Xi Liu, xl3504, Homework 5

Problem 1
since $X \in \{0,\ 1,\ ...,\ N\} = \{a_0,\ a_1,\ ...,\ a_N\}$
expectation of a discrete random variable X with the values $a_1, a_2,$ ...

$$\begin{aligned} E(X) &= \sum_i a_i P(X = a_i) \\ &= \sum_{n=0}^{N} a_n\, P(X = a_n) \\ &= \sum_{n=0}^{N} n\, P(X = n) \end{aligned}$$

	P(X = 0)	P(X = 1)	P(X = 2)	P(X = 3)	P(X = 4)	P(X = 5)	...	P(X = N)
row 1		1	1	1	1	1		1
row 2			1	1	1	1		1
row 3				1	1	1		1
row 4					1	1		1
row 5						1		1
...								...
row N								1

for each column of $P(X = n)$, its contribution to the expected value is $nP(X = n)$, in which n is equal to the sum of 1's in each column of $P(X = n)$

$$\sum_{n=0}^{N} n\, P(X = n) = \sum_{i=1}^{N} (\text{contribution from row } i)$$

using summation by parts

$$
\begin{aligned}
E(X) &= \sum_{n=0}^{N} n\, P(X = n) \\
&= \sum_{n=1}^{N} P(X = n) + \sum_{n=2}^{N} P(X = n) \\
&\quad + \sum_{n=3}^{N} P(X = n) + ... + \sum_{n=N}^{N} P(X = n) \\
&= \sum_{n=0}^{N-1} \left(\sum_{i=n}^{N} P(X = i) \right) \\
&= \sum_{n=0}^{N-1} P(X > n) \\
&/\text{* since } P(X > n) = \sum_{i=n}^{N} P(X = i) \text{ */}
\end{aligned}
$$

Problem 2

1.

$$
\begin{aligned}
a &:= x \\
F(a) &= P(X \leq a) \\
&= \int_{-\infty}^{a} f(x)dx \\
&= \int_{0}^{a} \left(2Kxe^{-Kx^2}\right) dx \\
/*\ u &:= -Kx^2 \\
du &= -2Kxdx \\
-du &= 2Kxdx \\
-\int e^u du &= -e^u\ */ \\
&= \left[-e^{-Kx^2}\right]_0^a \\
&= -\left[e^{-Kx^2}\right]_0^a \\
&= -\left(e^{-Ka^2} - e^0\right) \\
&= -\left(e^{-Ka^2} - 1\right) \\
&= 1 - e^{-Ka^2} \\
&= 1 - e^{-Kx^2}
\end{aligned}
$$

$$
\boxed{F(x) = \begin{cases} 1 - e^{-Kx^2} & \text{if } x > 0 \\ 0 & \text{otherwise} \end{cases}}
$$

2.
mean = $E[X]$

$$
\begin{aligned}
E[X] &= \int_{-\infty}^{\infty} x f(x) dx \\
&= \int_{0}^{\infty} x \left(2Kxe^{-Kx^2}\right) dx \\
&= \int_{0}^{\infty} \left(2Kx^2e^{-Kx^2}\right) dx \\
&= 2K \int_{0}^{\infty} x^2 e^{-Kx^2} dx \\
&\text{/* see calculations in next page */} \\
&= 2K(I_1) \\
&= 2K \left(\frac{1}{2K} I_2\right) \\
&= I_2 \\
&= \boxed{\sqrt{\frac{\pi}{4K}}}
\end{aligned}
$$

$$I_1 := \int_0^\infty x^2 e^{-kx^2} dx = \int_0^\infty x \cdot x e^{-kx^2} dx$$

$u := x; \quad du = dx$

$dv := x e^{-kx^2} dx$

$$V = \int dv = \int x e^{-kx^2} dx$$

$u_2 := -kx^2$

$du_2 = -2kx\,dx; \; x\,dx = \frac{du_2}{-2k}$

$$V = -\frac{1}{2k} e^{-kx^2}$$

$$\int u\,dv = uv - \int v\,du$$

$$\int u\,dv = x\left(-\frac{1}{2k} e^{-kx^2}\right) - \int \left(-\frac{1}{2k} e^{-kx^2}\right) dx$$

