Assignment 3

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Introduction

The purpose of this assignment is to model a credit-risky portfolio of corporate bonds. Consider a structural model for portfolio credit risk described in class. Using the data for 100 counterparties, simulate 1-year losses for each corporate bond. You will need to generate 3 sets of scenarios:

- Monte Carlo approximation 1 : 5000 in-sample scenarios ($N = 1000 \cdot 5 = 5000$ (1000 systemic scenarios and 5 idiosyncratic scenarios for each systemic), non-Normal distribution of losses)
- Monte Carlo approximation 2 : 5000 in-sample scenarios ($N = 5000$ (5000 systemic scenarios and 1 idiosyncratic scenario for each systemic), non-Normal distribution of losses)
- True distribution: 100000 out-of-sample scenarios ($N = 100000$ (100000 systemic scenarios and 1 idiosyncratic scenario for each systemic), non-Normal distribution of losses).

Evaluate VaR and CVaR at quantile levels 99% and 99.9% for the two portfolios:

(1) one unit invested in each of 100 bonds

(2) equal value (dollar amount) is invested in each of 100 bonds

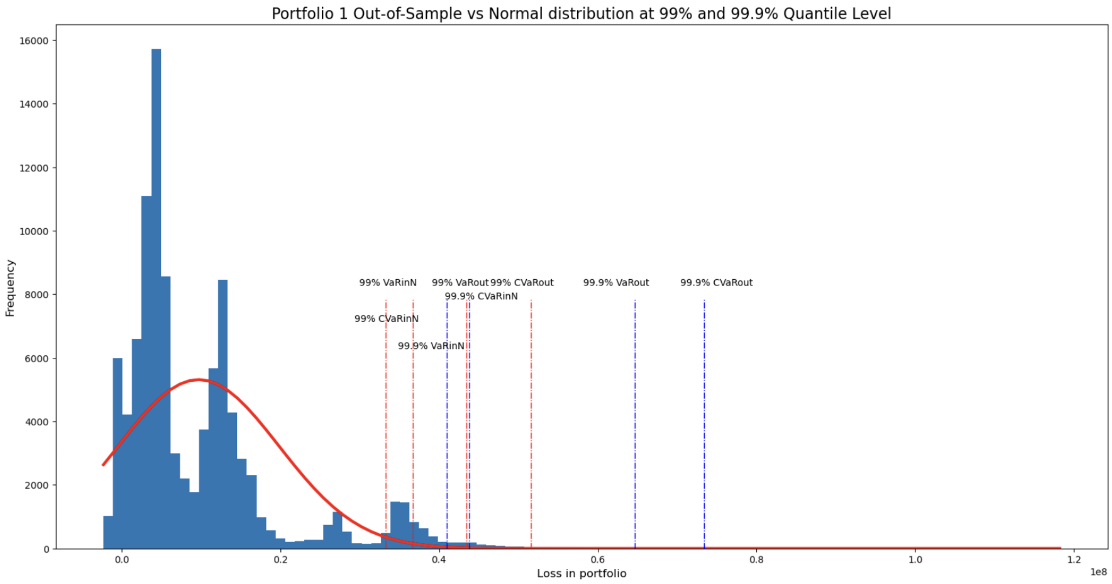
1.Implement portfolio credit risk simulation model in Python:

