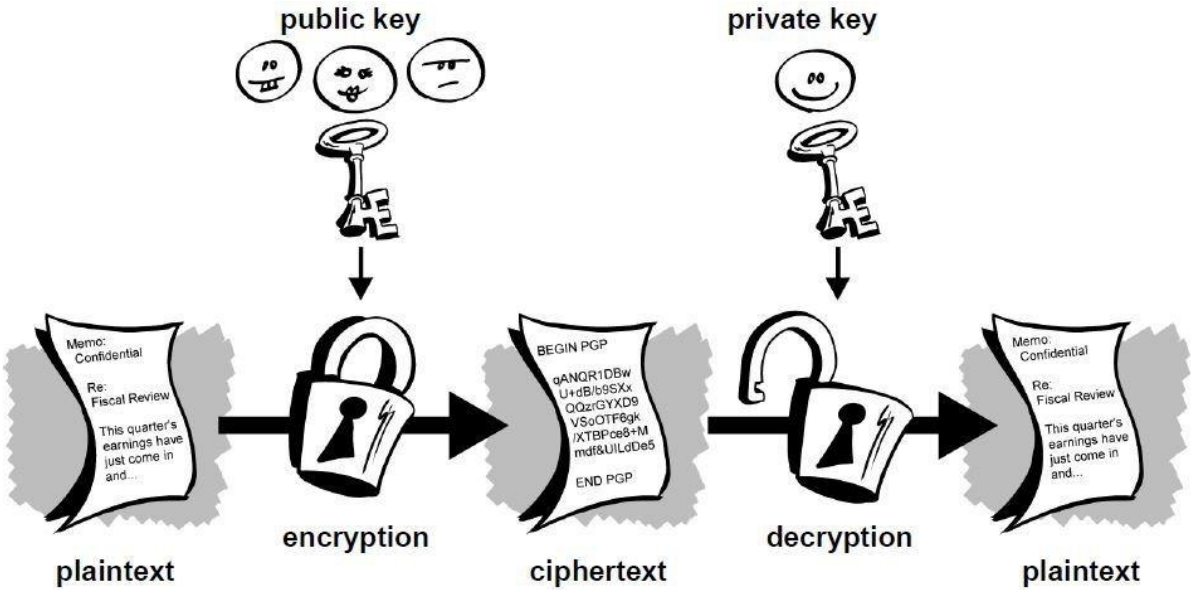
Introduction to Cryptography

By
Vipul Goyal



Some Basics

My Name: Vipul Goyal

Home page: http://www.cs.cmu.edu/~goyal/

Some resources for the class:

- Lecture notes from my (university) class at CMU: http://www.cs.cmu.edu/~goyal/s18/15503.html
- Katz and Lindell book: Introduction to Modern Cryptography
- Nice video lectures at coursera.org, lot of material on YouTube and Wikipedia

Course Basics

- Lecture sessions every week
- Mute your mic during the lectures
- Slides will be provided immediately after the lecture
- Go through the slides on your own before the next class
- If you have any questions during the lectures, please type them in the chat window
- We will take periodic breaks and I will answers the typed questions

Course Basics Contd..

- Students will be divided into groups later in the course
- Each group will take up a project. Examples:
  - Digital Signatures
  - RSA encryption
  - Bitcoin and cryptocurrencies
  - Applications of Blockchains to other areas
- At the end of the course: final presentation + final report by each group
- You will get a letter grade at the end of the course

Goal of this Course

- Life becoming more digital
  - Crypto: defines the rules of digital world
  - Imagine a society without rules

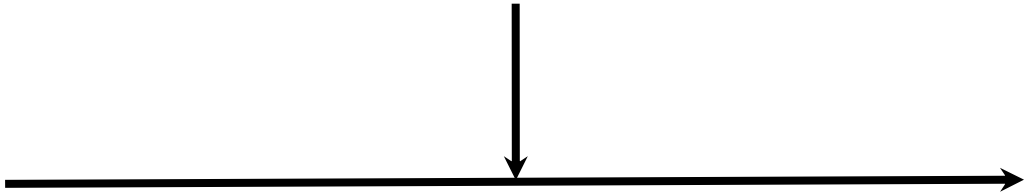
- My goal: give you an overview of the very basic concepts. Get you interested. Prepare you to dive deeper on your own.