$$= -\frac{1}{2k} x e^{-kx^2} + \frac{1}{2k} \int e^{-kx^2} dx$$

$$I_1 = \left[-\frac{1}{2k} x e^{-kx^2}\right]_0^\infty + \frac{1}{2k} \int_0^\infty e^{-kx^2} dx$$

$$= -\frac{1}{2k}(0-0) + \frac{1}{2k} \int_0^\infty e^{-kx^2} dx$$

$$= \frac{1}{2k} \int_0^\infty e^{-kx^2} dx$$

$$\boxed{I_2 := \int_0^\infty e^{-kx^2} dx}$$

$x^2 + y^2 = r^2, \quad x = r\cos\theta$

$y = r\sin\theta$

$$I_2^2 = \left(\int_0^\infty e^{-kx^2} dx\right)\left(\int_0^\infty e^{-ky^2} dy\right)$$

$$= \int_0^\infty \int_0^\infty e^{-kx^2 - ky^2} dx\,dy = \int_0^\infty \int_0^\infty e^{-k(x^2+y^2)} dx\,dy$$

$$= \int_0^{\pi/2} \int_0^\infty e^{-kr^2} r\,dr\,d\theta$$

$u_3 := -kr^2$

$du_3 = -2kr\,dr; \; r\,dr = \frac{du_3}{-2k}$

$$= \left(\frac{\pi}{2}\right)\left(-\frac{1}{2k}\right)\left[e^{-kr^2}\right]_0^\infty = \left(-\frac{\pi}{4k}\right)(0-1) = \frac{\pi}{4k}$$

$$I_2^2 = \frac{\pi}{4k}; \quad I_2 = \sqrt{\frac{\pi}{4k}}$$

alternatively, the calculation of I_2 can be done as follows:
a continuous random variable has a normal distribution with parameters μ and $\sigma^2 > 0$ if its probability density function is given by

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\begin{aligned}
\int_{-\infty}^{\infty} f(x)dx &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \\
&= \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx \\
/^* \text{ set } \mu = 0; \qquad K &= \frac{1}{2\sigma^2}; \qquad \sigma = \frac{1}{\sqrt{2K}} {}^*/ \\
&= \frac{1}{\sqrt{2\pi(1/\sqrt{2K})^2}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-0}{1/(\sqrt{2K})}\right)^2} dx \\
&= \frac{1}{\sqrt{2\pi}(1/\sqrt{2K})} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\sqrt{2K}x\right)^2} dx \\
&= \frac{\sqrt{2K}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(2Kx^2\right)} dx \\
&= \sqrt{\frac{K}{\pi}} \int_{-\infty}^{\infty} e^{-Kx^2} dx = 1
\end{aligned}$$

/* since $\int_{-\infty}^{\infty} f(x)dx = 1$ for a f that is the probability density function of a continuous random variable X */

