Data Communication & Networks
G22.2262-001

Session 10 - Main Theme
Performance in Queuing Systems

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Agenda

- Planning for Performance
- Queuing Analysis
- Queuing System Structure
- Queuing System Variables
- Queuing Models and Examples
- References
- Conclusion

Part I

Planning for Performance

Planning for Performance

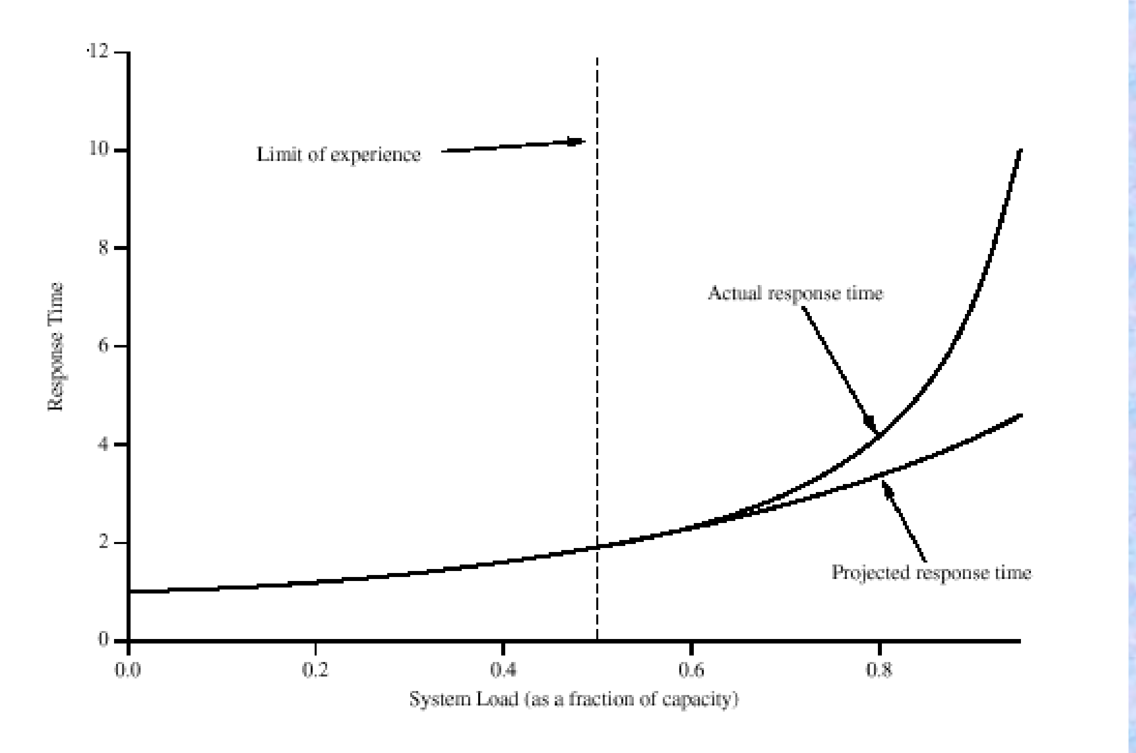
- When building Server apps, what can we do to insure acceptable performance?
- How do we answer questions like these:
  - What happens to response time when utilization goes up?
  - How many concurrent requests can be handled before response time is unacceptable?
  - What is the effect of adding (or removing) another server?

Planning for Performance

- A number of approaches are possible:
  1. Do an after-the-fact analysis based on actual values *("let's build something and see how it works!")*
  2. Make a simple projection by scaling up from existing experience to the expected future environment *("We expect double the number of users, so let's double the processor speed and memory!")*
  3. Develop an analytic model based on queuing theory *(subject of this lecture)*
  4. Program and run a simulation model *(this effort could be as big as the entire development effort!)*

The Problem with Approach 2

- The problem with this approach is that the behavior of most systems under a changing load is not what you might expect!