Please refer to credit_risk_simul.ipynb file

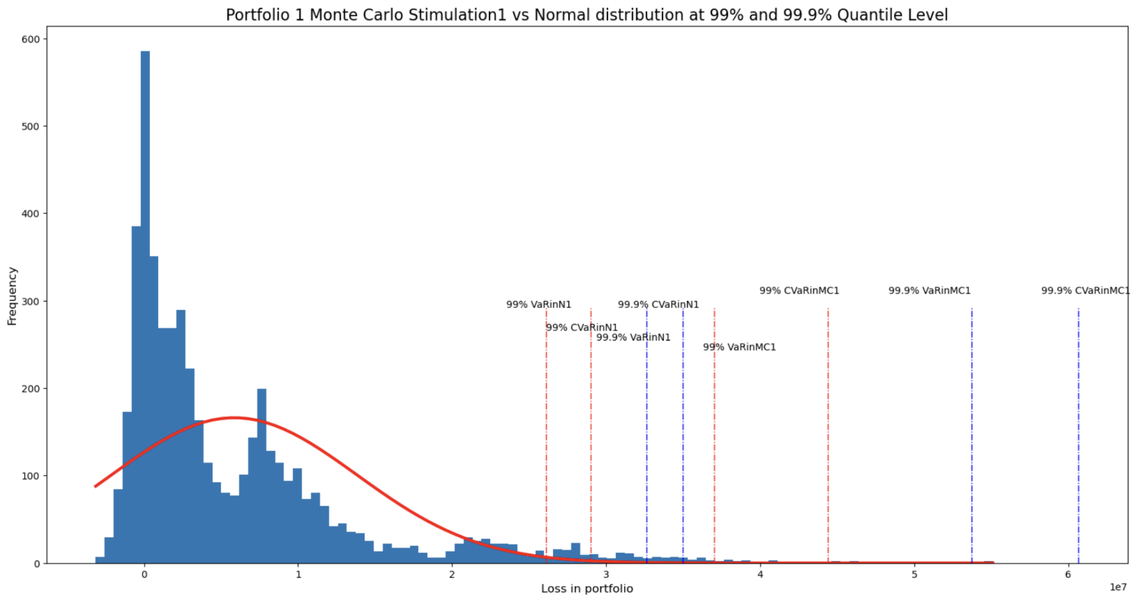
2. Analyze your results:

- Produce the following output from your Python code(refer to Appendix)
- Plot loss distributions in Python that illustrate both out-of-sample and in-sample results.Include plots that help illustrating your analysis in the report.