$$\int_{-\infty}^{\infty} e^{-Kx^2} dx = \sqrt{\frac{\pi}{K}}$$

/* since $f(x) = e^{-Kx^2}$ is an even function */

$$\int_{-\infty}^{\infty} e^{-Kx^2} dx = 2\int_{0}^{\infty} e^{-Kx^2} dx$$

$$\sqrt{\frac{\pi}{K}} = 2\int_{0}^{\infty} e^{-Kx^2} dx$$

$$\int_{0}^{\infty} e^{-Kx^2} dx = \frac{1}{2}\sqrt{\frac{\pi}{K}}$$

$$= \sqrt{\frac{1}{4}}\sqrt{\frac{\pi}{K}}$$

$$= \sqrt{\frac{\pi}{4K}} = I_2$$

variance $= Var(X)$

$$Var(X) = E[X^2] - E[X]^2$$

$$E[g(X)] = \int_{-\infty}^{\infty} g(x) f_X(x) dx$$

$$E[X^2] = \int_{-\infty}^{\infty} x^2 f(x) dx$$

$$= \int_{0}^{\infty} x^2 \left(2Kxe^{-Kx^2}\right) dx$$

$$= 2K \int_{0}^{\infty} x^3 e^{-Kx^2} dx$$

/* see calculations in next page */

$$= 2K(I_3)$$

$$= 2K\left(\frac{1}{2K^2}\right)$$

$$= \frac{1}{K}$$

$$
\begin{aligned}
I_3 &:= \int_0^\infty x^3 e^{-Kx^2} dx \\
&= \int_0^\infty x^2 \cdot x e^{-Kx^2} dx \\
u &:= x^2; \qquad du = 2xdx; \qquad dv := xe^{-Kx^2} dx \\
v &= \int dv = \int xe^{-kx^2} dx \\
u_2 &:= -Kx^2; \qquad du_2 = -2Kxdx; \qquad xdx = -\frac{1}{2K} du_2 \\
v &= -\frac{1}{2K} \int e^{u_2} du_2 = -\frac{1}{2K} e^{-Kx^2} \\
\int udv &= uv - \int vdu \\
&= x^2 \left(-\frac{1}{2K} e^{-Kx^2} \right) - \int \left(-\frac{1}{2K} e^{-Kx^2} \right) (2xdx) \\
&= -\frac{1}{2K} x^2 e^{-Kx^2} + \int \frac{1}{K} xe^{-Kx^2} dx \\
I_3 &= \left[-\frac{1}{2K} x^2 e^{-Kx^2} \right]_0^\infty + \int_0^\infty \frac{1}{K} xe^{-Kx^2} dx
\end{aligned}
$$

/* since $\forall a \in \mathbb{R},\ \lim_{x\to\infty} \frac{e^x}{x^a} > 0$

the exponential function e^x is an asymptotic upper bound on the polynomial function x^a */

$$\begin{aligned}
I_3 &= -\frac{1}{2K}(0-0) + \int_0^\infty \frac{1}{K} x e^{-Kx^2} dx \\
&= \int_0^\infty \frac{1}{K} x e^{-Kx^2} dx \\
&= \frac{1}{K}\int_0^\infty x e^{-Kx^2} dx
\end{aligned}$$

/* $u := -Kx^2; \qquad du = -2Kxdx; \qquad xdx = \frac{du}{-2K}$

$$\frac{1}{K}\int x e^{-Kx^2} dx = \frac{1}{K}\cdot\frac{1}{-2K}e^{-Kx^2} = \frac{1}{-2K^2}e^{-Kx^2}$$

*/

$$\begin{aligned}
I_3 &= \left[-\frac{1}{2K^2}e^{-Kx^2}\right]_0^\infty \\
&= -\frac{1}{2K^2}(0-1) \\
&= \frac{1}{2K^2}
\end{aligned}$$

$$\begin{aligned}
Var(X) &= E[X^2] - E[X]^2 \\
&= \frac{1}{K} - \left(\sqrt{\frac{\pi}{4K}}\right)^2 \\
&= \frac{1}{K} - \frac{\pi}{4K} \\
&= \boxed{\frac{4-\pi}{4K}}
\end{aligned}$$

3.

let $F_Y(a)$ be the cumulative distribution function of Y

$$\begin{aligned}
F_Y(y) &= P(Y \leq y) \\
&= P(KX^2 \leq y) \\
&= P(X \leq \sqrt{\frac{y}{K}}) \\
&= 1 - e^{-K\left(\sqrt{y/K}\right)^2} \\
&= 1 - e^{-K(y/K)} \\
&= \boxed{1 - e^{-y}}
\end{aligned}$$

let $f_Y(x)$ be the probability density function of Y

$$\begin{aligned}
f_Y(x) &= \frac{d}{dy}\left(F_Y(y)\right) \\
&= \frac{d}{dy}\left(1 - e^{-y}\right) \\
&= (-1)(-e^{-y}) \\
&= e^{-y}
\end{aligned}$$

$$\boxed{f_Y(x) = \begin{cases} e^{-y} & \text{if } y > 0 \\ 0 & \text{otherwise} \end{cases}}$$

Problem 3

1.