Part II

Queuing Analysis

Queuing Analysis: Basic Entities

- **Customers** (tasks, requests, etc)
  - Individual requests for service (e.g. a request for I/O, or a request by a customer in a bank, etc.)
- **Queues**
  - Waiting areas where requests for service wait for server(s) (e.g. the ready queue of processes waiting for CPU, or the waiting room at a doctor's office).
- **Servers**
  - Entities or resources that are capable of satisfying the service requests (e.g. CPU, disk, bank teller, etc.)

Queuing Analysis: Characterization

- **Dispatching Discipline**
  - When a server is done serving a customer, it must pick the next customer out of some queue. Algorithm used to do so is called the dispatching discipline. Examples are FCFS, FIFO, SJF, EDF, etc.
- **Distribution of Arrivals**
  - When do customers arrive? We will restrict our analysis to a Poisson process for the arrival of customers to the system.
- **Distribution of Service Time**
  - How long does it take a server to service a customer's request? The service time may be the same for all customers or, more realistically, the service time is likely to be variable.

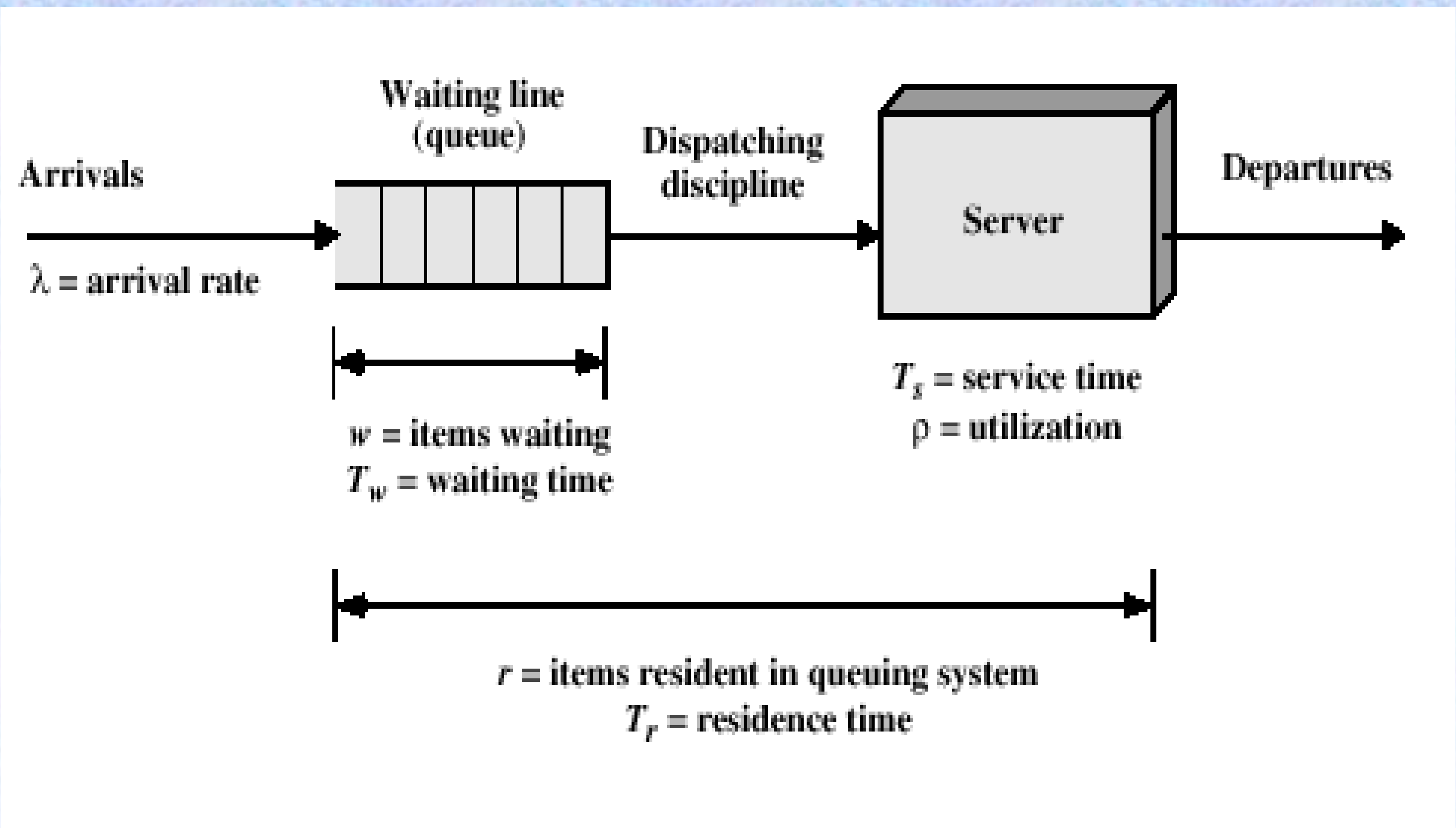
Simplifying Assumptions

- **Population**
  - Assume that Requests for service are generated from an infinite population. WHY? So that the arrival of a request to the system does not influence "future" arrivals
- **Queue Size**
  - Assume that queues have infinite capacity (unrealistic for many applications, but assume this for now)
- **Queue Service discipline** is FCFS

