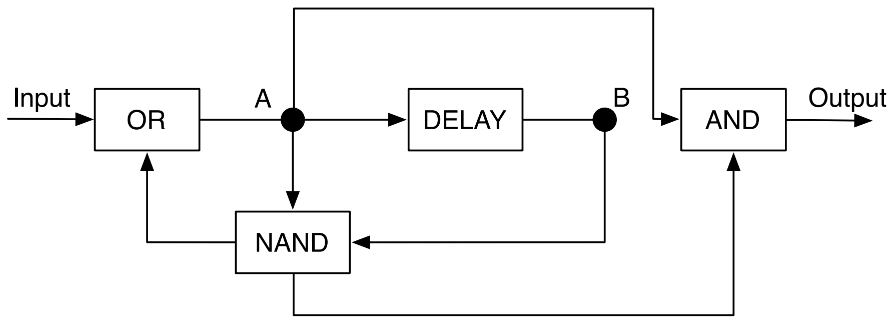
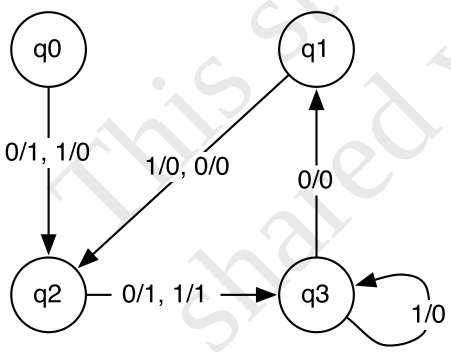
1. Construct Mealy machine that corresponds to the following circuit.



$B_{new} = A_{old}$
A_{new} = Input OR (A_{old} NAND B_{old})
Output = A_{old} AND (A_{old} NAND B_{old})

State			Input = 0				Input = 1			
	A_{old}	B_{old}	A_{new}	B_{new}	new state	Output	A_{new}	B_{new}	new state	Output
q_0	0	0	1	0	$= q_2$	0	1	0	$= q_2$	0
q_1	0	1	1	0	$= q_2$	0	1	0	$= q_2$	0
q_2	1	0	1	1	$= q_3$	1	1	1	$= q_3$	1
q_3	1	1	0	1	$= q_1$	0	1	1	$= q_3$	0



2. Construct a CFL which generates the language $L = \{a^m b^n, 3n \geq m \geq n \geq 1\}$.

S → aSb | aaSb | aaaSb | ab | aab | aaab

3. Prove that $L = \{a^m b^n, 3n \geq m \geq n \geq 1\}$ is a nonregular language.

Proof: Let A be a finite automaton for L with k states. Consider words for $m > k$. Let a^p and a^q finish in the same state, $p < q$, $p > m$, $q > m$, and let $a^p b^n$ be accepted (thus $3n \geq p \geq n$). Then $a^p, a^p a^{q-p}, a^p (a^{q-p})^2, \ldots, (a^{q-p})^t$ finish in the same state for any t. Thus $a^p (a^{q-p})^t b^n$ are accepted $\Rightarrow 3n \geq p + (q-p)t$ for any t, which is a contradiction.

Alternative Proof: Assume L is regular. Let p be the constant of the pumping lemma (which corresponds to the number of states of a finite automaton). Consider the word $w = a^p b^n$ with $3n \geq p \geq n$ which is in L and has a length greater than p. From the pumping lemma for regular languages follows that we can decompose w into xyz such that $|y| > 0$, $|xy| \leq p$ and for all $i \geq 0$, $xy^i z \in L$. From $|xy| \leq p$ follows that y consists only of 'a's and the decomposition of w has the following form:

$$\underbrace{a^{p-q}}_{x} \underbrace{(a^q)^i}_{y} \underbrace{b^n}_{z} \qquad \text{(for } q < p\text{)}$$