Cryptography

- Cryptography and computer security: very fast growing topics within computer science
- Online attackers are growing in sophistication. New websites are hacked every month.
- Severe shortage of talent in this area. Software companies simply cannot find enough people to hire.
- Very active area of research as well. Exciting new things being invented all the time. Latest Revolution: Bitcoin, cryptocurrencies, ...

First goal: Secret Communication

Adversary
Eavesdropper



"My credit card number is …"

First goal: Secret Communication

Adversary
Eavesdropper

“loru23n8uladjkfb!#@”

“My credit card number is …”

encryption

“loru23n8uladjkfb!#@”

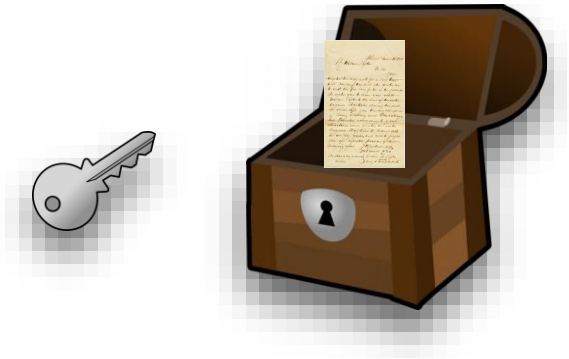
“loru23n8uladjkfb!#@”

decryption

“My credit card number is …”

Private Key Encryption (Ciphers)

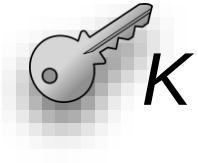
Private key cryptography



Parties must agree on a key pair beforehand.

Private key cryptography

C



K

M (plaintext)

K

K, M

Enc

C (ciphertext)

K, C

Dec

M

A more Formal Definition

A secret key encryption (SKE) consists of 3 algorithms:

1) Gen called key generation algorithm. It doesn't take any input. You can run Gen and it outputs a key K. (many times, K will just be a large number)

2) Enc called the encryption algorithm. Enc takes a message m, and, a key K as input. Enc outputs a ciphertext C

3) Dec called the decryption algorithm. Dec takes ciphertext C, and, a key K as input. Dec output the message m.

A note about security

Better to consider worst-case conditions

Assume the adversary knows everything except the key(s)

Adv might even know (part of) message (attack/defend)

Completely sees ciphertext C

Completely knows the algorithms Enc and Dec

History of Cipher Design

Is cryptography a branch of computer science?

YES and NO

- Computer Science = 50 year old
- Cryptography = 2000 year old

- Very useful in military conquests. Want to hide your strategy from the enemy.

History of Cipher Design

- A large number of ciphers designed over the last 2000 years
- Oldest recorded cipher: Caesar Cipher
- All ciphers designed prior to 1950: classical ciphers
- Every classical cipher: broken

Role of Encryption/Ciphers

- Very interesting movie: The Imitation Game (about WW2 and life of Alan Turing)
- Germans: using Enigma machine to encrypt their messages
- British tried but couldn’t break using traditional pen and paper approaches
- Turing: built a huge machine which was capable of trying thousands of keys every minute

Role of Encryption/Ciphers

- This machine: can be thought of as first ever computer

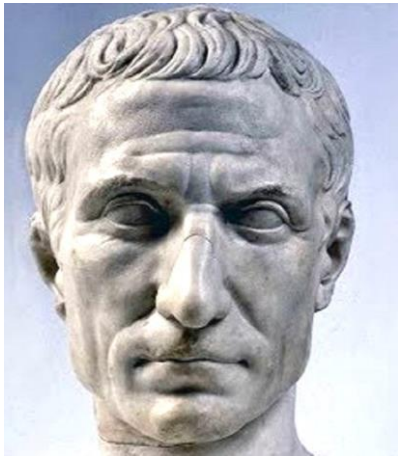
- Estimates: Breaking the ciphers resulted in war becoming shorter by 18 months. 9 million lives saved!

