Introduction to Machine Learning
[Fall 2022]

Logistic Regression

September 20, 2022

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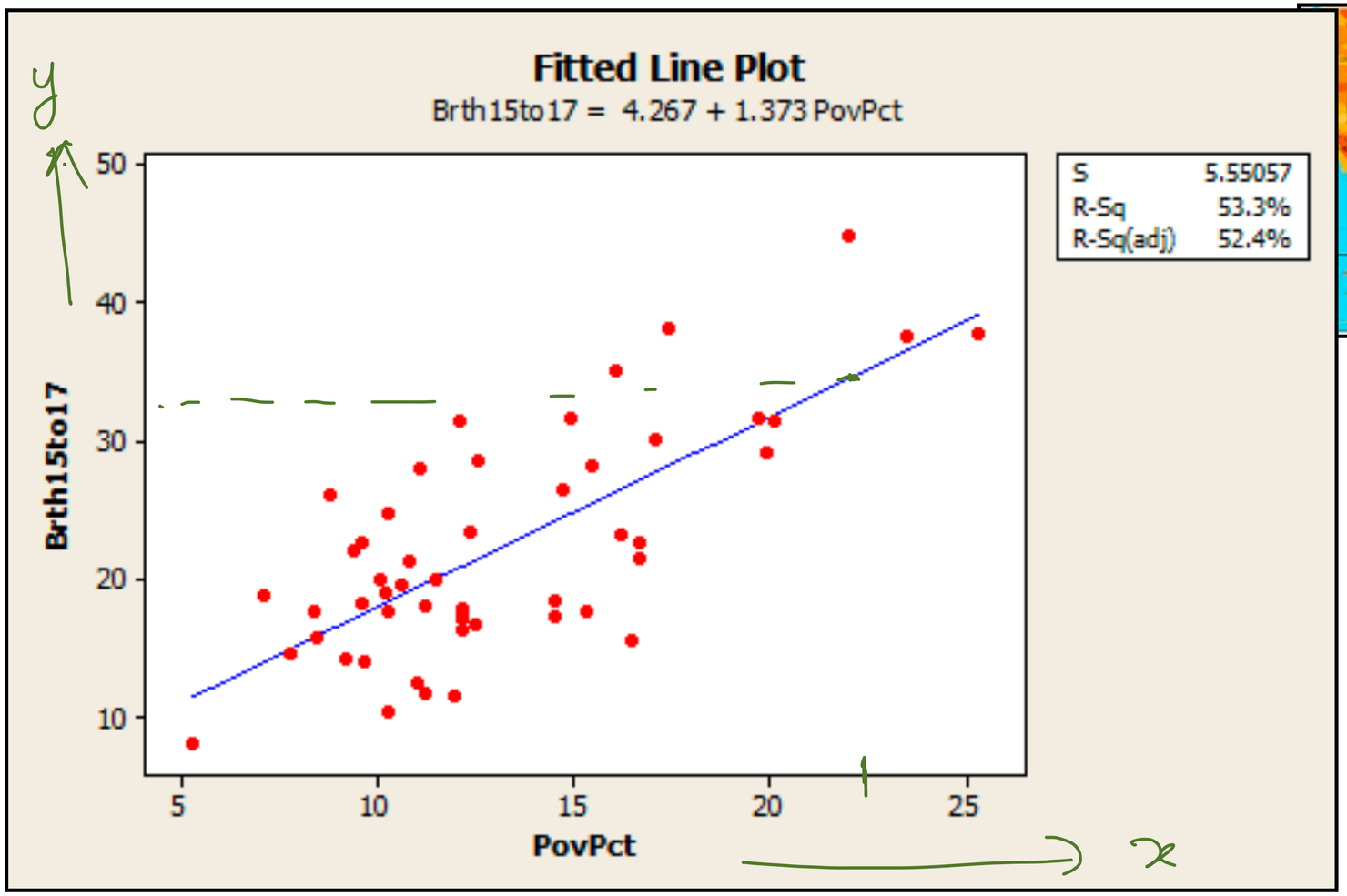
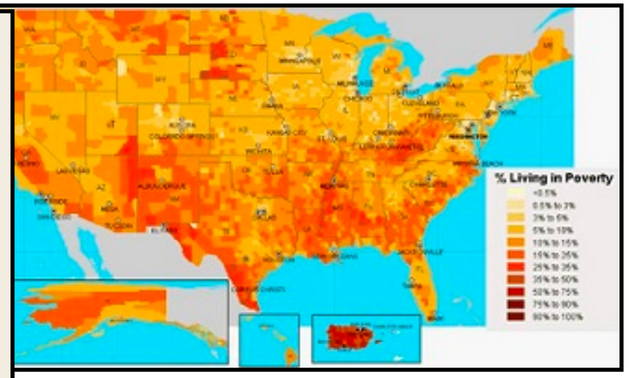
Topics for today

