

Probability and Statistics – Problem Set 8

March 31, 2022
April 7, 2022 in class

Problem 1

An insurance company selling homeowner's policies and auto policies releases its data. The deductibles on the homeowner's policies (variable X) and on the auto policies (variable Y) have the following joint probability mass function.

		X		
		0	100	200
Y	100	0.2	0.1	0.2
	250	0.05	0.15	0.3

Compute the covariance $\text{Cov}(X, Y)$ and the correlation coefficient $\rho(X, Y)$ of X and Y .

Problem 2

Let T be the interior of the triangle with vertices $O(0, 0)$, $I(1, 0)$, and $J(0, 1)$, and let (X, Y) be a couple of random variables corresponding to any point $P(X, Y)$ chosen at random (hence uniformly) in T .

1. What is the joint probability density function $f_{X,Y}$ of X and Y ?
2. Compute the marginal probability density functions f_X and f_Y .
3. Are X and Y independent random variables?
4. Compute $\text{Cov}(X, Y)$.

Problem 3

A bag contains n tokens, numbered from 1 to n . One draws two tokens out of the bag, without replacement. Let X be the random variable corresponding to the first number drawn, and Y be the random variable corresponding to the second number drawn.

1. Consider the random variable $Z = |X - Y|$. What is the probability mass function of Z ?
2. Compute $E[Z]$.

Problem 4

Queuing in a fast food restaurant is well represented by a Poisson process with rate $\lambda = 0.5$ customers per minute. Say we start the clock at $t = 0$.

1. Find the probability that no customer arrives in the time interval $(3, 8]$.
2. Find the probability that there is exactly one customer arriving in each of the following time intervals: $(0, 1]$, $(1, 2]$, $(2, 3]$, and $(3, 4]$.

Problem 5

Consider a Poisson process with rate λ . Compute

$$\text{Cov}(N_{t_1}, N_{t_2})$$

for times t_1 and t_2 such that $t_1, t_2 \in [0, +\infty)$ and $t_2 \geq t_1$.

Remember to justify your answers!