liu xi

multi choice
1
lec 12. 8
median of absolute deviation

$$mad = med(|x_i - med_n|, ..., \text{ for } i = 1 \text{ to } n)$$

$$\begin{aligned}
sort &= 38, 56, 59, 64, 74 \\
med_n &= 59 \\
|x_i - med| &= 21, 3, 0, 5, 15 \\
sort &= 0, 3, 5, 15, 21 \\
med &= 5
\end{aligned}$$

2

$$
\begin{aligned}
L(a) &= \prod_{i=1}^{4} f(x = x_i) \\
&= \frac{a^4}{4^a 2^a 5^a 3^a} \\
l(a) &= \ln(L(a)) \\
&= \ln\left(a^4\right) - \ln(4^a) - \ln(2^a) - \ln(5^a) - \ln(3^a) \\
&= 4\ln(a) - a\ln(4) - a\ln(2) - a\ln(5) - a\ln(3) \\
&= 4\ln(a) - a(\ln(4) + \ln(2) + \ln(5) + \ln(3)) \\
&= 4\ln(a) - a(\ln(120)) \\
\frac{dl}{da} &= \frac{4}{a} - \ln(120) = 0 \\
\frac{4}{a} &= \ln(120) \\
a &= \frac{4}{\ln(120)}
\end{aligned}
$$

3

$$
\begin{aligned}
mse(\hat{\theta}) &= var(\hat{\theta}) + (E[\hat{\theta}] - \theta)^2 \\
X or X_i &\sim uniform(0, \theta) \\
&\text{see formula page} \\
E[X] &= \frac{a+b}{2} = \frac{0+\theta}{2} = \frac{\theta}{2} \\
var(X) &= \frac{(b-a)^2}{12} = \frac{(\theta-0)^2}{12} = \frac{\theta^2}{12} \\
E[\overline{X_n}] &= E\left[\frac{\sum_{i=1}^{n} X_i}{n}\right] = \frac{1}{n} n E[X] = E[X]
\end{aligned}
$$

$$E[\hat{\theta}] = E[2\overline{X_n}] = 2E[X] = 2(\theta/2) = \theta$$

$$\begin{aligned} var(\hat{\theta}) &= var(2\overline{X_n}) = 4var(\overline{X_n}) = 4var\left(\frac{\sum_{i=1}^n X_i}{n}\right) \\ &= \frac{4}{n^2}var(\sum_{i=1}^n X_i) = \frac{4n}{n^2}var(X_i) = \frac{4}{n}var(X) \\ &= \frac{4}{n} \cdot \frac{\theta^2}{12} = \frac{\theta^2}{3n} \end{aligned}$$

$$\begin{aligned} mse(\hat{\theta}) &= var(\hat{\theta}) + (E[\hat{\theta}] - \theta)^2 \\ &= \frac{\theta^2}{3n} - (\theta - \theta)^2 \\ &= \frac{\theta^2}{3n} \end{aligned}$$

4

$$\begin{aligned} P(X \leq 2 | Y \leq 2) &= \frac{P(X \leq 2, Y \leq 2)}{P(Y \leq 2)} \\ &= \frac{1/4}{1/2} \\ &= \frac{1}{2} \end{aligned}$$

true false
1
true

$$
\begin{aligned}
F_{X,Y}(x,y) &= \int_0^a \int_0^b 12x^2y^3 dydx \\
&= 12\int_0^a \int_0^b x^2y^3 dydx \\
&= 12\int_0^a \left[x^2\frac{y^4}{4}\right]_{y=0}^{y=b} dx \\
&= 3\int_0^a x^2b^4 dx \\
&= a^3b^4
\end{aligned}
$$

$$
\begin{aligned}
f_X(x) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy \\
&= \int_0^1 12x^2y^3dy \\
&= 12x^2[y^4/4]_0^1 \\
&= 3x^2 \\
f_Y(y) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx \\
&= \int_0^1 12x^2y^3dx \\
&= 12[x^3/3]_0^1y^3 \\
&= 4y^3
\end{aligned}
$$

$$
f_X(x)f_Y(y) = (3x^2)(4y^3) = 12x^2y^3 = f_{X,Y}(x,y)
$$

2

true

$$\begin{aligned}
f_X(x) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy \\
&= \int_{0}^{\infty} 6e^{-(3x+2y)}dy \\
&= -3[e^{-(3x+2y)}]_{y=0}^{y=\infty} \\
&= -3(\frac{1}{e^{\infty}} - \frac{1}{e^{3x}}) \\
&= 3e^{-3x}
\end{aligned}$$

$$\begin{aligned}
f_Y(y) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx \\
&= \int_{0}^{\infty} 6e^{-(3x+2y)}dx \\
&= -2[e^{-(3x+2y)}]_{x=0}^{x=\infty} \\
&= -2(\frac{1}{e^{\infty}} - \frac{1}{e^{2y}}) \\
&= 2e^{-2y}
\end{aligned}$$

$$f_X(x)f_Y(y) = (3e^{-3x})(2e^{-2y}) = 6e^{-(3x+2y)} = f_{X,Y}(x,y)$$