

Lecture 9

Divergence, scheduling and floating point

Announcements

- Mac Mini lab (APM 2402)
 - ◆ Friday at 4pm to 6pm
 - ◆ Next week: Tues and Fri 4pm to 6pm
- Project proposals due on November 9

Projects

- Counts for 60% of your grade
- Complete in 3 weeks
- See the (growing) list of projects at cseweb.ucsd.edu/classes/fa12/cse260-b/Projects/ProjectList.html
- CUDA, MPI or CUDA + MPI
- Select your project from the list by 11/9
 - ♦ A limited number of self-proposed projects, requires a proposal
- Progress report: 11/21 (Weds)
- Final Report: 12/7 (Friday)

Project Proposals

- Due 11/9
- ♦ What are the goals of your project? Are they realistic?
- ♦ What are your hypotheses?
- ♦ What is your experimental method for proving or disproving your hypotheses?
- ♦ What experimental result(s) do you need to demonstrate?
- ♦ What would be the significance of those results?
- ♦ What code will you need to implement? What software packages or previously written software will use?
- ♦ A tentative division of labor among the team members
- ♦ A preliminary list of milestones—with completion dates

Projects!

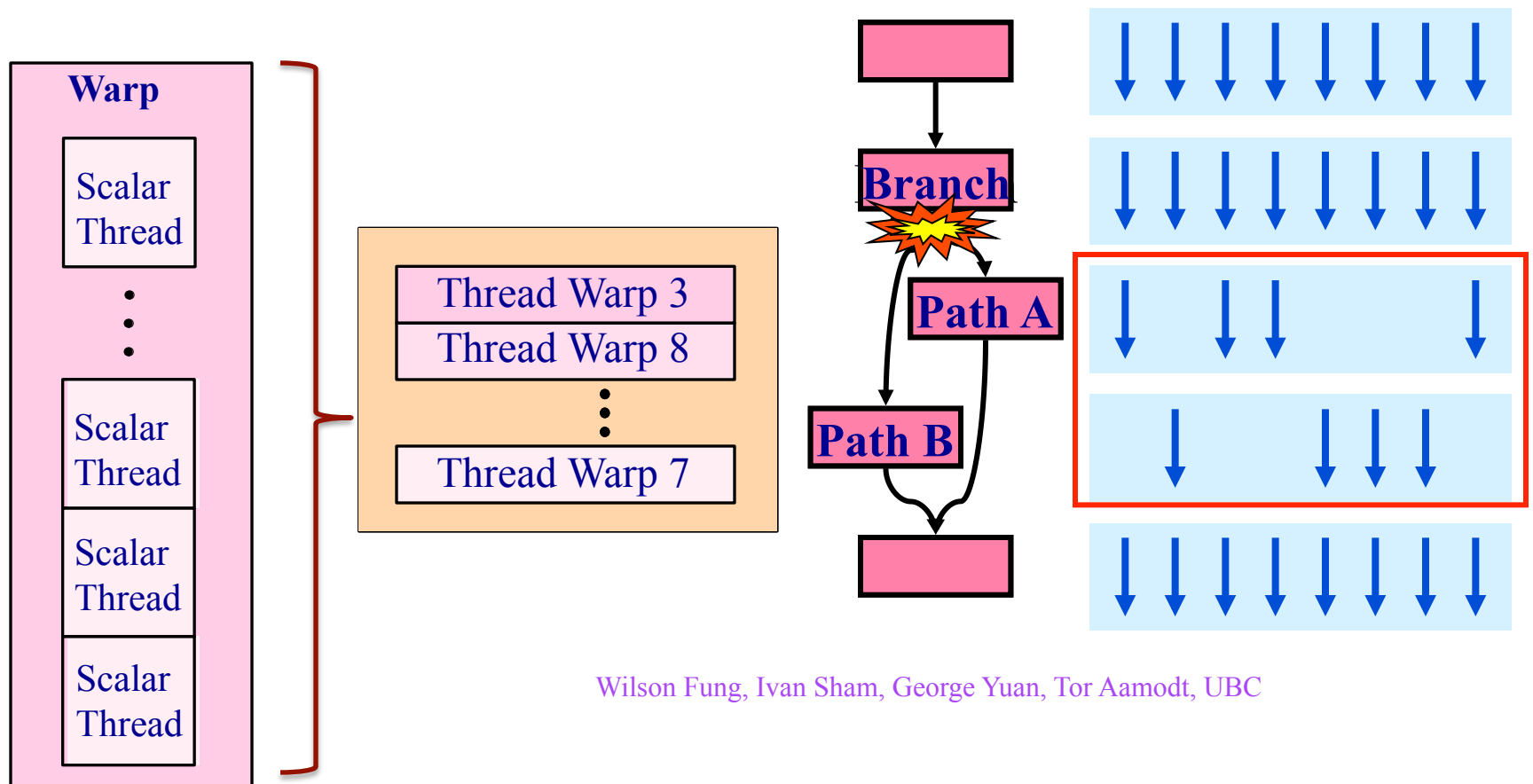
- Stencil method in 3 dimensions
- Multigrid
- Communication avoiding matrix multiplication (MPI)
- Algorithm based fault tolerance (MPI)
- 3D Fast Fourier Transform (MPI or CUDA)
- Particle simulation (MPI)
- Groups of 3 will do a more ambitious project
 - ♦ MPI projects can add communication overlap
 - ♦ MPI + CUDA
- Propose your own
- Make your choice by 11/9
www-cse.ucsd.edu/classes/fa12/cse260-b/Projects/ProjectList.html

