.4 Compute Bound

For a MacBook Pro with 2.7GHz Intel Core i5, 8G DDR3 memory, compute bound is achieved when the block size is over 40×40.

For a Lenovo with 2.4GHz Intel Core i7, running in a 64bit Red Hat virtual machine, compute bound is achieved when the block size is over 40×40.

For an Intel(R) Core(TM) i3-6100U CPU @ 2.30GHz running Ubuntu, compute bound was achieved when the block size is over 50×50.

The information is summarized in the following table:

Compute Bound Analysis	
System	Block Size
Intel(R) Core(TM) i7-4700MQ CPU @ 2.40Ghz GNU bash, version 4.2.46(1)-release ($x86-64$-redhat-linux-gnu), 4 threads	above 40 x 40
Intel(R) Core(TM) i3-6100U CPU @ 2.30GHz gcc version 5.4.0 20160609 (Ubuntu 5.4.0-6ubuntu1 16.04.2)	above 50 x 50
2.7 GHz Intel Core i5 gcc-6 (Homebrew gcc 6.2.0) 6.2.0, OS X Sierra, 4 threads	above 40 x 40

3 Step 4

3.1 Algorithm

In order to further optimize our code by making it cache aware, we used the information presented in "Anatomy of High-Performance Matrix Multiplication" (Goto, Van de Geijn)[1] to modify our algorithm.

3.2 Results

We proceeded to install the ATLAS software and compare the speeds of our code in step 3, the cache aware code, and ATLAS for multiplying two matrices. The results are presented in the following tables.

Multiplication Time Comparison				
Algorithm	Block Size	Block Matrix Size	Total Time (s) (1 Thread)	Total Time (s) (2 Threads)
Step 3 Code	1024x1024	1x1	2.493216	1.35267
Cache Aware	1024x1024	1x1	1.350122	6.953728×10^{-1}
ATLAS	1024x1024	1x1	1.91020×10^{-1}	

Multiplication Time Comparison				
Algorithm	Block Size	Block Matrix Size	Total Time (s) (1 Thread)	Total Time (s) (2 Threads)
Step 3 Code	1024x1024	2x2	19.22521	10.41158
Cache Aware	1024x1024	2x2	8.220221	4.329293
ATLAS	1024x1024	2x2	8.238590×10^{-1}	

Multiplication Time Comparison				
Algorithm	Block Size	Block Matrix Size	Total Time (s) (1 Thread)	Total Time (s) (2 Threads)
Step 3 Code	2048x2048	1x1	24.71400	13.03327
Cache Aware	2048x2048	1x1	10.68876	5.942914
ATLAS	2048x2048	1x1	9.003758×10^{-1}	

We note that, as expected, we see speed ups in the multiplication time, which become more noticeable as we increase the block size and the block matrix size.

3.3 Requirement

Basically same as section 2.3. In addition:

(i) In order to run step 2 code, type

```
$ make run TARGET=MMultiple2
```

change the number `2` to `3` or `4` to run step 3 or 4 instead.