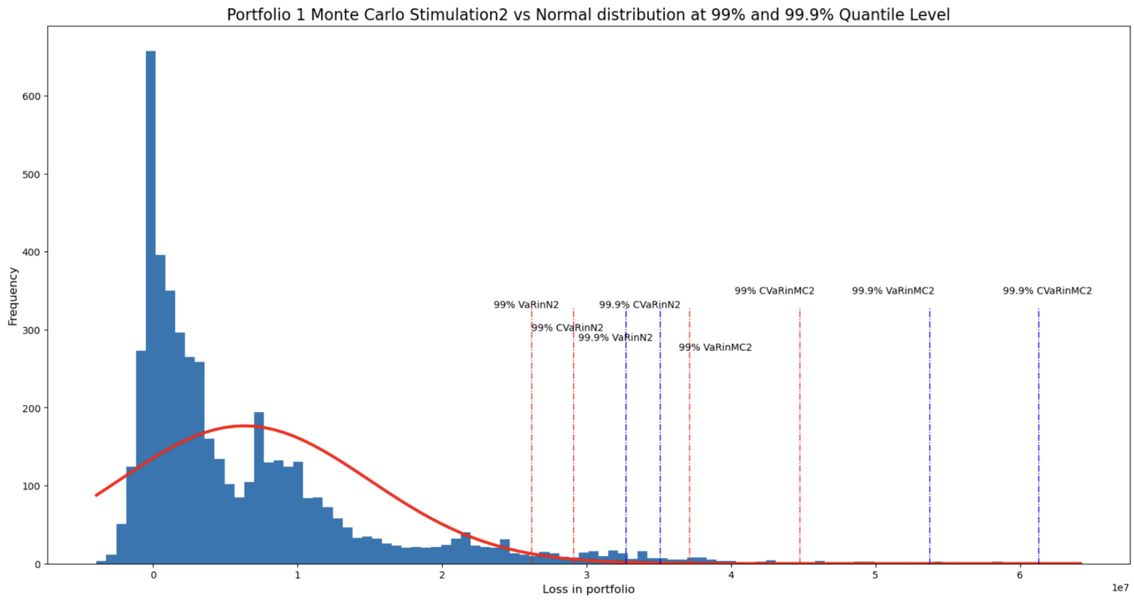
True loss distribution, portfolio 1



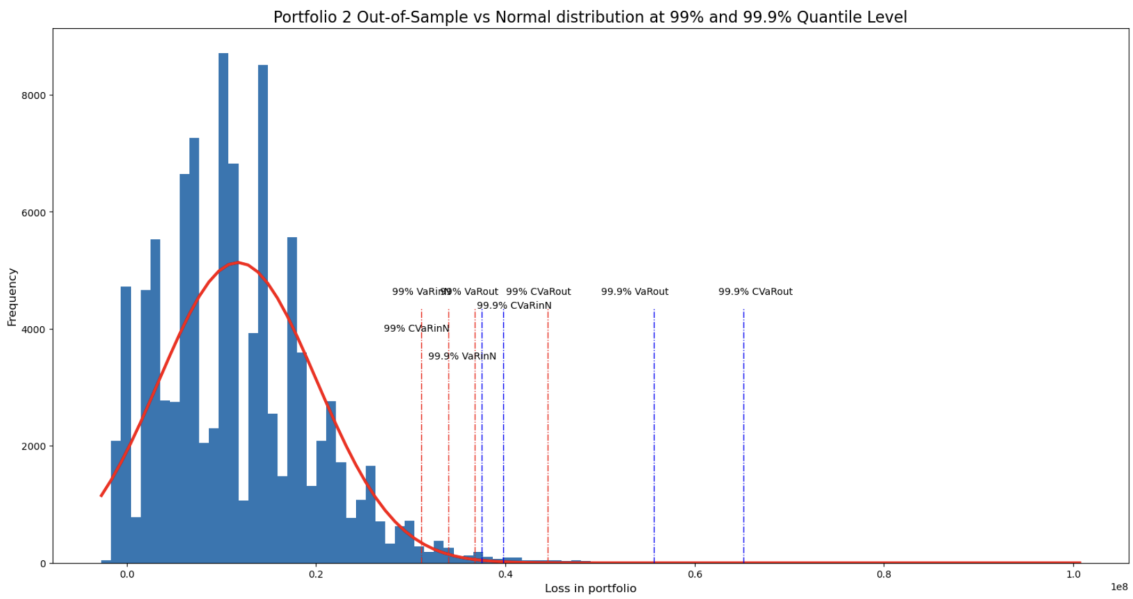
Loss distribution of MC1, portfolio 1



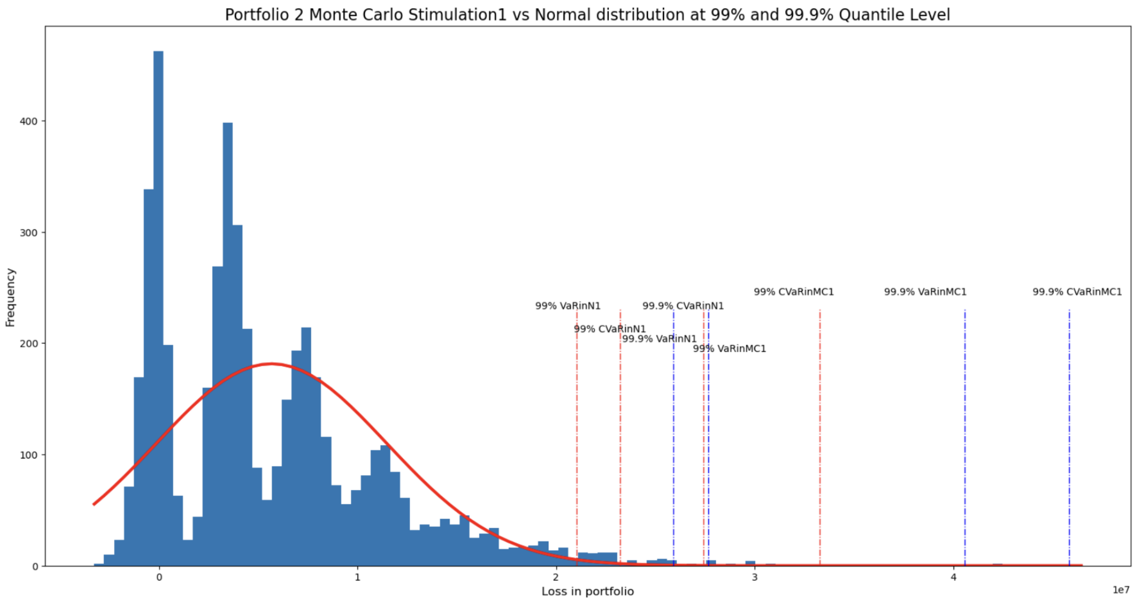
Loss distribution of MC2, portfolio 1



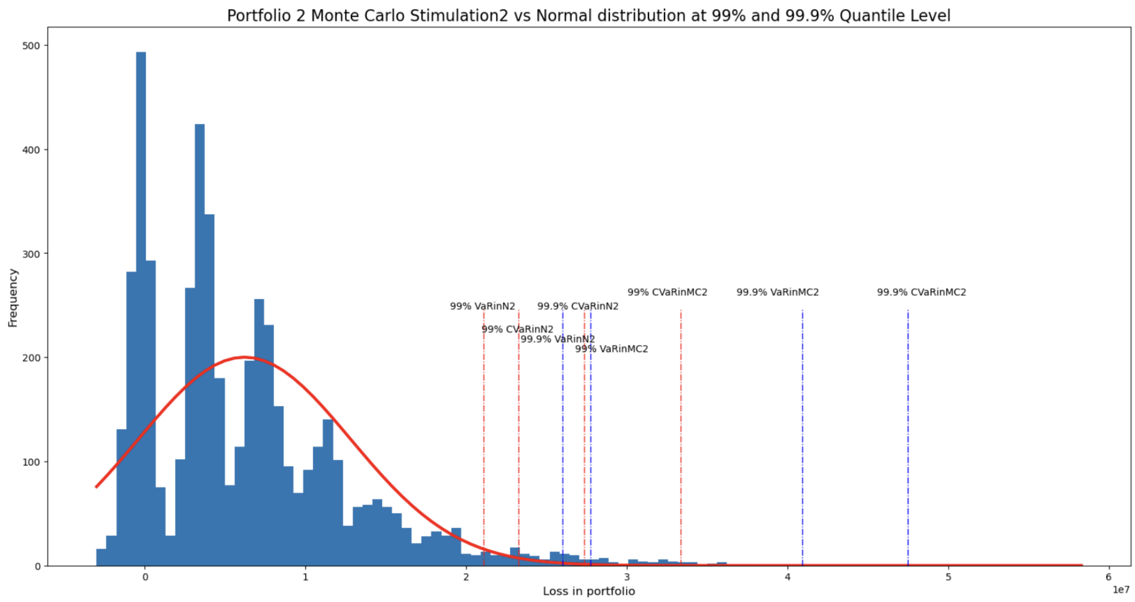
True loss distribution, portfolio 2



Loss distribution of MC1, portfolio 2



Loss distribution of MC2, portfolio 2



- Analyze sampling error when comparing non-Normal approximations to the true (out-of-sample) loss distribution. Analyze model error when comparing Normal approximations to the true (out-of-sample) loss distribution. Summarize the two types of errors in the tables.

Sampling Error

	Portfolio Distribution	Sampling Error(%)
0	Portfolio1 MC1(99% VaR)	14.745901
1	Portfolio1 MC1(99.9% VaR)	16.951261
2	Portfolio1 MC1(99% CVaR)	13.963657
3	Portfolio1 MC1(99.9% CVaR)	17.282284
4	Portfolio1 MC2(99% VaR)	14.526685
5	Portfolio1 MC2(99.9% VaR)	16.860004
6	Portfolio1 MC2(99% CVaR)	13.345626
7	Portfolio1 MC2(99.9% CVaR)	16.451276
8	Portfolio2 MC1(99% VaR)	25.328040
9	Portfolio2 MC1(99.9% VaR)	27.115125
10	Portfolio2 MC1(99% CVaR)	25.225104
11	Portfolio2 MC1(99.9% CVaR)	29.649952
12	Portfolio2 MC2(99% VaR)	25.582359
13	Portfolio2 MC2(99.9% VaR)	26.491724