Today's lecture

- Thread divergence
- Scheduling
- Floating Point

Thread divergence

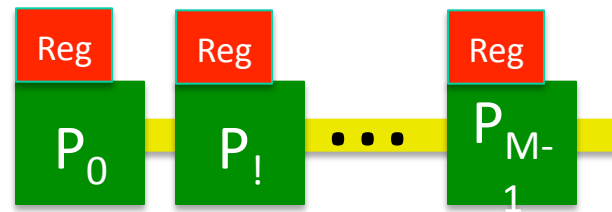
- All the threads in a warp execute the same instruction
- Different control paths are serialized



Wilson Fung, Ivan Sham, George Yuan, Tor Aamodt, UBC

Divergence example

```
if (threadIdx >= 2)
    a=100;
else
    a=-100;
```

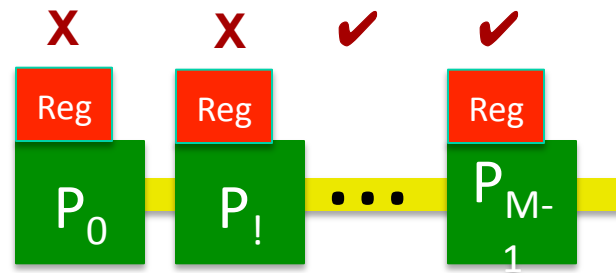


compare
threadIdx,2

Mary Hall

Divergence example

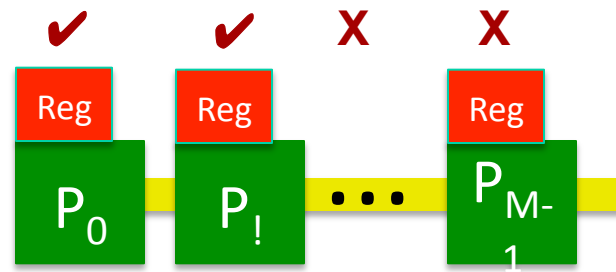
```
if (threadIdx >= 2)
    a=100;
else
    a=-100;
```



Mary Hall

Divergence example

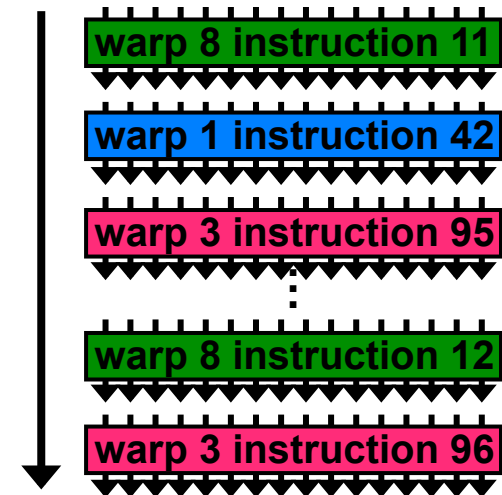
```
if (threadIdx >= 2)
    a=100;
else
    a=-100;
```



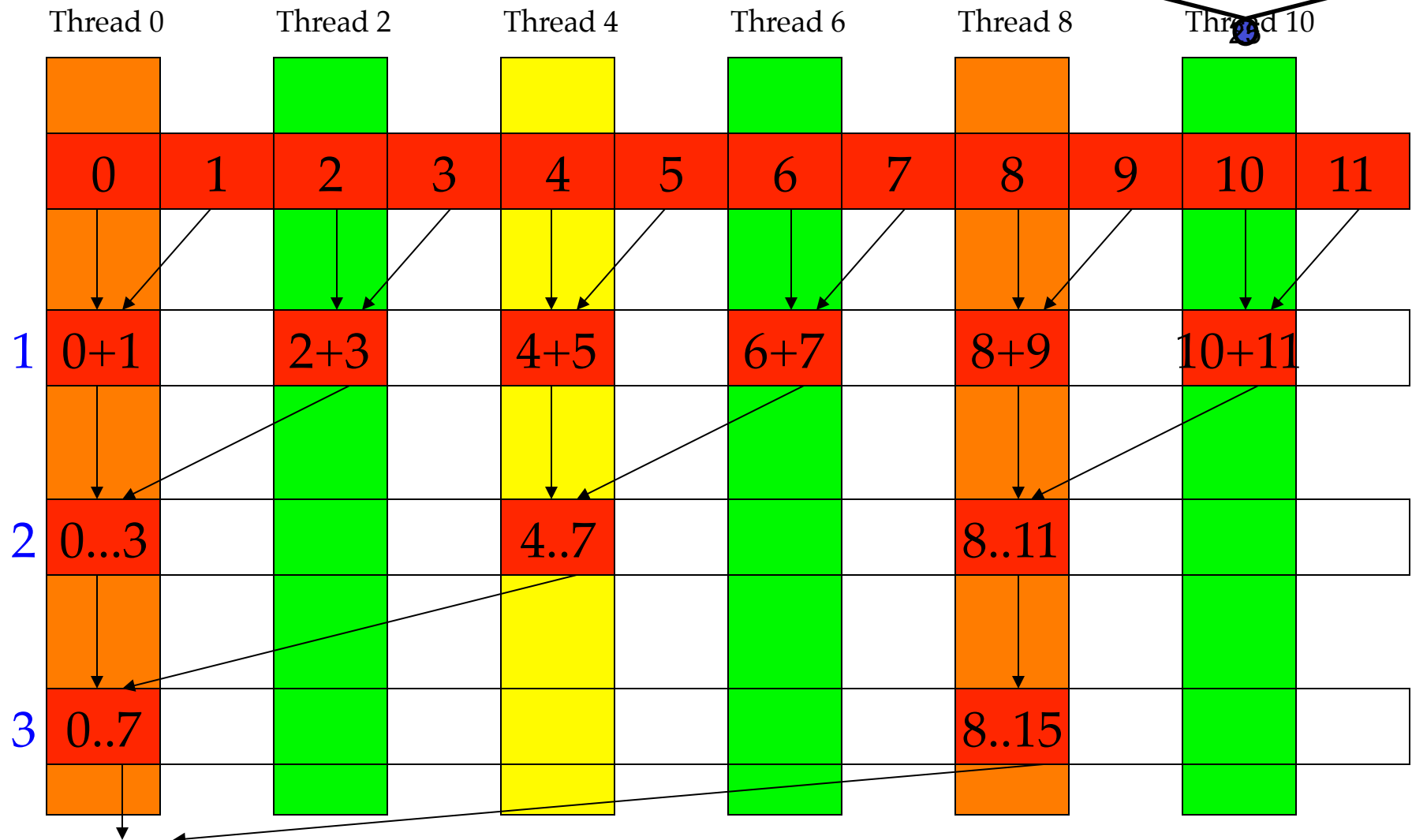
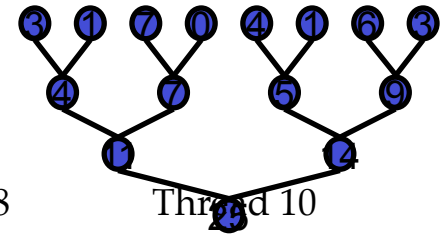
Mary Hall