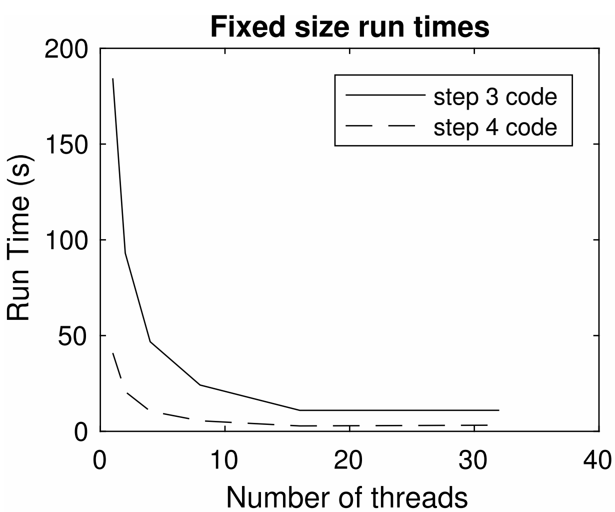
(ii) In order to run atlas, type

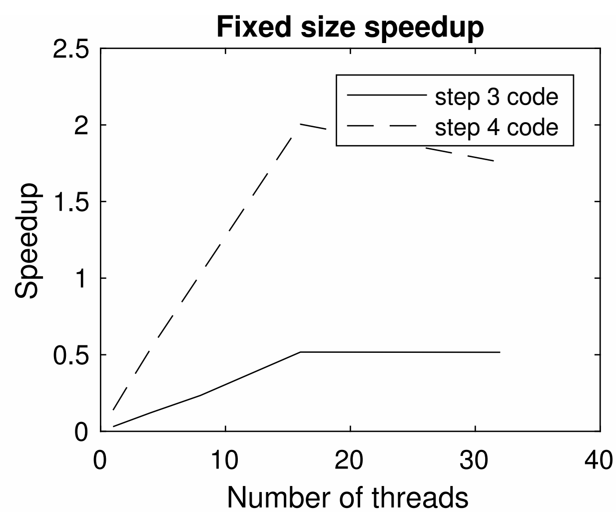
```
$ make atlas LPATH=/path/to/atlaslib
```

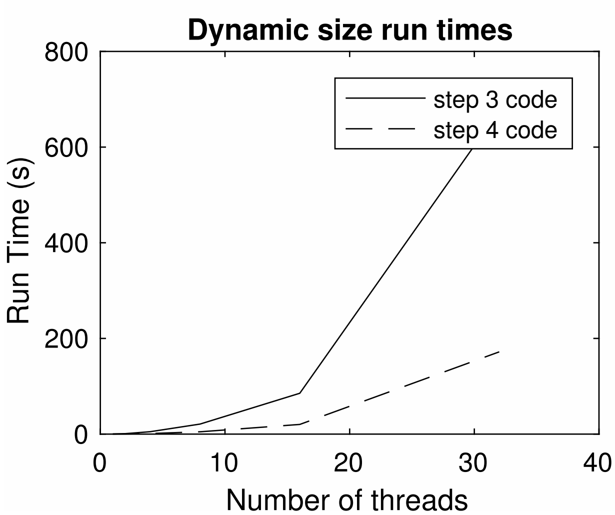
(iii) **Block has to be square and the number of rows/columns of one block has to be a multiplicity of 8**.

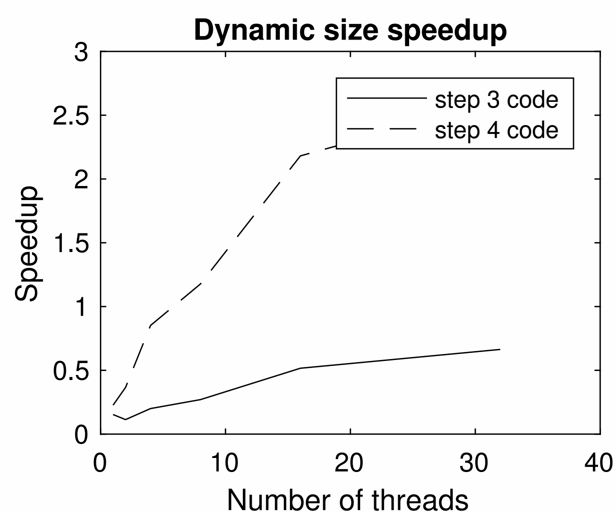
4 Step 5

The graphs are as follow:









	512×512	1024×1024	2048×2048	4096×4096	8192×8192	16384×16384
MKL	.0354595	.1060209	1.003119	5.671681	44.08856	449.2986
Step 2	1.286246	9.630381	292.011	2335.306	Beyond Patience	

Table 1: Run time of MKL and step 2 codes.

The fixed size run times and speedup are based on block size 4096×4096, with matrix size 1×1 block. The dynamic size starts at block size 512×512, and matrix size 1×1 block. And the number of rows and number of columns of one block are both doubled when the thread number is doubled, while the matrix size is always 1×1 block.

The speedup is based on MKL implementation. Table 1 shows the run times for MKL code and step 2 code. The matrix always contains only one block.

References

[1] Goto, K. and R. A. van de Geijn, *Anatomy of high-performance matrix multiplication*, ACM Transactions on Mathematical Software, 2008