

Probability and Statistics – Problem Set 6

March 10, 2022
March 24, 2022 in class

Problem 1

Consider two discrete random variables X and Y which are independent and have the following probability mass functions:

$$\begin{cases} p_X(1) = 0.7 \\ p_X(2) = 0.3 \\ p_X(x) = 0 \text{ if } x \neq 1, x \neq 2 \end{cases} \quad \begin{cases} p_Y(-2) = 0.3 \\ p_Y(5) = 0.5 \\ p_Y(8) = 0.2 \\ p_Y(y) = 0 \text{ if } y \neq -2, y \neq 5, y \neq 8 \end{cases}$$

1. What is the joint probability mass function of X and Y ? You may use a table to represent it.
2. What is the probability that both X and Y are even?
3. What is the probability that $X = 1$ given that Y is positive?

Problem 2

In this problem, you pick two cards from a standard deck of 52 cards, and consider the following random variables:

X is the number of diamonds picked.

Y is the number of queens picked.

1. What is the joint probability mass function of X and Y ? You may use a table to represent it.
2. Construct the marginal distribution p_X of X , and the marginal distribution p_Y of Y .
3. A friend of yours saw the cards before you, and tells you that at least one of the two cards is a diamond. What is the probability that you picked at least one queen?

Problem 3

Two independently running geolocation systems are subject to failures. The number of failures per day is given by the following table.

System X		System Y	
x_i	$p_X(x_i)$	y_i	$p_Y(y_i)$
0	0.07	0	0.1
1	0.35	1	0.2
2	0.34	2	0.5
3	0.18	3	0.17
4	0.06	4	0.03

1. Give the probability of the following events:
 - Y has at least two failures per day
 - The number of failures of X is strictly less than 2, and the number of failures of Y is greater than or equal to 3
 - There is only one failure in the day
 - X has the same number of failures as Y
2. What is the joint probability mass function of X and Y ? You may use a table to represent it.
3. Compute $E[X]$ and $E[Y]$.

Problem 4

You are given the following incomplete table for the joint probability mass function of X and Y . Complete the table, given the information that for any x and y , $P(X = x, Y = y)$ is either 0 or $1/14$.

		x					$p_Y(y)$
		1	2	3	4	5	
y	1						$5/14$
	2						$4/14$
	3						$2/14$
	4						$2/14$
	5						$1/14$
$p_X(x)$		$1/14$	$5/14$	$4/14$	$2/14$	$2/14$	1

Problem 5

Consider the function of two variables $f_{X,Y}$ given by

$$f_{X,Y}(x, y) = \begin{cases} K e^{-3x-2y} & \text{if } x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}$$

where K is a real constant.

1. For which value of K can $f_{X,Y}$ be a joint probability density function?
2. Compute the corresponding joint distribution function $F_{X,Y}(x, y)$ for all possible values of x and y .
3. Compute $P(X \leq 1, Y \leq 2)$

Remember to justify your answers!