Thread Divergence

- All the threads in a warp execute the same instruction
- Different control paths are serialized
- *Divergence* when a predicate is a function of the threadId
`if (threadId < 2) { }`
- No divergence if all follow the same path within a warp
`if (threadId / WARP_SIZE < 2) { }`
- We can have different control paths within the thread block



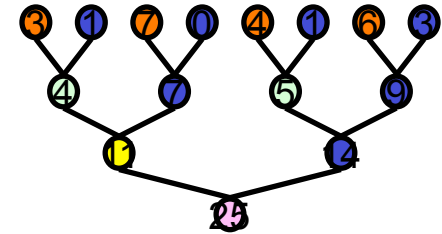
Example – reduction – thread divergence



DavidKirk/NVIDIA & Wen-mei Hwu/UIUC

The naïve code

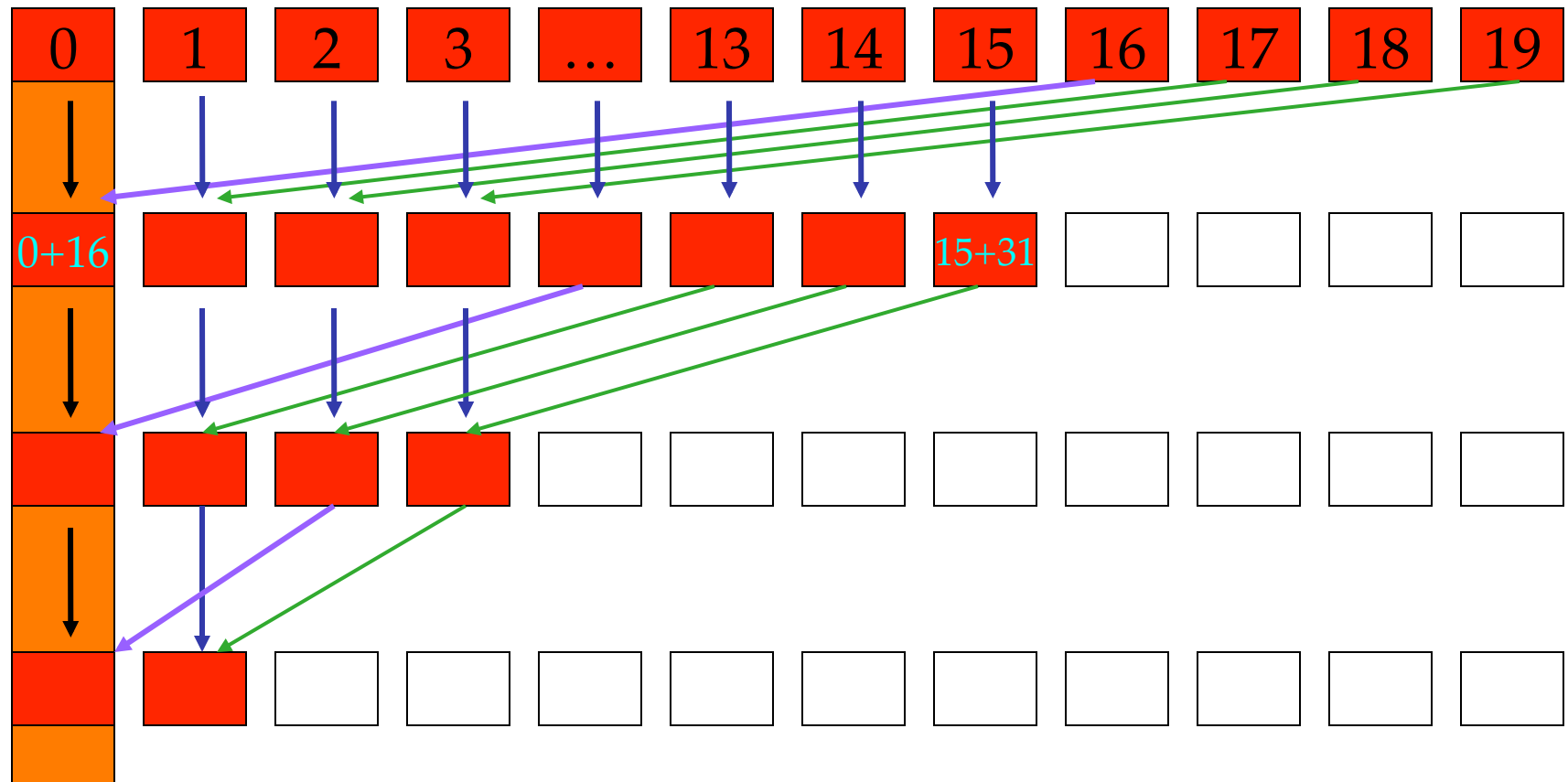
```
__global__ void reduce(int *input, unsigned int N, int *total){  
    unsigned int tid = threadIdx.x;  
    unsigned int i = blockIdx.x * blockDim.x + threadIdx.x;  
  
    __shared__ int x[BSIZE];  
    x[tid] = (i < N) ? input[i] : 0;  
    __syncthreads();
```



```
    for (unsigned int stride = 1; stride < blockDim.x; stride *= 2) {  
        __syncthreads();  
        if (tid % (2*stride) == 0)  
            x[tid] += x[tid + stride];  
    }  
  
    if (tid == 0) atomicAdd(total,x[tid]);  
}
```