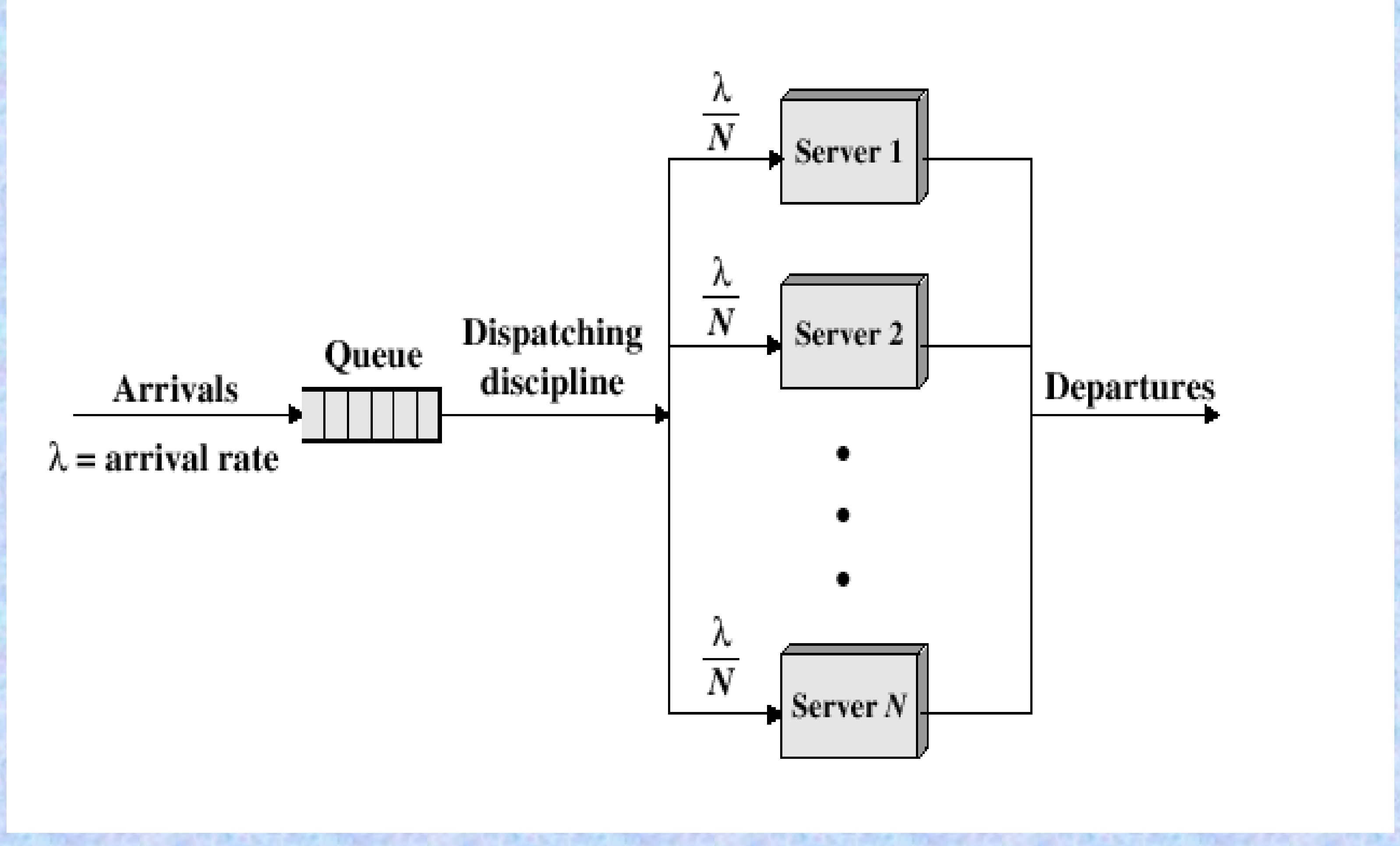
Part III

Queuing System's Structure

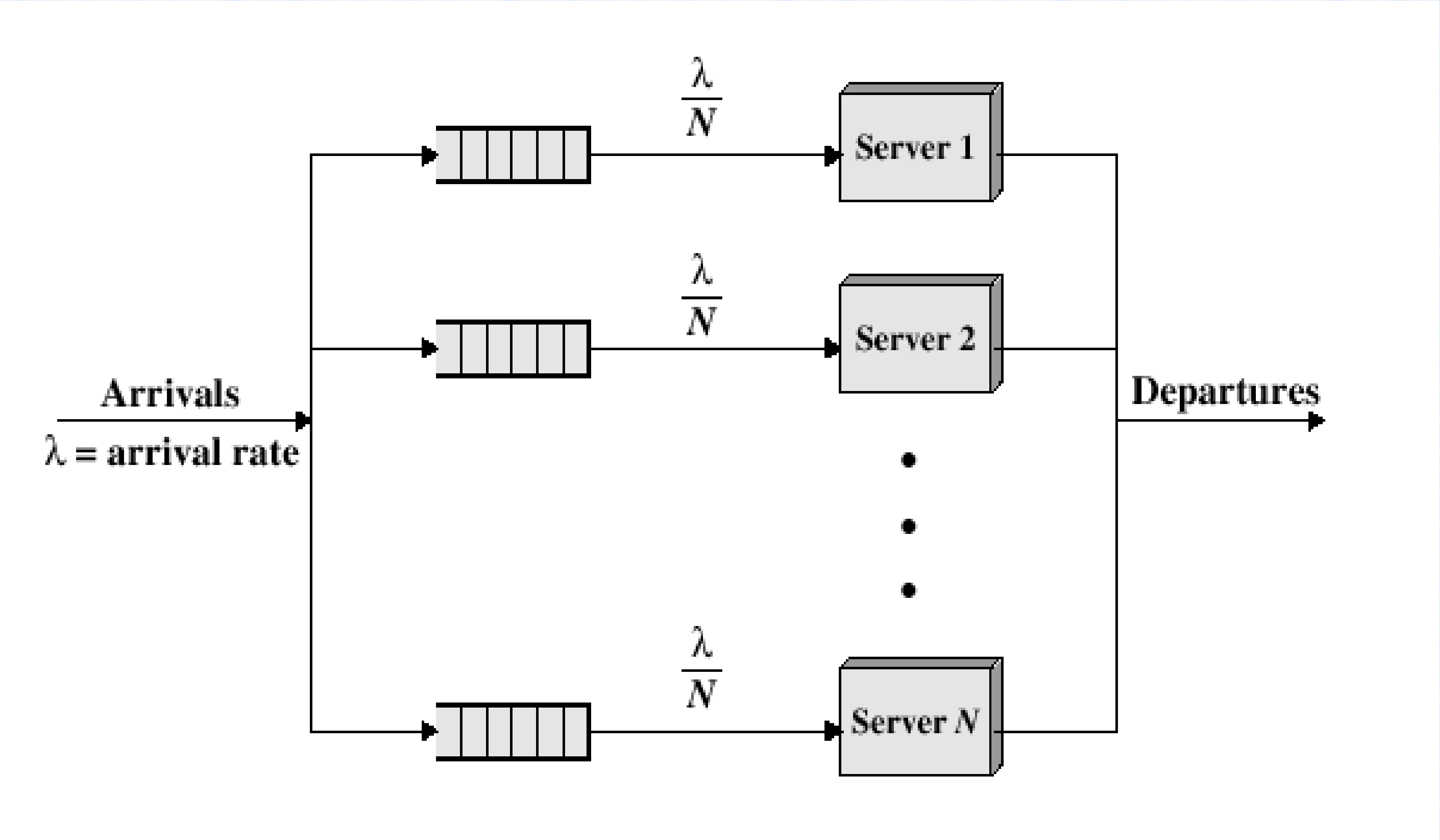
Queuing System Structure: Single Server



Single Queue, Multiple Servers



Multiple Single-Server Queues



Part IV

Queuing System Variables

Queuing System Variables

- If the average time it takes a server to service a request is $\mathbf{T_s}$, then it follows that the average rate of service (if the server has an infinite supply of requests to work on) would be:

$$\mu=1/T_s$$

- The utilization of the system, which is the ratio between the rate of arrivals and the rate of service is:

$$\rho=\lambda/\mu$$

- Obviously, in the steady state, the rate at which requests are queued cannot exceed the rate at which the server is able to serve them! Thus, we have:

$$\lambda < \mu$$
$$\rho < 1$$

Part V

Queuing Models and Examples

Little's Formulas

The following two relationships are true of any "steady state" queuing system (i.e. a queuing system in equilibrium).

$$\boxed{\begin{array}{c} \mathbf{r = \lambda\, T_r} \\ \mathbf{w = \lambda\, T_w} \end{array}}$$

In a queuing system, a customer's time is spent either waiting for service or getting service.

$$\boxed{\mathbf{T_r = T_w + T_s}}$$

Multiplying the above equation by the arrival rate λ and applying Little's formula, we get:

$$\mathbf{r = w + \lambda\, T_s = w + \lambda / \mu}$$

Remember, $\rho = \lambda / \mu$, so ...

$$\boxed{\mathbf{r = w + \rho}}$$

Notation for Queuing Systems

A/B/c/d

- A = the interarrival time distribution
- B = the service time distribution
- c = the number of servers
- d = the queue size limit
  - omitted if infinite

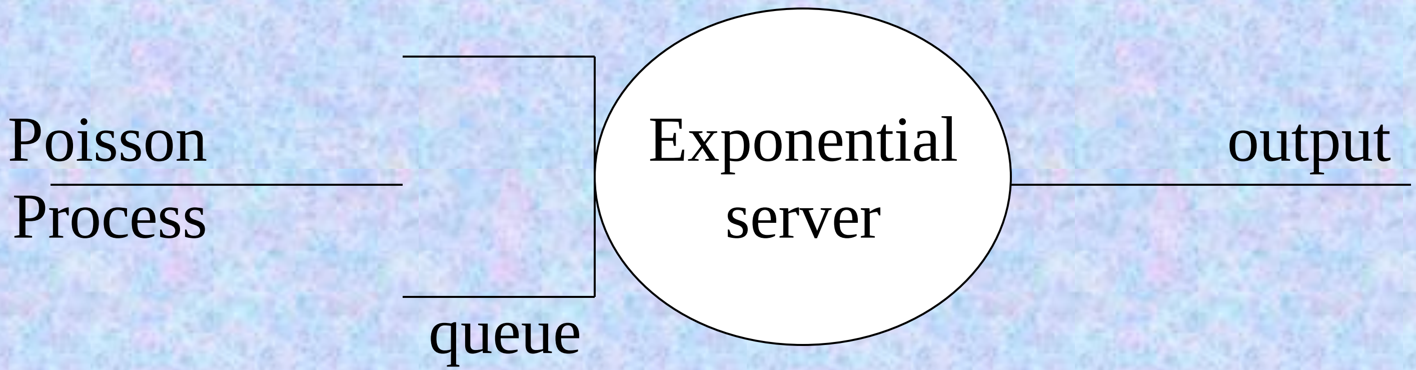
:Where **A** and **B** can be

D for Deterministic distribution

M for Markovian (exponential) distribution

G for General (arbitrary) distribution

The M/M/1 System



Understanding System Behavior

- To characterize the behavior of the system, we have to understand:
  - The frequency distribution of requests.
    - In any given time period, how many new request enter the system?
  - The distribution of service times
    - Does every request take equal time to process? (probably not).