- Regression vs. Classification
- Logistic regression

Recap: Linear Regression



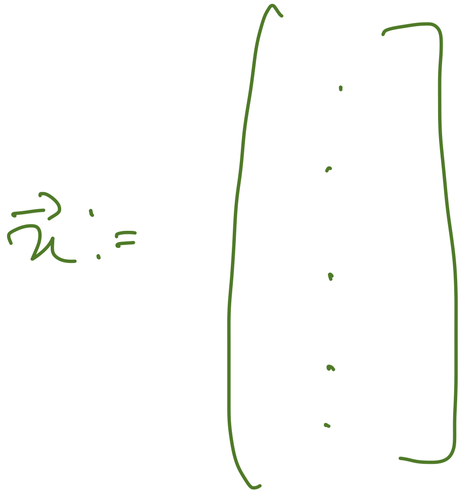


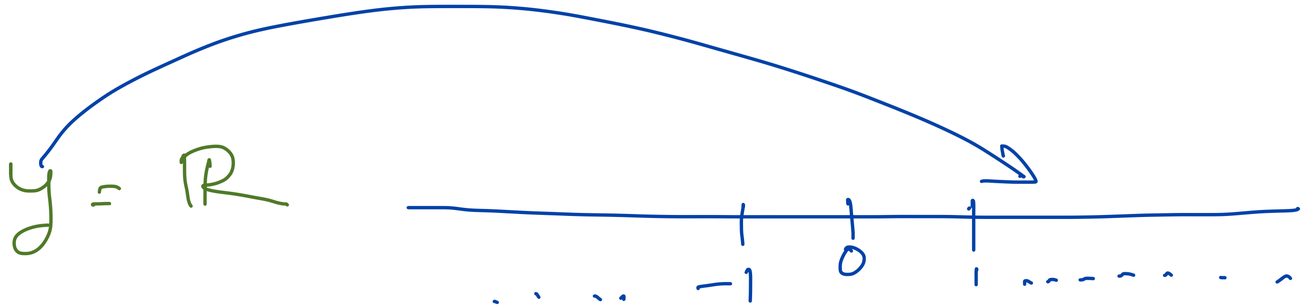
$y = w^T x$

https://online.stat.psu.edu/stat462/node/101/

Recap: Linear Regression

- Input data: $X \in \mathbb{R}^{d \times n}$, $Y \in \mathbb{R}^n$, where $(\vec{x} \in \mathbb{R}^d, y \in \mathbb{R}^1)$ corresponds to a data point.
  - $n \rightarrow$ # of data points, $d \rightarrow$ # of features / input dim.





Examples of Regression

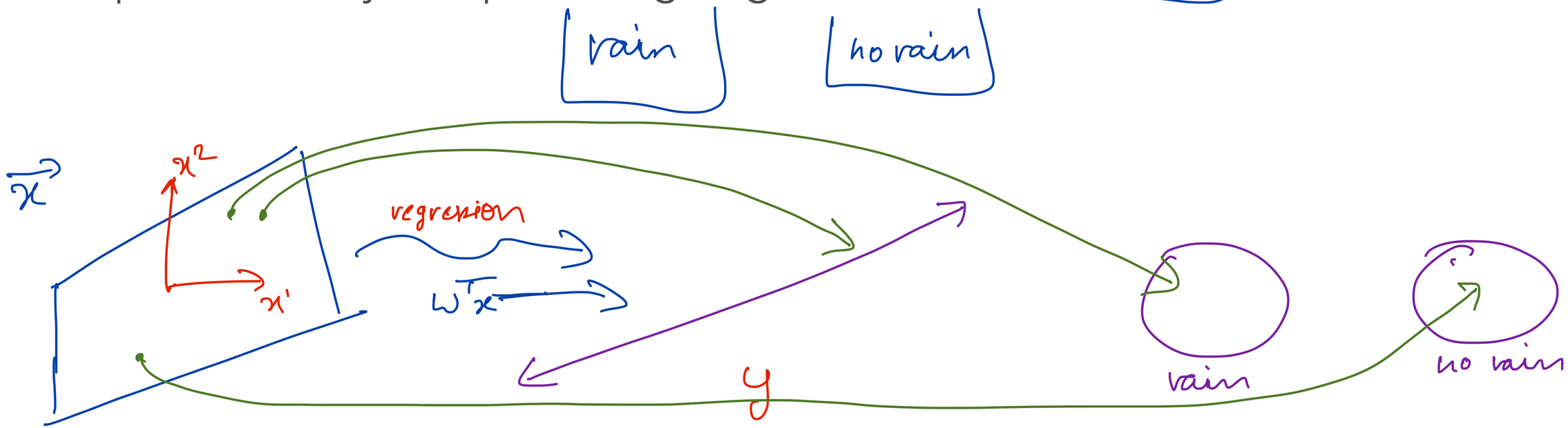
- Input: poverty rate. Output: birth rate
- Input: sensor reading. Output: robot torque
- Input: humidity. Output: cms of rainfall