Reducing divergence and avoiding bank conflicts

Thread 0



The improved code

- No divergence until $\text{stride} < 32$
- All warps active when $\text{stride} \geq 32$

```
for (stride = blockDim.x/2;  
      stride > 1;  
      stride /= 2) {  
    __syncthreads();  
  
    if (tid < stride)  
        x[tid] += x[tid + stride];  
}
```

```
for (stride = 1;  
      stride < blockDim.x;  
      stride *= 2) {  
    __syncthreads();  
  
    if (tid % (2*stride) == 0)  
        x[tid] += x[tid + stride];  
}
```

Predication on Fermi

- All instructions support predication in 2.x
- Condition code or *predicate* per thread:
set to true or false
- Execute only if the predicate is true
if (x>1)
y = 7;

test = (x>1)
test: y=7

- Compiler replaces a branch instruction with predicated instructions only if the number of instructions controlled by branch condition is not too large
- If the compiler predicts too many divergent warps....
threshold = 7, else 4

Concurrency – Host & Device

- Nonblocking operations
 - Kernel launch
 - Device \leftrightarrow {Host,Device}
 - Async memory copies
- Multiple kernel invocations: certain capability 2.x devices
- CUDA Streams (§3.2.6.5), with limitations

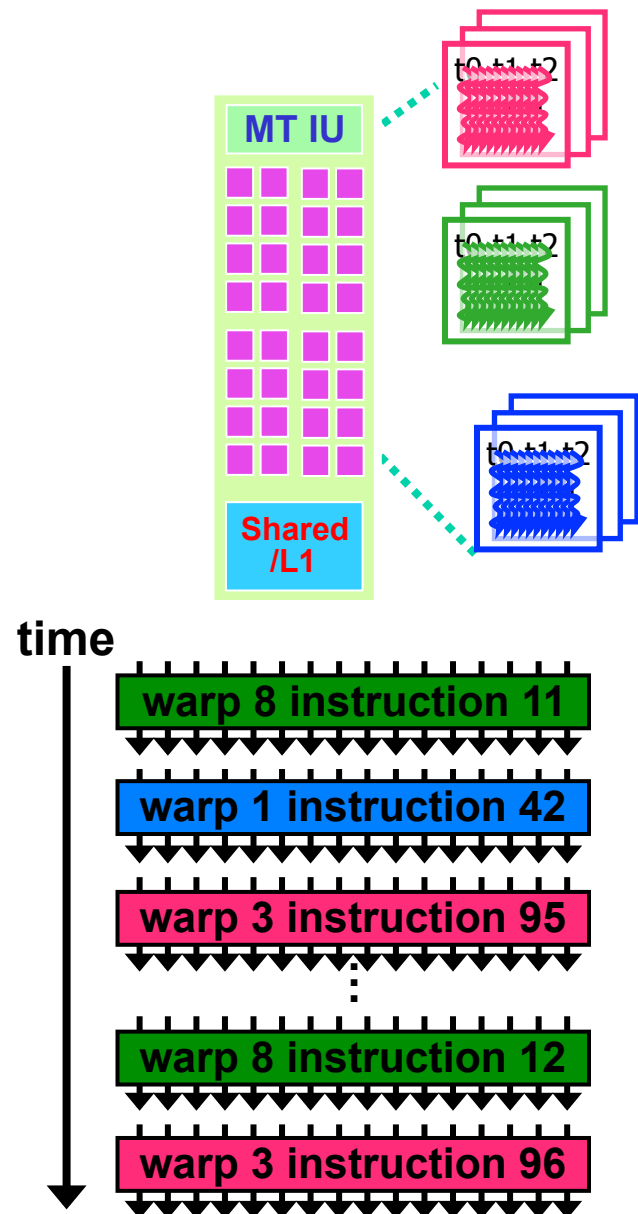
```
cudaStream_t stream[2];  
for (int i = 0; i < 2; ++i)  
    cudaStreamCreate(&stream[i]);  
for (int i = 0; i < 2; ++i)  
    Kernel<<<100, 512, 0, stream[i]>>> ( ... );
```

