	HW7
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[(a)	For $x \in (0, \frac{\pi}{2})$, $\sin(x) < x < \tan(x)$
	=> 1 2 × × × × × × × × × × × × × × × × × ×
· ·	Since him costs = 1,
	1 = \lim \frac{\sin(\pi)}{\pi} \leq
	50 min (20) - 1.
	United to have a second to the
(b)	1. $f(\infty) - f(c)$ 1. $sin(\infty) - sin(c)$
(7)	$\lim_{N \to c} \frac{f(x) - f(c)}{N - c} = \lim_{N \to c} \frac{\sin(x) - \sin(c)}{N - c}$
	$= \lim_{n \to \infty} \frac{2\cos\left(\frac{n+c}{2}\right)\sin\left(\frac{n-c}{2}\right)}{\sin\left(\frac{n-c}{2}\right)}$
	M-C N-C
	$=\lim_{\chi \to c} \cos\left(\frac{\chi + c}{2}\right) \lim_{\chi \to c} \frac{\sin\left(\frac{\chi - c}{2}\right)}{\chi \to c}$
	$n \rightarrow c$ $n \rightarrow c$ $n \rightarrow c$ $n \rightarrow c$
	$= \frac{\text{C+C}}{2} \cdot \frac{\text{C+C}}{2}$
	A STATE OF THE STA
	- cos (c)
	so $\forall n \in \mathbb{R}$, $f'(n) = cos(n)$.
2.(a)	L + 0, let &= 1= 1 > 0, then 35>0: \xe(c-5, c+5) \s\(\lambda\)
	h'(x) - L < \(\ \ \ = \frac{1}{2} \)
	> 1= > h(x) - L > h(x) - L
	>- = < h(∞) - L < =
	$\Rightarrow h(x) > = \Rightarrow h(x) \neq 0$

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(b)	Since h is continuous, h/A is continuous
	By part (a), 35>0, A=(c-8, c+8) 1/5/(c)), h/A(n) +0
	since c is a cluster point of S, c is a cluster point of A
	lim - xic
	xic hla(x) lim hla(x) h(c)
(c)	f(x) = f(x) - f(c)
	m = lim - since +(c)= g(c)= 0 x→c g(x) - g(c)
5	$\frac{f(x)-f(c)}{x-c}$
- 10 mg	$= \lim_{\chi \to c} \frac{\chi - c}{2(x) - g(c)}$ Since $\chi - c \neq 0$
	= 1 f(x)-f(c) / m f(x)-gc) x+c x-c x+c x-c x+c x-c
	270 7-C (im 3(x)-9C)
	= lim f'(x) lim g'(x)
	$=\lim_{N\to c}\frac{f'(\infty)}{g'(\infty)}$
	7 9'(x)
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ishter.	22 2) 22 V C 22 G and 1 10-1+1-12 1 1 12 1 1 1 2
	the state of the s
	- (+1249H=16802H=243E=8
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	$0 > 0 \Rightarrow f(\infty) \leq f(c)$
<u>}.(a)</u>	Suppose f is decreasing. Yx, CEI with N+C, x <c=) f(x)="">f(c)</c=)>
	$\int f(x) - f(c) = \int f(x) + \int f$
	N-C
	f(x) - f(c) = 0
	$=) f'(c) = \lim_{n \to c} f(n) - f(c) < 0$
	Suppose f'(n) = O YNEI. Take x, yEI with x=y.
	By MVT. 3 c E (a,b) st f(y)-f(x) = f'(c) (y-x)>0
	By MVT, $\exists c \in (a,b)$ st $f(y) - f(n) = f'(c)(y-x) \ge 0$ => $f(y) \ge f(x) \Rightarrow f$ is decreasing
	A Para Cara
(b)	Suppose f'(n) < 0 YXEI. Take x, yEI with x < y
(0)	By MVT, 3cc(a,b) s.t. f(y)-f(x)=f'(c)(y-x)>0
	$=) f(y) > f(\infty)$
	=) f is strictly decreasing
	1/20-x1/-=(019-(016)
4.(a)	Let $f(x) = x^n$, then $f(x) = hx^{n-1}$
	YP, O NEN X, YEL-R, R]
	by MVT Fc between 1 and 4,
	by MVT, Ic between N and y, $ x^n - y^n = f(x) - f(y) = f'(c) x - y \le n R^{n-1} x - y $
(b	Let $f(x) = \sqrt{x^2+1}$, then $f'(x) = \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$
	Yx, y ER, by MVT, Fic between X and y.
	\(\pi, y \in R, \text{ by MVT. = c between \(\chi\) and \(\gamma\). \[\lambda \frac{\pi}{\pi} - \lambda \frac{\pi}{\pi} = f'(c) \cdot \(\chi - \gamma \) \[\lambda \frac{\pi}{\pi} + \lambda - \frac{\pi}{\gamma} + \lambda = f'(c) \cdot \(\chi - \gamma \)
	= [-12-9]
	$\leq x-y $

5	Using Taylor's theorem at No=a,
	Using Taylor's theorem at xo=a, YXE[a,b], ICE(a,x) if X = a or C=a if X=a
	$f(x) = P_{n-1}^{\alpha}(x) + \frac{f^{n}(c)}{n!} (x-\alpha)^{n}$
	R. Min-Max Thorsen, since f'(x) is continuous.
	By Min-Max Theorem, since f'(x) is continuous. it achieves both an abs min M and abs max N on [a,b].
	Let De Da () M (m a)"
	Let P(n) = Pn (n) + M (x a)"
	0 Da N
	Q(x) = P= (x) + N (x a)"
	so P(n) = f(n) = Q(n) for all xe[a,b]
	For $\lambda = \frac{N}{n!} - \frac{M}{n!} > 0$.
	$Q(x)-P(x)=\lambda(x-\alpha)^n$
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L.S	