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Adaptive singularity-free controller design of constrained nonlinear systems with prescribed performance ☆

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ABSTRACT

This paper investigates the controller design for the tracking problem of uncertain nonlinear systems with constraints on input and output. First, to avoid the full-state measurement, the high-gain observer is designed to estimate the unmeasured state. Compared to existing observer design for nonlinear systems, the high-gain observer design only requires a modified version of the Lipschitz condition. The output feedback controller is further presented on this basis. Second, to consider the transient constraints on the tracking performance, a barrier Lyapunov function with the user-defined time-varying performance is developed. In addition, the proposed controller design is shown to be free of control direction singularity. The convergence of the learning scheme and the boundedness of all the closed-loop signals during the learning phase are discussed theoretically. Finally, two simulation examples are conducted to verify the advantage of the proposed adaptive output feedback controller design.

1. Introduction

Recently, the controller design for nonlinear systems with performance and safety considerations has gained much attention [1,2]. To deal with this problem, many control strategies have been yield, such as robust control [3,4], sliding mode control [5–7], iterative learning control [8], to name a few. Many engineering systems can be modeled as the nonlinear strict-feedback systems, which can be efficiently tackled by the backstepping methods [9]. In the backstepping design framework, the complex nonlinear system is divided into simple subsystems with low order by introducing the concept of virtual control, which breaks the stabilization problem of the large-scale nonlinear systems down into a series of simple sub-problems. However, as the system order increases, the virtual control design suffers from the ’complexity explosion’ due to the repeated differentiation [10]. Filter theory has been applied as a feasible solution to this issue, such as the dynamic surface control (DSC) method [11] and command filter design [12]. In this paper, we extend the DSC method to deal with nonlinear system tracking problem the tracking problem of the nonlinear systems in the strict-feedback form with both time-varying constraints on output tracking error and saturated input subject to the singularity of control direction.

In some scenarios, the complete measurement of the full-state is impossible or expensive. Therefore, a reliable state estimation design is desired. With the well-known Luenberger observer design [13–15], the Lipschitz condition has to be satisfied with the nonlinear dynamics captured by a Lipschitz constant. This might be a strong condition for some cases because it is difficult to obtain the Lipschitz constant for some functions. To relax this requirement, the high-gain observer design has been developed [16,17]. However, the constraints on the system output and control input are not considered. In this paper, we extend the high-gain observer design to the constrained nonlinear dynamical system.

Typical nonlinear dynamics have been investigated in existing results, such as time-delay [18,19], dead zone [20,21], quantization [22], and saturation [23,24]. The method of introducing an auxiliary control signal into the controller design [25,26] is usually utilized for

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the input saturation problem. However, these results are based on the non-singular requirement of the control directions. When the control direction is time-varying and can be singular at some instant, this method might be inefficient. In this paper, we tackle the input saturation subject to the singular control direction by introducing a positive parameter in the controller denominator design.

In addition to the stability concern, transient performance and safety consideration is also of interest [27,28]. The concept of barrier Lyapunov function (BLF) has been developed in [29,30] as an efficient method to deal with full/partial state constraints. However, the constraints considered in [29,30] are time-invariant. To achieve time-varying tracking performance, the prescribed performance adaptive control (PPAC) method has been proposed [31]. By employing a state transformation to the original system, another dynamical signal is yield to guarantee the original tracking error with a prescribed convergence rate. However, the error transformation depends on the initial condition when the constraints on the tracking error are asymmetric, which makes the controller design and stability analysis difficult. To tackle this issue, we combine BLF with PPAC, which can efficiently deal with asymmetric time-varying constraints using only one error transformation mapping regardless of the initial condition. In this paper, the combination of BLF with PPAC is investigated with the existence of the uncertain dynamics, constraints on input/output, and control direction singularity.

The objective of this paper is to design an observer-based adaptive output-feedback controller for the nonlinear strict-feedback systems with constraints on both input and output. With theoretical discussions, all the closed-loop signals are shown to be semi-global uniformly ultimately bounded (SGUUB). Compare to existing results, the main contributions of this paper are as follows.

(1) The high-gain observer is established to estimate the unmeasured states with a modified Lipschitz condition.
(2) In contrast to the existing results in [32,33], an asymmetric BLF combined with prescribed performance is employed to deal with the control problem of systems with output constraints.
(3) The input saturation is solved by introducing an auxiliary signal, and the singularity-freeness of the control input design is guaranteed by introducing a positive parameter in the controller denominator design.

The remainder is organized as follows. In Section 2, preliminaries background is provided. Section 3 gives the problem statement. In Section 4, a high-gain observer is established to estimate the unmeasured states. An asymmetric BLF with time-varying prescribed error bound is constructed in Section 5. Section 6 provides the simulation results. The concluding remark is shown in Section 7.

2. Preliminaries

Consider a nonlinear function $F(x):\mathbb{R}^n\to\mathbb{R}$. According to the universal approximation theorem, $F(x)$can be approximated by the following radial basis function neural network (RBF NN)

$$F_{nn}(x,\theta)=\theta^{\mathrm{T}}\phi(x)$$

where $x\in\Omega\in\mathbb{R}^n$ is the input vector, $\theta=[\theta_1,\theta_2,\ldots,\theta_N]^{\mathrm{T}}\in\mathbb{R}^N$ is the weight vector, with the NN node number $N>1$, and $\phi(x)=[\phi_1(x),\phi_2(x),\ldots,\phi_N(x)]^{\mathrm{T}}$, with $\phi_i(x)$ being selected as the following Gaussian function

$$\phi_i(x)=\exp\left[\frac{-(x-\mu_i)^{\mathrm{T}}(x-\mu_i)}{b^2}\right],\quad i=1,2,\ldots,N$$

where $\mu_i=[\mu_{i1},\mu_{i2},\ldots,\mu_{in}]^{\mathrm{T}}$ is the center of the receptive field and b is the width of the Gaussian function.

Then $F(x)$can be approximated by the RBF NN as

$$F(x)=\theta^{*\mathrm{T}}\phi(x)+\zeta^*(x)$$

where θ^* is the ideal weights, $\zeta^*(x)$ is the minimum approximation error.

Lemma 1. [34] *The RBF NN-basis function $\phi(\cdot)$ satisfies*

$$\|\phi(x)\|\leqslant N:=\sum_{k=0}^{\infty}3q(k+2)^{q-1}e^{\frac{-2\rho^2k^2}{b^2}}$$

where $\rho=\frac{1}{2}\min_{i\neq j}\left\|\mu_i-\mu_j\right\|$, μ_i and b are the center and width of the Gaussian function, respectively.

Lemma 2. [35] *Let $F(x)$ be a continuous function defined on a compact set Ω, then there exists a constant $\bar{\zeta}$ such that*

$$\sup_{x\in\Omega}\left|F(x)-\theta^{\mathrm{T}}\phi(x)\right|\leqslant\bar{\zeta}$$

where θ is the weight of the neural network, $\phi(x)$ is the basis function.

3. Problem formulation

Consider a nonlinear strict-feedback system as given below

$$\begin{aligned}\dot{x}_i&=f_i(\bar{x}_i)+x_{i+1}\\ \dot{x}_n&=f_n(\bar{x}_n)+G(\bar{x}_n)u(t)\\ y&=x_1\end{aligned}\tag{1}$$

where $\bar{x}_i=[x_1,x_2,\ldots,x_i]^{\mathrm{T}}\in\mathbb{R}^i(\bar{x}_n=x)$ is the state, $G(\cdot)$ is the control direction which is a known function, $f_i(\cdot):\mathbb{R}^i\to\mathbb{R}$ is unknown function, y and $u(t)$ are the system output and the system input. $u(t)$ is related to the designed control signal $v(t)$ as follows,

$$u(t)=sat(v(t))=\begin{cases}\mathrm{sign}(v(t))u_N, & |v(t)|\geqslant u_N\\ v(t), & |v(t)|<u_N\end{cases}$$

with the upper bound $u_N>0$. Note that $sat(\cdot)$ is a non-smooth function. In this paper, we use a smooth function to approximate $sat(\cdot)$ as

$$g(v(t))=u_N\tanh\left(\frac{v(t)}{u_N}\right)=u_N\frac{e^{v(t)/u_N}-e^{-v(t)/u_N}}{e^{v(t)/u_N}+e^{-v(t)/u_N}}.$$

Then the applied control can be rewritten as follows,

$$sat(v(t))=g(v(t))+d(v(t))$$

where $d(v(t))=u(t)-g(v(t))$is a bounded function and takes its maximum value at $v=u_N$. The bound of $d(v(t))$ can be described as

$$|d(v(t))|\leqslant u_N(1-\tanh(1))=\bar{d}.$$

For system (1), the following assumptions are given for the following theory analysis.

Assumption 1. Both the given reference signal y_d as well as its derivatives $\dot{y}_d$ and $\ddot{y}_d$ are continuous and bounded.

Assumption 2. The system (1) is bounded input bounded state.

Assumption 3. [16] There exists a known locally Lipschitz non-negative function $L(y)$ such that the following inequality hold for $i=2,\ldots,n$,

$$\left|f_i(\bar{x}_i)-f_i\left(\hat{\bar{x}}_i\right)\right|\leqslant L(y)(|x_2-\hat{x}_2|+\cdots+|x_i-\hat{x}_i|)$$

where $\hat{\bar{x}}_i$ is the estimation of state $\bar{x}_i$ designed later.

Remark 1. Existing results are usually based on the Lipschitz continuity with a constant [36–38], which might be a strong assumption. In this paper, the Lipschitz condition of the nonlinear function $f(\cdot)$ is modified as in Assumption 3. In addition, from Assumption 3, the Lipschitz constant is replaced with the non-negative function $L(y)$.

Assumption 4. The nonlinear function $G(\cdot)$ which is bounded satisfies $\|G(x_i) - G(\hat{x}_i)\| \leqslant \gamma_0 \|x_i - \hat{x}_i\|$ with a positive constant $\gamma_0 \in \mathbb{R}$.

Remark 2. In existing results on nonlinear strict-feedback systems, the control direction is either constant or lower-bounded by a positive constant. In contrast, in this paper, the control direction $G(x)$ is allowed to be singular for some x and only Lipschitz continuity of $G(x)$ is required.

Definition 1 [39]. If for any $\sum$, a compact subset of R^n and all $x(t_0) = x_0 \in \sum$, there exist a $\lambda > 0$ and a number $T(\lambda, x_0)$ such that $\|x(t)\| < \lambda$for all $t > t_0 + T$, then the system (1) is semi-globally uniformly ultimately bounded (SGUUB).