Today's lecture

- Thread divergence
- **Scheduling**
- Floating Point

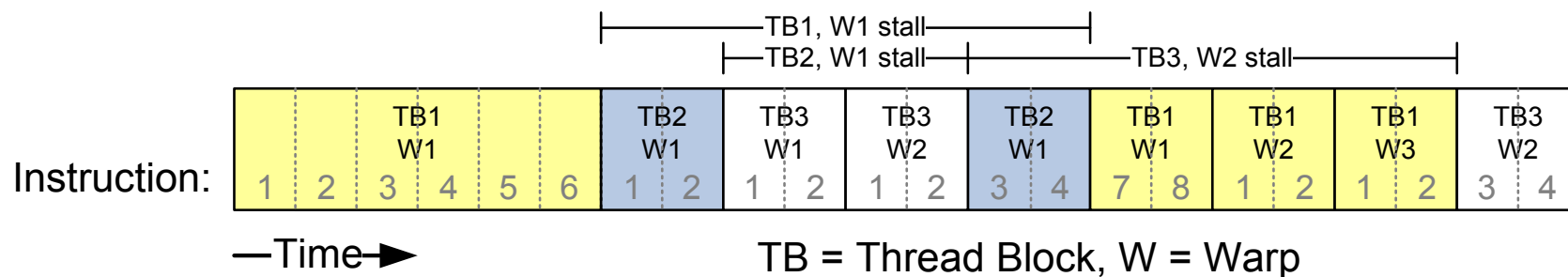
Warp scheduling on Fermi

- 2 schedulers find an eligible warp
 - ♦ Each issues 1 instruction per cycle
 - ♦ Dynamic instruction reordering: eligible warps selected for execution using a prioritized scheduling policy
 - ♦ Issue selection: round-robin/age of warp
 - ♦ Odd/even warps
 - ♦ Warp scheduler can issue instruction to $\frac{1}{2}$ the cores, each scheduler issues: 1 (2) instructions for capability 2.0 (2.1)
 - ♦ Scheduler must issue the instruction over 2 clock cycles for an integer or floating-point arithmetic instruction
 - ♦ Only 1 double prec instruction at a time
- All registers in all the warps are available, 0 overhead scheduling
- Overhead may be different when switching blocks



Scoreboarding

- Keep track of all register operands of all instructions in the Instruction Buffer
 - ◆ Instruction becomes ready after the needed values are written
 - ◆ Eliminates hazards
 - ◆ Ready instructions are eligible for issue
- Decouples the Memory/Processor pipelines
 - ◆ Threads can issue instructions until the scoreboard prevents issue
 - ◆ Allows Memory/Processor ops to proceed in parallel with other waiting Memory/Processor ops



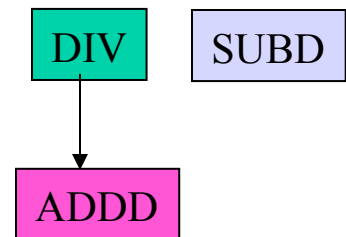
Static scheduling limits performance

- The ADDD instruction is stalled on the DIVide ..
- stalling further instruction issue, e.g. the SUBD

DIV **F0** , **F2** , **F4**

ADDD **F10** , **F0** , **F8**

SUBD **F12** , **F8** , **F14**



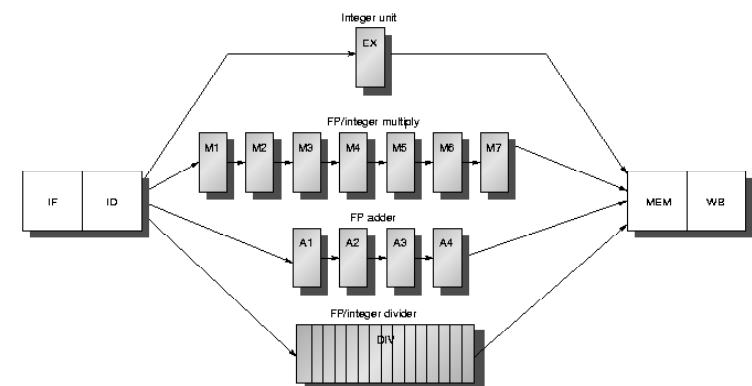
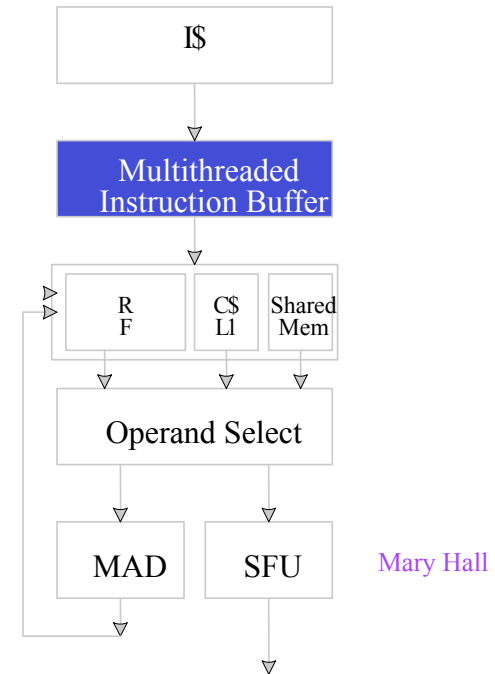
- But SUBD doesn't depend on ADDD or DIV
- If we have two adder/subtraction units, one will sit idle uselessly until the DIV finishes

Dynamic scheduling

- Idea: modify the pipeline to permit instructions to execute as soon as their operands become available
- This is known as *out-of-order execution (classic dataflow)*
- The SUBD can now proceed normally
- Complications: dynamically scheduled instructions also complete out of order

Dynamic scheduling splits the ID stage

- Issue sub-stage
 - ♦ Decode the instructions
 - ♦ Check for structural hazards
- Read operands substage
 - ♦ Wait until there are no data hazards
 - ♦ Read operands
- We need additional registers to store pending instructions that aren't ready to execute



Consequences of a split ID stage

- We distinguish between the time when an instruction **begins** execution, and when it **completes**
- Previously, an instruction stalled in the ID stage, and this held up the entire pipeline
- Instructions can now be in a suspended state, neither stalling the pipeline, nor executing
- They are waiting on operands

