Introduction to Machine Learning
[Fall 2022]

Dimensionality Reduction

November 10, 2022

Lerrel Pinto

Logistics

- We are nearing the end of the class!
  - HW5 & HW6.
  - Capstone Project.
- Syllabus changes:
  - https://nyu-robot-learning.github.io/ml-class/docs/lectures
- Want something covered in later classes? Mention on #class-discussion

What we have covered so far

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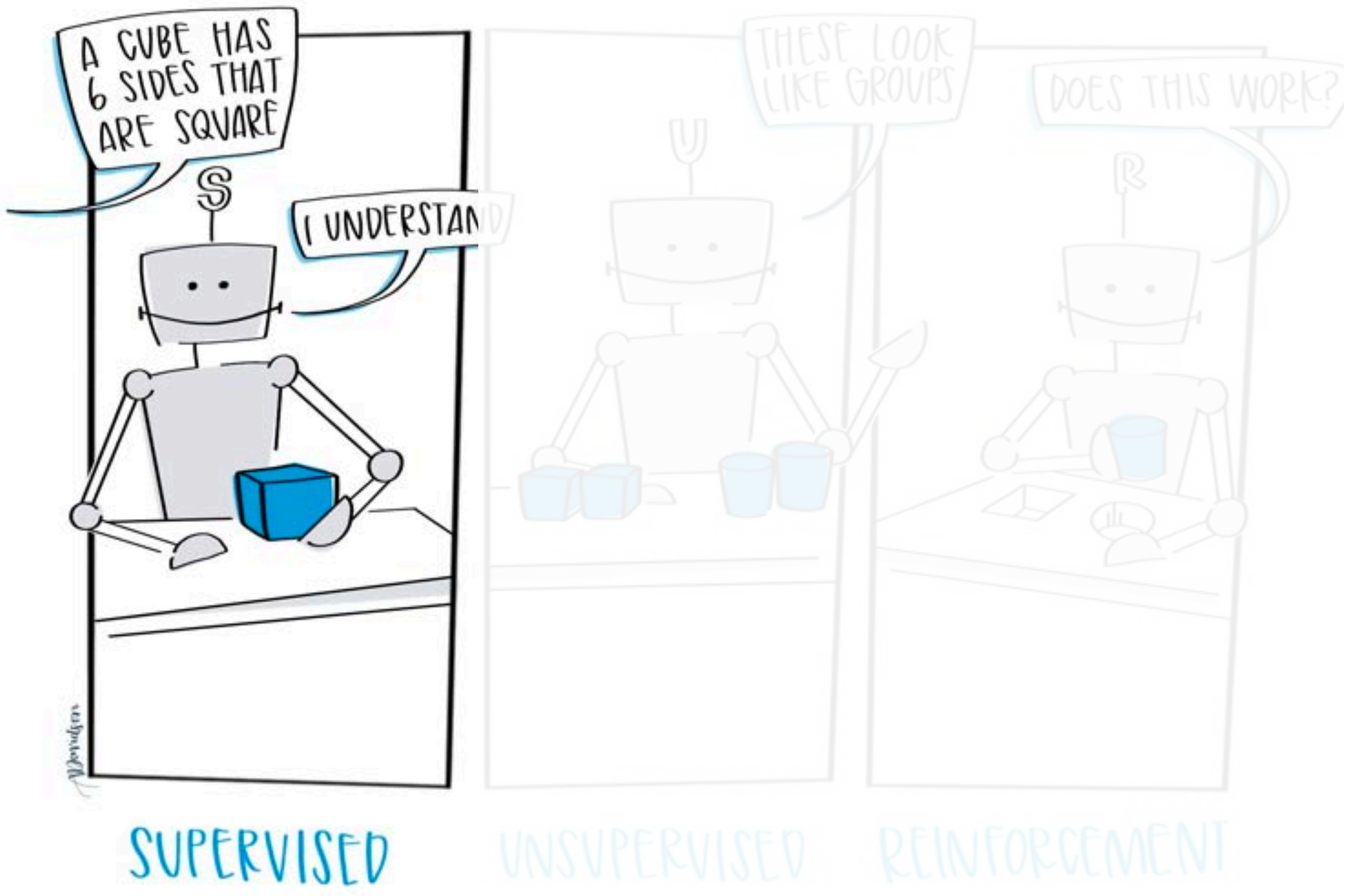

ceralytics

What we will cover in the remainder of the class

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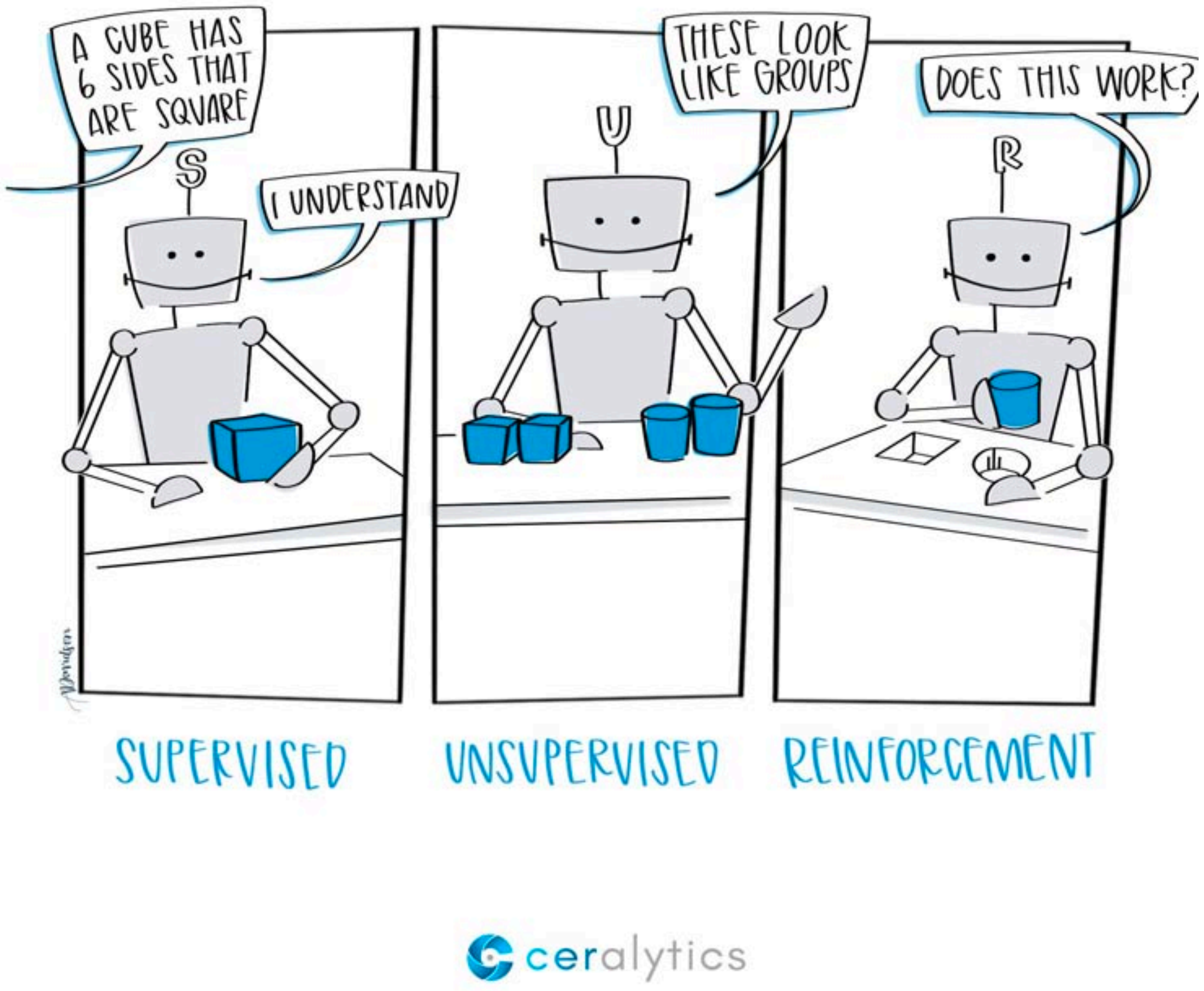