Problem 1. Given the reference signal y_d, determine an output feedback controller and adaptive laws such that all the signals in the closed-loop system and the tracking error $y - y_d$ are bounded.

In the following, Problem 1 is solved using an adaptive output-feedback controller to tackle the control direction singularity with the presence of constraints on both input and output.

4. High-gain observer design

The states except x_1 are unmeasured. To solve this problem, we design a novel type of high-gain observer to estimate the unmeasured states x_i for $i = 2, \ldots, n$. The dynamical system (1) can be rewritten as

$$\begin{aligned} \dot{x}_1 &= f_1(x_1) + x_2 \\ \dot{x}_i &= f_i\left(\hat{\bar{x}}_i\right) + x_{i+1} + \Delta f_i, \ \ i = 2, \ldots, n-1 \\ \dot{x}_n &= f_n\left(\hat{\bar{x}}_n\right) + G(\bar{x}_n) g(v(t)) + G(\bar{x}_n) d(v(t)) + \Delta f_n \\ y &= x_1 \end{aligned} \tag{2}$$

where $\hat{\bar{x}}_i = [\hat{x}_1, \hat{x}_2, \ldots, \hat{x}_i], \Delta f_i = f_i(\bar{x}_i) - f_i\left(\hat{\bar{x}}_i\right)$ and $\hat{\bar{x}}_i$ stands for the estimation of the state $\bar{x}_i$ provided by the observer. Using the neural network approximation, $f_i(\cdot)$ can be represented as

$$f_i(\cdot) = \omega_i^{*\mathrm{T}} \varphi_i(\cdot) + \varepsilon_i^*, \ i = 1, 2, \ldots, n$$

where ω_i^* is the ideal weights, ε_i^* is the approximation error. Then, system (2) can be equivalently described as

$$\begin{aligned} \dot{x}_1 &= \omega_1^{*\mathrm{T}} \varphi_1(x_1) + \varepsilon_1^* + x_2 \\ \dot{x}_i &= \omega_i^{*\mathrm{T}} \varphi_i\left(\hat{\bar{x}}_i\right) + \varepsilon_i^* + x_{i+1} + \Delta f_i, \ i = 2, \ldots, n-1 \\ \dot{x}_n &= \omega_n^{*\mathrm{T}} \varphi_n\left(\hat{\bar{x}}_n\right) + \varepsilon_n^* + G(\bar{x}_n) g(v(t)) + G(\bar{x}_n) d(v(t)) + \Delta f_n \\ y &= x_1. \end{aligned} \tag{3}$$

The high-gain observer is designed as

$$\begin{aligned} \dot{\hat{x}}_1 &= \omega_1^{\mathrm{T}} \varphi_1(x_1) + \hat{x}_2 + l_1 r(t)(y - \hat{x}_1) \\ \dot{\hat{x}}_i &= \omega_i^{\mathrm{T}} \varphi_i\left(\hat{\bar{x}}_i\right) + \hat{x}_{i+1} + l_i r^i(t)(y - \hat{x}_1) \\ \dot{\hat{x}}_n &= \omega_n^{\mathrm{T}} \varphi_n\left(\hat{\bar{x}}_n\right) + G\left(\hat{\bar{x}}_n\right) g(v(t)) + l_n r^n(t)(y - \hat{x}_1) \\ \dot{r} &= \psi(r, y) \end{aligned} \tag{4}$$

where $\omega = [\omega_1, \omega_2, \ldots, \omega_n]^{\mathrm{T}}$ is the adaptive law, $r(t)$ is an extra state to be designed later. For $i = 1, \ldots, n, l_i$ is a positive constant such that the polynomial $p(s) = s^n + l_1 s^{n-1} + \cdots + l_{n-1} s + l_n$ is Hurwitz. Define the state estimation error as

$$e = x - \hat{x} = [e_1, e_2, \ldots, e_n]^{\mathrm{T}}.$$

Then, the dynamics of e_i for $i = 1, \ldots, n$ can be written as

$$\begin{aligned} \dot{e}_1 &= e_2 + \tilde{\omega}_1^{\mathrm{T}} \varphi_1(x_1) + \varepsilon_1^* - l_1 r(t)(y - \hat{x}_1), \\ \dot{e}_i &= e_{i+1} + \tilde{\omega}_i^{\mathrm{T}} \varphi_i\left(\hat{\bar{x}}_i\right) + \varepsilon_i^* - l_i r^i(t)(y - \hat{x}_1) + \Delta f_i, \\ \dot{e}_n &= \tilde{\omega}_n^{\mathrm{T}} \varphi_n\left(\hat{\bar{x}}_n\right) + \varepsilon_n^* - l_n r^n(t)(y - \hat{x}_1) + \Delta G g(v(t)) + G(\bar{x}_n) d(v(t)) + \Delta f_n \end{aligned} \tag{5}$$

where $\tilde{\omega}_i = \omega_i^* - \omega_i, \Delta G = G(\bar{x}_n) - G\left(\hat{\bar{x}}_n\right)$.

For subsequent analysis, the following error transformation is introduced

$$\xi_i = \frac{e_i}{r(t)^{i-1+a}}, \quad a > 1, \ i = 1, \ldots, n \tag{6}$$

where $r(t)$ is the extra state to be designed later. To analysis the stability of the observer estimation error ξ, give the following lemma.

Lemma 3. [16] *There exist strictly positive numbers a, p and δ and a symmetric positive definite matrix P such that*

$$A^{\mathrm{T}} P + PA \leqslant -\delta P, \ pI \leqslant P \leqslant I (0 < p \leq 1) - aP \leqslant SP + PS \leqslant aP \tag{7}$$

where

$$A = \begin{bmatrix} -l_1 & & & & \\ -l_2 & & & & \\ \cdot & & I_{n-1} & & \\ \cdot & & & & \\ \cdot & & & & \\ -l_n & 0 & \cdot & \cdot & \cdot \ \ 0 \end{bmatrix}$$

and $S = \mathrm{diag}(0, \ldots, n-1)$ for $n \geqslant 2$.

Consider the following Lyapunov function

$$V_0 = \xi^{\mathrm{T}} P \xi. \tag{8}$$

Using (5) and (6), the dynamics of ξ is

$$\dot{\xi} = -\frac{\dot{r}(t)}{r(t)}(aI + S)\xi + \Lambda\left[\tilde{f} + \varepsilon^* + \Delta F + \Gamma + \tilde{G}\right] + r(t) A \xi \tag{9}$$

where $\varepsilon^* = [\varepsilon_1^*, \ldots, \varepsilon_n^*]^{\mathrm{T}}, \Delta F = [0, \Delta f_2, \ldots, \Delta f_n]^{\mathrm{T}}$, $\Lambda = diag\left(\frac{1}{r(t)^a}, \ldots, \frac{1}{r(t)^{a+n-1}}\right)$, $\tilde{G} = [0, \ldots, \Delta G g(v(t))]$, $\Gamma = [0, \ldots, G(\bar{x}_n) d(v(t))]. \tilde{f} = \left[\tilde{f}_1, \tilde{f}_2, \ldots, \tilde{f}_n\right]^{\mathrm{T}}$ with $\tilde{f}_1 = \tilde{\omega}_1^{\mathrm{T}} \varphi_1(x_1), \tilde{f}_i = \tilde{\omega}_i^{\mathrm{T}} \varphi_i\left(\hat{\bar{x}}_i\right)$ for $i = 2, \ldots, n$.

From Lemma 3, the following inequality can be obtained

$$-\frac{\dot{r}(t)}{r(t)} \xi^{\mathrm{T}} (SP + PS) \xi \leqslant a \frac{|\dot{r}(t)|}{r(t)} \xi^{\mathrm{T}} P \xi. \tag{10}$$

Using (7), (9) and (10), differentiating V_0 yields

$$\begin{aligned} \dot{V}_0 &= \dot{\xi}^{\mathrm{T}} P \xi + \xi^{\mathrm{T}} P \dot{\xi} \leqslant 2\xi^{\mathrm{T}} P \Lambda \left(\tilde{f} + \varepsilon^* + \Delta F + \Gamma + \tilde{G}\right) - \delta r(t) \xi^{\mathrm{T}} P \xi - 2a \frac{\dot{r}(t)}{r(t)} \xi^{\mathrm{T}} P \xi \\ &\quad - \frac{\dot{r}(t)}{r(t)} \xi^{\mathrm{T}} (SP + PS) \xi \leqslant -\left(\delta r(t) + 2a \frac{\dot{r}(t)}{r(t)} - a \frac{|\dot{r}(t)|}{r(t)}\right) \xi^{\mathrm{T}} P \xi \\ &\quad + 2\xi^{\mathrm{T}} P \Lambda \left(\tilde{f} + \varepsilon^* + \Delta F + \Gamma + \tilde{G}\right). \end{aligned} \tag{11}$$

In the light of Assumptions 4–3, $\|\Lambda\| \leqslant 1$ and Young's inequality, we have

$$2\xi^{\mathrm{T}}P\Lambda\Delta F \leqslant 2\sum_{i=2}^{n}\left|\xi^{\mathrm{T}}P\right|L(y)(|\xi_2|+\cdots+|\xi_i|) \leqslant 2(n-1)L(y)\xi^{\mathrm{T}}P\xi$$
$$\leqslant \tfrac{2(n-1)L(y)}{\sqrt{p}}\xi^{\mathrm{T}}P\xi 2\xi^{\mathrm{T}}P\Lambda\widetilde{G} \leqslant \left(1+\gamma_0^2\|P\|^2u_N^2\right)\|\xi\|^2.$$

Then we can obtain

$$\dot{V}_0 \leqslant \underbrace{-\left[\delta r(t)+2a\frac{\dot{r}(t)}{r(t)}-a\frac{|\dot{r}(t)|}{r(t)}-\frac{2(n-1)L(y)}{\sqrt{p}}\right]\xi^{\mathrm{T}}P\xi}_{:=Q}+2\xi^{\mathrm{T}}P\Lambda\left(\tilde{f}+\varepsilon^*+\Gamma\right)$$
$$+\left(1+\gamma_0^2\|P\|^2u_N^2\right)\|\xi\|^2. \tag{12}$$

The auxiliary signal $r(t)$ is designed with the dynamics as

$$\dot{r}(t) = -\tfrac{r(t)}{a}\left[\frac{\delta}{3}(r(t)-1)-\frac{2(n-1)L(y)}{\sqrt{p}}\right], \quad r(0)=1. \tag{13}$$

It can be shown that $r(t) \geqslant 1$ which can be proved by contradiction. Suppose that $r(t) < 1$, it follows that $\dot{r}(t) > 0$. Furthermore, we can obtain $r(t) \geqslant r(0) = 1$, which contradicts with the fact $r(t) < 1$.