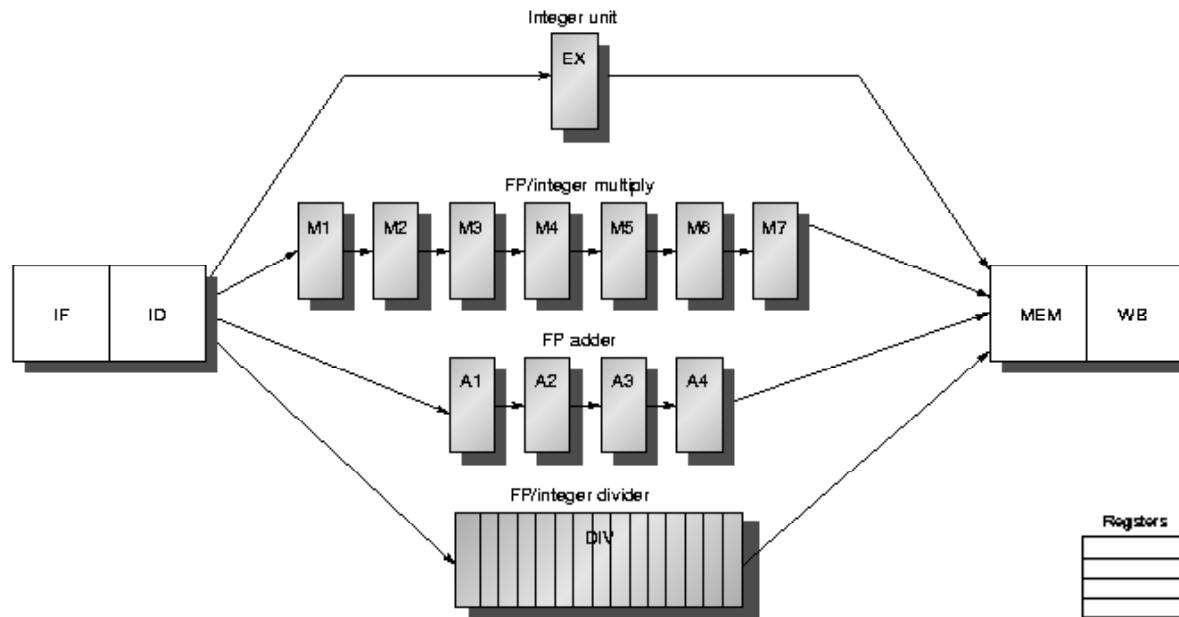
Two schemes for dynamic scheduling

- Scoreboard
 - ♦ CDC 66000
- Tomasulo's algorithm
 - ♦ IBM 360/91
- We'll vary the number of functional units, their latency, and functional unit pipelining

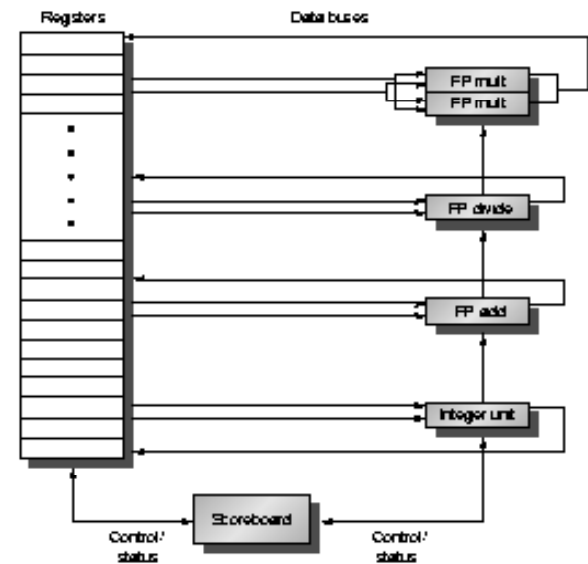
What is a scoreboarding?

- A technique that allows instructions to execute out of order...
 - ♦ So long as there are sufficient resources and
 - ♦ No data dependencies
- The goal of scoreboarding
 - ♦ Maintain an execution rate of one instruction per clock cycle

Multiple execution pipelines in DLX with scoreboarding



Unit	Latency
Integer	0
Memory	1
FP Add	2
FP Multiply	10
FP Div	40



What are the requirements?

- Responsibility for instruction issue and execution, including hazard detection
- Multiple instructions must be in the EX stage simultaneously
- Either through pipelining or multiple functional units
- DLX has: **2** multipliers, **1** divider, **1** integer unit (memory, branch, integer arithmetic)

How is a scoreboard implemented?

- A centralized bookkeeping table
- Tracks instructions, along with register operand(s) they depend on and which register they modify
- Status of result registers (who is going to write to a given register)
- Status of the functional units

How does it work?

- As each instruction passes through the scoreboard, construct a description of the data dependencies (Issue)
- Scoreboard determines when the instruction can read operands and begin execution
- If the instruction can't begin execution, the scoreboard keeps a record, and it listens for one the instruction *can* execute
- Also controls when an instruction may write its result
- All hazard detection is centralized

Stripmining

A perspective on programming

- Vector length is not built into the instruction: we can run a program on a GPUs supporting different vector lengths
- A thread block is a single thread of vector instructions with a programmable vector length (the block size)
- The number of warps in a block is configurable

Strip mining

- Partitioning the iteration space into chunks

for $i = 0$ **to** $N-1$

$a[i] = b[i] + c[i];$

for $j = 0$ **to** $N-1$ **by** VL

for $i = j$ **to** $\min(N, j+VL) - 1$

$a[i] = b[i] + c[i];$

int $idx = blockIdx.x * blockDim.x + threadIdx.x;$

if ($idx < N$) $a[idx] = a[idx] + 1.f;$

Strip mining on the GPU

- Partitioning long vectors into warps corresponds to strip-mining into *independent* instruction streams
 - Traditionally: render independent instructions in the *same* instruction stream
- ```
int idx = blockIdx.x*blockDim.x + threadIdx.x;
if (idx<N) a[idx] = a[idx]+1.f;
```

```
for j = 0 to N-1 by VL
 for i = j to min(N, j+VL) - 1
 a[i] = b[i] + c[i];
```

