| | HW3 |
|--------------|--|
| | Collaborators: Xi Lin, Cerina Yas |
| 1 | (;) ∀ € > 0, ∃ M ∈ N st. Y n>M, 1xn-x1 < € |
| | (1) V2, 0, 3 MCN st. 7 N3/V, 1Xn-X1<2 |
| | (ii) \(\frac{1}{2} \sigma 0, \text{AMEN s.t. } \(\frac{1}{2} \text{M} \) \(\frac{1}{2} \text{M} \) |
| | (i)⇒(ii) |
| 715 | $\forall \xi > 0$, given (i), let $\xi = \alpha \xi'$, |
| | JM=M'st. Hn>M, xn-x < \ = \x \x' |
| | $(ii) \Rightarrow (i)$ |
| | ∀ ε>0, given (ii), let ≤'= ± ε |
| | 7 M=M st Un>M, 1xn-x) <x 2'="x</td"></x> |
| (0) | Therefore, the two statements are equivalent. |
| 0-3 | to be a second of the second o |
| 2. (a) | Take <> 0. Define Z = ax + by, where x = lim xn, y = lim yn. |
| | Since MA = X = AMIEN: YNZMI, IM-201< 2 |
| | yn > y = 7 M2∈N + Hn2M2, 14n-4 < \(\frac{\xi}{2b}\) |
| | Take M= max (M, M2), then for all nzM, Zn=axn+bxn |
| | $ Z_n - Z = \alpha x_n + b y_n - \alpha x - b y $ |
| | $\leq \alpha \gamma_n - \alpha \gamma + b \gamma_n - b \gamma $ |
| | $= \alpha \chi_n - \chi + b \gamma_n - \gamma $ |
| | $\langle \frac{\xi}{2} + \frac{\xi}{2} \rangle = \xi$ |
| 143 | $\Rightarrow \lim_{n \to \infty} (\alpha x_n + b y_n) = \alpha (\lim_{n \to \infty} x_n) + b (\lim_{n \to \infty} y_n)$ |
| | $h \rightarrow \infty$ $h \rightarrow \infty$ |
| (<i>P</i>) | Let xn → x as n → ∞. Take Zn:=xn, Z=x2. |
| (0) | $ Z_n - Z = \chi_n^2 - \chi^2 $ |
| | $= \left \left(\chi_{n} - \chi + \chi \right)^{2} - \chi^{2} \right $ |
| | |
| | $= 2(x_n - x) \cdot x + (x_n - x)^2 $ $= 2(x_n - x) \cdot x + x_n - x ^2$ |

| | Let 200 be given. Take k = max (12), [\frac{2}{3} \] |
|-----|--|
| | 3 MiEN: Yn 3 Mi, 1xn-x1< \frac{5}{3k} (\$1) |
| | Take M:= Mi. |
| | for all no M. |
| | $ Z_{n}-Z \leq 2 \chi_{n}-\chi \cdot \chi + \chi_{n}-\chi ^{2}$ |
| | $<2\cdot\frac{\xi}{3k}\cdot k+\left(\frac{\xi}{3k}\right)^2$ |
| | $\leq \frac{2}{3} \leq + \frac{1}{3} \leq = \leq$ |
| | $= \lim_{n \to \infty} (\chi_n^2) = \left(\lim_{n \to \infty} \chi_n\right)^2$ |
| | the the there are the same that the same tha |
| (c) | Let xn > x as n > x. Take Zn = \frac{1}{2}, Z = \frac{1}{2} Let \(\frac{1}{2} > 0. \) |
| \ | $ Z_n - Z = \left \frac{1}{x_n} - \frac{1}{x_n} \right $ |
| | $= \frac{ x_0 - x }{ x_0 - x }$ |
| | $=\frac{12n-x}{12n(1x)}$ |
| | 3MEN: Vn3M, 1xn-x1< min (1x12 & 1x1) |
| | For all n3M, 1X-Xn < 1x1 |
| | => x = x-xn+xn = x-xn + xn < x + xn |
| | => 1xn |
| | $\Rightarrow \frac{ x_n }{ x } < \frac{ x }{ x }$ |
| | $\frac{ Z_n-Z = x_n-x }{ x_n \cdot x }=\frac{ x_n-x }{ x }\cdot\frac{2}{ x }$ |
| | |
| | $\frac{ x ^2 \frac{\xi}{2}}{2} = \xi$ |
| | X X |
| | => lim x = lim x |
| | min An lim An |