mean = $E[X]$

$$
\begin{aligned}
E[X] &= \int_{-\infty}^{\infty} x f(x) dx \\
&= \int_{0}^{\infty} x \left(\lambda e^{-\lambda x}\right) dx \\
&= \lambda \int_{0}^{\infty} x e^{-\lambda x} dx \\
&\text{/* see calculations in next page */} \\
&= \lambda (I_6) \\
&= \lambda \left(\frac{1}{\lambda^2}\right) \\
&= \boxed{\frac{1}{\lambda}}
\end{aligned}
$$

$$
\begin{aligned}
I_6 &:= \int_0^\infty xe^{-\lambda x}dx \\
u &:= x; \qquad du = dx; \qquad dv := e^{-\lambda x}dx \\
v &= \int dv = \int e^{-\lambda x}dx \\
u_2 &:= -\lambda x; \qquad du_2 = -\lambda dx; \qquad dx = \frac{du_2}{-\lambda} \\
v &= -\frac{1}{\lambda}e^{-\lambda x} \\
\int udv &= uv - \int vdu \\
&= x\left(-\frac{1}{\lambda}e^{-\lambda x}\right) + \int \left(-\frac{1}{\lambda}e^{-\lambda x}\right)dx \\
&= -\frac{1}{\lambda}xe^{-\lambda x} - \frac{1}{\lambda}\int e^{-\lambda x}dx \\
u &= -\lambda x; \qquad du = -\lambda dx; \qquad dx = \frac{du}{-\lambda} \\
I_6 &= \left[-\frac{1}{\lambda}xe^{-\lambda x}\right]_0^\infty + \left(\frac{1}{\lambda^2}\right)\left[e^{-\lambda x}\right]_0^\infty \\
&= \left(-\frac{1}{\lambda}\right)(0-0) + \left(\frac{1}{\lambda^2}\right)(0-1) \\
&= \frac{1}{\lambda^2}
\end{aligned}
$$

2.
variance = $Var(X)$

$$Var(X) = E[X^2] - E[X]^2$$

$$\begin{aligned}
E[g(X)] &= \int_{-\infty}^{\infty} g(x) f_X(x) dx \\
E[X^2] &= \int_{-\infty}^{\infty} x^2 f(x) dx \\
&= \int_{0}^{\infty} x^2 \left(\lambda e^{-\lambda x}\right) dx \\
&= \lambda \int_{0}^{\infty} x^2 e^{-\lambda x} dx \\
&\text{/* see calculations in next page */} \\
&= \lambda (I_7) \\
&= \lambda \left(\frac{2}{\lambda^3}\right) \\
&= \frac{2}{\lambda^2}
\end{aligned}$$

$$\begin{aligned}
Var(X) &= E[X^2] - E[X]^2 \\
&= \frac{2}{\lambda^2} - \left(\frac{1}{\lambda}\right)^2 \\
&= \frac{2}{\lambda^2} - \frac{1}{\lambda^2} \\
&= \boxed{\frac{1}{\lambda^2}}
\end{aligned}$$

$$I_7 := \int_0^\infty x^2 e^{-\lambda x} dx$$

$$u := x^2; \qquad du = 2x dx; \qquad dv := e^{-\lambda x} dx$$

$$v = \int dv = \int e^{-\lambda x} dx$$

$$u_2 := -\lambda x; \qquad du_2 = -\lambda dx; \qquad dx = \frac{du_2}{-\lambda}$$

$$v = -\frac{1}{\lambda} e^{-\lambda x}$$

$$\begin{aligned}
\int u dv &= uv - \int v du \\
&= x^2 \left(-\frac{1}{\lambda} e^{-\lambda x} \right) + \int \left(-\frac{1}{\lambda} e^{-\lambda x} \right) (2x dx) \\
&= -\frac{1}{\lambda} x^2 e^{-\lambda x} - \frac{2}{\lambda} \int x e^{-\lambda x} dx \\
&\qquad \text{/* from previous calculation of } I_6\text{, we know} \\
&\qquad \int x e^{-\lambda x} dx = -\frac{1}{\lambda} x e^{-\lambda x} + \frac{1}{\lambda^2} e^{-\lambda x} \text{ */} \\
&= -\frac{1}{\lambda} x^2 e^{-\lambda x} - \frac{2}{\lambda} \left(-\frac{1}{\lambda} x e^{-\lambda x} + \frac{1}{\lambda^2} e^{-\lambda x} \right) \\
&= -\frac{1}{\lambda} x^2 e^{-\lambda x} + \frac{2}{\lambda^2} x e^{-\lambda x} - \frac{2}{\lambda^3} e^{-\lambda x} \\
&= e^{-\lambda x} \left(-\frac{1}{\lambda} x^2 + \frac{2}{\lambda^2} x - \frac{2}{\lambda^3} \right) \\
I_7 &= \left[e^{-\lambda x} \left(-\frac{1}{\lambda} x^2 + \frac{2}{\lambda^2} x - \frac{2}{\lambda^3} \right) \right]_0^\infty \\
&= \left(0 - (1) \left(-0 + 0 - \frac{2}{\lambda^3} \right) \right) \\
&= \frac{2}{\lambda^3}
\end{aligned}$$