# Today's lecture

- Thread divergence
- Scheduling
- Floating Point

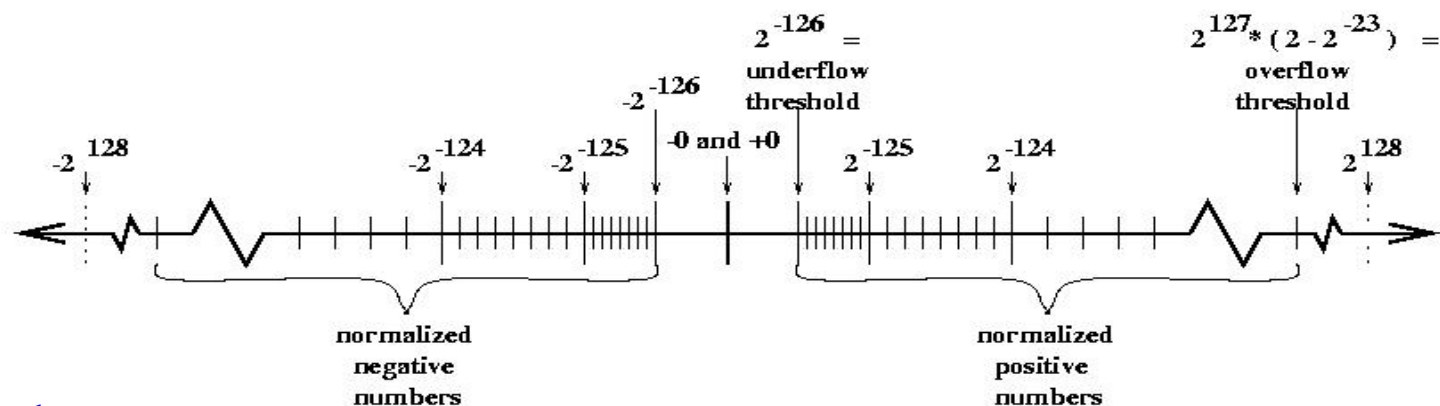
# What is floating point?

- A representation
  - ♦  $\pm 2.5732... \times 10^{22}$
  - ♦ NaN  $\infty$
  - ♦ Single, double, extended precision
- A set of operations
  - ♦  $+$   $=$   $*$   $/$   $\sqrt{\phantom{x}}$  rem
  - ♦ Comparison  $<$   $\leq$   $=$   $\neq$   $\geq$   $>$
  - ♦ Conversions between different formats, binary to decimal
  - ♦ Exception handling
- IEEE Floating point standard P754
  - ♦ Universally accepted
  - ♦ W. Kahan received the Turing Award in 1989 for design of IEEE Floating Point Standard
  - ♦ Revision in 2008

# IEEE Floating point standard P754

- Normalized representation  $\pm 1.d\dots d \times 2^{\text{exp}}$ 
  - Macheps** = Machine epsilon =  $\epsilon = 2^{-\#\text{significant bits}}$   
relative error in each operation
  - OV** = overflow threshold = largest number
  - UN** = underflow threshold = smallest number
- $\pm\text{Zero}$ :  $\pm\text{significant}$  and exponent = 0

| Format | # bits    | #significant bits | macheps                            | #exponent bits | exponent range                                  |
|--------|-----------|-------------------|------------------------------------|----------------|-------------------------------------------------|
| Single | 32        | 23+1              | $2^{-24}$ ( $\sim 10^{-7}$ )       | 8              | $2^{-126} - 2^{127}$ ( $\sim 10^{+-38}$ )       |
| Double | 64        | 52+1              | $2^{-53}$ ( $\sim 10^{-16}$ )      | 11             | $2^{-1022} - 2^{1023}$ ( $\sim 10^{+-308}$ )    |
| Double | $\geq 80$ | $\geq 64$         | $\leq 2^{-64}$ ( $\sim 10^{-19}$ ) | $\geq 15$      | $2^{-16382} - 2^{16383}$ ( $\sim 10^{+-4932}$ ) |



# What happens in a floating point operation?

- Round to the nearest representable floating point number that corresponds to the exact value(correct rounding)
- Round to nearest value with the lowest order bit =0 (rounding toward nearest even)
- Others are possible
- We don't need the exact value to work this out!
- Applies to  $+$   $=$   $*$   $/$   $\sqrt{\phantom{x}}$  rem
- Error formula:  $\text{fl}(a \text{ op } b) = (a \text{ op } b) * (1 + \delta)$  where
  - $\text{op}$  one of  $+$  ,  $-$  ,  $*$  ,  $/$
  - $|\delta| \leq \epsilon$
  - assuming no overflow, underflow, or divide by zero
- Addition example
  - ♦  $\text{fl}(\sum x_i) = \sum_{i=1:n} x_i * (1 + e_i)$
  - ♦  $|e_i| \sim < (n-1)\epsilon$

# Exception Handling

- An exception occurs when the result of a floating point operation is not representable as a normalized floating point number
  - ♦  $1/0$ ,  $\sqrt{-1}$
- P754 standardizes how we handle exceptions
  - ♦ **Overflow**: - exact result  $> OV$ , too large to represent
  - ♦ **Underflow**: exact result nonzero and  $< UN$ , too small to represent
  - ♦ **Divide-by-zero**: nonzero/0
  - ♦ **Invalid**:  $0/0$ ,  $\sqrt{-1}$ ,  $\log(0)$ , etc.
  - ♦ **Inexact**: there was a rounding error (common)
- Two possible responses
  - ♦ Stop the program, given an error message
  - ♦ Tolerate the exception