Some Classical Ciphers (and why they are bad)

Caesar cipher

Example: shift by 3

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | | |
d e f g h i j k l m n o p q r s t u v w x y z a b c
```



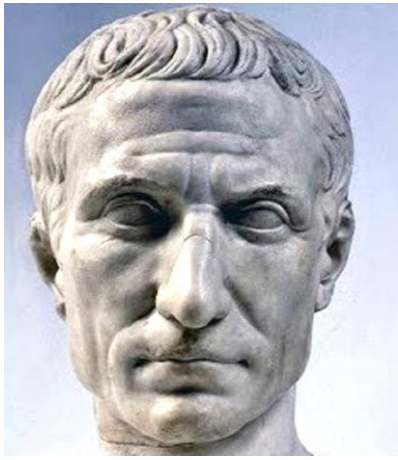
(similarly for capital letters)

“Dear Math, please grow up and solve your own problems.”

“Ghdu Pdwk, sohdvh jurz xs dqg vroyh brxu rzq sureohpv.”

Caesar cipher

Example: shift by 3



```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | |
d e f g h i j k l m n o p q r s t u v w x y z a b c
```

(similarly for capital letters)

 : the shift number

If key = 3:
To encrypt: simply shift forward each letter by 3
To decrypt: shift backward each letter by 3

Breaking Caesar Cipher

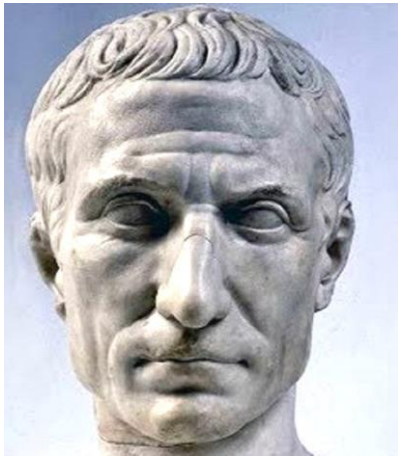
Total number of possible keys?

- Only 26

How to break:

- Adversary sees a ciphertext
- Tries every possible key
- Sees if the decrypted text makes sense

Moral of the story: keys should be large



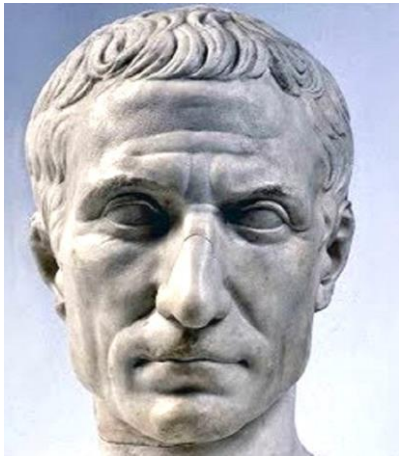
Other type of characters?

- What about space, comma, semi-colon?
- What about another language such as Chinese?

Basic principal remains the same:

- Write down all possible characters
- To encrypt, shift by a certain amount

: the shift number



Substitution cipher


: a table mapping the input letter to an output letter

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | | |
j k b d e l m c f g n o x y r s v w z a t u p q h i
```

Say:

- message = acd
- Ciphertext = jbd

Substitution cipher


: a table mapping the input letter to an output letter

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | | |
j k b d e l m c f g n o x y r s v w z a t u p q h i
```

To encrypt:
Replace each letter of the message by the corresponding letter from the table

To decrypt:
Simply read back what each letter means from the table

Substitution cipher

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | | |
j k b d e l m c f g n o x y r s v w z a t u p q h i
```

- Total number of keys: 26! (where ! = factorial function)

Why?

- First entry in table = 26 possible letters
- Second entry in table = 25 possible letters
- ….
- Total ways of constructing the table = 26.25.24……1

- This is too large! Hard to try every possible key

Breaking Substitution cipher?