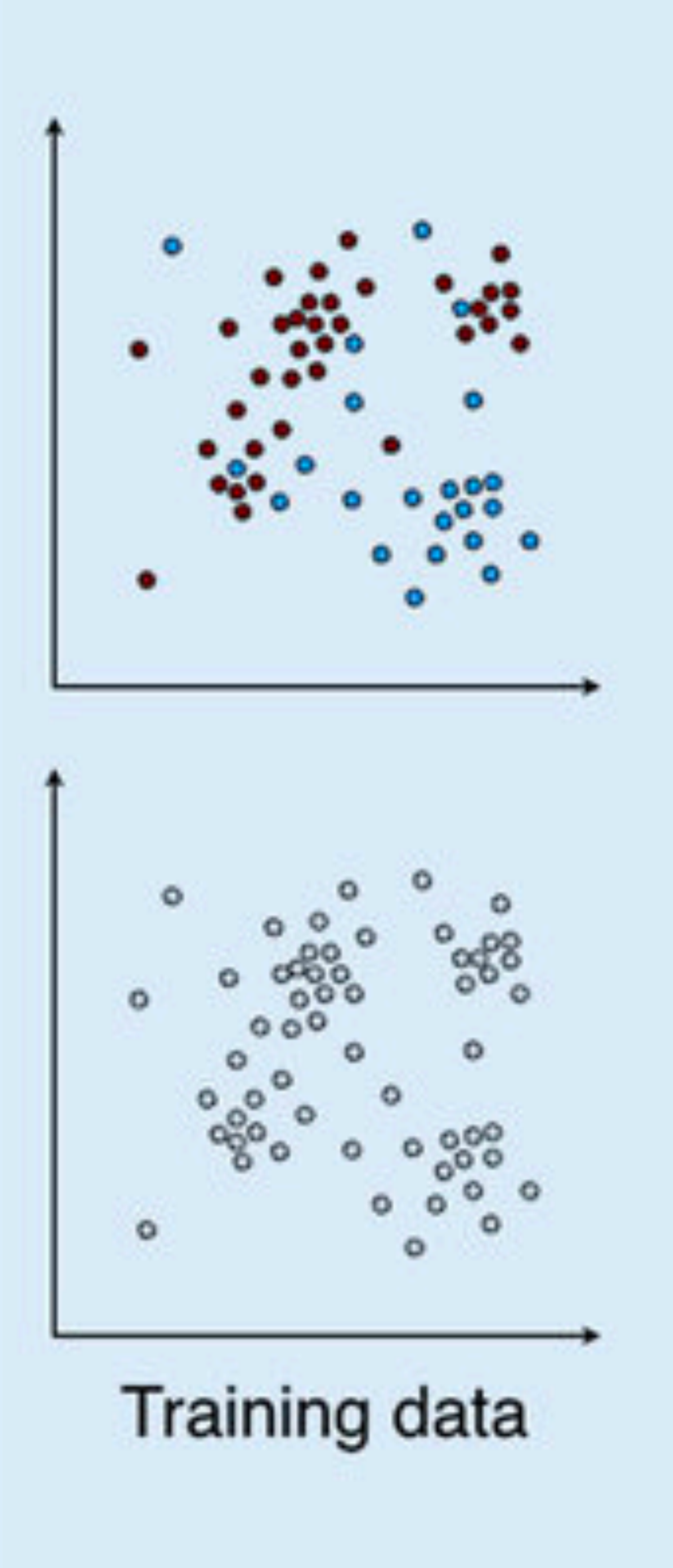
Unsupervised Learning

Credits: Langs et al. 2018

Unsupervised Learning

Supervised Learning

Unsupervised Learning



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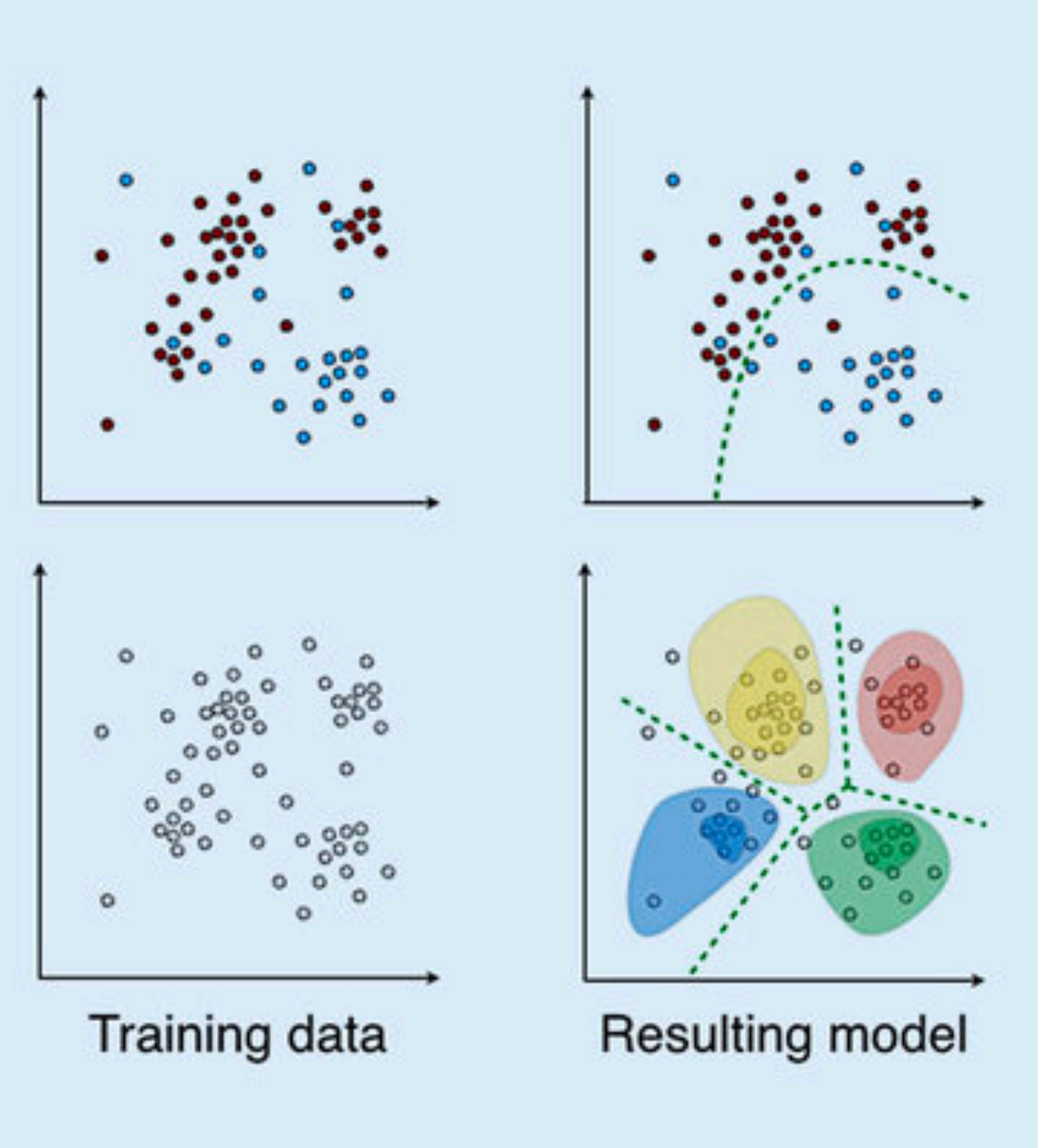

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Dimensionality Reduction

Dimensionality Reduction

- High-dimensional data can be repetitive:
  - Document classification - Thousands of words per document

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Dimensionality Reduction

- High-dimensional data can be repetitive:
