	HW2
	Collaborators: Xi Liu, Cerina Yao
(1.)	For any XEIR and any £20,
1 d)-(a)	let y= x+E, 50 YER
	By Theorem 1.2.4.ii, x, y ER and X & Y,
(4	then there exists an rEQ s.t. XCXCY => XCXCX+E
	$\Rightarrow \mathcal{N} - \mathcal{E} < \mathcal{N} - (\mathcal{N} + \mathcal{E}) = \mathcal{E}$
	deint b. RES in X mediatridus apart 500 att x 71-8
	Hadarbaxaldstinos solf Mr. Ild Le lith Le lith Le lith Res
2.	Assume x + y, so x-y+0, so x-y >0
	let &= 1x-y1, 50 270
	Since $ X-Y \leq \xi = \frac{ X-Y }{2}$
	=> x-y =0 => contradicts with x-y >0
	Therefore, X=y
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3. (a)	f,g are bounded $\Rightarrow \exists B_1, B_2 \in \mathbb{R}$ s.t. $ f(x) \leq B_1, g(x) \leq B_2$
	for all MED
,	$\Rightarrow (f+g)(n) = f(n)+g(n) $
	< f(x) + g(x) (by to iangle inequality)
	= B1+B2 for all NED
	Therefore, f+9 is bounded by B1+B2.
	The state of the s
(P)	$ (f_g)(n) = f(x)g(n) $
	$= f(x) \cdot g(x) $
	= BiBz for all xED

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Therefore, to is bounded by Bi+Bz
(c) |(f/h)(x)| = |f(x)/h(x)|
                    = |f(x)| / |h(x)|
                    = B./(h(x)) (since |f(x)|= B.)
= B./c for YMED (since |h(x)|zc, c>0)
     Therefore, f/h is bounded by Bi/c.
(d) h. E → D => ∀xeE, h(x) = y ∈ D => |f(y)| ≤ B,
      foh(x) = |f(h(x))|
     Therefore, foh is bounded by B.
4. 1° YNED, supg(N) = g(N)
         f(x) \leq g(x) =) \sup_{x \in P} g(x) \geq f(x)
=> \sup_{x \in P} g(x) is an upper bound for f(x)
By definition, \sup_{x \in P} f(x) \leq \sup_{x \in P} g(x)
    2° \forall x \in D, \inf f(x) \leq f(x)

f(x) \leq g(x) \Rightarrow \inf f(x) \leq g(x)

\Rightarrow \inf f(x) \text{ is a lower bound for } g(x)

By definition, \inf f(x) \leq \inf g(x)

g(x) \leq \inf g(x)
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	3° By definition, inf f(x) < f(x) < sup f(x) for all x(E),
	Since $f(x) \leq f(x) $ for all $\chi(E)$,
	$ f(x) ^2$, $ f(x) $
	by 1°, sup $f(x) \leq \sup f(x) $ Since $- f(x) \leq f(x)$ for all $x \in D$,
	by 2°, inf- fro = inf fro)
	by 2°, inf- fro = inff(x) Since M is a bound for f
	=> for all $x \in D$, $ f(x) < M => - f(x) > -M$
2	=>-M=int-If(x), sup/f(x)=M
	In conclusion, I the find Joxef = (16)
	$-M \le \inf - f(x) \le \inf f(x) \le \sup f(x) \le \sup f(x) \le M$
	AND KEY KEY KEY
t. (a)	When & < 2, X = 4 = 3 = 1
	$\forall M \in \mathbb{N}$, there exists $n = 2M + 1 > M$,
	$ \chi_{h}-1 = (-1)^{2M+1}-1 = (-1)-1 =2>5$
	so 1xn-1/2 & holds infinitely often
	When 2>2. (2) (2) (2) (2) (2) (3)
	AM=IEN s.t. for all nzM=1,
	1×n-1 ≤ ×n + -1 (by triangle inequality)
	= +1 = () = = = = () = = = = = = = = = = =
	= 2 < 2 - 1 ml -M. / Mar.
	so xn-1/2 & holds at most finitely often
	Therefore, 2>2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
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	1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1

