1. a) Prove that $L=\{(ab)^n a(ab)^n, n=1,2,3,...\}$ is a non-regular language.

SUPPOSE THAT L IS A REGULAR LANGUAGE. LET THE CORRESPONDING AUTOMATA HAVE k STATES.
CONSIDER WORDS $(ab)^n$, $n=1,2,3....k,k+1,...$
LET $(ab)^p$ AND $(ab)^q$ FINISH IN THE SAME STATE, $p\neq q$
THEN $X=(ab)^p(ab)^{n-p}a(ab)^n$ AND $Y=(ab)^q(ab)^{n-p}a(ab)^n$
ALSO FINISH IN THE SAME STATE. BUT $X=(ab)^n a(ab)^n \in L$
WHILE $Y=(ab)^{n-p+q}a(ab)^n \notin L$ WHICH IS A CONTRADICTION.

b) Find a context free grammar that accepts the above language L.

$$S \to XSX \mid a$$
$$X \to ab$$

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SOLUTIONS

2. Is the language $\{(ab)^n(ba)^{2n}(ab)^n$ for n=1,2,3,... $\}$ context free? If so, find a context free grammar for it. If not, prove so.

SUPPOSE THAT THE LANGUAGE IS CF, AND THAT ITS CNF HAS p LIVE PRODUCTIONS.

LET $W = (ab)^{2^p}(ba)^{2^{p+1}}(ab)^{2^p} \in L$.

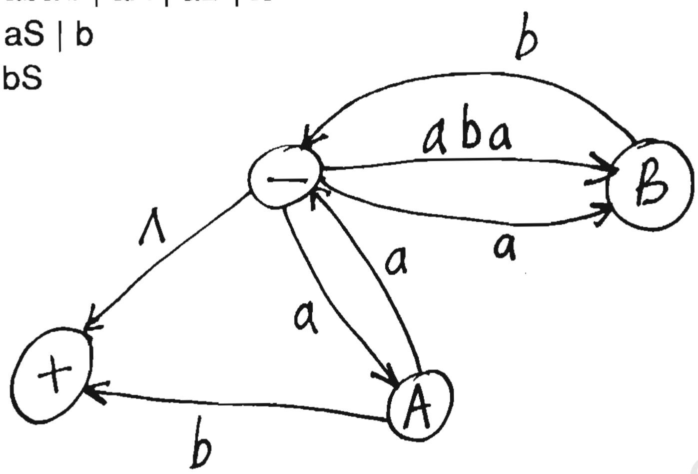
THERE IS ONLY ONE aa AND ONLY ONE bb IN ANY WORD OF L. BY PUMPING LEMA, ~~LET~~ $W = uvxyz$ SUCH THAT $uv^kxy^kz \in L$ FOR ANY k AND LENGTH $(vxy) < 2^p$.

V AND Y DO NOT CONTAIN ANY SUBWORDS aa OR bb. THUS W CAN BE SPLIT INTO THREE PIECES: $(ab)^{2^p}$, $(ba)^{2^{p+1}}$, $(ab)^{2^p}$ SUCH THAT V AND Y ARE EACH COMPLETELY INSIDE ONE OF THESE PIECES.

THEREFORE ONE OR TWO OF THESE PIECES WILL "GROW" WITH INCREASING k WHILE AT LEAST ONE OF THEM REMAINS FIXED. THIS WILL CREATE INBALANCE IN THE SIZES OF THE PIECES, WHICH CONTRADICTS THE DEFINITION OF L.

