S6 An evaporating water droplet

Concentration: $\dfrac{\text{amount of material}}{\text{unit volume}}$ ← moles

(e.g., → Concentration of water vapor per unit volume of air

flux of mass $\equiv J_A = \underbrace{-D\nabla C_A}_{\text{Fick's law}} + u C_A$

diffusivity coefficient

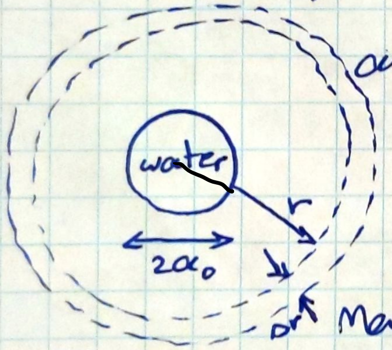
← $q = -k\nabla T + \rho c_p u T$.

mass/mole of a species (A) in a medium (e.g., water vapor in air)

velocity

diffusion

convection



← an elemental volume

Question: How long does it take for the radius to drop to some $a_c < a_0$ due to evaporation?

area of a sphere

Mass balance: $(J_{A_r} 4\pi r^2)_r - (J_{A_r} 4\pi r^2)_{r+\Delta r} = 0$

(*) pseudo steady state

(*) Clearly a (radius of the droplet) is changing due to evaporation. But we assume that happens very slowly so that the concentration profile around the droplet is almost steady-state at all times. → diffusion time scale << evaporation time scale

$$\frac{d}{dr}\left(J_{A_r} 4\pi r^2\right) = 0 \rightsquigarrow -D\frac{dC_A}{dr}(4\pi r^2) = k_1$$

$J_{A_r} = -D\dfrac{dC_A}{dr}$

$$\Rightarrow C_A = \frac{k_1}{4\pi D r} + k_2$$

BCs: at $r = a \Rightarrow C_A = C_A^*$ ← saturation concentration (function of temp)

as $r \to \infty \Rightarrow C_A \to C_A^\infty$ ← concentration of water vapor in the ambient

$$\Rightarrow \boxed{\frac{C_A - C_A^\infty}{C_A^* - C_A^\infty} = \frac{a}{r}}$$

moles of water in droplet

input is 0, only output so is negative

Now we write a balance for the droplet:

$$\frac{d}{dt}\left(\frac{4}{3}\pi a^3 \rho/_{MW}\right) = -\left(J_{A_r} 4\pi r^2\right)_{r=a} = \left[\frac{D a (C_A^* - C_A^\infty)}{\cancel{r^2}} \cancel{4\pi r^2}\right]_{r=a}$$

$3a^2 \frac{da}{dt}\left(\frac{4}{3}\pi \cdot \frac{\rho}{MW}\right)$

time derivative of the total moles of water in the droplet

$$\Rightarrow 4\cancel{\pi} a^{\cancel{2}} \frac{\rho}{MW}\frac{da}{dt} = -D \cancel{a} (C_A^* - C_A^\infty)(\cancel{4\pi})$$

$$\boxed{a\frac{da}{dt} = \frac{-D\,MW\,(C_A^* - C_A^\infty)}{\rho}}$$

Integrate $\rightsquigarrow$ $a^2 = a_0^2\left(1 - \underbrace{\frac{2D\,MW\,(C_A^* - C_A^\infty)}{\rho a_0^2}}\, t\right)$

$$\tau_e = \frac{\rho a_0^2}{2D\,MW\,(C_A^* - C_A^\infty)}$$

has dimensions $\frac{1}{\text{time}}$

time scale of evaporation

$$\tau_D = \frac{a_0^2}{D}$$

time scale of diffusion

$\tau_e \gg \tau_D$

$$\frac{\rho \cancel{a_0^2}}{2\cancel{D}\,MW\,(C_A^* - C_A^\infty)} \gg \frac{\cancel{a_0^2}}{\cancel{D}}$$

$$\rightarrow \frac{\rho}{2MW\,(C_A^* - C_A^\infty)} \gg 1 \checkmark$$

reasonable

How much does it take for the droplet radius to decrease to $a = a_c$

$$\Rightarrow \boxed{t_c = \left(1 - \left(\frac{a_c}{a_0}\right)^2\right)\frac{\rho a_0^2}{2D\,MW\,(C_A^* - C_A^\infty)}}$$

Let $\rho = 1000\ \frac{kg}{m^3}$, $D = 2.5\times10^{-5}\ \frac{m^2}{s}$, $MW = 18\times10^{-3}\ \frac{kg}{mol}$

$C_A^* = 1.2\ \frac{mol}{m^3}$, $C_A^\infty = \frac{1}{2}C_A^*$ $\rightsquigarrow$ $\left.\begin{array}{l} a_0 = 5\ \mu m \\ a_c = 2.5\ \mu m \end{array}\right\} \rightarrow \underline{t_c = 34\ ms}$

$\frac{P_A^*}{RT} \leftarrow 3\times10^3\ Pa$; % 50 Relative Humidity

respiratory droplet size for breathing and talking

How long does it take for the respiratory drop to fall on the ground?

Stokes? $V = \frac{4a_0^2(\rho - \rho_{air})g}{18\mu_{air}} \approx 2.7\ \frac{mm}{s} \rightarrow Re = \frac{\rho_{air}V(2a_0)}{\mu_{air}} \approx 0.0014 \ll 1 \checkmark$

falling time $\sim \frac{H}{V} \approx 740\ s$

$\sim 2m$

it is actully more than this because $a < a_0$ and $\rho < \rho_{water}$