$\mathbb{R}^d \rightarrow \mathbb{R}$

$\vec{x}$ y

Examples of Classification

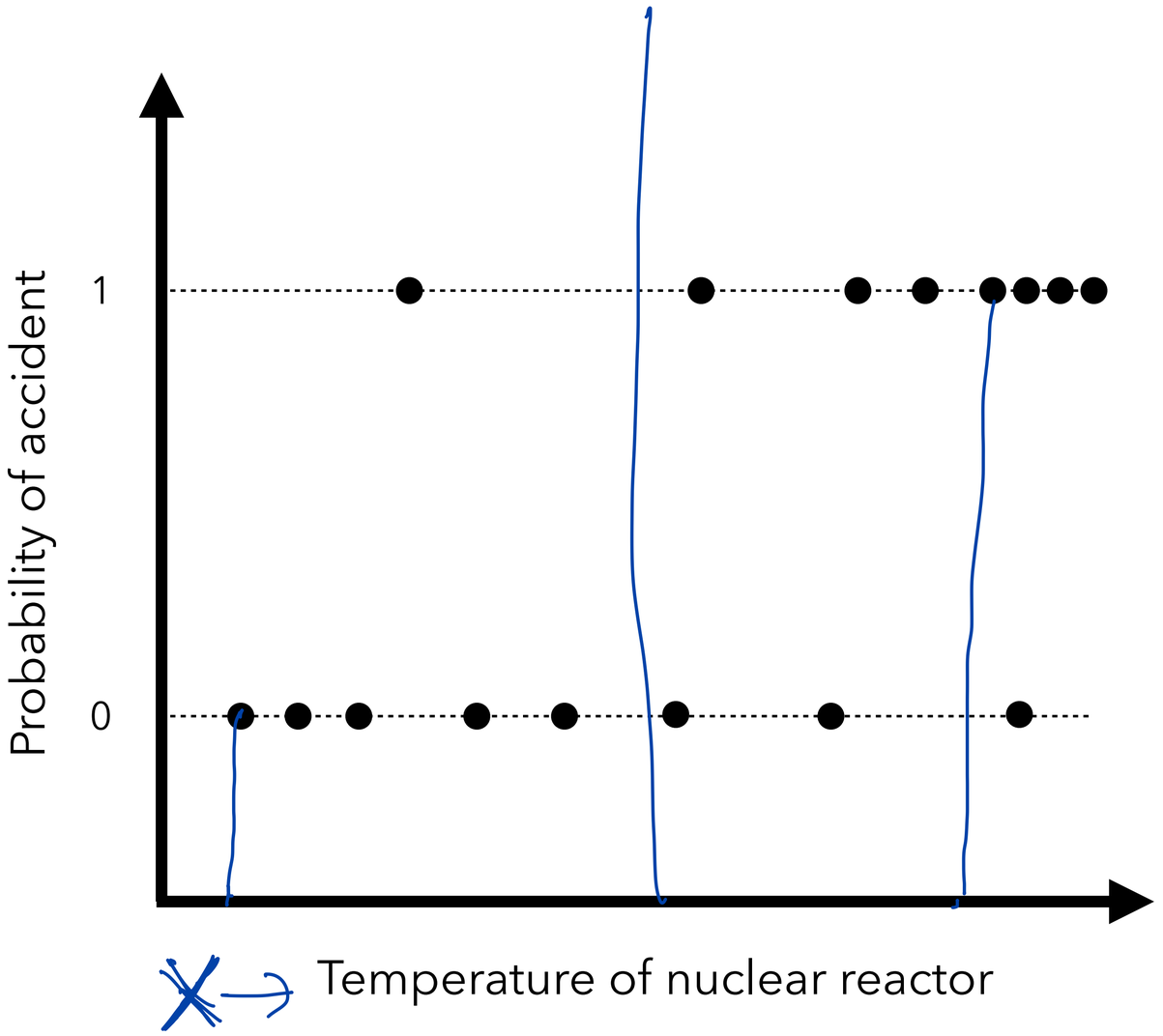
- Input: poverty rate. Output: Is the birthrate > 2?
- Input: sensor reading. Output: Is the torque low (0-1Nm) or high (>1Nm)?
- Input: humidity. Output: Is it going to rain?

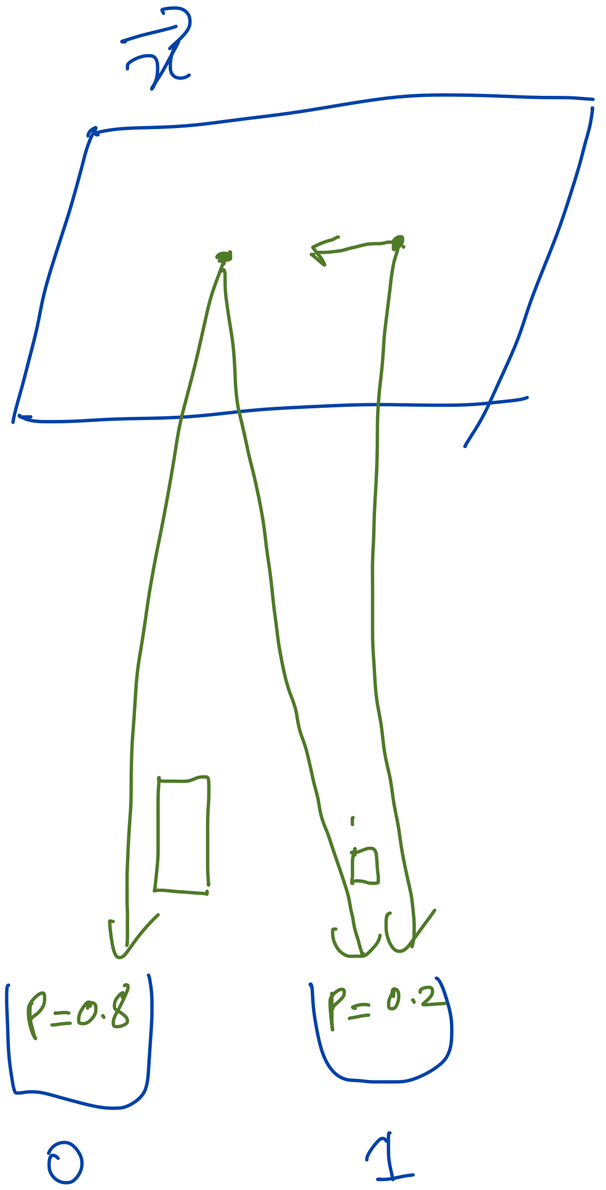


How do you input continuous and output discrete?

