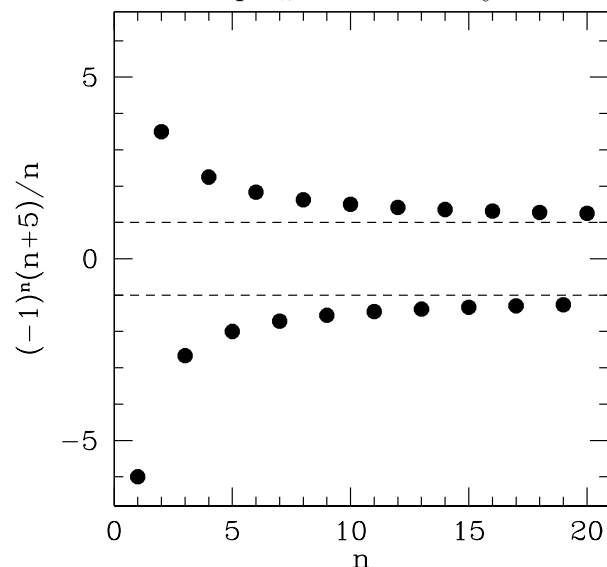# Handout: Examples of lim sup and lim inf

Example Calculate lim sup a_n and lim inf a_n for $a_n = (-1)^n(n+5)/n$.

Solution Define $\alpha_n = \sup\{a_k | k \geq n\}$. Then

$$
\begin{aligned}
\alpha_n &= \sup\left\{(-1)^n(n+5)/n, (-1)^{n+1}(n+6)/(n+1), \ldots\right\} \\
&= (n+5)/n \text{ for } n \text{ even, and} (n+6)/(n+1) \text{ for } n \text{ odd} \\
&\to 1 \text{ as } n \to \infty.
\end{aligned}
$$

Therefore lim sup $a_n = 1$. Similarly lim inf $a_n = -1$.

Example Calculate lim sup a_n and lim inf a_n for $a_n = (-1)^n n/(n+8)$.

Solution Define $\alpha_n = \sup\{a_k | k \geq n\}$. Then

$$
\begin{aligned}
\alpha_n &= \sup\left\{3 + (-1)^n n/(n+8), 3 + (-1)^{n+1}(n+1)/(n+9), \ldots\right\} \\
&= 4 \\
&\to 4 \text{ as } n \to \infty.
\end{aligned}
$$

Therefore lim sup $a_n = 4$. Similarly lim inf $a_n = 2$.