CSCI-UA.0310-005 Midterm

Xi Liu

TOTAL POINTS

80 / 100

QUESTION 1

1 Problem 1 **25 / 30**

+ **30 pts** Correct
✓ + **25 pts** **Mistake in one part**
+ **20 pts** Mistake in two parts
+ **15 pts** Mistake in three parts

c is incorrect.

QUESTION 2

2 Problem 2 **30 / 35**

+ **35 pts** Correct
✓ + **30 pts** **Mistake in one part**
+ **25 pts** Mistake in two parts
+ **0 pts** No answer

Note that we must not recursively call the function on BOTH left and right subarrays in part c, since it gives a linear time algorithm.

QUESTION 3

3 Problem 3 **25 / 35**

+ **35 pts** Correct
+ **30 pts** Mistake in the recursion
✓ + **25 pts** **Major mistake in the recursion**
+ **17 pts** The recursion is not well-defined
+ **0 pts** No answer

T(n)=max(T(n-1), C[n]+T(n-k-1)) for n > k+1.

Xi Liu, xl3504, midterm, N15017945

“I understand the ground rules and agree to abide by them. I will not share ideas, solutions, or assist

another student during this exam, nor will I seek assistance from another student or attempt to view

their ideas or solutions.”

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Problem 1

(a)

$f(n) = O(g(n))$

since polynomial functions are asymptotically greater than logarithm functions

$\lim_{n \to \infty} n^5 (\log n)^{50} / (n^{5.1} (\log n)) = 0$

which means the function at the denominator ($n^{5.1} (\log n)$) is asymptotically greater than the function at the numerator ($n^5 (\log n)^{50}$).

(b)