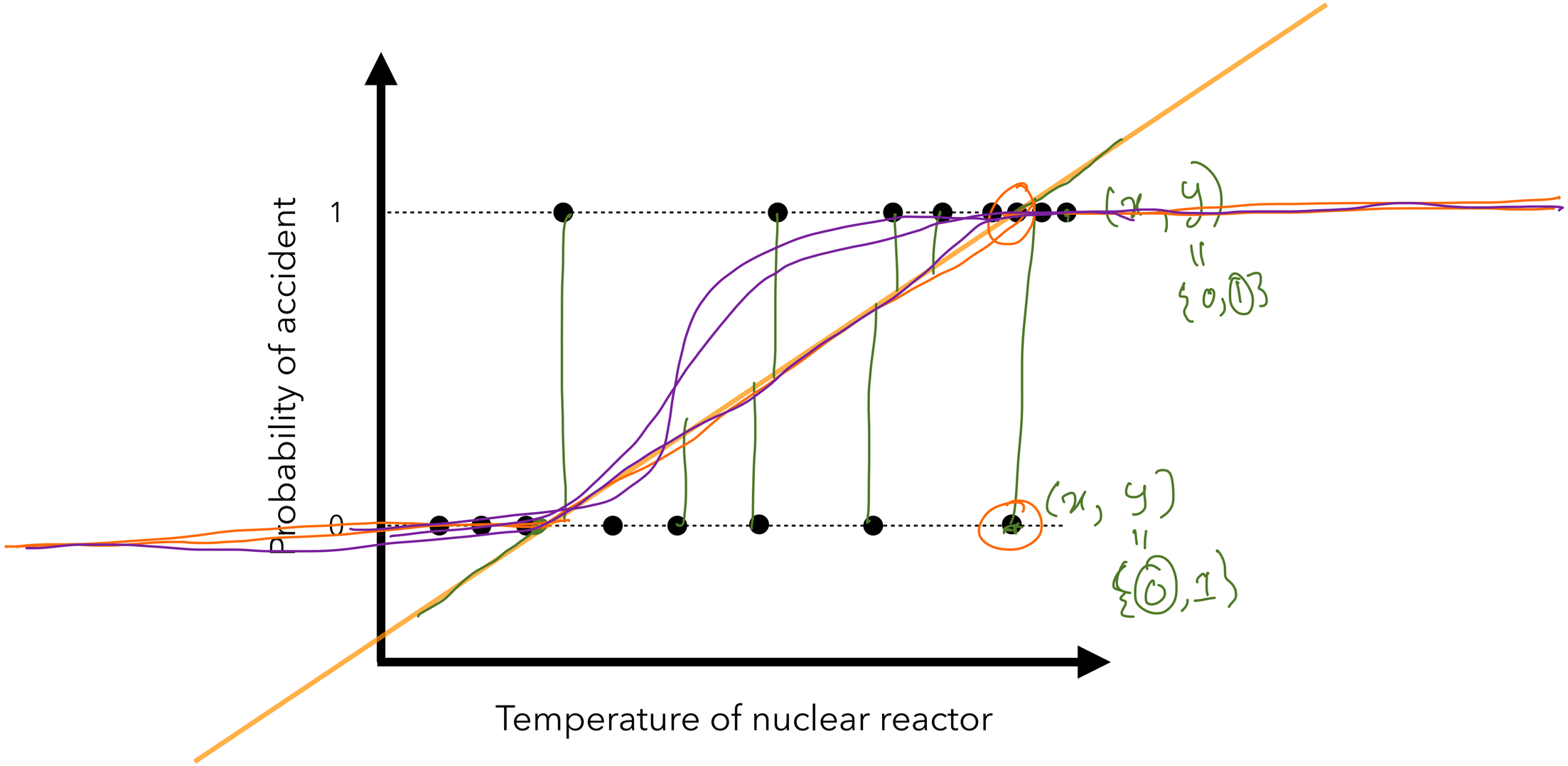
- Input data: $X \in \mathbb{R}^{d \times n}$, $Y \in \{0,1\}^n$, where $(\vec{x} \in \mathbb{R}^d, y \in \{0,1\})$ corresponds to a data point.