Understanding Arrival of Requests

```
Time ----->

| | | | | | | | | | | | | | | | |

t0 t1 t2      ....               tn
```

In any one time period ti, some number of requests can arrive. The number that arrive is a random variable.

Arrivals Follow a Poisson Process

- $a(t)$ = # of arrivals in time interval $[0,t]$

$$\Pr(a(t) = n) = e^{-\lambda t} (\lambda t)^n / n!$$

Models for Interarrivals and Service Times

- Customers arrive at times $t_0 < t_1 < ....$ - Poisson distributed
- The differences between consecutive arrivals are the **interarrival times** : $\tau_n = t_n - t_{n-1}$
- τ_n in Poisson process with mean **arrival rate** λ, are exponentially distributed,

$$\boxed{\Pr(\tau_n \leq t) = 1 - e^{-\lambda t}}$$

- **Service times** are exponentially distributed, with mean **service rate** μ: $\boxed{\Pr(S_n \leq s) = 1 - e^{-\mu s}}$

M/M/1 Performance

- Average number of customers in a M/M/1 System

$$\boxed{\mathbf{r} = \rho/(1-\rho)}$$

- Average number of customers waiting for service in a M/M/1 system

$$\boxed{\mathbf{w} = \rho^2 /(1-\rho)}$$

- Average Time waiting in a M/M/1 queue

$$\boxed{\mathbf{T_w} = \rho/\mu(1-\rho)}$$

M/M/1 Performance

- Probability that the number of customers in the system = **N**

$$\Pr[R=N] = (1-\rho)\, \rho^{N}$$

- Probability that the number of customers in the system ☯ **N**

Pr[R ☯ N] = $\sum_{i=0}^{N} (1-\rho)\, \rho^{i}$

Examples

- **Given**
  - Arrival rate of 50 requests/sec
  - Service rate of 60 requests/sec
- **Find**
  - Utilization
  - Probability of having 10 requests in the system
  - How big should we make the queue?

ρ **=** λ/μ **=** 50/60 = 0.833

Pr[r=12] **=** $(1-\rho)\,\rho^N = (0.166)(0.833)^{10} = 0.026$

Pr[r=50] **=** $(1-\rho)\,\rho^N = (0.166)(0.833)^{50} = 0.00002$

Pr[r=100] **=** $(1-\rho)\,\rho^N = (0.166)(0.833)^{100} = 2*10^{-9}$

So, it looks like 100 buffers will suffice (50 will not)

Examples (continued)

- What is the average in-queue wait time?

  $\mathbf{T_w =}$ $\rho/\mu(1-\rho) = 0.833/(60*0.167) = .083 = 83$ msec

And finally, what is the average response time (T_r) ?

$\mathbf{T_r = T_w + T_s}$

$\mathbf{T_s = 1/}\mu = 1/60 = 0.0167 = 17$ msec

$\mathbf{T_r = 83 + 17 = 100}$ **msec**

What happens if we increase service rate to 75/sec?

Examples (continued)

- Service rate is now 75, so

  $\rho = \lambda/\mu = 50/75 = 0.67$

  $T_w = \rho/\mu(1-\rho) = 0.67/(75*0.33) = .027 = 27$ msec

- And finally, what is the average response time (Tr) ?

  $T_s = 1/\mu = 1/75 = 0.013 = 13$ msec
  $T_r = 27 + 13 = 40$ msec!!

- **We increased service rate by only 25% yet response time dropped by 60%!**

Part VI

Conclusion

Assignment & Readings

- Readings:
  - ***Queuing Analysis*** paper by William Stallings (recommended):

    ftp://shell.shore.net/members/w/s/ws/Support/QueuingAnalysis.pdf
  - **Tom Slater's Queuing Theory Tutor**

    http://www.dcs.ed.ac.uk/home/jeh/Simjava/queueing/
  - **Myron Hlynka's Queueing Theory Page**

    http://www2.uwindsor.ca/~hlynka/queue.html

Next Session:

Multimedia Networking
Network Security
Network Management