  - Document classification - Thousands of words per document
  - Netflix / Amazon review - number of users * number of shows

	movie 1	movie 2	movie 3	movie 4	movie 5	movie 6
Tom	5	?	?	1	3	?
George	?	?	3	1	2	5
Susan	4	3	1	?	5	1
Beth	4	3	?	2	4	2

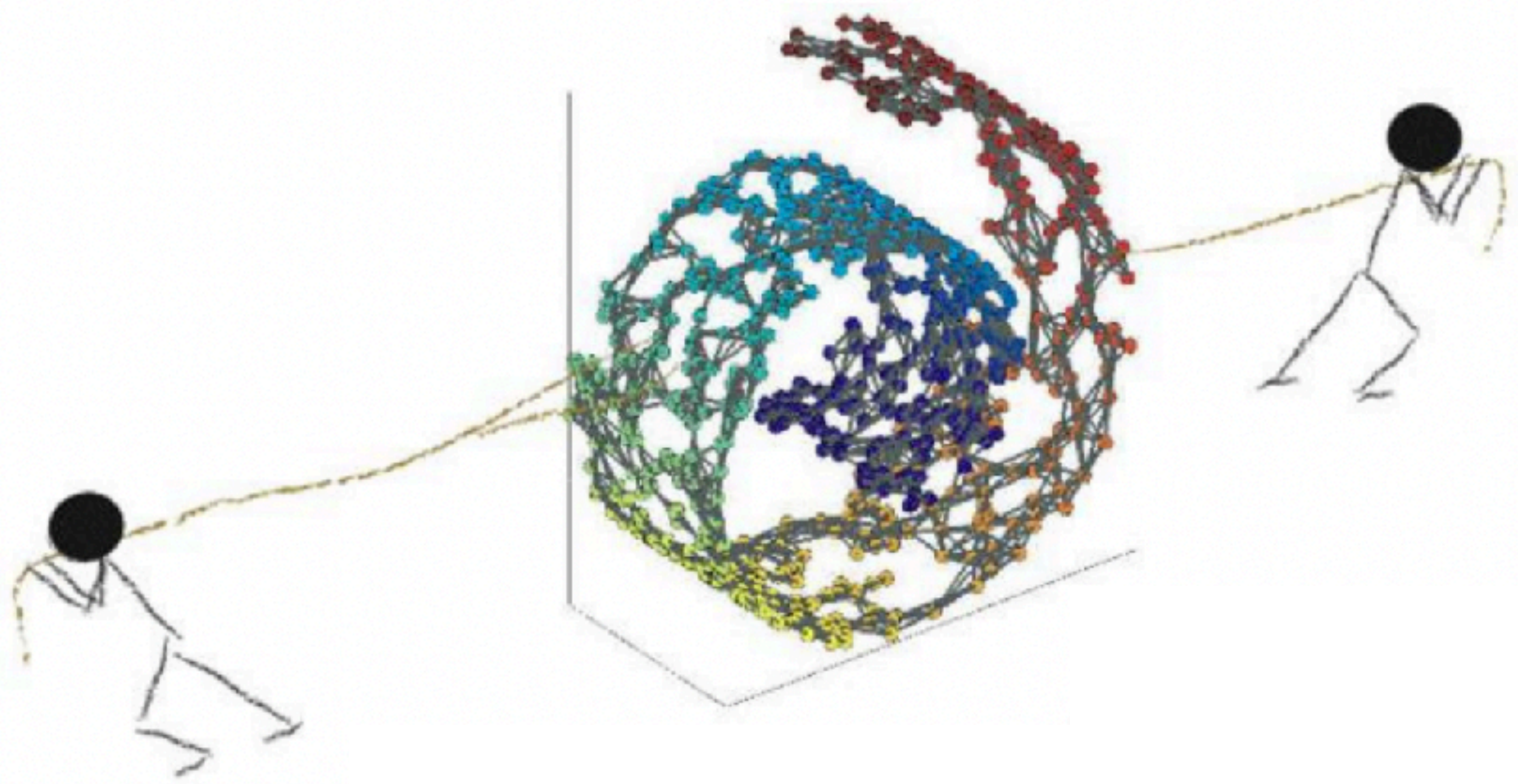
Credits: Aarti Singh

Dimensionality Reduction

- Curse of dimensionality:
  - Redundant / Irrelevant features degrade performance. (Signal to noise)
  - Difficult to interpret and visualize.
  - Computation becomes infeasible for some algorithms.
  - Hard to store high dimensional data.