Logistic Regression

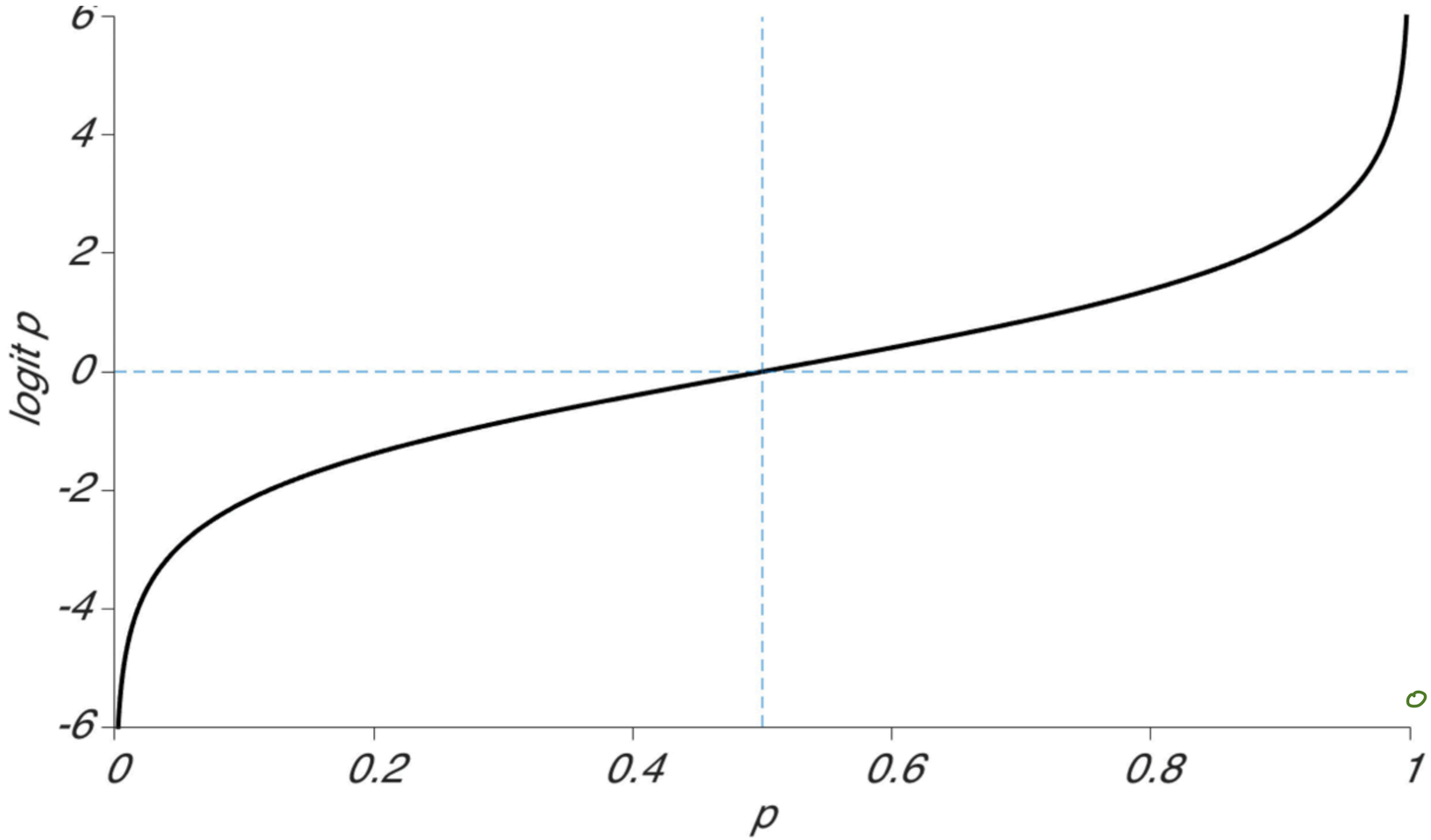




Logistic Regression



The logit function



Prob. reactor having an accident

$$logit(p(y)) = \ln\left(\frac{p(y)}{1-p(y)}\right)$$

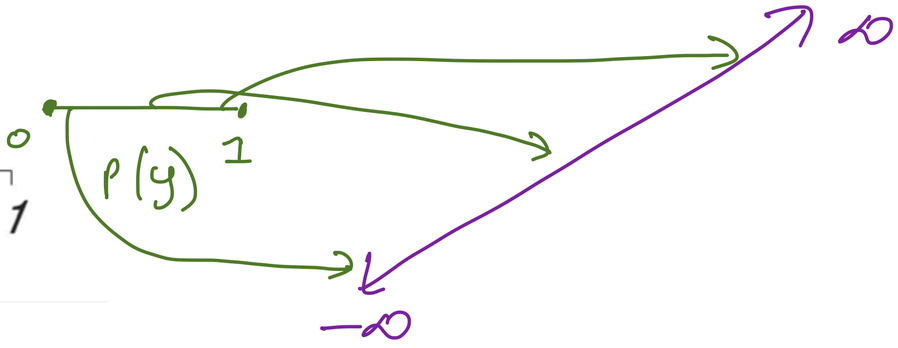
1

0

log Odds

0 — 1

$-\infty$ ∞



Deriving the logistic function

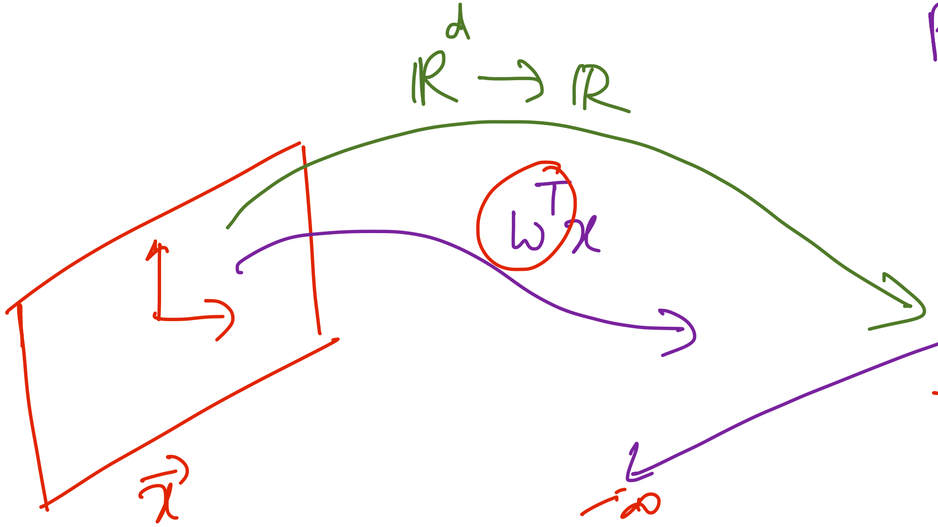