Consider the Lyapunov function $V_r = \frac{1}{2}r(t)^2$, then

$$\dot{V}_r = -\frac{r(t)^2}{a}\left[\frac{\delta}{3}(r(t)-1)-\frac{2(n-1)L(y)}{\sqrt{p}}\right]$$
$$= -\frac{\delta}{3a}r(t)^3+\left(\frac{\delta}{3a}+\frac{2(n-1)L(y)}{a\sqrt{p}}\right)r(t)^2.$$

Since $\dot{V}_r$ is a cubic function with $-\frac{\delta}{3a} < 0$. Combined with the fact that $r(t) \geqslant 1$, then, $\dot{V}_r$ is negative when $r(t)$ is sufficient large, which implies that $r(t)$ is bounded for any bounded $L(y)$.

Since $L(y)$ is strictly positive, this implies that for $\dot{r}(t) \geqslant 0$,

$$Q = -\left(\delta r(t)+a\frac{\dot{r}(t)}{r(t)}-\frac{2(n-1)L(y)}{\sqrt{p}}\right)\xi^{\mathrm{T}}P\xi = -\frac{\delta}{3}(2r(t)+1)\xi^{\mathrm{T}}P\xi$$
$$\leqslant -\delta\xi^{\mathrm{T}}P\xi.$$

For $\dot{r}(t) < 0$, we can obtain

$$Q = -\left(\delta r(t)+3a\frac{\dot{r}(t)}{r(t)}-\frac{2(n-1)L(y)}{\sqrt{p}}\right)\xi^{\mathrm{T}}P\xi$$
$$= -\left(\delta r(t)+\frac{4(n-1)L(y)}{\sqrt{p}}\right)\xi^{\mathrm{T}}P\xi-\delta\xi^{\mathrm{T}}P\xi.$$

To summarize, for any $\dot{r}(t)$, the conclusion $Q \leqslant -\delta\xi^{\mathrm{T}}P\xi$ holds.

In the light of Young's inequality, Lemma 1 and Lemma 2, the second term in (12) satisfies

$$\begin{aligned}
&2\xi^{\mathrm{T}}P\Lambda\tilde{f} \leqslant \|\xi\|^2+\|P\|^2\sum_{i=1}^{n}N_i\|\tilde{\omega}_i\|^2\\
&2\xi^{\mathrm{T}}P\Lambda\varepsilon^* \leqslant \|\xi\|^2+\|P\|^2\|\bar{\varepsilon}\|^2\\
&2\xi^{\mathrm{T}}P\Lambda\Gamma \leqslant \|\xi\|^2+\|P\|^2\|\bar{\Gamma}\|^2
\end{aligned} \tag{14}$$

where $\bar{\varepsilon}, \bar{\Gamma}$ is the upper bound of ε^* and $\Gamma, \bar{\Gamma} = \left[0,\ldots,\overline{G}\bar{d}\right]^{\mathrm{T}}$ with $|G(\bar{x}_n)| \leqslant \overline{G}, N_i$ is the number of neurons.

Note the fact that $\lambda_{\min}(P)\|\xi\|^2 \leqslant \xi^{\mathrm{T}}P\xi \leqslant \lambda_{\max}(P)\|\xi\|^2$, where $\lambda_{\min}, \lambda_{\max}$ represent the minimum eigenvalue and the maximum eigenvalue of matrix P. Combined with (12) and (14), we have

$$\dot{V}_0 \leqslant -\left(\delta\lambda_{\min}(P)-4-\gamma_0^2\|P\|^2u_N^2\right)\|\xi\|^2+M_0 \tag{15}$$

where $M_0 = \|P\|^2\left(\sum_{i=1}^{n}N_i\tilde{\omega}_i^{\mathrm{T}}\tilde{\omega}_i+\|\bar{\varepsilon}\|^2+\|\bar{\Gamma}\|^2\right)$.

5. Adaptive Neural Control Design Using Output Feedback

In this section, an adaptive output feedback controller based on the backstepping technique is developed.

Define the coordinates change as

$$z_1 = y-y_d \tag{16}$$
$$z_i = \hat{x}_i-\alpha_{i-1}, \quad 2 \leqslant i \leqslant n-1 \tag{17}$$
$$z_n = \hat{x}_n-\alpha_{n-1}-v \tag{18}$$
$$\chi_i = \alpha_i-s_i, \quad 1 \leqslant i \leqslant n-1 \tag{19}$$

where s_i is the virtual control law, α_i is the output of a first-order filter which is designed to avoid the repeated differentiation of s_i, χ_i is the output error of the first-order filter, v is the additional control signal to be designed in the final step.

5.1. Barrier Lyapunov Function with Prescribed Performance

In this subsection, a novel asymmetric BLF with time-varying boundary is designed to consider both the transient and steady state performance.

To guarantee the prescribed performance of the tracking error $z_1 = y_1-y_d$, a function $\rho(t)$is chosen as

$$\rho(t) = (\rho_0-\rho_\infty)e^{-kt}+\rho_\infty \tag{20}$$

where $k > 0$ and $\rho_0 > \rho_\infty > 0$. According to (20), $\rho(t)$ satisfies $\rho(0) = \rho_0$ and $\lim_{t\to\infty}\rho(t) = \rho_\infty$. In this paper, the following prescribed transient performance is exerted on the output tracking error z_1 as

$$-k_1\rho(t) \leqslant z_1 \leqslant k_2\rho(t) \tag{21}$$

where $k_1 > 0$ and $k_2 > 0$ are constants, $\{-k_1\rho_0, k_2\rho_0\}$and $\{-k_1\rho_\infty, k_2\rho_\infty\}$represent the constraints on the initial tracking error $z_1(0)$ and steady-state tracking error $z_1(\infty)$, respectively and $\{-k_1\rho(t), k_2\rho(t)\}$represents the constraints on the transient tracking error $z_1(t)$.

To consider the predefined constraints, the asymmetric BLF is employed based on the properties as

$$V_\eta = \frac{q(z_1)}{2}\frac{z_1^2}{k_b^2(t)-z_1^2}+\frac{1-q(z_1)}{2}\frac{z_1^2}{k_a^2(t)-z_1^2} \tag{22}$$

where $k_b(t) = k_1\rho(t), k_a(t) = k_2\rho(t), q(z_1)$ is defined as

$$q(z_1) = \begin{cases} 1, & \text{if } z_1 > 0\\ 0, & \text{if } z_1 \leqslant 0 \end{cases}.$$

Then, the BLF (22) can be rewritten as

$$V_\eta = \frac{\eta^2}{2(1-\eta^2)} \tag{23}$$

where

$$\eta = q(z_1)\eta_b+[1-q(z_1)]\eta_a, \ |\eta| < 1,$$
$$\eta_a = \frac{z_1}{k_a(t)}, \ \eta_b = \frac{z_1}{k_b(t)},$$
$$\dot{\eta}_a = \frac{\dot{z}_1k_a(t)-z_1\dot{k}_a(t)}{k_a^2(t)}, \ \dot{\eta}_b = \frac{\dot{z}_1k_b(t)-z_1\dot{k}_b(t)}{k_b^2(t)}.$$

It is worthwhile to highlight that V_η is a continuous function, as shown below.

$$\lim_{z_1\to 0^-}V_\eta(z_1) = \lim_{z_1\to 0^-}\frac{1-q(z_1)}{2}\frac{z_1^2}{k_a^2(t)-z_1^2} = 0$$
$$\lim_{z_1\to 0^+}V_\eta(z_1) = \lim_{z_1\to 0^+}\frac{q(z_1)}{2}\frac{z_1^2}{k_b^2(t)-z_1^2} = 0$$
$$V_\eta(z_{10}) = \frac{1}{2}\frac{z_{10}^2}{k_a^2(t)-z_{10}^2} = 0, \ z_{10} = 0.$$

Then, $\lim_{z_1 \to 0^-} V_\eta(z_1) = \lim_{z_1 \to 0^+} V_\eta(z_1) = V_\eta(z_{10})$. Therefore, $V_\eta$is a continuous function.

Lemma 4. *For all* $|\eta| < 1$, *the following inequality holds*

$$\frac{\eta^2}{1-\eta^2} \leqslant \frac{\eta^2}{(1-\eta^2)^2}.$$

Proof.

$$\frac{\eta^2}{1-\eta^2} - \frac{\eta^2}{(1-\eta^2)^2} = \frac{(1-\eta^2)\eta^2 - \eta^2}{(1-\eta^2)^2} = \frac{-\eta^4}{(1-\eta^2)^2} \leqslant 0.$$

Remark 3. In PPAC, the Lyapunov candidate is still in the quadratic form. In BLF, the Lyapunov candidate is in the composite form of logarithmic and fractional function. In this paper, we first modify the BLF as a pure fractional function, which renders the stability analysis easier. In addition, the boundary of the BLF is determined by the prescribed performance function $\rho(t)$, which makes the BLF able to deal with the time-varying transient performance.

5.2. Backstepping design

In this subsection, an adaptive controller is designed by using the backstepping technique.

5.2.1. Step 1

The virtual control law s_1 and the adaptive law ω_1 are designed as

$$s_1 = -c_1 z_1 - \omega_1^{\mathrm{T}} \varphi_1(x_1) + \dot{y}_d - \mu z_1 + z_1 \left\{ q(z_1)\frac{\dot{k}_b(t)}{k_b(t)} + [1-q(z_1)]\frac{\dot{k}_a(t)}{k_a(t)} \right\} \quad (24)$$

$$\dot{\omega}_1 = \varphi_1(x_1)\mu z_1 - \sigma\omega_1 \quad (25)$$

where $\sigma > 0$ is a constant parameter. Define a first-order filter as follows

$$\tau_1 \dot{\alpha}_1 + \alpha_1 = s_1 \quad (26)$$

where $\alpha_1(0) = s_1(0), \tau_1 > 0$ is the time constant. Because $\chi_1 = \alpha_1 - s_1$, we have

$$\dot{\alpha}_1 = -\frac{\chi_1}{\tau_1}. \quad (27)$$

Then,

$$\dot{\chi}_1 = \dot{\alpha}_1 - \dot{s}_1 = -\frac{\chi_1}{\tau_1} + H_1(\cdot) \quad (28)$$

where

$$H_1(\cdot) = c_1\dot{z}_1 + \dot{\omega}_1^{\mathrm{T}}\varphi_1(x_1) + \frac{\omega_1^{\mathrm{T}}\partial\varphi_1(x_1)}{\partial x_1}\dot{x}_1 - \ddot{y}_d + \mu\dot{z}_1 + \frac{\partial\mu}{\partial z_1}\dot{z}_1 z_1$$
$$-\dot{z}_1\left\{ q(z_1)\frac{\dot{k}_b}{k_b} + [1-q(z_1)]\frac{\dot{k}_a}{k_a} + q(z_1)\frac{\ddot{k}_b k_b - \dot{k}_b^2}{k_b^2} + [1-q(z_1)]\frac{\ddot{k}_a k_a - \dot{k}_a^2}{k_a^2} \right\}$$

is a continuous function of variables $z_1, \omega_1, y_d, \dot{y}_d, \ddot{y}_d$ and ρ.