## An example

- Graph the function

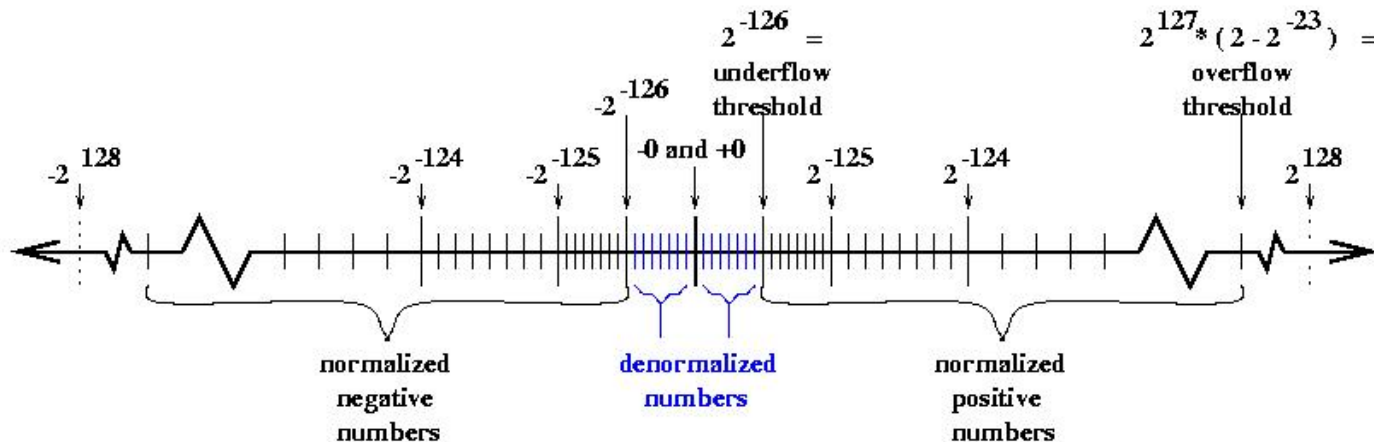
$$f(x) = \sin(x) / x$$

- $f(0) = 1$
- But we get a singularity @  $x=0$ :  $1/x = \infty$
- This is an “accident” in how we represent the function (W. Kahan)
- We *catch* the exception (divide by 0)
- Substitute the value  $f(0) = 1$



# Denormalized numbers

- We compute if  $(a \neq b)$  then  $x = a/(a-b)$
- We should never divide by 0, even if  $a-b$  is tiny
- *Underflow* exception occurs when exact result  $a-b < \text{underflow threshold UN}$
- We return a *denormalized number* for  $a-b$ 
  - ◆  $\pm 0.d\dots d \times 2^{\text{min\_exp}}$
  - ◆ sign bit, **nonzero** significand, minimum exponent value
  - ◆ Fills in the gap between 0 and UN



Jim

# NaN (Not a Number)

- Invalid exception
  - ♦ Exact result is not a well-defined real number
- We can have a quiet NaN or an sNaN
  - ♦ Quiet –does not raise an exception, but propagates a distinguished value
    - E.g. missing data:  $\max(3, \text{NaN}) = 3$
  - ♦ Signaling - generate an exception when accessed
    - Detect uninitialized data

# Exception handling

- An important part of the standard, 5 exceptions
  - ♦ Overflow and Underflow
  - ♦ Divide-by-zero
  - ♦ Invalid
  - ♦ Inexact
- Each of the 5 exceptions manipulates 2 flags
- Sticky flag set by an exception
  - ♦ Remains set until explicitly cleared by the user
- Exception flag: should a trap occur?
  - ♦ If so, we can enter a trap handler
  - ♦ But requires precise interrupts, causes problems on a parallel computer
- We can use exception handling to build faster algorithms
  - ♦ Try the faster but “riskier” algorithm
  - ♦ Rapidly test for accuracy  
(possibly with the aid of exception handling)
  - ♦ Substitute slower more stable algorithm as needed

# When compiler optimizations alter precision

- Let's say we support 79+ bit extended format in registers
- When we store values into memory, values truncated to the lower precision format, e.g. 64 bits
- Compilers can keep things in registers and we may lose *referential transparency*

- An example

```
float x, y, z;
int j;
....
 x = y + z;
 if (x >= j) replace x by something smaller than j
 y=x;
```

- With optimization turned on,  $x$  is computed to extra precision; it is not a float
- If  $x < j$  and held in a register....  
no guarantee the condition  $x < j$  will be preserved ...  
when  $x$  is stored in  $y$ , i.e.  $y \geq j$

# P754 on the GPU

- Cuda Programming Guide (4.1)  
“All compute devices follow the IEEE 754-2008 standard for binary floating-point arithmetic with the following deviations”
  - ♦ There is no mechanism for detecting that a floating-point exception has occurred and all operations behave as if the exceptions are always masked... SNaN ... are handled as quiet
- Cap. 2.x: FFMA ... is an IEEE-754-2008 compliant fused multiply-add instruction ... the full-width product ... used in the addition & a single rounding occurs during generation of the final result
  - ♦  $\text{rnd}(A \times A + B)$  with FFMA (2.x) vs  $\text{rnd}(\text{rnd}(A \times A) + B)$  FMAD for 1.x
- FFMA can avoid loss of precision during subtractive cancellation when adding quantities of similar magnitude but opposite signs
- Also see *Precision & Performance: Floating Point and IEEE 754 Compliance for NVIDIA GPUs,* by N. Whitehead and A. Fit-Florea