$$\text{logit}(P(y)) = \ln\left(\frac{P(y)}{1-P(y)}\right) = \omega^T x$$

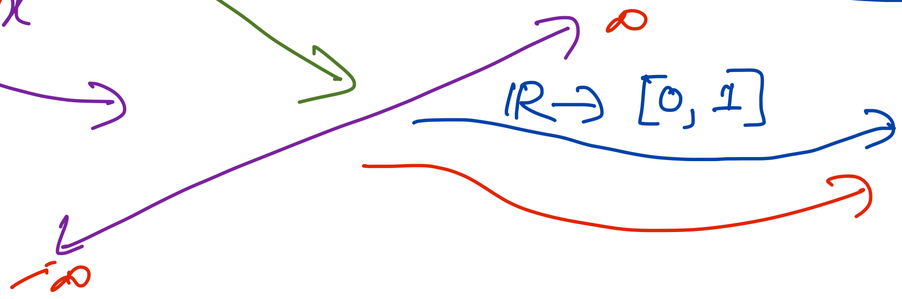
$$\frac{P(y)}{1-P(y)} = e^{\omega^T x}$$

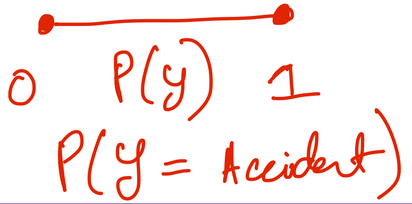
$$P(y) = e^{\omega^T x} - P(y)\left(e^{\omega^T x}\right)$$

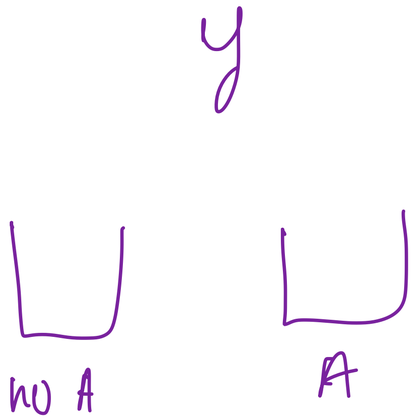
$$P(y) = \frac{e^{\omega^T x}}{1+e^{\omega^T x}}$$