5.2.2. Step 2

The virtual control law s_2 and the adaptive law ω_2 are designed as

$$s_2 = -c_2 z_2 - \omega_2^{\mathrm{T}}\varphi_2\left(\hat{\bar{x}}_2\right) - \frac{1}{2}z_2 - l_2 r^2(t)(y - \hat{x}_1) + \dot{\alpha}_1 \quad (29)$$

$$\dot{\omega}_2 = z_2\varphi_2\left(\hat{\bar{x}}_2\right) - \sigma\omega_2, \ \sigma > 0. \quad (30)$$

The first-order filter is designed as follows

$$\tau_2\dot{\alpha}_2 + \alpha_2 = s_2 \quad (31)$$

where $\alpha_2(0) = s_2(0), \tau_2 > 0$. Similar to Step 1, we have

$$\dot{\alpha}_2 = -\frac{\chi_2}{\tau_2}. \quad (32)$$

Then,

$$\dot{\chi}_2 = \dot{\alpha}_2 - \dot{s}_2 = -\frac{\chi_2}{\tau_2} + H_2(\cdot) \quad (33)$$

where

$$H_2(\cdot) = c_2\dot{z}_2 + \dot{\omega}_2^{\mathrm{T}}\varphi_2\left(\hat{\bar{x}}_2\right) + \frac{\omega_2^{\mathrm{T}}\partial\varphi_2\left(\hat{\bar{x}}_2\right)}{\partial\hat{\bar{x}}_2}\dot{\hat{\bar{x}}}_2 + \frac{1}{2}\dot{z}_2 - \frac{\dot{\chi}_1}{\tau_1}$$
$$+ 2l_2 r(t)(y - \hat{x}_1) + l_2 r^2(t)\left(\dot{y} - \dot{\hat{x}}_1\right).$$

is a continuous function of z_2, ω_2, χ_1.

5.2.3. Step i ($3 \leqslant i \leqslant n-1$)

The virtual control law s_i and the adaptive law ω_i are designed as

$$s_i = -c_i z_i - \omega_i^{\mathrm{T}}\varphi_i\left(\hat{\bar{x}}_i\right) - \frac{1}{2}z_i - l_i r^i(t)(y - \hat{x}_1) + \dot{\alpha}_{i-1} - z_{i-1} \quad (34)$$

$$\dot{\omega}_i = z_i\varphi_i\left(\hat{\bar{x}}_i\right) - \sigma\omega_i, \ \sigma > 0. \quad (35)$$

The first-order filter is designed as follows

$$\tau_i\dot{\alpha}_i + \alpha_i = s_i \quad (36)$$

where $\alpha_i(0) = s_i(0), \tau_i > 0$. Similar to Step 1, we have

$$\dot{\alpha}_i = -\frac{\chi_i}{\tau_i}. \quad (37)$$

Then,

$$\dot{\chi}_i = \dot{\alpha}_i - \dot{s}_i = -\frac{\chi_i}{\tau_i} + H_i(\cdot) \quad (38)$$

where

$$H_i(\cdot) = c_i\dot{z}_i + \dot{\omega}_i^{\mathrm{T}}\varphi_i\left(\hat{\bar{x}}_i\right) + \frac{\omega_i^{\mathrm{T}}\partial\varphi_i\left(\hat{\bar{x}}_i\right)}{\partial\hat{\bar{x}}_i}\dot{\hat{\bar{x}}}_i + \frac{1}{2}\dot{z}_i - \frac{\dot{\chi}_{i-1}}{\tau_{i-1}}$$
$$+ i l_i r^{i-1}(t)(y - \hat{x}_1) + l_i r^i(t)\left(\dot{y} - \dot{\hat{x}}_1\right) + \dot{z}_{i-1}.$$

is a continuous function of $z_i, \omega_i, \chi_{i-1}$.

5.2.4. Step n

In this step, the control signal v and the adaptive law ω_n are designed as

$$v = \frac{G\left(\hat{\bar{x}}_n\right)}{G^2\left(\hat{\bar{x}}_n\right) + \tau}$$
$$\left(-c_n z_n - \omega_n^{\mathrm{T}}\varphi_n\left(\hat{\bar{x}}_n\right) + \dot{\alpha}_{n-1} - z_{n-1} - \frac{1}{4}z_n - l_n r^n(t)(y - \hat{x}_1)\right) - \mathrm{sign}\left(G\left(\hat{\bar{x}}_n\right)\right)\nu \quad (39)$$

$$\dot{\omega}_n = z_n\varphi_n\left(\hat{\bar{x}}_n\right) - \sigma\omega_n, \sigma > 0 \quad (40)$$

where τ is a positive parameter such that

$$0 < \frac{G^2\left(\hat{\bar{x}}_n\right)}{G^2\left(\hat{\bar{x}}_n\right) + \tau} = 1 - \frac{\tau}{G^2\left(\hat{\bar{x}}_n\right) + \tau} < 1. \quad (41)$$

Therefore, the control input is singularity-free. ν is the additional signal which is designed as

$$\dot{v} = -\left|G\left(\hat{\bar{x}}_n\right)\right| v + G\left(\hat{\bar{x}}_n\right)(g(v(t)) - v(t)) + \frac{\tau}{G^2\left(\hat{\bar{x}}_n\right) + \tau} \times \left[c_n z_n + \omega_n^{\mathrm{T}} \varphi_n\left(\hat{\bar{x}}_n\right) - \dot{\alpha}_{n-1} + z_{n-1} + \frac{1}{4} z_n + l_n r^n(t)(y - \hat{x}_1)\right].$$

5.3. Stability analysis

This section shows the stability analysis process of the closed-loop system for the adaptive backstepping design in Section 5.2. The following theorem states the main results.

Theorem 1. Applying the virtual control input (24), (29), (34), the adaptive laws (25), (30), (35), (40), the dynamic surface filter parameter design (26), (31), (36), and the control signal (39) to the nonlinear system (1) under Assumptions 1–3, all the signals of the closed-loop system are SGUUB. Also, the tracking error satisfies the prescribed performance (21).

Proof. The proof consists of n steps.

Step 1: Consider the following Lyapunov function

$$V_1 = V_0 + V_\eta + \frac{\tilde{\omega}_1^{\mathrm{T}} \tilde{\omega}_1}{2} + \frac{\chi_1^2}{2}. \tag{42}$$

Based on (5), we have $x_2 = e_2 + \hat{x}_2$. Then,

$$\dot{z}_1 = z_2 + \chi_1 + s_1 + \omega_1^{\mathrm{T}} \varphi_1(x_1) + \tilde{\omega}_1^{\mathrm{T}} \varphi_1(x_1) + \varepsilon_1^* + e_2 - \dot{y}_d. \tag{43}$$

From (43), the time derivative of V_1 satisfies

$$\begin{aligned} \dot{V}_1 = & \ \dot{V}_0 + \left(\tfrac{\eta^2}{2(1-\eta^2)}\right)' + \tilde{\omega}_1^{\mathrm{T}} \dot{\tilde{\omega}}_1 + \chi_1 \dot{\chi}_1 \\ \leqslant & -\left(\delta \lambda_{\min}(P) - 4 - \gamma_0^2 \|P\|^2 u_N^2\right) \|\xi\|^2 + \tfrac{\eta \dot{\eta}}{1-\eta^2} - \tilde{\omega}_1^{\mathrm{T}} \dot{\omega}_1 + \chi_1 \dot{\chi}_1 + M_1 \end{aligned} \tag{44}$$

with

$$\begin{aligned} \left(\tfrac{\eta^2}{2(1-\eta^2)}\right)' = & \ \tfrac{2\eta(1-\eta^2) + 2\eta^3}{2(1-\eta^2)^2} \dot{\eta} = \tfrac{\eta}{(1-\eta^2)^2} \dot{\eta} \\ = & \ \tfrac{q(z_1)\eta_b}{(1-\eta_b^2)^2} \dot{\eta}_b + \tfrac{(1-q(z_1))\eta_a}{(1-\eta_a^2)^2} \dot{\eta}_a. \end{aligned}$$

Based on (43) and (44), we can obtain

$$\begin{aligned} \dot{V}_1 \leqslant & -\left(\delta \lambda_{\min}(P) - 4 - \gamma_0^2 \|P\|^2 u_N^2\right) \|\xi\|^2 + M_1 - \tilde{\omega}_1^{\mathrm{T}} \dot{\omega}_1 + \chi_1 \dot{\chi}_1 + \tfrac{q(z_1)\eta_b}{k_b(t)(1-\eta_b^2)^2}\left(\dot{z}_1 - z_1 \tfrac{\dot{k}_b(t)}{k_b(t)}\right) + \tfrac{(1-q(z_1))\eta_a}{k_a(t)(1-\eta_a^2)^2}\left(\dot{z}_1 - z_1 \tfrac{\dot{k}_a(t)}{k_a}\right) \\ = & -\left(\delta \lambda_{\min}(P) - 4 - \gamma_0^2 \|P\|^2 u_N^2\right) \|\xi\|^2 - \tilde{\omega}_1^{\mathrm{T}} \dot{\omega}_1 + \chi_1 \dot{\chi}_1 + M_1 + \mu z_1 \left(\dot{z}_1 - q z_1 \tfrac{\dot{k}_b(t)}{k_b(t)} - (1-q) z_1 \tfrac{\dot{k}_a(t)}{k_a(t)}\right) \\ = & -\left(\delta \lambda_{\min}(P) - 4 - \gamma_0^2 \|P\|^2 u_N^2\right) \|\xi\|^2 - \tilde{\omega}_1^{\mathrm{T}} \dot{\omega}_1 + \chi_1 \dot{\chi}_1 + M_1 + \mu z_1 \left[\omega_1^{\mathrm{T}} \varphi_1(x_1) + \tilde{\omega}_1^{\mathrm{T}} \varphi_1(x_1) + \varepsilon_1^* + e_2 - \dot{y}_d - q z_1 \tfrac{\dot{k}_b(t)}{k_b(t)} - (1-q) z_1 \tfrac{\dot{k}_a(t)}{k_a(t)} + z_2 + \chi_1 + s_1\right] \end{aligned} \tag{45}$$

