Basic Algorithms CSCI-UA.0310

Additional Practice Problems

Remark: The following problems will be solved during the recitation class.

Problem 1

You are given n balls and m boxes. Each ball has a diameter and each box has a length. You may assume you are given two arrays $D[1 \dots n]$ and $L[1 \dots m]$ where the diameter of the i^{th} ball is $D[i]$ and the length of the j^{th} box is $L[j]$.
The i^{th} ball can be stored in the j^{th} box if $D[i] \leq L[j]$. You cannot store more than one ball in each box. The goal is to store the balls in the boxes in such a way that the number of balls stored is maximized. Develop a greedy algorithm and analyze its running time.

Problem 2

Describe a point set with n points that is the worst-case input for Graham's scan algorithm.

Problem 3

Suppose in a weighted undirected graph $G = (V, E)$, each edge has weight 1 or 2. Give an algorithm to find the least weight path from some vertex s to another vertex t in G in $O(|V| + |E|)$ time.
Hint: If all edges had weight 1, we could just use BFS. Try to modify the input graph so that this approach works.

Problem 4

Let $G = (V, E)$ be an undirected connected graph where each edge has a weight from the set $\{1, 10, 25\}$. Describe an $O(|V| + |E|)$ time algorithm to find an MST of G.

Problem 5

Let e be a maximum-weight edge on some cycle of the connected graph $G = (V, E)$. Show that there is a minimum spanning tree of G that does not include e.

Problem 6

Let $G = (V, E)$ be a weighted DAG, i.e., G is a directed acyclic graph that is also weighted. Develop an algorithm to find the length of the shortest paths from one vertex s to all other vertices in G in $O(|V| + |E|)$ time.
Hint: Use topological sorting.

Problem 7

Find a weighted graph with some edges assigned a negative weight on which Dijkstra's Algorithm fails. Justify your answer.

Problem 8

Recall the outer while loop in Dijkstra's Algorithm: *while(Q is nonempty)*. Does the algorithm still output the correct answer if we change this loop to *while($|Q| > 1$)*?

Problem 9

Give an algorithm that determines whether or not a given undirected graph $G = (V, E)$ contains a cycle. Your algorithm should run in $O(|V|)$ time, independent of $|E|$.

Problem 10

A mother vertex in a directed graph $G = (V, E)$ is a vertex from which there are paths to all other vertices in the graph. Give an $O(|V| + |E|)$ time algorithm to find a mother vertex of G, if it exists.

Problem 11

Let G be an undirected connected graph with a distinct weight $w(e)$ on every edge e. Suppose e_0 is the edge with the least weight in G, i.e. $w(e_0) < w(e)$ for every edge $e \neq e_0$. Show that any minimum spanning tree T of G contains the edge e_0.