3.
part 1

$$
\begin{aligned}
a &:= x \\
F(a) &= P(X \leq a) \\
&= \int_{-\infty}^{a} f(x)dx \\
&= \int_{0}^{a} \left(\lambda e^{-\lambda x}\right) dx \\
&= \lambda \int_{0}^{a} e^{-\lambda x} dx \\
& u := -\lambda x; \qquad du = -\lambda dx; \qquad dx = \frac{du}{-\lambda} \\
&= - \left[e^{-\lambda x}\right]_{0}^{a} \\
&= - \left(e^{-\lambda a} - 1\right) \\
&= 1 - e^{-\lambda a} \\
&= 1 - e^{-\lambda x}
\end{aligned}
$$

$$
\boxed{F(x) = \begin{cases} 1 - e^{-\lambda x} & \text{if } x \geq 0 \\ 0 & \text{otherwise} \end{cases}}
$$

complementary CDF:

$$
\begin{aligned}
P(X > x) &= 1 - (1 - e^{-\lambda x}) \\
&= e^{-\lambda x}
\end{aligned}
$$

part 2

$$
\begin{aligned}
P(X > s + t | X > t) &= \frac{P((X > s + t) \cap (X > t))}{P(X > t)} \\
&\text{/* since } s > 0,\ t > 0 \text{ */} \\
&= \frac{P(X > s + t)}{P(X > t)} \\
&= \frac{1 - P(X \leq s + t)}{1 - P(X \leq t)} \\
&= \frac{1 - (1 - e^{-\lambda(s+t))}}{1 - (1 - e^{-\lambda t})} \\
&= \frac{e^{-\lambda(s+t)}}{e^{-\lambda t}} \\
&= e^{-\lambda s} \\
&= P(X > s)
\end{aligned}
$$

Problem 4

X is the random variable corresponding to the time of collapse (in million years)

1.

$$\begin{aligned}
P(X \leq 1) &= 0.00000002 \\
&= 1 - e^{-\lambda(1)} \\
&= 1 - e^{-\lambda} \\
e^{-\lambda} &= 1 - 0.00000002 = 0.99999998 \\
-\lambda &= \ln(0.99999998) \\
\lambda &= -\ln(0.99999998) \\
\lambda &= \boxed{2.00000002 \cdot 10^{-8}}
\end{aligned}$$

2.

$$\begin{aligned}
E[X] &= \frac{1}{\lambda} \\
&= \frac{1}{2.00000002 \cdot 10^{-8}} \\
&= \boxed{49999999.5}
\end{aligned}$$

3.

$$\begin{aligned}
billion &= 10^9 \\
million &= 10^6 \\
1\ billion &= (10^3)(10^6) = 1000\ million \\
3\ billion &= 3000\ million \\
P(X \leq 3000) &= 1 - e^{-\lambda(3000)} \\
&= 1 - e^{-3000\lambda} \\
&= 1 - e^{-3000(2.00000002 \cdot 10^{-8})} \\
&= 1 - 0.99994000179 \\
&= \boxed{0.00005999821}
\end{aligned}$$

4.

$$\begin{aligned}
10\ billion &= 10000\ million \\
P(X > 10000) &= e^{-\lambda(10000)} \\
&= e^{-(10000)\lambda} \\
&= e^{-(10000)(2.00000002 \cdot 10^{-8})} \\
&= \boxed{0.99980001999}
\end{aligned}$$