with

$$\mu z_1 = \frac{q(z_1)\eta_b}{k_b(t)(1-\eta_b^2)^2} + \frac{(1-q(z_1))\eta_a}{k_a(t)(1-\eta_a^2)^2}$$

and $\mu = \frac{q(z_1)}{(k_b^2(t) - z_1^2)^2} + \frac{1-q(z_1)}{(k_a^2(t) - z_1^2)^2}$. Applying Young's inequality, we have

$$\begin{aligned} \mu z_1 z_2 \leqslant & \ \tfrac{1}{4}(\mu z_1)^2 + z_2^2 \\ \mu z_1 \chi_1 \leqslant & \ \tfrac{1}{4}(\mu z_1)^2 + \chi_1^2 \\ \mu z_1 e_2 \leqslant & \ \tfrac{1}{4}(\mu z_1)^2 + e_2^2 \leqslant \tfrac{1}{4}(\mu z_1)^2 + \left\|\Lambda^{-1}\right\|^2 \|\xi\|^2 \\ \mu z_1 \varepsilon_1^* \leqslant & \ \tfrac{1}{4}(\mu z_1)^2 + \|\bar{\varepsilon}_1\|^2. \end{aligned} \tag{46}$$

Substituting (46) into (45) yields

$$\begin{aligned} \dot{V}_1 \leqslant & -\left(\delta \lambda_{\min}(P) - 4 - \gamma_0^2 \|P\|^2 u_N^2\right) \|\xi\|^2 + M_1 + z_2^2 \\ & + \left\|\Lambda^{-1}\right\|^2 \|\xi\|^2 + \|\bar{\varepsilon}_1\|^2 + \tilde{\omega}_1^{\mathrm{T}}(\varphi_1(x_1)\mu z_1 - \dot{\omega}_1) + \chi_1 \dot{\chi}_1 + \mu z_1 \\ & \left[\omega_1^{\mathrm{T}} \varphi_1(x_1) - \dot{y}_d + s_1 + \mu z_1 - \left(q z_1 \frac{\dot{k}_b(t)}{k_b(t)} + (1-q) z_1 \frac{\dot{k}_a(t)}{k_a(t)}\right)\right] + \chi_1^2. \end{aligned} \tag{47}$$

Applying (28) and Young's inequality, we have

$$\begin{aligned} \chi_1 \dot{\chi}_1 = & -\tfrac{\chi_1^2}{\tau_1} + H_1(\cdot)\chi_1 \leqslant -\tfrac{\chi_1^2}{\tau_1} + |H_1(\cdot)\chi_1| \\ \leqslant & -\tfrac{\chi_1^2}{\tau_1} + \tfrac{\chi_1^2}{2\pi_1} + \tfrac{\pi_1 H_1^2(\cdot)}{2}. \end{aligned} \tag{48}$$

Inserting (24), (25) and (48) into (47) yields

$$\begin{aligned} \dot{V}_1 \leqslant & -\left(\delta \lambda_{\min}(P) - 4 - \gamma_0^2 \|P\|^2 u_N^2\right) \|\xi\|^2 + \sigma \tilde{\omega}_1^{\mathrm{T}} \omega_1 \\ & - c_1 \mu z_1^2 + z_2^2 - \left(\frac{1}{\tau_1} - \frac{1}{2\pi_1} - 1\right) \chi_1^2 + M_1 \end{aligned} \tag{49}$$

where $M_1 = \left\|\Lambda^{-1}\right\|^2 \|\xi\|^2 + M_0 + \|\bar{\varepsilon}_1\|^2 + \frac{\pi_1 H_1^2(\cdot)}{2}$.

Step 2: Define the Lyapunov function candidate as follows

$$V_2 = V_1 + \frac{1}{2} z_2^2 + \frac{\tilde{\omega}_2^{\mathrm{T}} \tilde{\omega}_2}{2} + \frac{\chi_2^2}{2}. \tag{50}$$

According to (17), the dynamics of z_2 is

$$\dot{z}_2 = \omega_2^{\mathrm{T}} \varphi_2\left(\hat{\bar{x}}_2\right) + \hat{x}_3 + l_2 r^2(t)(y - \hat{x}_1) - \dot{\alpha}_1. \tag{51}$$

According to (19) and (51), the time derivative of V_2 is

$$\begin{aligned} \dot{V}_2 = & \ \dot{V}_1 - \tilde{\omega}_2^{\mathrm{T}} \dot{\omega}_2 + \chi_2 \dot{\chi}_2 \\ & + z_2\left[\omega_2^{\mathrm{T}} \varphi_2\left(\hat{\bar{x}}_2\right) + \chi_2 + s_2 + z_3 + l_2 r^2(t)(y - \hat{x}_1) - \dot{\alpha}_1\right] \\ & - \tilde{\omega}_2^{\mathrm{T}} \varphi_2\left(\hat{\bar{x}}_2\right) z_2 + \tilde{\omega}_2^{\mathrm{T}} \varphi_2\left(\hat{\bar{x}}_2\right) z_2. \end{aligned} \tag{52}$$

Using Young's inequality and Lemma 1, we have

$$\begin{aligned} & z_2 \chi_2 \leqslant \frac{1}{4} z_2^2 + \chi_2^2 \chi_2 \dot{\chi}_2 \leqslant -\frac{\chi_2^2}{\tau_2} + \frac{H_2^2(\cdot)}{2\pi_2} \chi_2^2 + \frac{\pi_2}{2} \\ & - z_2 \tilde{\omega}_2^{\mathrm{T}} \varphi_2\left(\hat{\bar{x}}_2\right) \leqslant \frac{1}{4} z_2^2 + N_2 \tilde{\omega}_2^{\mathrm{T}} \tilde{\omega}_2. \end{aligned} \tag{53}$$

Substituting (29), (30) and (53) into (52) yields

$$\begin{aligned} \dot{V}_2 \leqslant & -\left(\delta \lambda_{\min}(P) - 4 - \gamma_0^2 \|P\|^2 u_N^2\right) \|\xi\|^2 + \sigma \sum_{j=1}^{2} \tilde{\omega}_j^{\mathrm{T}} \omega_j \\ & - \sum_{j=1}^{2} \left(\tfrac{1}{\tau_j} - \tfrac{1}{2\pi_j} - 1\right) \chi_j^2 - c_1 \mu z_1^2 - c_2 z_2^2 + M_2 + z_2^2 + z_2 z_3 \end{aligned}$$

where $M_2 = M_1 + N_2 \tilde{\omega}_2^{\mathrm{T}} \tilde{\omega}_2 + \frac{\pi_2 H_2^2(\cdot)}{2}$.

Step i $(3 \leqslant i \leqslant n-1)$**:** Define the Lyapunov function candidate as follows

$$V_i = V_{i-1} + \frac{1}{2} z_i^2 + \frac{\tilde{\omega}_i^{\mathrm{T}} \tilde{\omega}_i}{2} + \frac{\chi_i^2}{2}. \tag{55}$$

According to (17), the dynamics of z_i is

$$\begin{aligned}\dot{z}_i =& \ \omega_i^{\mathrm{T}}\varphi_i\left(\hat{\bar{x}}_i\right) + \chi_i + s_i + z_{i+1} + l_i r^i(t)(y-\hat{x}_1) - \dot{\alpha}_{i-1} \\ =& \ -c_i z_i - \tfrac{1}{2}z_i - z_{i-1} + z_{i+1} + \chi_i\end{aligned} \tag{56}$$

According to (19) and (56), the time derivative of V_i is

$$\begin{aligned}&\dot{V}_i = \dot{V}_{i-1} - \tilde{\omega}_i^{\mathrm{T}}\dot{\omega}_i + \tilde{\omega}_i^{\mathrm{T}}\varphi_i\left(\hat{\bar{x}}_i\right)z_i + \chi_i\dot{\chi}_i - c_i z_i^2 \\ &+z_i\left[-\frac{1}{2}z_i - z_{i-1} + z_{i+1} + \chi_i - \tilde{\omega}_i^{\mathrm{T}}\varphi_i\left(\hat{\bar{x}}_i\right)\right]\end{aligned} \tag{57}$$

Using Young's inequality and Lemma 1, we have

$$\begin{aligned}&z_i\chi_i \leqslant \frac{1}{4}z_i^2 + \chi_i^2 \\ &\chi_i\dot{\chi}_i \leqslant -\frac{\chi_i^2}{\tau_i} + \frac{H_i^2(\cdot)}{2\pi_i}\chi_i^2 + \frac{\pi_i}{2} - z_i\tilde{\omega}_i^{\mathrm{T}}\varphi_i\left(\hat{\bar{x}}_i\right) \leqslant \frac{1}{4}z_i^2 + N_i\tilde{\omega}_i^{\mathrm{T}}\tilde{\omega}_i.\end{aligned} \tag{58}$$

Substituting (34), (35) and (58) into (57) yields

$$\begin{aligned}\dot{V}_i \leqslant& \ \dot{V}_{i-1} + \sigma\tilde{\omega}_i^{\mathrm{T}}\omega_i - \left(\frac{1}{\tau_i} - \frac{H_i^2(\cdot)}{2\pi_i} - 1\right)\chi_i^2 + \frac{\pi_i}{2} - c_i z_i^2 \\ &+ N_i\tilde{\omega}_i^{\mathrm{T}}\tilde{\omega}_i - z_i z_{i-1} + z_i z_{i+1} \\ \leqslant& -\left(\delta\lambda_{\min}(P) - 4 - \gamma_0^2\|P\|^2 u_N^2\right)\|\xi\|^2 + \sigma\sum_{j=1}^{i}\tilde{\omega}_j^{\mathrm{T}}\omega_j - \sum_{j=1}^{i}\left(\frac{1}{\tau_j} - \frac{1}{2\pi_j} - 1\right) \\ &\chi_j^2 - c_1\mu z_1^2 - \sum_{j=2}^{i}c_j z_j^2 + M_i + z_2^2 + z_i z_{i+1}\end{aligned} \tag{59}$$

where $M_i = M_{i-1} + \frac{\pi_i}{2} + N_i\tilde{\omega}_i^{\mathrm{T}}\tilde{\omega}_i + \frac{\pi_j H_j^2(\cdot)}{2}$.