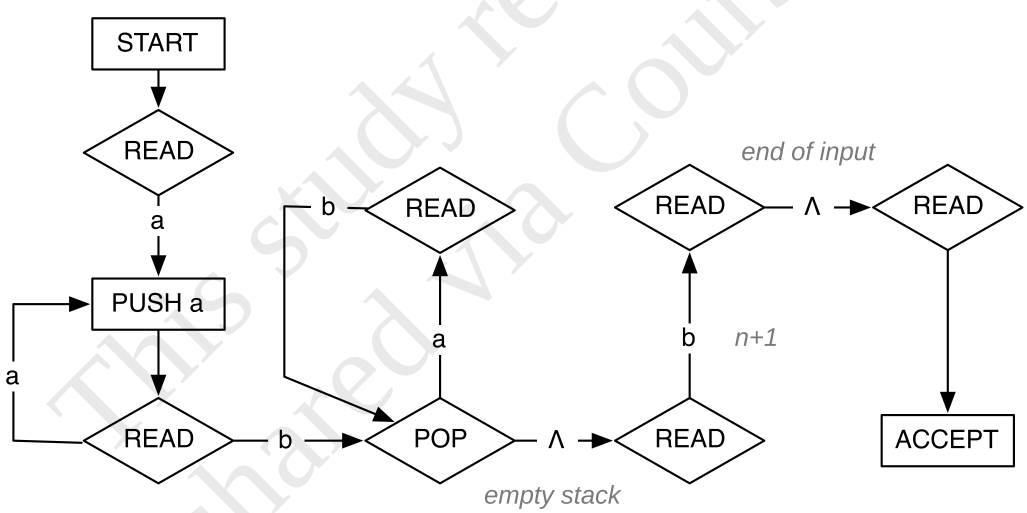
Now, w can be "pumped up" such that $a^{p-q}(a^q)^i b^n \in L$ for all $i \geq 0$. Choosing $i = 3n$ violates the condition $3n \geq (p-q) + q \cdot i$ because $p \geq n$ and $q > 0$, which is a contradiction.

4. For the following grammar, decide whether the corresponding language is finite or infinite. Justify your answer.

S → XY
X → AA | XY | b
A → BC
B → AC
C → BA
Y → a

The language is infinite, because it has useful self-embedded non-terminals. X is useful, i.e. it is used in another production (S → XY) and it is self-embedded (X → XY). Therefore, it can be used infinitely often.

5. Construct a PDA which accepts the language $L = \{a^n b^{n+1}, n = 1, 2, ...\}$.



(if a transition is not specified, it means reject)

6. Is the language $L = \{a^{2n}b^{3n}a^{4n}, n = 1, 2, 3, ...\}$ context free? If not, prove. If yes, give a CFL that accepts it.

L is not context-free.

Proof (by contradiction): Assume L is context-free language. Let p be an sufficiently large constant. From the pumping lemma for context-free languages follows that we can decompose a word w with $|w| \geq p$ into $w = uvxyz$ with $|vxy| \leq p$ and $|vy| > 0$, such that $uv^ixy^iz \in L$ for all $i \geq 0$.

For words $w = a^{2n}b^{3n}a^{4n} \in L$ with $|w| > p$ there is no way of finding a decomposition into $uvxyz$, such that $uv^ixy^iz \in L$. If in the decomposition the parts v or y contain both 'a's and 'b's, then they would be in the wrong order after "pumping up" the word to uv^ixy^iz. Otherwise, if v or y contain only either 'a's or 'b's, then the "pumping" would cause an imbalance in the number of the letters: Either the number of the 'a's on the left and/or the number 'b's in the middle increases, but there is no chance to increase the number of the 'a's on the right at the same time.

Example: $\underbrace{a}_{u}\underbrace{(aa)^i}_{v}\underbrace{abb}_{x}\underbrace{(bbb)^i}_{y}\underbrace{baaaaaaaa}_{z}$

7. Convert the following CFL into CNF.

$S \to ABABAB$
$A \to a \mid \Lambda$
$B \to b \mid \Lambda$

Shrink the right-hand side:
$S \to A\ R_1$
$R_1 \to B\ R_2$
$R_2 \to A\ R_3$
$R_3 \to B\ R_4$
$R_4 \to A\ B$
$A \to a \mid \Lambda$
$B \to b \mid \Lambda$

Remove null productions:
$S \to R_1 \mid A\ R_1$
$R_1 \to R_2 \mid B\ R_2$
$R_2 \to R_3 \mid A\ R_3$
$R_3 \to R_4 \mid B\ R_4$
$R_4 \to A \mid B \mid A\ B$
$A \to a \mid \Lambda$
$B \to b \mid \Lambda$

Remove unit productions:
$S \to A\ R_1 \mid B\ R_2 \mid A\ R_3 \mid B\ R_4 \mid A\ B \mid a \mid b$
$R_1 \to B\ R_2 \mid A\ R_3 \mid B\ R_4 \mid A\ B \mid a \mid b$
$R_2 \to A\ R_3 \mid B\ R_4 \mid A\ B \mid a \mid b$
$R_3 \to B\ R_4 \mid A\ B \mid a \mid b$
$R_4 \to A \mid B \mid A\ B$
$A \to a$
$B \to b$
($S \to \Lambda$ required to obtain the original grammar)

8. Build a TM that accepts the language $L = \{(ab)^n(ba)^n, n = 1, 2, 3, ...\}$.