Problem 5

let A be the event: "a machine has a failure within the first 5 years of operation"

let B be the event: "a machine goes permanently out of order"

$$\begin{aligned} P(A) &= 0.3 \\ P(A^C) &= 1 - 0.3 = 0.7 \\ P(B|A) &= 0.75 \\ P(B|A^C) &= 0.4 \end{aligned}$$

1.

$$\begin{aligned} P(B) &= \sum_i P(B|A_i)P(A_i) \\ &= P(B|A)P(A) + P(B|A^C)P(A^C) \\ &= (0.75)(0.3) + (0.4)(0.7) \\ &= \boxed{0.505} \end{aligned}$$

2.

$$\begin{aligned} P(A^C|B) &= \frac{P(A^C \cap B)}{P(B)} \\ P(A^C \cap B) &= P(B \cap A^C) = P(B|A^C)P(A^C) \\ P(A^C|B) &= \frac{P(B|A^C)P(A^C)}{P(B)} \\ &= \frac{(0.4)(0.7)}{0.505} \\ &= \boxed{0.55445544554} \end{aligned}$$

3.
let $p_X(a)$ be the probability mass function for X
$p := P(A) = 0.3$
$1 - p = 1 - 0.3 = 0.7$

$$\binom{10}{a}(1-p)^{10-a}p^a = \binom{10}{a}(0.7)^{10-a}(0.3)^a$$

$$p_X(a) = \begin{cases} \binom{10}{a}(0.7)^{10-a}(0.3)^a & \text{if } a \in [0, 10] \cap \mathbb{N} \\ 0 & \text{otherwise} \end{cases}$$

4.
part 1
expected value $= E[X]$
let n be the number of trials

$$\begin{aligned} E[X] &= \sum_i a_i P(X = a_i) \\ &= \sum_{a=0}^{10} a \binom{10}{a}(0.7)^{10-a}(0.3)^a \end{aligned}$$

/* see calculations in next page */

$$= \boxed{3}$$

alternatively:

$$\begin{aligned} E[X] &= np \\ &= 10(0.3) \\ &= \boxed{3} \end{aligned}$$

```
/* calculation of sum_{a=0}^{10} a C(10,a) (0.7)^(10-a) (0.3)^a */

#include <stdio.h>
#include <math.h>

int fac(int n)
{
    int ret = 1;
    for(int i = 2; i <= n; i++)
        ret = ret * i;
    return ret;
}

int ncr(int n, int r)
{
    return fac(n) / (fac(r) * fac(n - r));
}

void exp_x()
{
    double sum = 0;
    for(int a = 0; a <= 10; a++)
    {
        sum += a * ncr(10, a)
                    * pow(0.7, 10 - a) * pow(0.3, a);
    }
    printf("E[X] = %0.10f\n", sum);
}

int main()
{
    exp_x();
    return 0;
}
```

part 2
variance = $Var(X)$

$$Var(X) = E[X^2] - E[X]^2$$

$$\begin{aligned}
E[g(X)] &= \sum_i g(a_i)P(X = a_i) \\
E[X^2] &= \sum_i a_i^2 P(X = a_i) \\
&= \sum_{a=0}^{10} a^2 \binom{10}{a} (0.7)^{10-a}(0.3)^a \\
&\text{/* see calculations below */} \\
&= 11.1
\end{aligned}$$

```
/* the calculation of ∑_{a=0}^{10} a^2 (10 choose a)(0.7)^{10−a}(0.3)^a
uses the exp_x_squared() function,
which calls the same fac()
and ncr() described in part 1*/

void exp_x_squared()
{
    double sum = 0;
    for(int a = 0; a <= 10; a++)
    {
        sum += a * a * ncr(10, a)
                 * pow(0.7, 10 - a) * pow(0.3, a);
    }
    printf("E[X^2] = %0.10f\n", sum);
}
```

$$
\begin{aligned}
Var(X) &= E[X^2] - E[X]^2 \\
&= 11.1 - (3)^2 \\
&= 11.1 - 9 \\
&= \boxed{2.1}
\end{aligned}
$$

$$
\begin{aligned}
&\text{alternatively:} \\
Var(X) &= np(1-p) \\
&= (10)(0.3)(1-0.3) \\
&= (3)(0.7) \\
&= \boxed{2.1}
\end{aligned}
$$