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | | |
j k b d e l m c f g n o x y r s v w z a t u p q h i
```

- Still possible to break! No need to try all keys.
- Idea: frequency analysis
- Some letters in English more frequent than others
  e = 12.5 %
  t = 9.28 %
  a = 8.04 %
  …

Frequency Analysis

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | | |
j k b d e l m c f g n o x y r s v w z a t u p q h i
```

- Some letters in English more frequent than others
  e = 12.5 %
  t = 9.28 %
  a = 8.04 % …

- Now say in ciphertext, frequency of b = 12%
  => This means e was mapped to b

- Repeating this: we can learn many entries of the table

Frequency Analysis Contd..

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | | |
j k b d e l m c f g n o x y r s v w z a t u p q h i
```

- Frequency of Bigrams
  th = 3.56 %
  in = 2.43 % …

- If we see lot of rj in ciphertext
  => t mapped to r, h mapped to j

- Frequency of double letters
  ee = 0.38
  oo = 0.21

Frequency Analysis Contd..

```
a b c d e f g h i j k l m n o p q r s t u v w x y z
| | | | | | | | | | | | | | | | | | | | | | | |
j k b d e l m c f g n o x y r s v w z a t u p q h i
```

- Attack requires the adversary to collect a large amount of ciphertext. If you have encryption of a single sentence, not enough to break.

- Excellent post. Read as homework: http://norvig.com/mayzner.html

Vigenere cipher


: a random string (say BAE)

Encryption:

- Repeat key to make it equal to message length
- Shift each letter of message by the letter of the key
- Sometimes easier to write each character of the key as a number rather than letter

M = ATTACK
K = BAEBAE
C = CUYCDP

Vigenere cipher

 : a random string (say BAE)

Decryption:

- Repeat key to make it equal to ciphertext length
- Shift back each letter of message by the letter of the key

C = CUYCDP
K = BAEBAE
M = ATTACK

Vigenere cipher security?


: a random string (say BAE)

Total number of keys: 26.26.26….

$= 26^n$ (assuming key has n letters)

- Could be huge if n is large (say n = 50)
- Hard to try all possible keys!
- Secure?

Vigenere cipher (in)security


: a random string (say BAE)

Again: frequency analysis to the rescue

Step 1: guess length of the key (try again if fail). Say length = 3.

Step 2: divide ciphertext into 3 parts
CT = FWJUOIFDSLKFSFSLJ……
First part = FUFLSL

Step 3: Apply frequency analysis separately on each part

Vigenere cipher (in)security

 : a random string (say BAE)

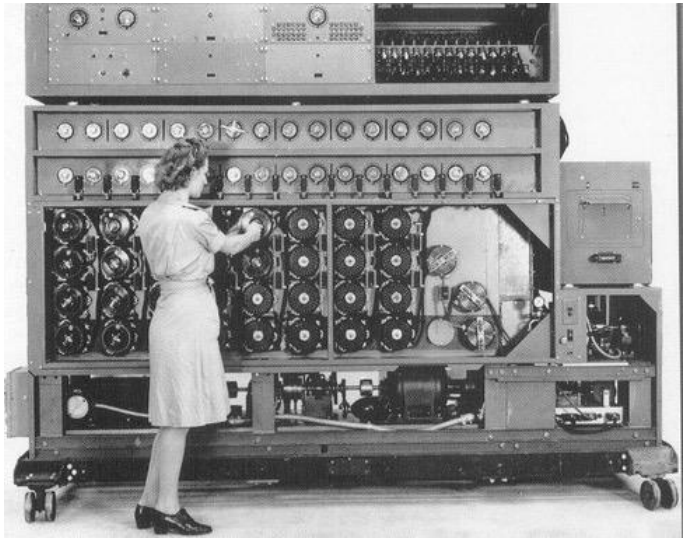
CT = FWJUOIFDSLKFSFSLJ……
First part = FUFLSL

Step 3: Apply frequency analysis separately on each part

- Observation: each letter in first part is shifted by the same letter (i.e. by B)
- Say we notice frequency of G is about 12%
- Then: E was mapped to G
- Hence: key = B (shift of 2)
- Repeat to find each letter of the key

Enigma (WW2)

A much more complex cipher: Still broken



Questions so far?