Credits: Aarti Singh, Jeff Howbert

Dimensionality Reduction



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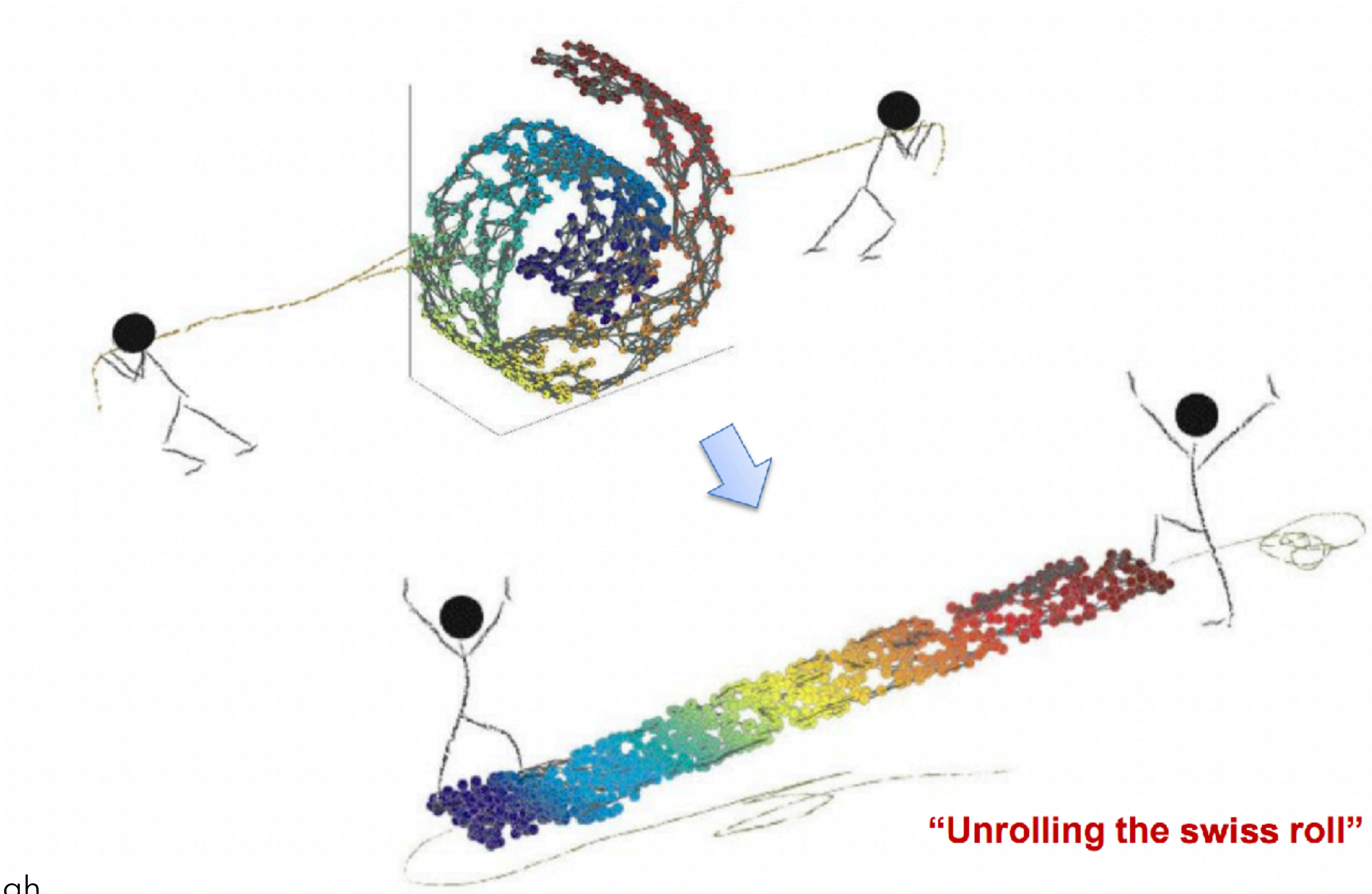
Dimensionality Reduction



Credits: Aarti Singh

Dimensionality Reduction

Credits: Aarti Singh, Jeff Howbert

Dimensionality Reduction

- Option 1: Feature selection
  - Cons: Requires domain knowledge.

Credits: Aarti Singh, Jeff Howbert

Dimensionality Reduction

- Option 1: Feature selection
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- Option 2: Model Regularization
  - Cons: Requires supervised learning

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Dimensionality Reduction

- Option 1: Feature selection
    - Cons: Requires domain knowledge.
- Option 2: Model Regularization
    - Cons: Requires supervised learning
- Option 3: Learn mapping from high-D to low-D space
    - Either linear or non-linear mapping

Credits: Aarti Singh, Jeff Howbert

Linear Dimensionality Reduction

Credits: Aarti Singh, Jeff Howbert

Linear Dimensionality Reduction

- Goal: given a dataset: $X \in \mathbb{R}^{d \times n}$, where $\vec{x} \in \mathbb{R}^d$ corresponds to a data point. Project $\vec{x} \rightarrow \vec{z}$, where $\vec{z} \in \mathbb{R}^k$ and $k < d$

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  - Example: Project 1000 word features to 3 dimensions (horror, fiction, reading level)