| 3 (a) | False. |
|-------------------|--|
| 3 | Counterexample: Th= (-1) , yn= (-1) n+1, then (xn) (yn) diverge |
| | Counterexample: $\chi_n = (-1)^n$, $\chi_n = (-1)^{n+1}$, then $\{\chi_n\}_{\{\chi_n\}}$ diverges $\{\chi_n\}_{\{\chi_n\}}$ on verges to 0 |
| | Contract and Alterial artification of March |
| (b) | Folse. |
| | Counterexample : Xn=(-1)h, Yn=(-1)ht then (xn). (Yn) diverge |
| | YneN, xnyn=-1, then {xnyn} converges to -1 |
| | 12 LO + STONE / Many A. Many . |
| (c) | |
| | Pf. Take ≤>0. Let xn → x, xn+yn → Z, y=Z-X |
| | 3 Mi EN: Ynz Mi, IXn-X/< = |
| | 3 M2 EN: Yn2M2, 1(xn+yn) - (x+y) < = |
| | Take M= max (M, Mz), for all nz M, |
| and of the second | $ y_n - y = (x_n + y_n) - (x_1 + y_1) - x_n + x_1 $ |
| | $\leq (\chi_n + \gamma_n) - (\chi_+ \gamma_n) + \chi_n - \chi_n $ |
| • | $<\frac{\xi}{2} + \frac{\xi}{2} = \xi$ |
| | so { yn} converges. |
| | INTELL MONTE |
| (q) | True. |
| 1 100 | Pf Take 5 > 0. |
| | BER st. Xn = B for all n |
| | AMEN st. AnoM, 14n-01< & |
| | So ynzM, 1xnyn-0 = xn . yn < B. \frac{\xx}{B} = \xi |
| | so {Xn/n} converges to 0. |

| (e) | False. |
|-----|---|
| | Countergrande: Xn=n2, Yn= n |
| | then (Mn) is un sounded, (M) converge + 1 |
| | YneN, xnyn=n, then (xnyn) diverges |
| | |
| 4, | |
| | han yn |
| | ⇒MEN s.t. ∀nzM, xm - 0 < 1xh < |
| | 1xh1 < 1 |
| | 12/2/201 |
| | since 1/20 => 1/2 / 1/21 |
| | Line Aller Are Mil Month Vision |
| | 2° Take (/n) = (nd), (xn) = (p(n) - nd) |
| | 1. Xn - 1. Cd-1 n a-1 + + Ca |
| | 1. $\frac{\chi_n}{h^{-1}\infty} = \frac{1}{h^{-1}} \frac{C_{d-1} n^{d-1} + \cdots + C_n}{h^{-1}\infty}$ |
| | - 1. (Cd-1,, Co) |
| | h-> m hd = |
| | => 3MEN st. YnzM. Ynz (Xn) |
| | hd> Cn-1 nd-1++ Co |
| | $2n^{d} > n^{d} + (n_1 n^{d-1} + \dots + C_n = p(n))$ |
| | n to it as in the second |
| | 业 × (0-10年) 株本 / / → (Andrew) |
| | # Halastan Later Contract Manual Street |
| | Det manual Extension |
| | |

| | The sequence converges. For all E>0, there exists MEN st. M. E>1 |
|-----|--|
| | $\frac{ h\cos h - 0 = h \cos h }{ h^2 + 1 } \leq \frac{n}{ h$ |
| | So hit as home. |
| | |
| (b) | The sequence diverges. |
| | The Half to the water of the first to the same |
| | |
| | 3/x-x1 |
| | MEAN - Neighborn |
| | SS/7-WX/E |
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| <u></u> | Let hi = i . {Xni} i=1 = {Xni} = 1 |
|---------|--|
| 0. | $ A + h = i + \langle x_n \rangle_{i=1}^{\infty} = \langle x_n \rangle_{n=1}^{\infty}$ |
| | Then {Xn} = converges to X. |
| | Man transfer that the text that |
| | 2° => " If $\{N_n\}_{n=1}^{\infty}$ converges to X , for any subsequence $\{N_n\}_{i=1}^{\infty}$, |
| | for any subsequence (Mni) i=1, |
| | for i= hizi= |
| | suppose for i=k. Nizi |
| | for i= k+1, ni= nk+1 3 nk+1 3 k+1=1 |
| | So nizi for all i EN. |
| | For every 5>0, there exists MEN st. for all NZM, |
| | 12n-21< E |
| | Since hizi, i>M => ni>M |
| | $\Rightarrow \chi_{n_i} - \chi < \xi$ |
| | => {Ni;} = converges to N. |
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