

Probability and Statistics – Problem Set 3

February 10, 2022
Due February 27, 2022 in class

Problem 1

1. Consider an urn with 12 balls, numbered between 1 and 12. One randomly draws one of the balls, and considers the following two events
A = “An even number is drawn”
B = “A multiple of 3 is drawn”
Are the events A and B independent?
2. Same question for the case for which the urn contains 13 balls.

Problem 2

A forest has three species of trees: 30% of the trees are maple trees, 50% of the trees are oak trees, and 20% of the trees are birch trees. Following a powerful storm, there is a disease outbreak; 10% of the maple trees have the disease, as do 4% of the oak trees, and 25% of the birch trees.

If you know that a given tree in the forest has a disease, what is the probability it is a maple tree? What is the probability it is an oak tree? What is the probability it is a birch tree?

Problem 3

Let A_1, A_2, \dots, A_n be n independent events in a sample space Ω , with respective probability $p_i = P(A_i)$. Give a simple expression for the probability $P(A_1 \cup A_2 \cup \dots \cup A_n)$ in terms of p_1, p_2, \dots, p_n .

Let us now apply your result in a practical setting: a robot undergoes n independent tests, which are such that for each test the probability of failure is p . What is the probability that the robot fails at least one of the tests?

Problem 4

Suppose that 5 people, including you and a friend, line up at random. Let the random variable X denote the number of people standing between you and a friend. Determine the probability mass function of X and sketch the corresponding cumulative distribution function F .

Problem 5

In a factory, computer hard drives are collected in boxes containing 40 hard drives each. A box has 6 defective hard drives, and 34 hard drives which are not defective. A quality control inspector randomly selects 3 hard drives in that box. Consider the discrete random variable X defined as the number of defective hard drives

the inspector selects. Find the probability mass function p_X of X . Sketch the corresponding cumulative distribution function F .

Remember to justify your answers!