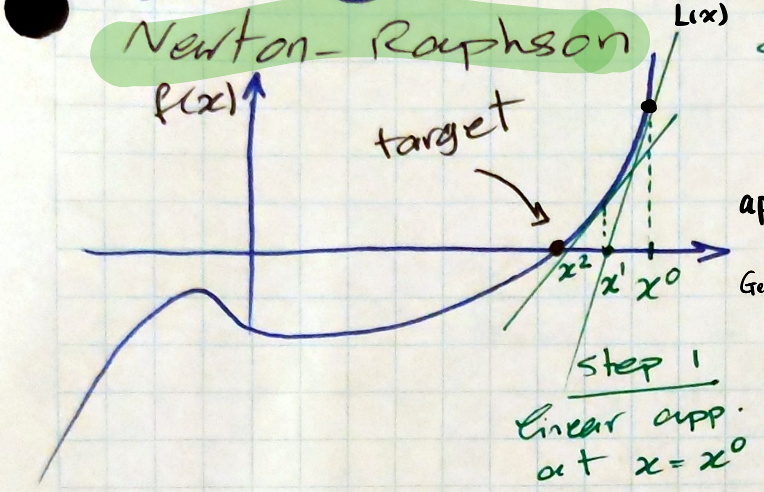
S8

Newton-Raphson
Projectile motion with drag
A radiating body

Newton-Raphson



step 1: $$f'(x^0) = \frac{\overset{f(x^0)}{L(x^0)} - \overset{0}{L(x^1)}}{x^0 - x^1}$$

$$\Rightarrow \underset{\text{approximate the target}}{x^1} = x^0 - \frac{f(x^0)}{f'(x^0)}$$

General formula $$\rightsquigarrow x^{k+1} = x^k - \frac{f(x^k)}{f'(x^k)}$$ in each iteration

x^0 (initial guess) $\to x^1, x^2, \dots$

$|f(x^k)| < \epsilon$ (tolerance) small value

Recall from last session:

$$r^2 \frac{dr}{dt} = -\gamma\sqrt{r-a} \to \int \frac{r^2}{\sqrt{r-a}} dr = -\gamma t + C$$

from the IC $r=r_0$ at $t=0$

$$\Rightarrow \frac{2}{15}\sqrt{r-a}\,(8a^2 + 4ar + 3r^2) + \gamma t - C = 0$$

$\to$ at each time $t = t_i$ $(t_0, t_1, t_2, \dots)$

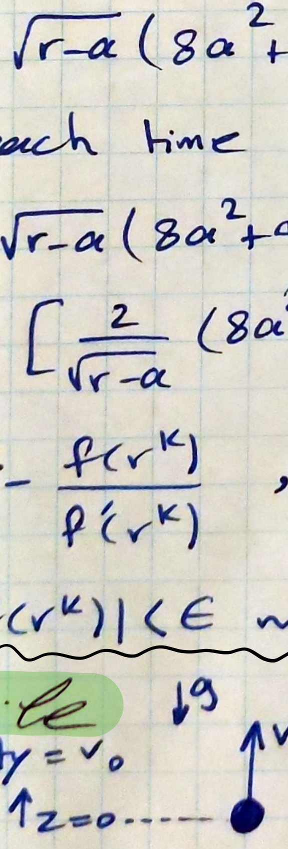
$$f(r) = \frac{2}{15}\sqrt{r-a}\,(8a^2+4ar+3r^2) + \gamma t_i - C = 0$$

$$f'(r) = \frac{2}{15}\left[\frac{2}{\sqrt{r-a}}(8a^2+4ar+3r^2) + \sqrt{r-a}\,(4a+6r)\right]$$

$$r^{k+1} = r^k - \frac{f(r^k)}{f'(r^k)}, \quad r^0 = r_{i-1}$$ value from the previous time

when $|f(r^k)| < \epsilon \rightsquigarrow r_i = r^k$ see code drainingReservoirNewoton.py

Projectile



initial velocity $= v_0$

$\ddot{z} = \dot{v}$

$$m\ddot{z} = \underset{\text{gravitatial force}}{-mg} - \underset{\text{drag force}}{\epsilon \dot{z}^3}$$ $\ddot{z} = \frac{d^2z}{dt^2}$

drag force can be linear, square, cube ...., we take cube here.

$$\rightsquigarrow m\dot{v} = -mg - \epsilon v^3$$

If $\epsilon = 0 \Rightarrow m\dot{v} = -mg \to \dot{v} = -g \to v = -gt + v_0$

$$\rightsquigarrow z = -\frac{1}{2}gt^2 + v_0 t$$

Note: max height happens when $v=0 \Rightarrow t = \frac{v_0}{g}$

when $\epsilon \neq 0 \rightarrow$ numerical (or perturbation)
focus of another session

$$m \frac{dv}{dt} = -mg - \epsilon v^3$$

Euler's method: $\frac{v_{i+1} - v_i}{\Delta t} = -g - \frac{\epsilon}{m} v_i^3$ $\quad \frac{dv}{dt} \approx \frac{V(i+1) - V_i}{\Delta t}$ when Δt is small

$$\Rightarrow \boxed{v_{i+1} = v_i - \Delta t \left(g + \frac{\epsilon}{m} v_i^3 \right)}$$

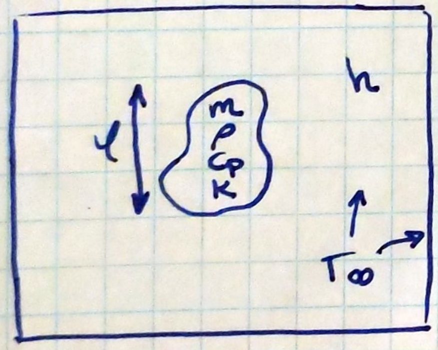
$$\frac{dz}{dt} = v \Rightarrow \frac{z_{i+1} - z_i}{\Delta t} = v_i \Rightarrow \boxed{z_{i+1} = z_i + \Delta t v_i}$$

see code projectile.py

A radiating body

temperature is T_∞ everywhere in the room.

Consider a body of material (e.g., a sphere, cylinder, cubes etc.) in a large room.



If the conduction time-scale within the body is small, i.e., heat transfers fast in the body

we can assume the T is uniform inside the body

Energy balance

$\sigma \approx 5.7 \times 10^{-8} \frac{W}{m^2 \cdot K^4}$ $\quad \tau_{cond} \sim \frac{\ell^2}{\alpha}$ = time, α ← thermal diffusivity $= \frac{K}{\rho c_p}$

$$\frac{d}{dt}(m c_p T) = -hA(T - T_\infty) - \sigma \epsilon (T^4 - T_\infty^4) A$$

both constant

energy change in the object

energy change in the system

emissivity =1 for black body

radiation

Stefan-Boltzmann equation

$$\Rightarrow \frac{dT}{dt} = -\frac{hA}{mc_p}(T - T_\infty) - \frac{\sigma \epsilon A}{mc_p}(T^4 - T_\infty^4)$$

Euler? $T_{i+1} = T_i - \Delta t \left[\frac{hA}{mc_p}(T - T_\infty) + \frac{\sigma \epsilon A}{mc_p}(T^4 - T_\infty^4) \right]$

see code radiatingBody.py

$\tau_{conv} \sim \frac{mc_p}{hA}$