Step n: Consider the Lyapunov candidate as

$$V_n = V_{n-1} + \frac{1}{2}z_n^2 + \frac{\tilde{\omega}_n^{\mathrm{T}}\tilde{\omega}_n}{2}. \tag{60}$$

According to (4) and (18), we have

$$\dot{z}_n = \omega_n^{\mathrm{T}}\varphi_n\left(\hat{\bar{x}}_n\right) + G\left(\hat{\bar{x}}_n\right)g(v(t)) + l_n r^n(t)(y-\hat{x}_1) - \dot{\alpha}_{n-1} - \dot{v}. \tag{61}$$

According to (59) and (61), we can obtain

$$\begin{aligned}\dot{V}_n =& \ \dot{V}_{n-1} - \tilde{\omega}_n^{\mathrm{T}}\varphi_n\left(\hat{\bar{x}}_n\right)z_n + \tilde{\omega}_n^{\mathrm{T}}\varphi_n\left(\hat{\bar{x}}_n\right)z_n - \tilde{\omega}_n^{\mathrm{T}}\dot{\omega}_n \\ &+ z_n\left\{\omega_n^{\mathrm{T}}\varphi_n\left(\hat{\bar{x}}_n\right) - \dot{\alpha}_{n-1} - \dot{v} + G\left(\hat{\bar{x}}_n\right)g(v(t)) + l_n r^n(t)(y-\hat{x}_1)\right\}.\end{aligned} \tag{62}$$

Using Young's inequality and Lemma 1, we have

$$-z_n\tilde{\omega}_n^{\mathrm{T}}\varphi_n\left(\hat{\bar{x}}_n\right) \leqslant \frac{1}{4}z_n^2 + N_n\tilde{\omega}_n^{\mathrm{T}}\tilde{\omega}_n. \tag{63}$$

Inserting (39) and (63) into (62) yields

$$\begin{aligned}\dot{V}_n \leqslant& \ \dot{V}_{n-1} + \sigma\tilde{\omega}_n^{\mathrm{T}}\omega_n - c_n z_n^2 + N_n\tilde{\omega}_n^{\mathrm{T}}\tilde{\omega}_n \\ \leqslant& -\left(\delta\lambda_{\min}(P) - 4 - \gamma_0^2\|P\|^2 u_N^2\right)\|\xi\|^2 - c_1\mu z_1^2 \\ &-\sum_{j=2}^{n}c_j z_j^2 - \sum_{j=1}^{n-1}\left(\frac{1}{\tau_j} - \frac{H_j^2(\cdot)}{2\pi_j} - 1\right)\chi_j^2 + z_2^2 + \sum_{j=1}^{n}\sigma\tilde{\omega}_j^{\mathrm{T}}\omega_j + M_{n+1}\end{aligned} \tag{64}$$

where $M_{n+1} = M_n + N_n\tilde{\omega}_n^{\mathrm{T}}\tilde{\omega}_n$. Note that

$$\tilde{\omega}_i^{\mathrm{T}}\omega_i = \tilde{\omega}_i^{\mathrm{T}}(\omega_i^* - \tilde{\omega}_i) \leqslant -\frac{1}{2}\tilde{\omega}_i^{\mathrm{T}}\tilde{\omega}_i + \frac{1}{2}\omega_i^{*\mathrm{T}}\omega_i^*, \quad i = 1,2,\ldots,n. \tag{65}$$

Substituting (65) into (64) yields

$$\begin{aligned}\dot{V}_n \leqslant& -\left(\delta\lambda_{\min}(P) - 4 - \gamma_0^2\|P\|^2 u_N^2\right)\|\xi\|^2 - c_1\mu z_1^2 - \sum_{j=2}^{n}c_j z_j^2 \\ &-\sum_{j=1}^{n-1}\left(\frac{1}{\tau_j} - \frac{1}{2\pi_j} - 1\right)\chi_j^2 + M_{n+1} - \frac{1}{2}\sum_{j=1}^{n}\sigma\tilde{\omega}_j^{\mathrm{T}}\omega_j + z_2^2\end{aligned}$$

where
$M_{n+1} = \frac{1}{2}\sum_{j=1}^{n}\sigma_j\omega_j^{*\mathrm{T}}\omega_j^* + M_n$.
For $i = 1,2,\ldots,n$, let

$$\begin{aligned}&\mathscr{A}_i = \{\vartheta | \kappa(\vartheta) \leqslant k\} \theta = \begin{bmatrix}\xi & z_1 & \cdots & z_i & \tilde{\omega}_1 & \cdots & \tilde{\omega}_i & \chi_1 & \cdots & \chi_i\end{bmatrix} \\ &\kappa(\vartheta) = \frac{1}{2}\sum_{j=1}^{i}z_j^2 + \frac{1}{2}\sum_{j=1}^{i}\tilde{\omega}_j^{\mathrm{T}}\tilde{\omega}_j + \frac{1}{2}\sum_{j=1}^{i}\chi_j^2 + \xi^{\mathrm{T}}P\xi\end{aligned}$$

where k is a positive constant. Note that $\kappa(\vartheta)$ is a continuous function, then, for arbitrary constant k, the set $\mathscr{A}_i$ is compact [11]. In addition, based on the fact that $H_j(\cdot)$ is a continuous function, there exists a positive constant B_j such that $|H_j(\cdot)| \leqslant B_j$ on the compact set $\mathscr{A}_i$ [37]. Therefore, we have

$$\begin{aligned}\dot{V}_n \leqslant& -\left(\delta\lambda_{\min}(P) - 4 - \gamma_0^2\|P\|^2 u_N^2\right)\|\xi\|^2 - c_1\mu z_1^2 - \sum_{j=2}^{n}c_j z_j^2 \\ &-\sum_{j=1}^{n-1}\left(\frac{1}{\tau_j} - \frac{1}{2\pi_j} - 1\right)\chi_j^2 + M_{n+1} - \frac{1}{2}\sum_{j=1}^{n}\sigma\tilde{\omega}_j^{\mathrm{T}}\omega_j + z_2^2.\end{aligned} \tag{66}$$

From (42), (50), (55) and (60), V_n can be equivalently expressed as

$$V_n = \underbrace{\xi^{\mathrm{T}}P\xi}_{V_\xi} + \underbrace{\frac{\eta^2}{2(1-\eta^2)}}_{V_\eta} + \underbrace{\frac{1}{2}\sum\nolimits_{j=2}^{n}z_j^2}_{V_z} + \underbrace{\frac{1}{2}\sum\nolimits_{j=1}^{n-1}\chi_j^2}_{V_\chi} + \underbrace{\frac{1}{2}\sum\nolimits_{j=1}^{n}\tilde{\omega}_j^{\mathrm{T}}\tilde{\omega}_j}_{V_\omega}.$$

Accordingly, $\dot{V}_n$ satisfies

$$\begin{aligned}\dot{V}_n \leqslant& \underbrace{-\left(\delta\lambda_{\min}(P) - 4 - \gamma_0^2\|P\|^2 u_N^2 + \left\|\Lambda^{-1}\right\|^2\right)\|\xi\|^2}_{\dot{V}_\xi}\underbrace{-c_1\mu z_1^2}_{\dot{V}_\eta} \\ &+ \underbrace{\quad}\left(\|P\|^2 N_1 - \frac{\sigma}{2}\right)\tilde{\omega}_1^{\mathrm{T}}\tilde{\omega}_1 + \sum\nolimits_{j=2}^{n}\left(\|P\|^2 N_j - \frac{\sigma}{2} + N_j\right)\tilde{\omega}_j^{\mathrm{T}}\tilde{\omega}_{j\dot{V}_\omega} \\ &+ \underbrace{z_2^2 - \sum\nolimits_{j=2}^{n}c_j z_j^2}_{\dot{V}_z} \underbrace{-\sum\nolimits_{j=1}^{n-1}\left(\frac{1}{\tau_j} - \frac{1}{2\pi_j} - 1\right)\chi_j^2}_{\dot{V}_\chi} + D\end{aligned}$$

where
$D = \frac{\sigma}{2}\sum_{j=1}^{n}\omega_j^{*\mathrm{T}}\omega_j^* + \|P\|^2\|\bar{\varepsilon}\|^2 + \|P\|^2\left\|\bar{\Gamma}\right\|^2 + \|\bar{\varepsilon}_1\|^2 + \frac{1}{2}\sum_{j=1}^{n-1}\pi_j B_j^2$.

It is noted that

$$\|\xi\|^2 \leqslant \frac{\xi^{\mathrm{T}}P\xi}{\lambda_{\min}(P)}, \ \delta > 0, \ \Lambda^{-1} = \mathrm{diag}\left(r^a, \ldots, r^{a+n-1}\right).$$

Then,

$$\begin{aligned}\dot{V}_\xi \leqslant& -\underbrace{\frac{\delta\lambda_{\min}(P) - 4 - \gamma_0^2\|P\|^2 u_N^2 - r^{2(a+n-1)}}{\lambda_{\min}(P)}}_{K_\xi}\xi^{\mathrm{T}}P\xi \\ =& -K_\xi V_\xi.\end{aligned} \tag{67}$$

According to Lemma 4, $\dot{V}_\eta$ can be rewritten as

$$\begin{aligned}\dot{V}_\eta =& -c_1\mu z_1^2 = -c_1\frac{\eta^2}{\left(1-\eta^2\right)^2} \\ \leqslant& -\underbrace{2c_1}_{K_\eta}\frac{\eta^2}{2(1-\eta^2)} = -K_\eta V_\eta.\end{aligned} \tag{68}$$

Similarly, we can obtain

$$\begin{aligned}
\dot{V}_z &= -\sum_{j=2}^{n} c_j z_j^2 + z_2^2 \\
&\leqslant \underbrace{-2\min\{c_2-1, c_3, \ldots, c_n\}}_{K_z} \tfrac{1}{2}\sum_{j=2}^{n} z_j^2 = -K_z V_z \\
\dot{V}_\chi &= -\sum_{j=1}^{n-1}\left(\tfrac{1}{\tau_j} - \tfrac{1}{2\pi_j} - 1\right)\chi_j^2 \\
&\leqslant \underbrace{-2\min\left\{\frac{1}{\tau_j} - \frac{1}{2\pi_j} - 1\right\}_{j=1}^{n-1}}_{K_\chi} \tfrac{1}{2}\sum_{j=1}^{n-1}\chi_j^2 = -K_\chi V_\chi \\
\dot{V}_\omega &= \left(\|P\|^2 N_1 - \tfrac{\sigma}{2}\right)\tilde{\omega}_1^{\mathrm{T}}\tilde{\omega}_1 + \sum_{j=2}^{n}\left(\|P\|^2 N_j - \tfrac{\sigma}{2} + N_j\right)\tilde{\omega}_j^{\mathrm{T}}\tilde{\omega}_j \\
&\leqslant \underbrace{-2\min\left(-\|P\|^2 N_j + \frac{\sigma}{2} - N_j, -\|P\|^2 N_1 + \frac{\sigma}{2}\right)}_{K_\omega} \tfrac{1}{2}\sum_{j=1}^{n}\tilde{\omega}_j^{\mathrm{T}}\tilde{\omega}_j \\
&= -K_\omega V_\omega
\end{aligned}$$

