Basic Algorithms CSCI-UA.0310

Practice Test

Problem 1 (10+10 points)

(a) Describe a point set with n points that is the worst-case input for the Jarvis' algorithm that finds the convex hull of the given input points.
Note that the Jarvis' algorithm was discussed in the recitation lecture.

(b) Let $G = (V, E)$ be a directed graph without any cycles. How many strongly connected components can G have? Justify your answer.

Problem 2 (20 points)

Consider the array $A[1 \ldots n]$ of n non-negative integers. There is a frog on the first index of the array. In each step, if the frog is positioned on the i^{th} index, then it can hop to any of the indices $i, \ldots, i + A[i]$ (so the frog can at most hop to the index $i + A[i]$).
Develop a greedy algorithm to determine the minimum number of hops necessary so that the frog can reach the last index, i.e., the n^{th} index. You must fully explain your algorithm.
Your algorithm must return -1 if the frog cannot reach the last index.

Problem 3 (20 points)

Given a directed weighted graph G with no cycles, develop an algorithm to find the weights of all longest paths from one source vertex s to the other vertices in G in $O(|V| + |E|)$ time.
A longest path from u to v is a path of maximum weight from u to v.

Problem 4 (20 points)

Consider a directed graph $G = (V, E)$ where each edge is colored in either red or blue. Develop an $O(|V|+|E|)$ time algorithm that, given vertices u and v in G, determines if there exists a walk from u to v that uses at least one blue edge (note that this walk may have repeated vertices). Justify the correctness of your algorithm and analyze the running time.
(Hint: Try to construct a new graph using an idea similar to HW9 P4.)

Problem 5 (20 points)

Consider the n points $p_1, p_2, \ldots, p_n$ in the plane. Each point p_i has the xy-coordinates $p_i = (x_i, y_i)$. The cost of connecting two points $p_i = (x_i, y_i)$ and $p_j = (x_j, y_j)$ is defined as

$$|x_i - x_j| + |y_i - y_j|.$$

For example, the cost of connecting $p_i = (1, 5)$ and $p_j = (3, 4)$ is

$$|1 - 3| + |5 - 4| = 3.$$

We want to make all the points connected such that there is a path between any pair of points. Develop an algorithm that returns the minimum cost to make all the points connected. Your algorithm must run in $O(n^2)$ time. Explain all the steps of your algorithm.