14	Portfolio2 MC2(99% CVaR)	25.011434
15	Portfolio2 MC2(99.9% CVaR)	27.130695

We conducted an analysis to compare the sampling error of non-Normal approximations to the actual loss distribution in out-of-sample datasets. Our findings indicate that Portfolio 2 has greater sampling errors compared to Portfolio 1 in two in-sample datasets. Additionally, for both VaR and CVaR, Monte Carlo approximation 1 has slightly larger sampling errors than Monte Carlo approximation 2 for Portfolio 1, while Monte Carlo approximation 2 has slightly larger sampling errors than Monte Carlo approximation 1 for Portfolio 2.

Model Error

	Portfolio Normal Distribution	Modeling Error
0	Portfolio1 N1(99% VaR)	39.859967
1	Portfolio1 N1(99.9% VaR)	49.577280
2	Portfolio1 N1(99% CVaR)	43.838028
3	Portfolio1 N1(99.9% CVaR)	52.338677
4	Portfolio1 N2(99% VaR)	39.656649
5	Portfolio1 N2(99.9% VaR)	49.412375
6	Portfolio1 N2(99% CVaR)	43.651249
7	Portfolio1 N2(99.9% CVaR)	52.184225
8	Portfolio2 N1(99% VaR)	42.713527
9	Portfolio2 N1(99.9% VaR)	53.462422
10	Portfolio2 N1(99% CVaR)	63.839197
11	Portfolio2 N1(99.9% CVaR)	57.504882
12	Portfolio2 N2(99% VaR)	42.556463
13	Portfolio2 N2(99.9% VaR)	69.439469
14	Portfolio2 N2(99% CVaR)	63.698522
15	Portfolio2 N2(99.9% CVaR)	69.980515