Then,

$$\begin{aligned}
\dot{V}_n &\leqslant \dot{V}_\xi + \dot{V}_\eta + \dot{V}_z + \dot{V}_\chi + \dot{V}_\omega + D \\
&\leqslant -K_\xi V_\xi - K_\eta V_\eta - K_z V_z - K_\chi V_\chi - K_\omega V_\omega + D \\
&\leqslant -\min\{K_\xi, K_\eta, K_z, K_\chi, K_\omega\}(V_\xi + V_\eta + V_z + V_\chi + V_\omega) + D.
\end{aligned} \tag{69}$$

Select the design parameters τ_j, c_i such that $c_1 > 0, c_2 > 1, c_i > 0, \frac{1}{\tau_j} - \frac{1}{2\pi_j} - 1 > 0$. Choose a positive and large value for the design parameter δ such that

$$\lambda_{\min}(P) > \frac{4 + \gamma_0^2\|P\|^2 u_N^2 + r^{2(a+n-1)}}{\delta}. \tag{70}$$

Denote $C = \min\{K_\xi, K_\eta, K_z, K_\omega, K_\chi\}$. Then, (69) can be further expressed as

$$\dot{V}_n \leqslant -CV_n + D. \tag{71}$$

Integrating of the differential inequality (71) yields

$$V_n \leqslant V_n(0)e^{-ct} + \frac{D}{C}\left(1 - e^{-ct}\right) \tag{72}$$

where c is a constant.

Note that upper bound of $H_j(\cdot)$ on the compact set $\mathscr{A}_j, B_j$, is a positive constant and is bounded even it is unknown. In addition, the unknown bound B_j is only involved in the analysis and does not show up in the controller design procedure. According to (72) and Lemma 1.2 in [40], $V_n(t)$ is bounded. This implies $z_i, \omega_i, i = 2, \ldots, n, \xi, \eta, \chi_j, j = 1, \ldots, n-1$ are bounded. From Assumption 2, there exists a class-$\mathscr{K}$ function [41] $\phi(\cdot)$ such that

$$\sup_{0\leqslant\tau\leqslant t}\|x(\tau)\| \leqslant K + \phi\left(\sup_{0\leqslant\tau\leqslant t}\|u(\tau)\|\right) \leqslant K + \phi(u_N^2)$$

where K is a constant. Thus, the global boundedness of $x_1, x_2, \ldots, x_n$ is established. Based on above discussions, we can have following inferences.

(1) According to (5), we can obtain that $\hat{x}_2, \ldots, \hat{x}_n$ are bounded due to the boundedness of e and $x_i,\ i = 1, 2, \ldots, n$.
(2) According to (17), $\alpha_1, \ldots, \alpha_{n-2}$ are bounded due the boundedness of z_i and $\hat{x}_i, i = 2, \ldots, n-1$. Then $s_1, \ldots, s_{n-2}$ are bounded according to (19) and the boundedness of $\chi_j, j = 1, \ldots, n-2$.
(3) According to (37), $\dot{\alpha}_{n-2}$is bounded due to the boundedness of χ_{n-2}. According to (34), s_{n-1} is bounded due to the boundedness of z_{n-1} and z_n. Then α_{n-1} is bounded due to the boundedness of χ_{n-1} and (19).
(4) According to (19), v is bounded due to the boundedness of $z_n, \hat{x}_n$ and α_{n-1}. Then v is bounded according to39).

Thus all the closed-loop signals are bounded and the output tracking error satisfies the prescribed performance. In this paper, the initial states of the control systems belong to a compact set instead of an entire space. Therefore, the signals of systems are SGUUB. This completes the proof.

Remark 4. From the discussion in Section 4, the parameter $r(t)$ is bounded. Therefore, the right hand side of (70) is upper-bounded. Then, from Lemma 3 and the proof of Theorem 1, the combination of (7) and (70) is a set of well-defined linear matrix inequalities, where the matrix P can be efficiently obtained by using the well-known optimization YALMIP [42].

6. Simulation study

To prove the above control scheme, two examples are considered with the uncertain nonlinear dynamics

$$\begin{aligned}
\dot{x}_1 &= x_2 + f_1(x_1) \\
\dot{x}_2 &= G(x_1, x_2)u + f_2(x_1, x_2) \\
y &= x_1
\end{aligned}$$

where

$$\begin{aligned}
f_1 &= (x_1 - x_1^3)/(1 + x_1^4), f_2 = -e^{-x_1^2}\sin(5x_1) \\
G &= 3\sin(x_1)
\end{aligned}$$

The control objective is to design a control signal u such that the system output y tracks the reference $y_d = sin(t) + cos(t)$. For the prescribed performance function, the parameters are selected as $k = 0.3, \rho_0 = 2.6, \rho_\infty = 0.4, k_1 = 2, k_2 = 1.8$.

6.1. Example 1: Classical design

In this example, we employ the classical backstepping framework with the Luenberger observer and quadratic Lyapunov function design for the tracking controller design.

To handle the nonlinear functions, we employ the RBFNN with five hidden nodes to compensate for the unknown dynamics $f_1(\cdot)$ and $f_2(\cdot)$ with the adaptive parameter adaptation as

$$\dot{\omega}_1 = z_1\varphi_1(x_1) - \sigma\omega_1,\ \dot{\omega}_2 = z_2\varphi_2\left(\hat{\bar{x}}_1\right) - \sigma\omega_2,\ \sigma = 2.$$

The observer is designed as

$$\begin{aligned}
&\dot{\hat{x}}_1 = \omega_1^{\mathrm{T}}\varphi_1(x_1) + \hat{x}_2 + l_1(y - \hat{x}_1), \\
&\dot{\hat{x}}_2 = \omega_2^{\mathrm{T}}\varphi_2\left(\hat{\bar{x}}_2\right) + u + l_2(y - \hat{x}_1), \\
&l_1 = 2, l_2 = 2,
\end{aligned}$$

The virtual control law and the control signal takes the forms

$$\begin{aligned}
s_1 &= -c_1 z_1 - \omega_1^{\mathrm{T}}\varphi_1(x_1) + \dot{y}_d \\
u &= -c_2 z_2 - \omega_2^{\mathrm{T}}\varphi_2(x_2) + \dot{s}_1 - l_2(y - \hat{x}_1) - \tfrac{5}{4}z_2
\end{aligned}$$

where $c_1 = 5, c_2 = 5$. The dynamic surface filter is designed as

$$\tau_1\dot{\alpha}_1 + \alpha_1 = s_1, \tau_1 = 0.05.$$

By applying the proposed adaptive controller to the system, simulation results are shown in Figs. 1–4. The tracking performance of the proposed control scheme is shown in Fig. 1. One can observe that classical method does not guarantee the prescribed tracking performance due to the singular control direction. The evolution of states x_1, x_2 with their estimation $\hat{x}_1, \hat{x}_2$ are shown

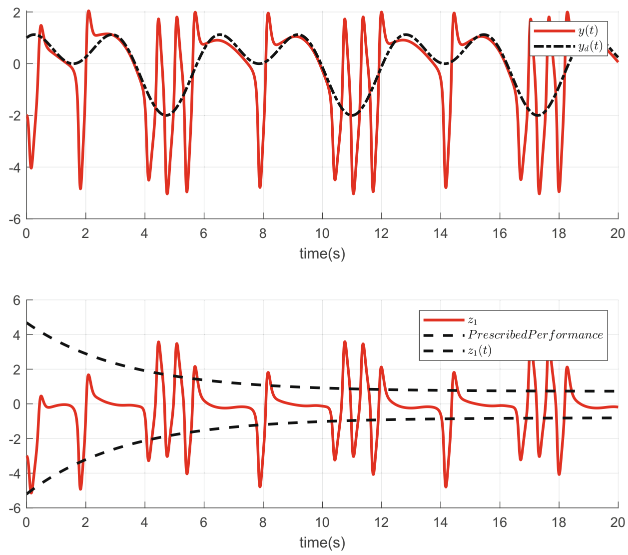

Fig. 1. The output tracking performance.

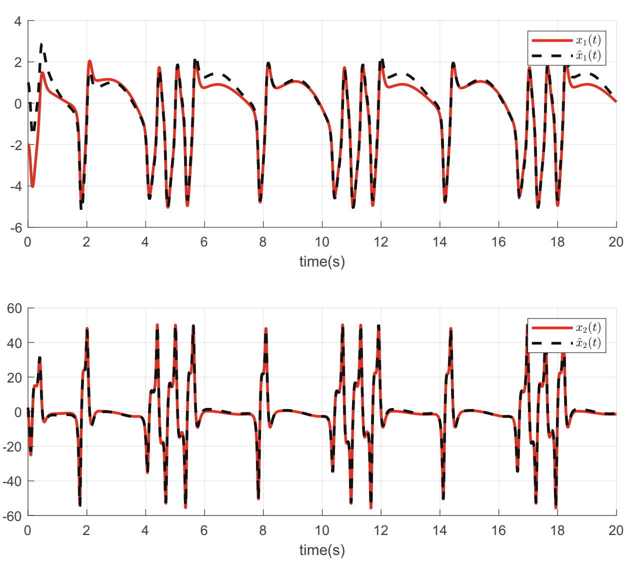

Fig. 2. State estimation.