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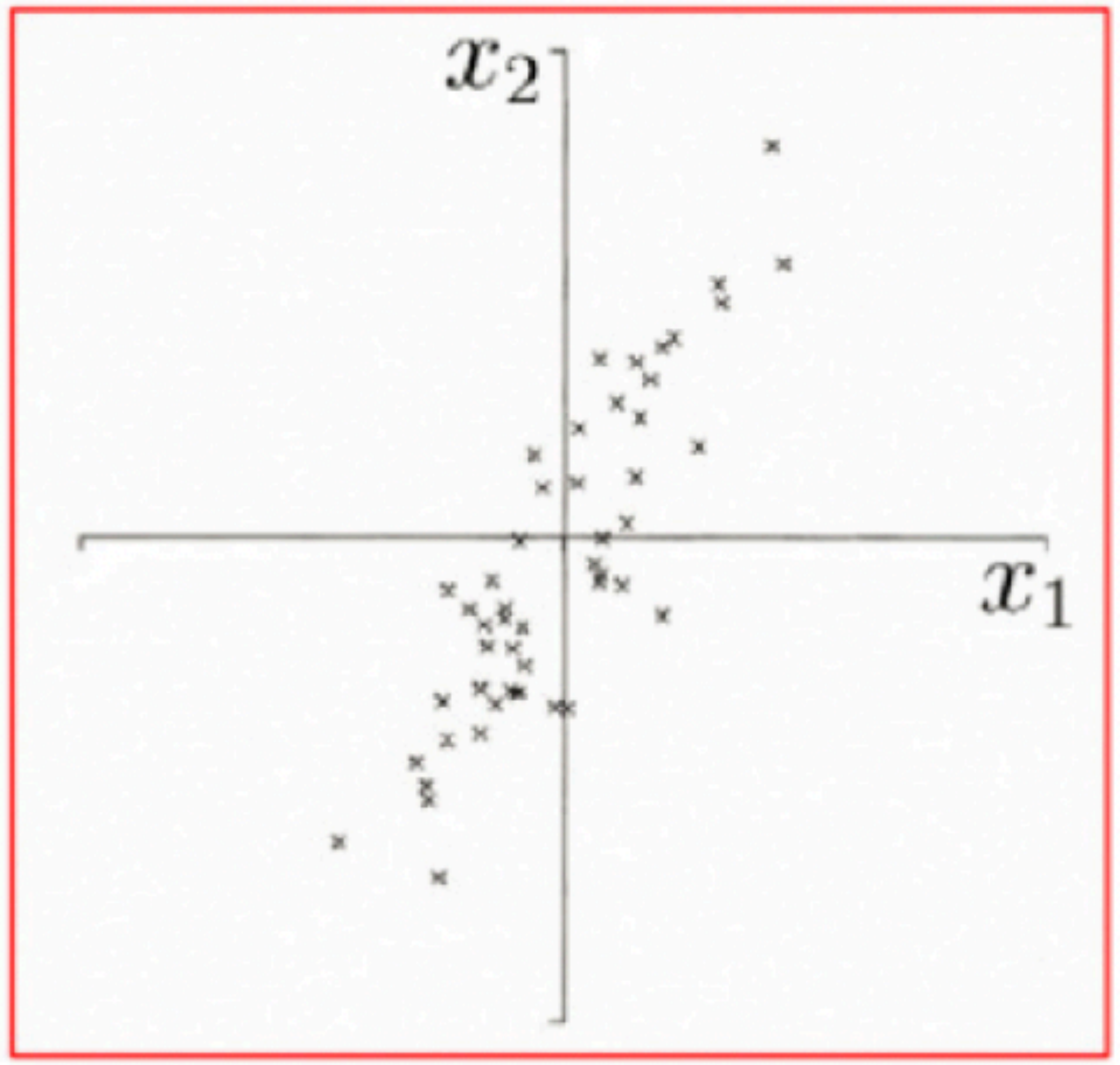
  - Usually $k << d$
  - Example: Project 1000 word features to 3 dimensions (horror, fiction, reading level)
  - Linear: $\vec{z} = A\vec{x}$

Credits: Aarti Singh, Jeff Howbert

How to solve this? (Intuitively)

Principal Component Analysis (PCA)



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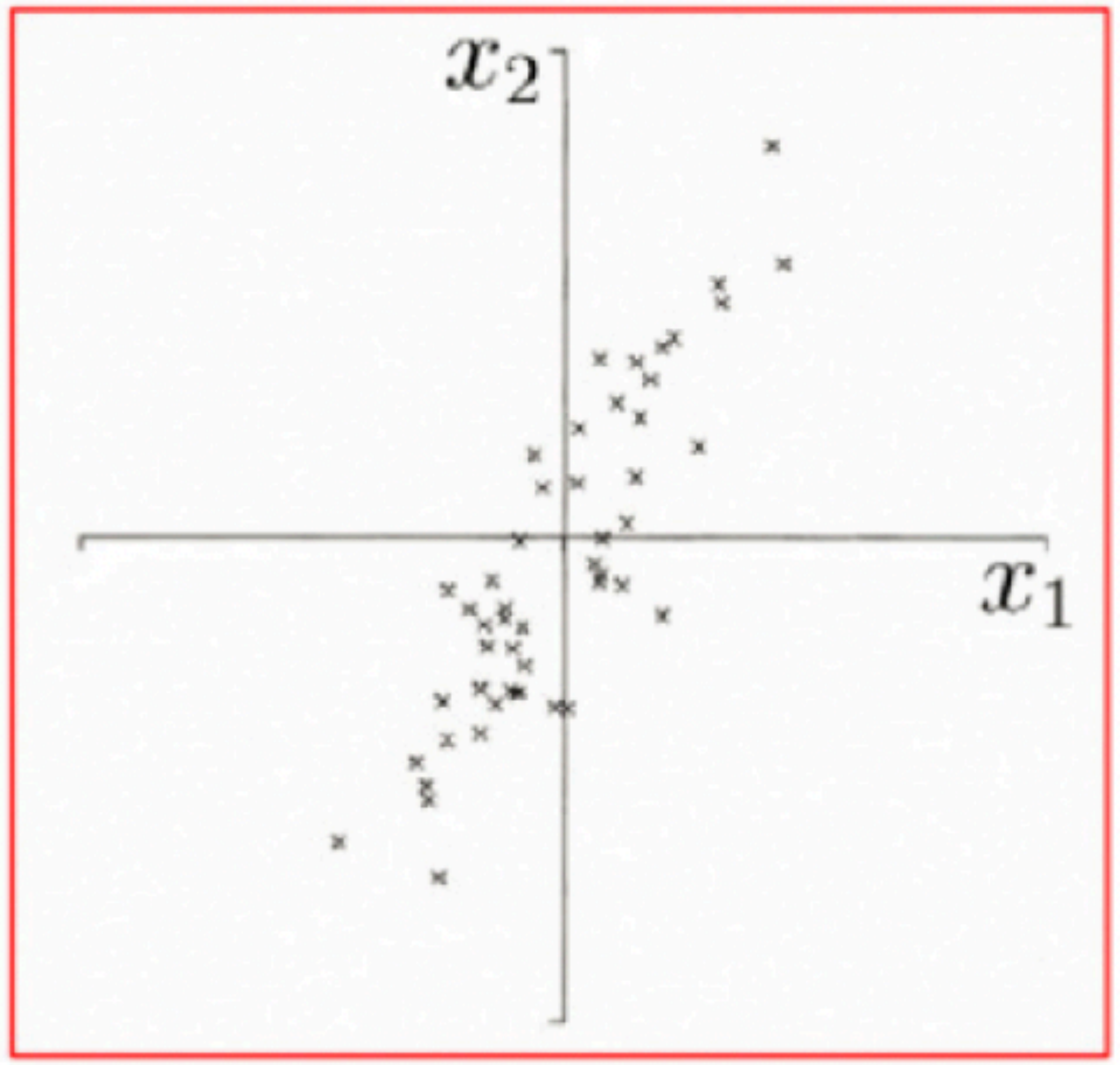

- First PC: Projection direction that maximizes variance of data.
- Second PC: Orthogonal to first and maximizes variance of data.