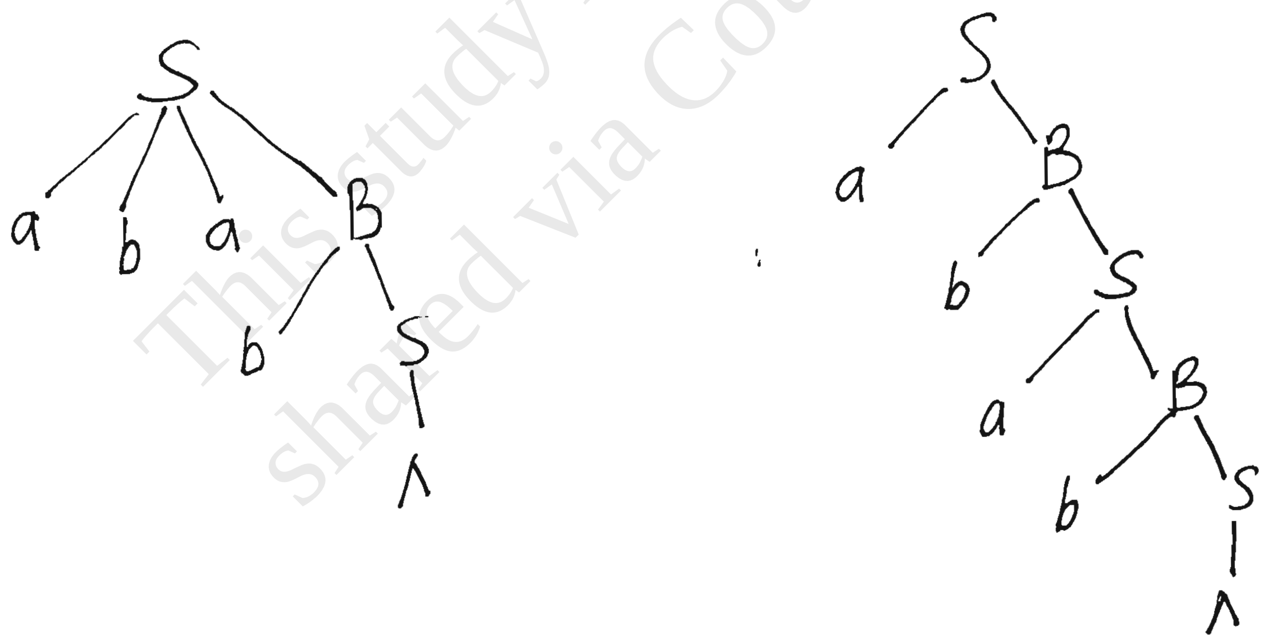
3. a) Construct a transition graph that accepts the language produced by the following context free grammar.

$S \rightarrow abaB \mid aA \mid aB \mid \Lambda$
$A \rightarrow aS \mid b$
$B \rightarrow bS$



b) Prove that the above context free grammar is ambiguous.

AMBIGUOS BECAUSE OF TWO DERIVATIONAL TREES OF abab (FOR EXAMPLE).



4. Is the language $L=\{a^n b^m a^m b^n$ for $n,m=1,2,3,\ldots\}$ context free? If so, find a context free grammar for it. If not, prove so.

L IS CONTEXT FREE.

$S \rightarrow aSb \mid X$

$X \rightarrow bXa \mid ba$

5. a) Reduce the following context free grammar to Chomsky normal form.

$$S \rightarrow SaS \mid SaSbS \mid \Lambda$$

ELIMINATE Λ: $S \rightarrow SaS \mid SaSbS \mid aS \mid Sa \mid a \mid aSbS \mid SabS$

$S \rightarrow abS \mid aSb \mid Sab \mid ab$

CNF: $S \rightarrow SR_1 \mid SQ \mid AS \mid SA \mid a \mid R_1R_2 \mid R_3R_2 \mid R_3R_4$

$S \rightarrow AR_2 \mid AR_4 \mid R_3B \mid AB$

$R_1 \rightarrow AS \quad R_2 \rightarrow BS \quad R_3 \rightarrow SA \quad R_4 \rightarrow SB$

$A \rightarrow a \quad B \rightarrow b$

b) If L is a CFL that contains the word Λ and we reduce it into CNF and then add on the sole extra production $S \rightarrow \Lambda$, do we now generate all of L and only L? If so then prove it, otherwise give a counterexample and propose another way of returning Λ to L.

IT IS NOT ALWAYS TRUE THAT WE HAVE ALL L AND ONLY L.

FOR EXAMPLE, $L = \{\Lambda, a, aaa, aaaaa, \ldots\} = \{\Lambda \text{ OR ODD } \# \text{ OF } a\text{'s}\}$

CNF FOR $L \setminus \{\Lambda\}$: $S \rightarrow AR \mid a \quad R \rightarrow SA \quad A \rightarrow a$

I.E. $S \rightarrow AR \rightarrow aR \rightarrow aSA \rightarrow aaa \in L$

BUT IF WE ADD $S \rightarrow \Lambda$ THEN $S \rightarrow AR \rightarrow aR \rightarrow aSA \rightarrow aa \notin L$

IN ORDER TO FIX THAT, WE CAN CHANGE ALL 'OLD' S INTO S_1 AND ADD $S \rightarrow S_1 \mid \Lambda$

ABOVE EXAMPLE: $S_1 \rightarrow AR \mid a$

$R \rightarrow S_1A \quad A \rightarrow a$

6. Build a TM that accepts the language $L=\{ab^nab^na,\ n=1,2,3,\dots\}$.