We conducted a study to compare the accuracy of Normal approximations to the actual loss distribution in out-of-sample datasets. Our findings reveal that Portfolio 2 has higher modelling errors compared to Portfolio 1 in two in-sample datasets. Furthermore, compare both VaR and CVaR for 99% and 99.9% quantile accordingly, Normal distribution 1 has lower modelling error than Normal distribution 2 for portfolio 1. However, for portfolio 2, Normal distribution 1 has higher modelling error for 99% VaR, but lower modelling error for 99% CVaR compared to Normal distribution 2. In addition, Normal distribution 2 has higher modelling error for both 99.9% VaR and CVaR than Normal distribution 1.

3. Discuss possible strategies for minimizing impacts of sampling and model errors:

- If you report the in-sample VaR and CVaR to decision-makers in your bank, what consequences for the bank capital requirements it may have?

  By reporting in-sample VaR and CVaR to decision-makers in the bank, it may result in underestimating the actual risk, leading to insufficient capital reserves to cover potential losses. This can jeopardize the financial stability of the bank and result in regulatory non-compliance and penalties. To avoid such risks, it is crucial to use accurate risk measures based on out-of-sample data and to consider the limitations of in-sample data when making decisions about risk management and capital requirements.

- Can you suggest techniques for minimizing impacts of sampling and model errors?

  To reduce the impacts of sampling and model errors in risk management, one could consider increasing the sample size to minimize sampling errors, thereby obtaining a more accurate representation of the true distribution. Furthermore, implementing machine learning algorithms can help reduce model errors and provide more accurate risk estimates. Additionally, using multiple risk models could also be beneficial in reducing errors.

Appendix

Outputs

Portfolio 1:

Out-of-sample: VaR 99.0% = $43408181.86, CVaR 99.0% = $51611512.56

In-sample MC1: VaR 99.0% = $37007254.43, CVaR 99.0% = $44404657.90

In-sample MC2: VaR 99.0% = $37102411.94, CVaR 99.0% = $44723633.12

In-sample No: VaR 99.0% = $33261479.09, CVaR 99.0% = $36692437.53

In-sample N1: VaR 99.0% = $26105695.06, CVaR 99.0% = $28986043.43

In-sample N2: VaR 99.0% = $26193951.43, CVaR 99.0% = $29082442.80

Out-of-sample: VaR 99.9% = $64650772.14, CVaR 99.9% = $73334002.09

In-sample MC1: VaR 99.9% = $53691650.81, CVaR 99.9% = $60660211.68

In-sample MC2: VaR 99.9% = $53750649.44, CVaR 99.9% = $61269623.10

In-sample No: VaR 99.9% = $40995666.23, CVaR 99.9% = $43798799.43

In-sample N1: VaR 99.9% = $32598677.62, CVaR 99.9% = $34951955.94

In-sample N2: VaR 99.9% = $32705290.23, CVaR 99.9% = $35065221.49

Portfolio 2:

Out-of-sample: VaR 99.0% = $36737045.28, CVaR 99.0% = $44496393.51

In-sample MC1: VaR 99.0% = $27432271.76, CVaR 99.0% = $33272131.76

In-sample MC2: VaR 99.0% = $27338842.56, CVaR 99.0% = $33367207.41

In-sample No: VaR 99.0% = $31119089.66, CVaR 99.0% = $33943084.02

In-sample N1: VaR 99.0% = $21045357.65, CVaR 99.0% = $23205372.05

In-sample N2: VaR 99.0% = $21103058.28, CVaR 99.0% = $23267967.39

Out-of-sample: VaR 99.9% = $55685188.78, CVaR 99.9% = $65135235.69

In-sample MC1: VaR 99.9% = $40586080.12, CVaR 99.9% = $45822669.60

In-sample MC2: VaR 99.9% = $40933222.42, CVaR 99.9% = $47463593.53

In-sample No: VaR 99.9% = $37485037.04, CVaR 99.9% = $39792273.49

In-sample N1: VaR 99.9% = $25914538.06, CVaR 99.9% = $27679295.11

In-sample N2: VaR 99.9% = $25983272.51, CVaR 99.9% = $27752028.59