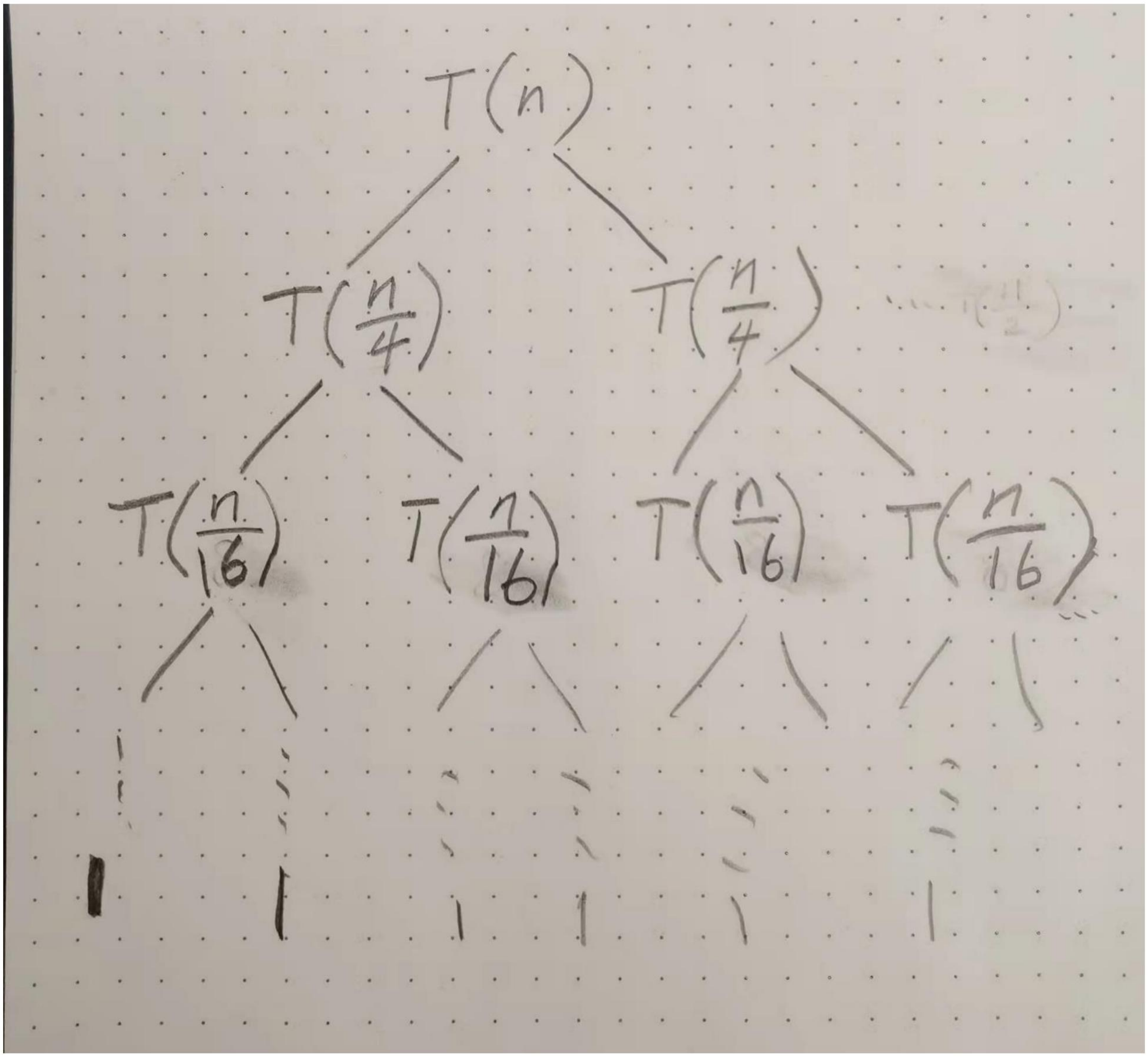
$f(n) = \Omega(g(n))$

since exponential functions (with a bigger exponent) are asymptotically greater than polynomial functions

$\lim_{n \to \infty} 5^n / (125^{\sqrt{n}} + n^2) = \infty$

which means the function at the denominator ($125^{\sqrt{n}} + n^2$) is asymptotically less than the function at the numerator (5^n).

(c)



cost at depth i is equal to n / 2^i

number of levels = log_2 n

T(n) = sum_{i = 0}^{n - 1} (n / 2^i)

= n * sum_{i = 0}^{n - 1} (1 / 2^i)

= Theta(n / 2^n)

1 Problem 1 **25 / 30**

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c is incorrect.

Problem 2

(a)

initially left = 1, right = A.length

calculate mid = (left + right) / 2

recursively call pos_after_neg(A, temp, left, mid) and pos_after_neg(A, temp, mid + 1, right), array A is divided into A[left…mid] and A[mid+1…right], then copy the negative elements of A[left…mid] and A[mid+1…right] to array temp, then copy the positive elements of A[left…mid] and A[mid+1…right] to array temp. finally copy the elements in array temp back to A

```
def merge(a, temp, left, mid, right):
    t_i = left
    for i in range(left, mid + 1):
        # negative from left
        if a[i] < 0:
            temp[t_i] = a[i]
            t_i += 1

    for i in range(mid + 1, right + 1):
        # negative from right
        if a[i] < 0:
            temp[t_i] = a[i]
            t_i += 1

    for i in range(left, mid + 1):
        # positive from left
        if a[i] >= 0:
            temp[t_i] = a[i]
            t_i += 1
```

```
    for i in range(mid + 1, right + 1):
        # positive from right
        if a[i] >= 0:
            temp[t_i] = a[i]
            t_i += 1

    for i in range(left, right + 1):
        a[i] = temp[i]

def pos_after_neg(a, temp, left, right):
    if left < right:
        mid = (left + right) // 2 # integer division
        pos_after_neg(a, temp, left, mid)
        pos_after_neg(a, temp, mid + 1, right)
        merge(a, temp, left, mid, right)
```

(b)

use i and j as 2 pointers, i initially holds the index of the beginning of the array, j initially holds the index of the end of the array.

iteratively find the positive numbers at a[i] and negative numbers at a[j],

if a positive number is encountered at a[i], swap a[i] with the element with current a[j], then i = i + 1, j = j - 1

(c)

initially left = 1, right = A.length

calculate mid = (left + right) / 2

recursively call the function with A[left…mid] and A[mid+1…right],

if A[left] > 0, return A[left]

test if A[left] <= 0 and A[mid] <= 0, everything is negative or 0 in between, then exit out of recursion

if A[left] <= 0 and A[mid] > 0, recursively call the function again if A[mid + 1] <= 0 and A[right] > 0, recursively call the function again

if A[mid + 1] <= 0 and A[right] <= 0, then there is no positive number in the array

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Note that we must not recursively call the function on BOTH left and right subarrays in part c, since it gives a linear time algorithm.

Problem 3

(a)

base case: T(0) = 0

number of cookies from 0 boxes is 0

```
T(n) =
{
	max_{k <= i <= n} (c[i] + T(n - i))
}
```

boxes must be at least k distance apart, so i >= k, once chosen the box c[i], the remaining amount of box is n – i, so need to add c[i] with the optimal solution of the subproblem with size n – i

(b)

```
int memo[n];
for(int i = 1; i < n; ++i)
{
	memo[i] = c[i];
	for(int j = 1; j < i; ++j)
	{
		if(i – j >= k) /* boxes must be more than k distance apart */
			memo[i] = max(memo[i], c[j] + memo[i - j]);
	}
}
return memo[n];
```

(c)

time complexity = Theta(n^2)

2 levels of for loop

operations within the innermost level are constant, values in c and memo are readily available to be fetched

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T(n)=max(T(n-1), C[n]+T(n-k-1)) for n > k+1.