Read each question carefully and be sure to SHOW ALL WORK. Correct answer without proper justification will not receive a "Complete" grade. Paç fat! Good luck!

Name: _____

LO 5. Integrals Challenge. I deeply understand the concepts behind Riemann sums, definite integrals, and their connection to antiderivatives and indefinite integrals through the Fundamental Theorem of Calculus.

Criteria for Success: I can solve conceptual questions related to Riemann sums, definite integrals, and the Fundamental Theorem of Calculus that lie on the top half of Bloom's Taxonomy (analyze, evaluate, and create).

Question: The goal of this question is to use substitution in combination with geometry to solve a definite integral.

(a) Circle all that apply and for each option explain why you chose to circle it or not. Clearly state u and du. If $f(x)$ is a continuous function, then $\int_0^1 x^2 f(x^3+1)dx$ is

(I) $\int_1^2 f(u)du$

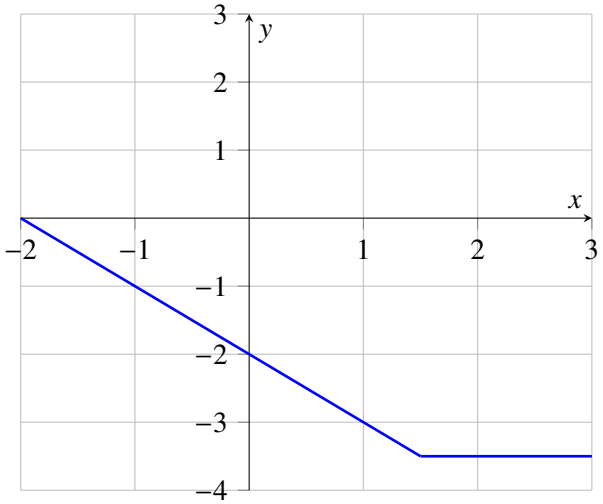
(II) $\frac{1}{3}\int_1^2 f(u)du$

(III) $\frac{1}{3}\int_0^1 f(u)du$

(IV) $\frac{1}{3}\int_0^1 f(u+1)du$

(V) none of the above

(b) Use the sketch below of the graph of $y = f(x)$ on the interval $[-2, 3]$ to find the exact numerical value of $\int_0^1 x^2 f(x^3 + 1)dx$ using the result of the substitution in the previous question, and geometric arguments such as formulas of areas from geometry and properties of definite integrals. Avoid using the Fundamental Theorem of Calculus.



(c) Now verify the above integral computation using the Fundamental Theorem of Calculus and properties of definite integrals. Note that $f(x) = -2 - x$ for $-2 \leq x \leq 1.5$ and $f(x) = -3.5$ for $1.5 \leq x \leq 3$.