Credits: Jeff Howbert

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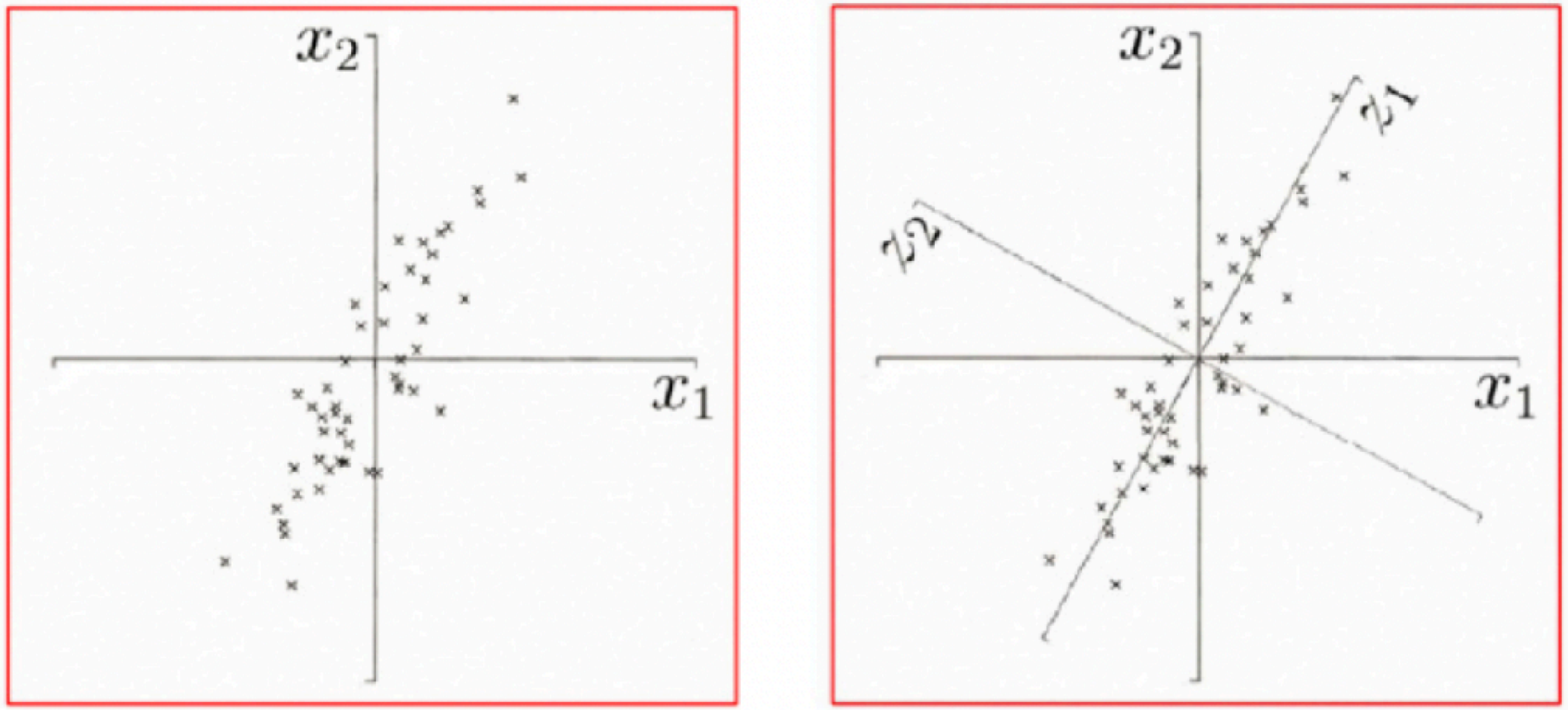

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Principal Component Analysis (PCA)

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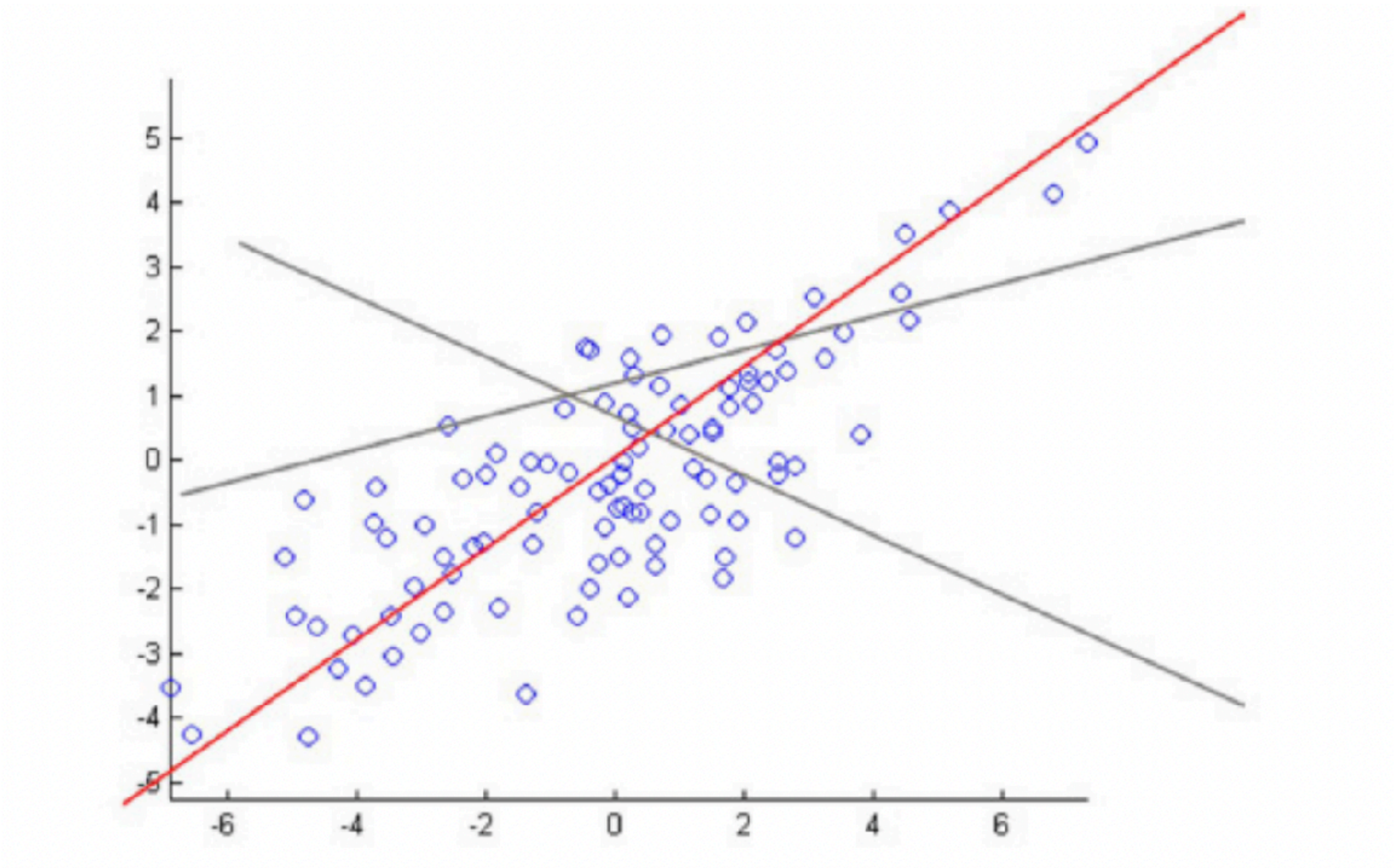