(b)	When $\xi \leq 0$,
	YMEN, there exists n=MZM,
4 1-0	$ X_{n}-1 = \frac{n+1}{n}-1 = \frac{1}{n} =\frac{1}{n}=\frac{1}{m}>\epsilon$
	50 /2n-1/2 & holds infinitely often
	When 270,
	by Archimedean Property of R,
	E, IER and E>D, then IMEN st. M. E> => E> th
	Forall nzM
Prop 1905	$ \chi_{n-1} = \left \frac{n+1}{n} - 1\right = \left \frac{1}{n}\right = \frac{1}{n} \leq \frac{1}{M} \leq \frac{1}{2}$
	so 1×n-1/≥ & holds at most finitely often
	Therefore &> 0.
F)	Tf 2x, } and for the same ER
(c)	1 1 1 M 1 M M M M M M M M M M M M M M M
	4520 JMEN. YnzM, Mn-LI <e< th=""></e<>
	= HCOD JMEN. MnzM. Xn-L/32 is talse.
	=> for all E>0, 1×n-L > E holds at most finitely often.
	If for all 270, xn-L 22 holds at most finitely often,
	C 11520 - FMEN. HnzM, IXn-LIZE is talse
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	=> {7/n} converges to some LER
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	1 - ± squares (This 2 c)
egise.	

6.0	(a) {n'} does not converge
U.	pf. Assume (n') converges to some LER
	Then for $\xi = \frac{1}{2} > 0$, there exists MEN s.t. 4 nzM, $ X_h - L < \epsilon$
	For n=M, n=M+1, this implies
	M2-L < &= 1
	$ (M+1)^2-L <\xi=\frac{1}{2}$
- N	Then we would have
	$2M+1 = \bar{L}(M+1)^2 - L] - (M^2 - L)$
	< (M+1)2- L + M2- L < 28= => M<0 contradiction!
	= Therefore, {n} does not converge.
	Thousand I to the second of th
	b) {xn} where xh = { sin(n²) n<1000000 converges to 0 n > 1000000
	3 m) where M) 0 n > 1000000
	For all E>0, there exists M=1000000EN such that
	Mn-L = 0-0 = 0 < & for all n=M=
	50 (Mn) converges to 0
(0	(1) {4nt1} converges to 1
	For all 2>0, by Archimedian Property of R.
	there exists MEN st. M.(42)>1 => M>715
	$ \chi_n - L = \frac{\mu_n}{\mu_{n+1}} - 1 = \frac{\mu_n}{\mu_{n+1}} \leq \frac{\mu_n}{\mu_n}$
	< 4.45+1 = \frac{1}{5} = \frac{1}{5} \text{ for all \$N > N}
	50 {4n } converges to 1

(d)	$\left(\frac{2h}{h^2+1}\right)$ converges $+2$
(0()	For all ≥ 0 , by Archimedian Property of IR, there exists MEN s.t. $M.\leq >2 \Rightarrow m.\leq 2$ $ X_{h}-L = \frac{2n}{n+1}-0 =\frac{2n}{n+1}<\frac{2n}{n^{2}}=\frac{2}{n}\leq \frac{2}{m}\leq 2$ for all $n\geq M$
	there exists MEN st M.S >> => m < 5
	$ X_{h}-L =\frac{2n}{n+1}-0 =\frac{2n}{n+1}<\frac{2n}{n^{2}}=\frac{2}{n}<\frac{2}{m}<\frac{2}{m}<\frac{2}{m}$
	50 \\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\\
	The state of the s
7.	(ii) ⇒ (i)
	For every E>0, there exists MEIN st. Xn-LI < E for all nzM
	For every \$>0, there exists MEN st. Xn-LI < & for all n2M => For every £'70, let £=£', so £70, there exist MEN s.t. Xn-LI < £ for all n2M
	there exist MEIN s.t. Xn-L < E for all nZM
	$\xi = \xi' \Rightarrow \chi_n - L < \xi' \Rightarrow \chi_n - L \leq \xi'$
	2°(;) ⇒(;;)
	- 1 A A A I LA A I LA A A A A A A A A A A A
	For every $\xi > 0$, there exists $ NC \sqrt{1 + 1} = 2 + 1 = 2 +$
	there exists MEIN st My-LISE' for all n3M
	$\Sigma' = \frac{\Sigma}{2} \Rightarrow \chi_n - L \leq \frac{\Sigma}{2} < \Sigma$
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8.	For any given NER, let En=t,
	from problem we know that for N, En,
	there is a $xn \in Q$ st. $ x-x_n < \xi_n$
NA:	for all hEN, we choose In in this way
	$50 \forall n - \infty = \times - \forall n < \Sigma_n = \frac{1}{n}$
	By Archimedian Property of IR, 75>0,
	there exists MEN st. M. E > 1 => E > M
Miss long	50 8n-n < \(\frac{1}{h} \) ≤ \(\frac{1}{h}
	50 lim &n = N.
	when the + fall-and standard matt
	3 = 11 - 12 = 13 = 13 = 3 = 3
14	
140 CA 190 Y	
	AARA HALLI YASAN WELLAMA AA
	$2 \times \frac{1}{2} = 1 \cdot 1 - 1 \times 1 = \frac{1}{2} = 2$