→ logistic function

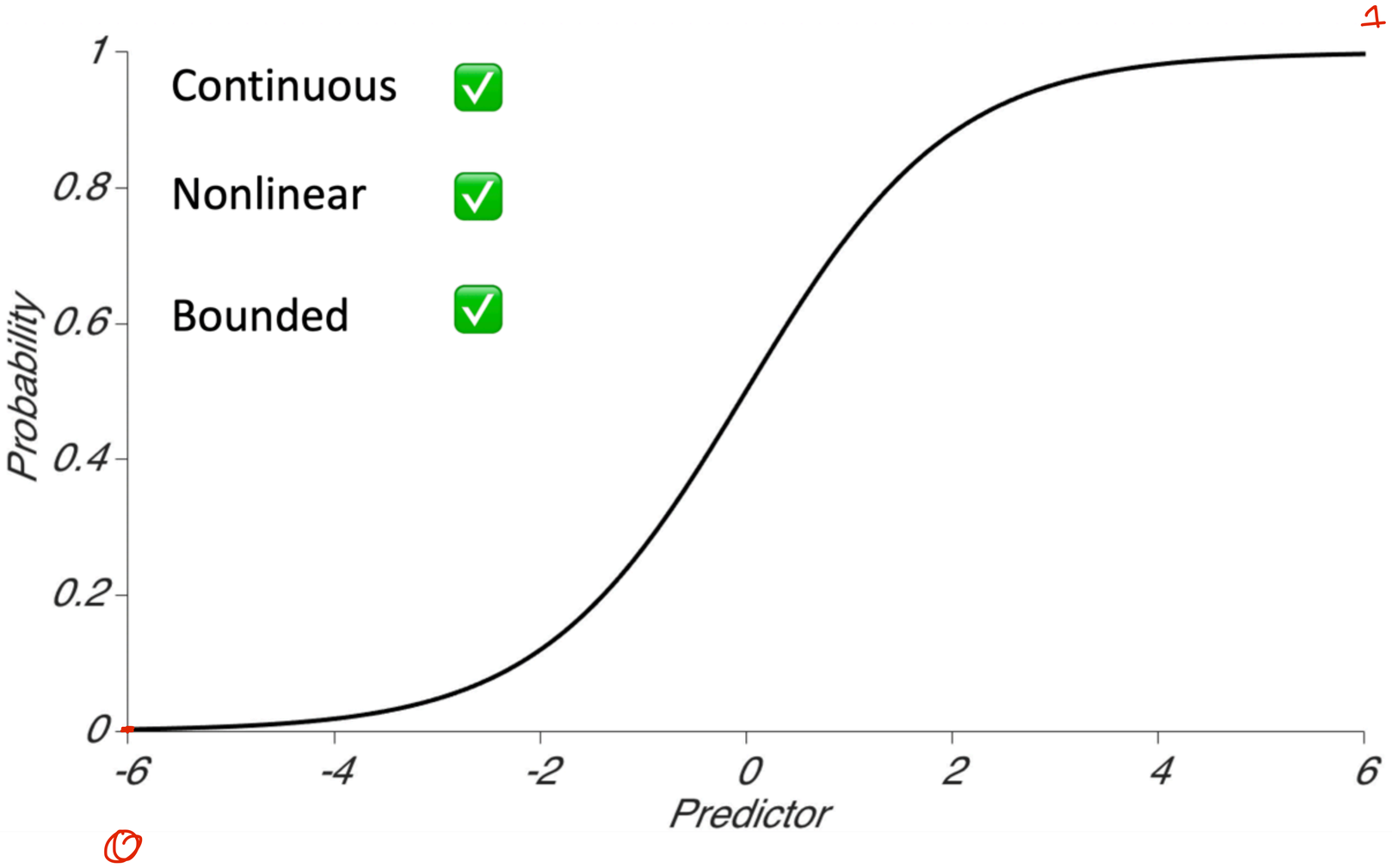








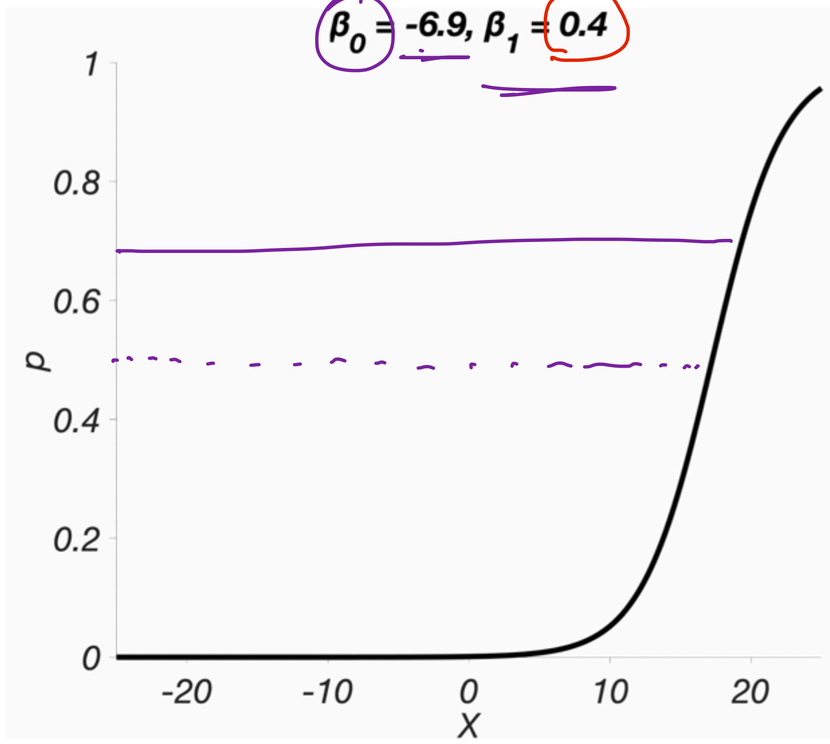
The inverse logit / logistic function

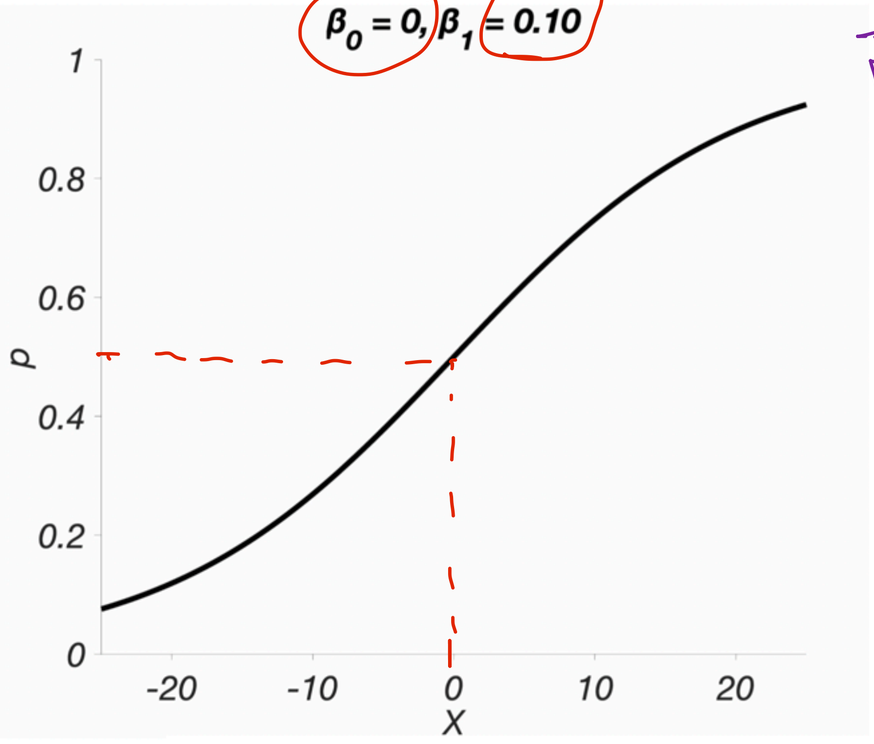


$$p(y = 1) = \frac{e^{w^T x}}{1 + e^{w^T x}}$$