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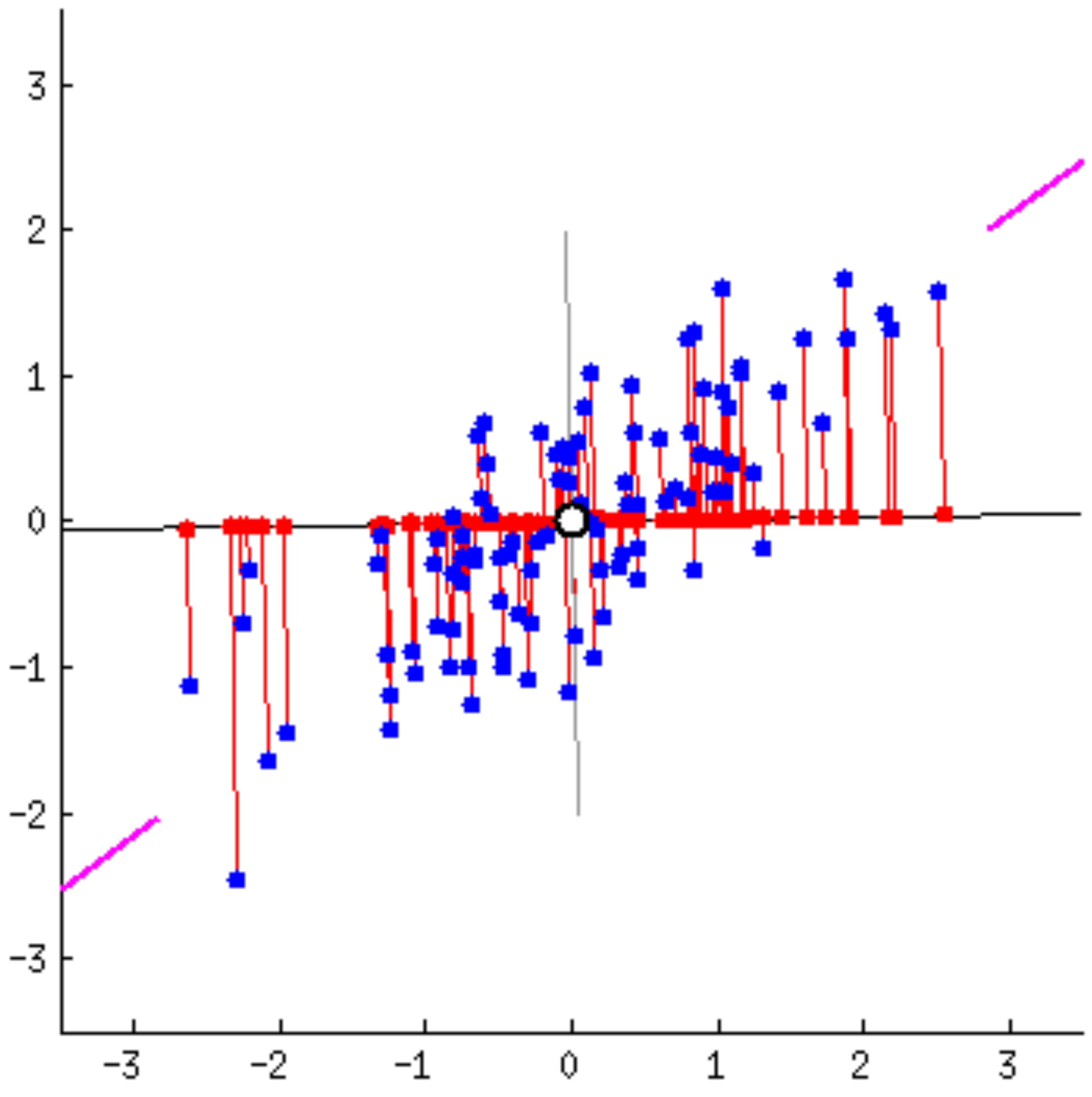

Credits: https://builtin.com/data-science/step-step-explanation-principal-component-analysis

