Read each question carefully and be sure to SHOW ALL WORK. Correct answer without proper justification will not receive a “Complete” grade. Paç fat! Good luck!

Name: ____________________

LO 9. Power Series [CORE]. I can manipulate and analyze power series. Power series are mainly used to represent functions, store an infinite sequence of numbers (in which case they're called generating functions), and solve recurrence relations (important in computer science).

Criteria for Success: I can

- determine the radius and interval of convergence of a power series
- add, subtract, differentiate, integrate power series

Question: A power series of some function $f(x)$ is $\sum_{k=7}^{\infty} a_k(x+2)^k$, for some real number constants a_k for $k \geq 7$. Assuming that it **diverges** at $x = -5$ and $x = 3$, answer the following questions. No need for explanations here, but a picture of the interval may help you out.

(a) What is the center of the given power series?

(b) For what values of x it is guaranteed that the series converges?

(c) For what values of x it is guaranteed that the series diverges?

(d) How big and small could the radius of convergence possibly be?

LO 10. Power Series Representations [CORE]. I can represent functions as power series and vice versa represent power series as functions in order to solve various application questions.

Criteria for Success: I can

- use geometric series, Taylor series, binomial series, and formulas derived from them to go back and forth between a function and a power series representation
- find a series exact sum by plugging into the appropriate power series representation
- find a Taylor polynomial of a specified degree to approximate a given function
- solve limit or integral questions using power series

Question: A common mistake in Calculus 1 is to use power rule when finding the derivative of $f(x) = 2^x$. In fact, $f'(x) = 2^x \ln(2)$, which is quite surprizing. The goal of this question is to use power series to shed light on why this is so.

(a) Find the power series centered at 0 for $f(x) = 2^x$ given the power series

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots + \frac{x^n}{n!} + \ldots.$$

Hint: $a^x = e^{\ln(a^x)} = e^{x\ln(a)}$.

(b) Find the derivative of the power series representation you found in the previous part.

(c) Simplify the derivative power series and show that $f'(x) = 2^x \ln(2)$.

(d) Another way to derive this derivative formula is to directly use the limit definition of the derivative:

$$f'(x) = \lim_{h\to 0} \frac{2^{x+h} - 2^x}{h} = \lim_{h\to 0} \frac{2^x \cdot 2^h - 2^x}{h} = 2^x \lim_{h\to 0} \frac{2^h - 1}{h}.$$

Now use the power series in part (a) to compute the above limit and therefore show that

$$f'(x) = 2^x \ln(2).$$