Probability and Statistics – Problem Set 9

April 7, 2022
April 14, 2022 in class

Problem 1

Consider a Poisson process with rate λ. What is the probability that there are two arrivals in the time interval $(0, 2]$ and three arrivals in the time interval $(1, 4]$?

Problem 2

Let X be a Poisson distributed random variable with parameter λ and Y be a Poisson distributed random variable with parameter μ. What is the probability mass function of the random variable Z given by $Z = X + Y$, given that X and Y are assumed to be independent random variables?

Application: A room has two smoke detectors. The number X of alpha particles emitted by the first smoke detector is a random variable with Poisson distribution and parameter $\lambda = 8.392$ emissions per 10-second interval. The number Y of alpha particles emitted by the second smoke detector is a random variable with Poisson distribution and parameter $\mu = 7.854$ emissions per 10-second interval.

What is the probability that we observe 10 emissions in total in the room in any given 10-second interval?

Problem 3

Let X be a binomial distribution with parameters n and p, and k a number such that $p < k < 1$.

1. Find an upper bound for $P(X \geq kn)$ using Markov's inequality.
2. Find an upper bound for $P(X \geq kn)$ using Chebyshev's inequality.
3. Let $p = \frac{1}{2}$ and $k = \frac{3}{4}$. Which inequality gives the tighter bound?

Problem 4

You are trying to determine the melting point of a new material, of which you have a large number of samples. For each sample that you measure, you find a value close to the actual melting temperature T but corrupted with a measurement error. You decide to model this with random variables:

$$T_i = T + U_i$$

where T_i is the measured temperature in degree Kelvin, and U_i is the random measurement error.

Imagine that you know that your measurement apparatus is such that for all i, $E[U_i] = 0$ and $\mathrm{Var}(U_i) = 3$, and that the random variables T_i are independent. How many measurements do you need to make to be 90% sure that the average of the measurements is within half a degree Kelvin of T?

Problem 5

A casino offers the following game: a fair dice is rolled 100 times, and you can bet on the average of the 100 results. In order to put the odds in your favor, you decide to use your math skills to better understand the experiment. Give as accurate a lower bound as you can for the probability that the average of the 100 results is between 3.2 and 3.8?

Remember to justify your answers!