Probability and Statistics – Problem Set 7

March 24, 2022
March 31, 2022 in class

Problem 1

Consider two discrete random variables X and Y with joint probability mass function as given in the table below.

		x		
		1	2	4
y	1	$\frac{1}{3}$	$\frac{1}{6}$	$\frac{1}{12}$
	2	$\frac{1}{12}$	0	$\frac{1}{3}$

Let $Z = X - 2Y$.

1. Find the probability mass function of Z
2. Compute $E[Z]$
3. Calculate $P(X = 2|Z = 0)$.

Problem 2

Let us assume that the number N of children in a given family follows a Poisson distribution with parameter λ. Let us also assume that for each birth, the probability that the child is a girl is $p \in [0, 1]$, and that the probability that the child is a boy is $q = 1 - p$. Let us finally assume that the genders of the successive births are independent from one another.

Let X be the discrete random variable corresponding to the number of girls per family. Give the joint probability mass function of N and X.

Problem 3

Consider two dice which each only have the numbers one, two and three on their faces, such that each number appears on two faces. One rolls these dice, and let X be the face value of the first dice, and Y be the face value of the second dice. We define $W = X + Y$ and $Z = X - Y$.

1. Compute the joint probability mass function of W and Z.
2. Are W and Z independent?
3. Compute $E[W]$ and $E[Z]$.

Problem 4

Let X and Y be two continuous random variables with the following joint probability density function:

$$f_{X,Y}(x,y) = \begin{cases} abe^{-ax-by} & \text{if } x \geq 0\ ,\ y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

1. Determine the marginal probability density functions f_X and f_Y.
2. What is the expected value of X? What is the expected value of Y?
3. What is the probability that $X < Y$?

Problem 5

Two continuous random variables X and Y have the following joint probability density function:

$$f_{X,Y}(x,y) = \begin{cases} e^{-y} & \text{if } 0 < x < y < \infty \\ 0 & \text{otherwise} \end{cases}$$

Are X and Y independent?

Remember to justify your answers!