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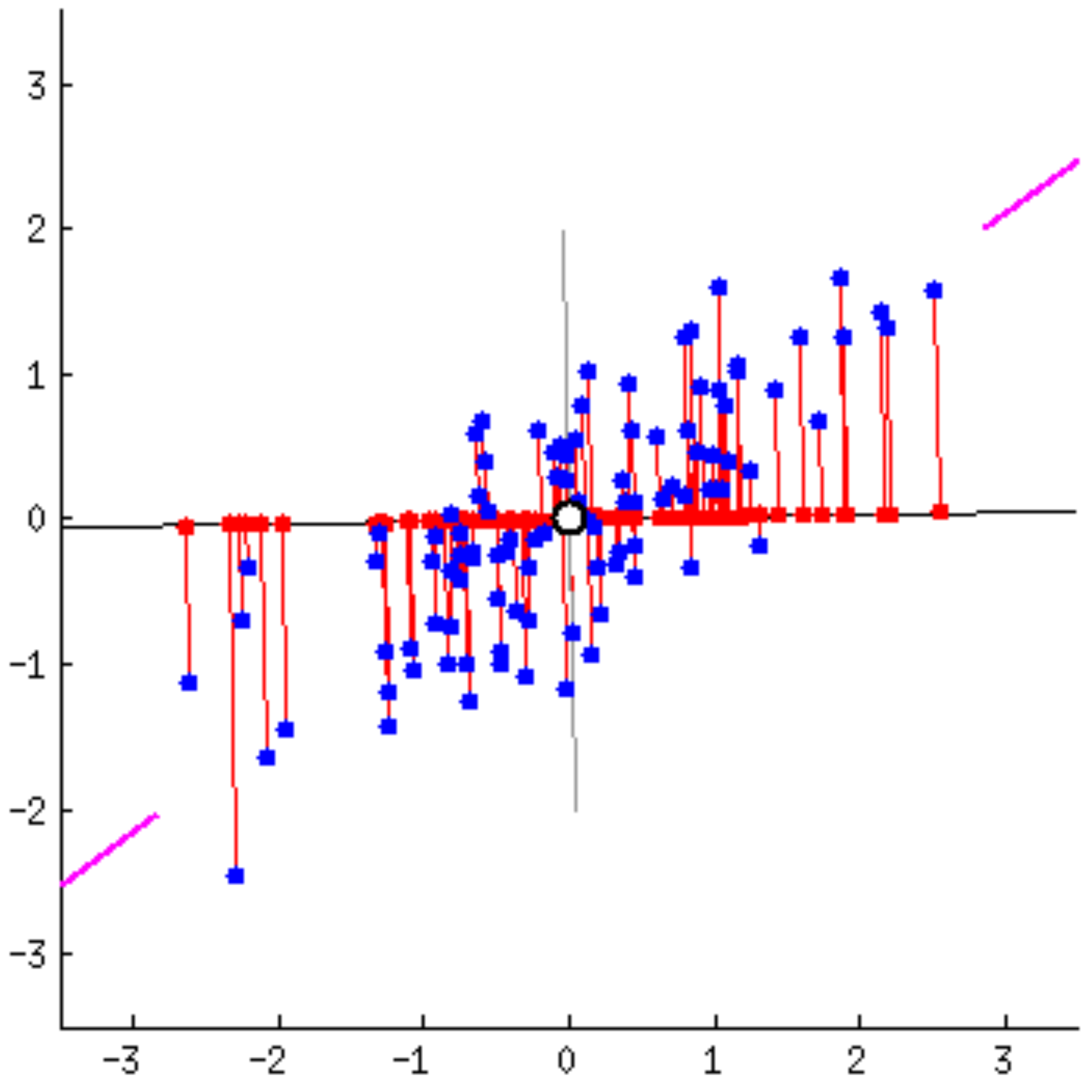

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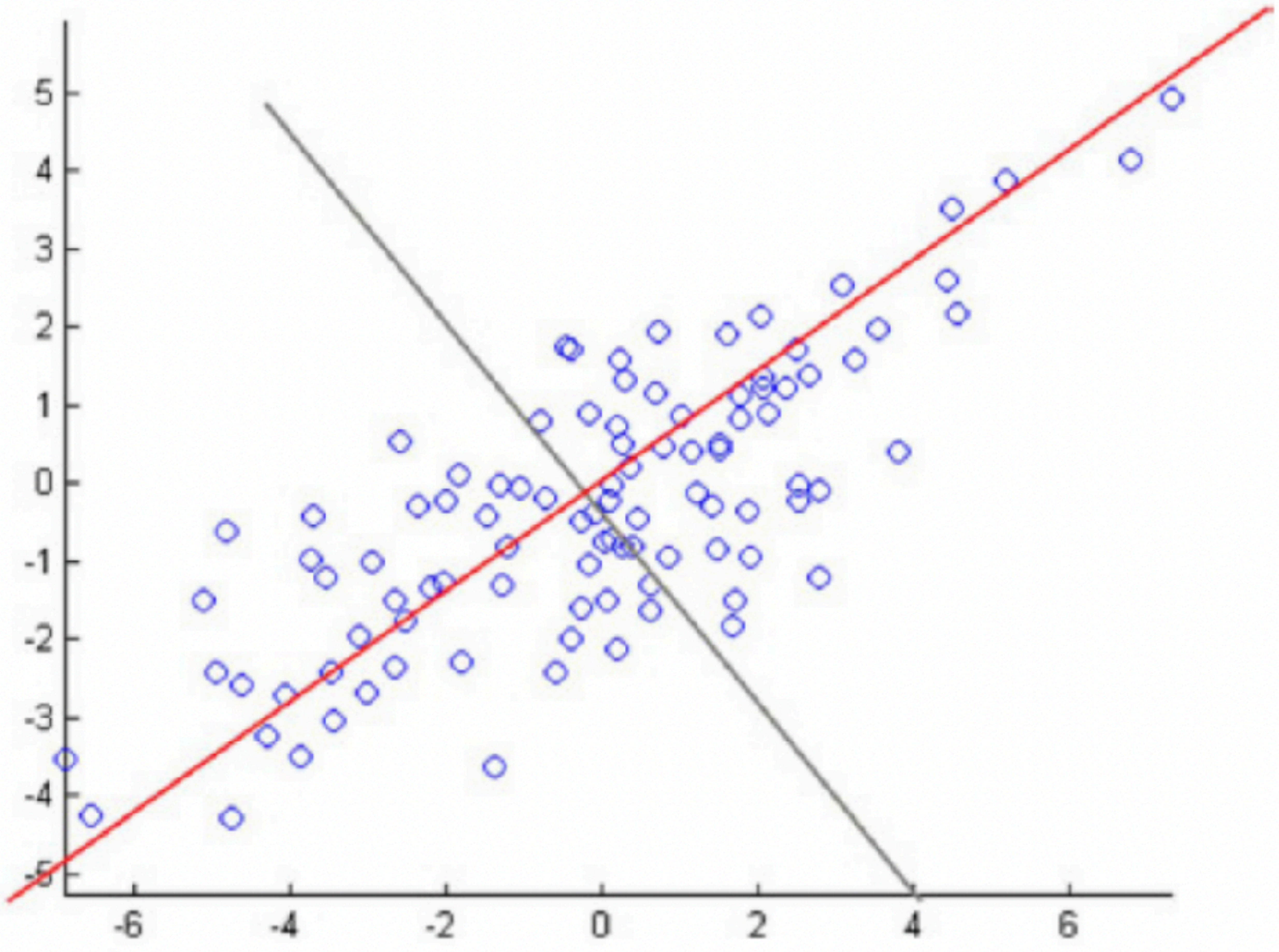
Principal Component Analysis (PCA)

Step 2



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Principal Component Analysis (PCA)

Interactive tool:

https://setosa.io/ev/principal-component-analysis/

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- Step 1: Make the mean of the data zero (Standardize your data)
- Step 2: Compute the Covariance Matrix C
- Step 3: Calculate eigenvectors and eigenvalues of C
  - Eigenvector with largest eigenvalue $\lambda_1$corresponds to the 1st PC (why?)
  - Eigenvector with k^{th} largest eigenvalue λ_k corresponds to k^{th} PC
  - $\frac{\lambda_k}{\sum_i \lambda_i}$ = proportion of variance captured by k^{th} PC

Questions?