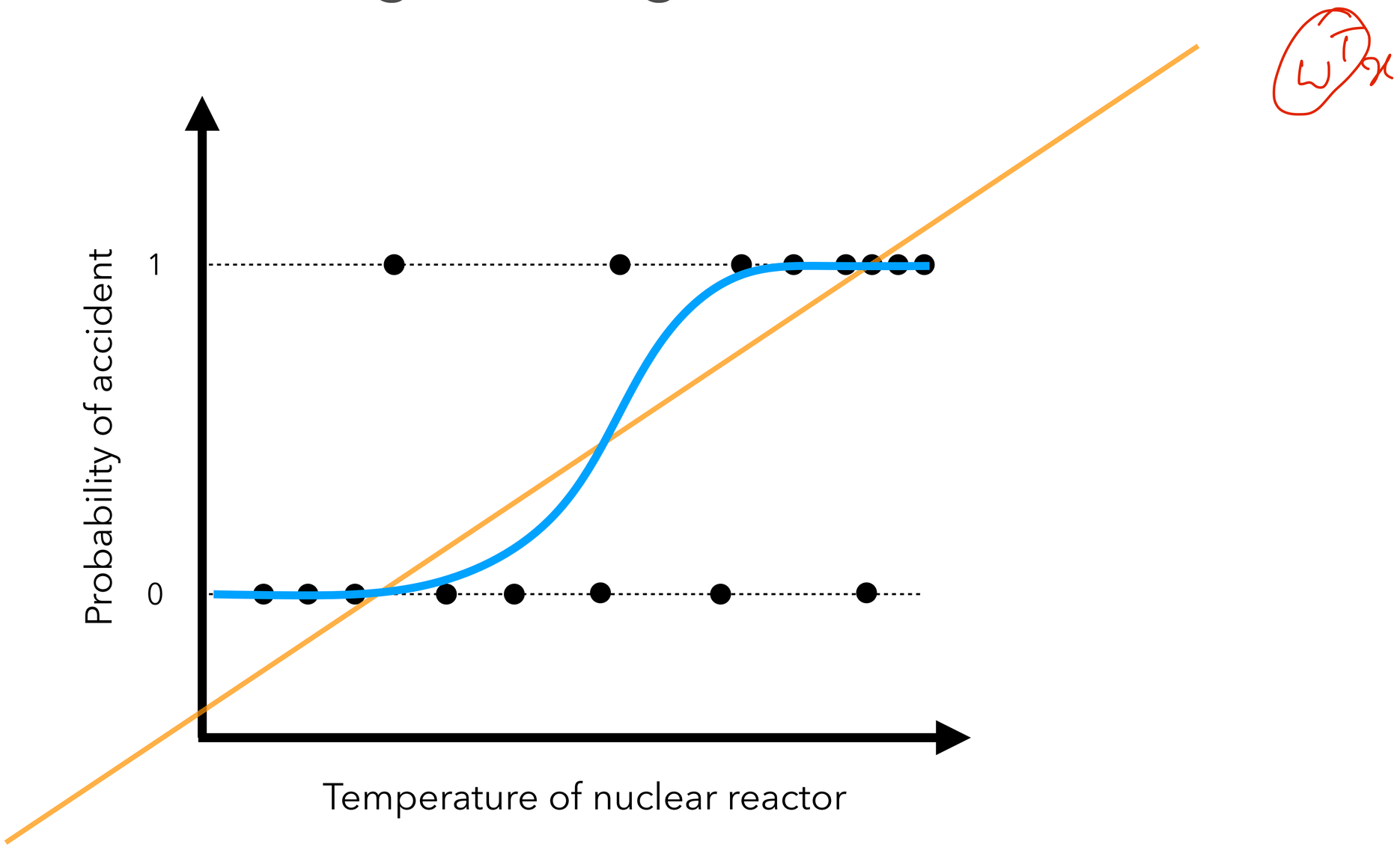
The inverse logit / logistic function

$$p = \frac{e^{\beta_0+\beta_1 x_1}}{(1+e^{\beta_0+\beta_1 x_1})}$$





Logistic Regression



Additional Reading

- https://nhorton.people.amherst.edu/ips9/IPS_09_Ch14.pdf

Questions?