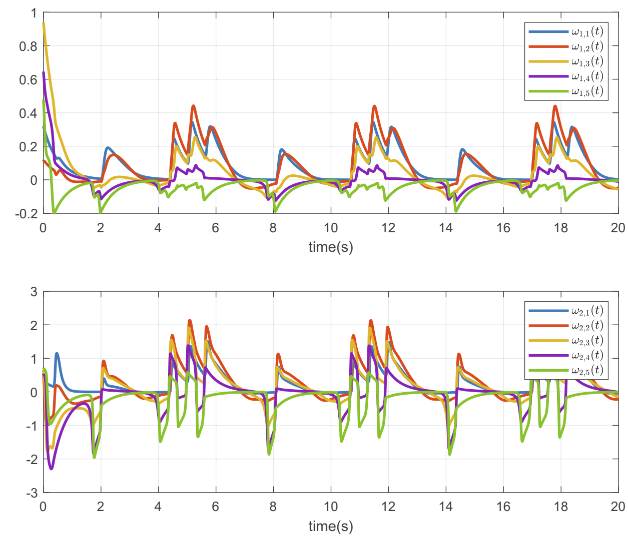

Fig. 3. Adaptive laws ω_{1j}, ω_{2j}, for $j = 1, \ldots, 5$.

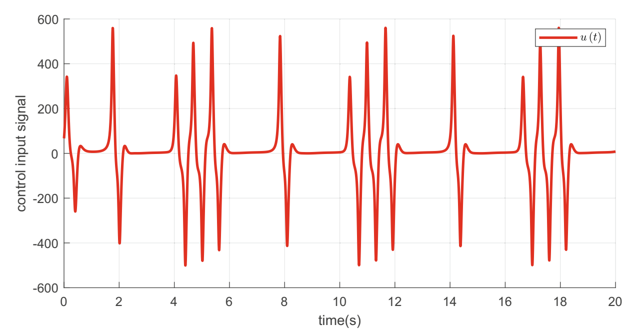

Fig. 4. Control signal $u(t)$.

in Fig. 2. The adaptive laws ω_1, ω_2 are shown in Fig. 3. Fig. 4 shows the change of the control input signal $u(t)$, which has quite a large norm.

6.2. Example 2: Presented design

In this example, we employ the presented design scheme with the high-gain observer and BLF with perscribed performance design for the tracking controller design.

To handle the nonlinear functions, we employ the RBFNN with five hidden nodes to compensate for the unknown dynamics $f_1(\cdot)$ and $f_2(\cdot)$ with the adaptive parameter adaptation as

$$\dot{\omega}_1 = \varphi_1(x_1)\mu z_1 - \sigma\omega_1,\ \omega_1(0) = 0.1,$$

$$\dot{\omega}_2 = z_2\varphi_2\left(\hat{\bar{x}}_2\right) - \sigma\omega_2,\ \sigma = 2.$$

The high-gain observer is designed as

$$\begin{aligned}
\dot{\hat{x}}_1 &= \omega_1^{\mathrm{T}}\varphi_1(x_1) + \hat{x}_2 + l_1 r(t)(y - \hat{x}_1) \\
\dot{\hat{x}}_2 &= \omega_2^{\mathrm{T}}\varphi_2\left(\hat{\bar{x}}_2\right) + Gg(v) + l_2 r^2(t)(y - \hat{x}_1) \\
l_1 &= 2, l_2 = 2 \\
\dot{r} &= -\tfrac{r}{a}\left[\tfrac{\delta}{3}(r-1) - \tfrac{2(n-1)L(y)}{\sqrt{p}}\right], \\
a &= 4, \delta = 8,\ p = 0.4, L(y) = 6.4,
\end{aligned}$$

The virtual control law and the control signal are designed as

$$\begin{aligned}
s_1 = &\ -c_1 z_1 - \omega_1^{\mathrm{T}}\varphi_1(x_1) + \dot{y}_d - \mu z_1 \\
&+ z_1\left(q(z_1)\tfrac{\dot{k}_b(t)}{k_b(t)} + (1 - q(z_1))\tfrac{\dot{k}_a(t)}{k_a(t)}\right) \\
s_2 = &\ -c_2 z_2 - \omega_2^{\mathrm{T}}\varphi_2\left(\hat{\bar{x}}_2\right) - \tfrac{1}{2}z_2 - l_2 r^2(t)(y - \hat{x}_1) + \dot{\alpha}_1 \\
v = &\ \tfrac{G\left(\hat{\bar{x}}_3\right)}{\left(G^2\left(\hat{\bar{x}}_3\right) + \tau\right)}\left(-c_3 z_3 - \omega_3^{\mathrm{T}}\varphi_3\left(\hat{\bar{x}}_3\right) + \dot{\alpha}_2 - z_2 - \tfrac{1}{4}z_3 - l_3 r^3(t)(y - \hat{x}_1)\right) \\
&- sign\left(G\left(\hat{\bar{x}}_3\right)\right)v
\end{aligned}$$

where $c_1 = 5, c_2 = 5, \tau = 3.8$. The dynamic surface filter is designed as

$$\tau_1\dot{\alpha}_1 + \alpha_1 = s_1, \tau_1 = 0.05.$$

The additional signal is designed as

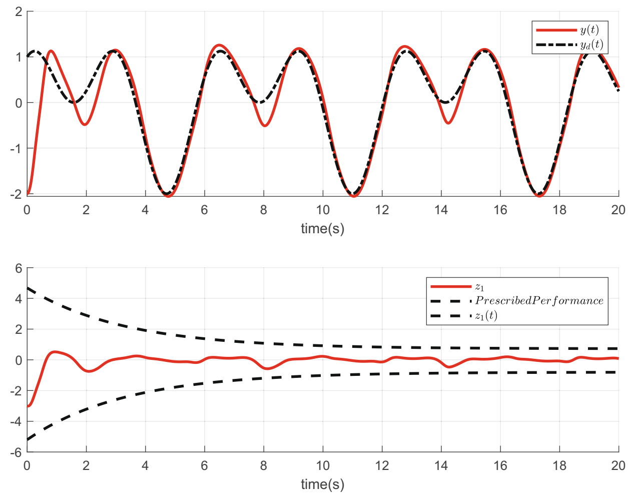

Fig. 5. Output tracking performance.

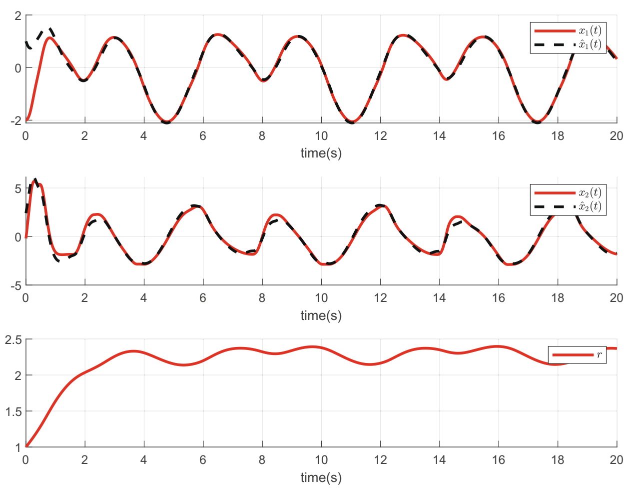

Fig. 6. State estimation with the high-gain observer design.

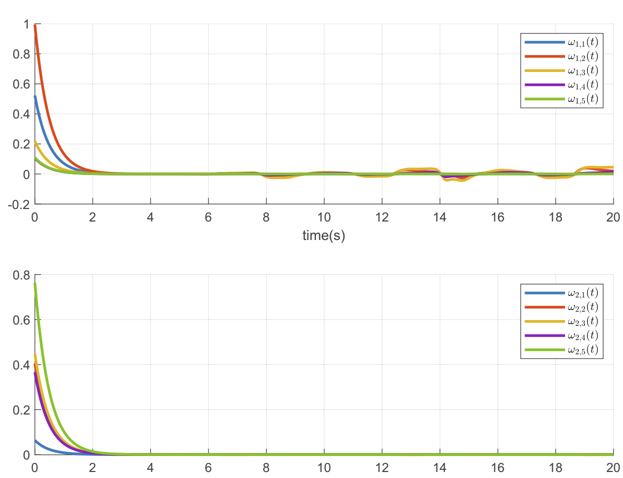

Fig. 7. Adaptive laws ω_{1j}, ω_{2j}, for $j = 1, \ldots, 5$.

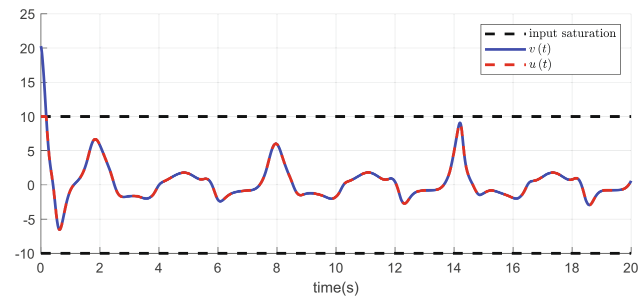

Fig. 8. Input Saturation Design: $v(t)$: auxiliary input; $u(t)$: control signal.

$$\dot{v} = -\left|G\left(\hat{\bar{x}}_3\right)\right| v + G\left(\hat{\bar{x}}_3\right)(g(v) - v) + \frac{\tau}{G^2\left(\hat{\bar{x}}_3\right) + \tau} \times \left[c_3 z_3 + \omega_3^{\mathrm{T}} \varphi_3\left(\hat{\bar{x}}_3\right) - \dot{\alpha}_2 + z_2 + \frac{1}{4} z_3 + l_3 r^3(t)(y - \hat{x}_1)\right]$$

By applying the proposed adaptive controller to the system, simulation results are shown in Figs. 5–8. The tracking performance of the proposed control scheme is shown in Fig. 5. One can observe that the combination of PPAC with BLF guarantees the prescribed tracking performance with the existence of the singular control direction. The evolution of states x_1, x_2 with their estimation $\hat{x}_1, \hat{x}_2$ using high-gain observer design are shown in Fig. 6. The adaptive laws ω_1, ω_2 are shown in Fig. 7. Fig. 8 shows the input saturation design of control input $u(t)$ with auxiliary input $v(t)$.

7. Conclusion

In this work, the tracking problem for strict-feedback systems with uncertain nonlinearity is solved by designing an adaptive output-feedback controller. The unmeasured states are estimated by establishing a high-gain observer, which can relax the Lipschitz condition of the system dynamics. The output constraints are solved by an asymmetric BLF with the time-varying prescribed performance. The control scheme is free of control direction singularity by introducing a positive parameter into the controller denominator design. Additionally, the presented controller design can guarantee the prescribed tracking performance with the saturation constraints. It is shown that with the presented design scheme, all the closed-loop signals are guaranteed to be bounded. Future work aims to extend the results in this paper to the distributed synchronization protocol design of multi-agent systems.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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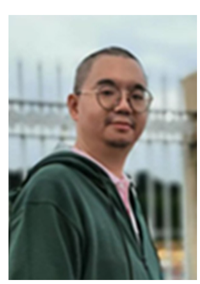

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