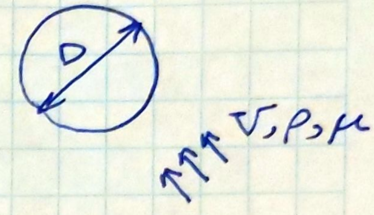
S2′

Clarifications re S2:

1) For the drag force on a smooth sphere, we showed

$$\frac{F_D}{\rho V^2 D^2} = \phi\left(\frac{\rho V D}{\mu}\right)$$



Equivalently, one can write

$$\frac{F_D}{\frac{1}{2}\rho V^2 \frac{\pi D^2}{4}} = \psi\left(\frac{\rho V D}{\mu}\right) \quad (*)$$

where the difference between functions ϕ and ψ is the sole constant $\frac{\pi}{8}$.

Now, the LHS of eqn (*) is termed (by convention) as the drag coefficient C_D:

$$C_D = \psi\left(\frac{\rho V D}{\mu}\right), \text{ where } C_D = \frac{F_D}{\frac{1}{2}\rho V^2 \frac{\pi D^2}{4}}$$

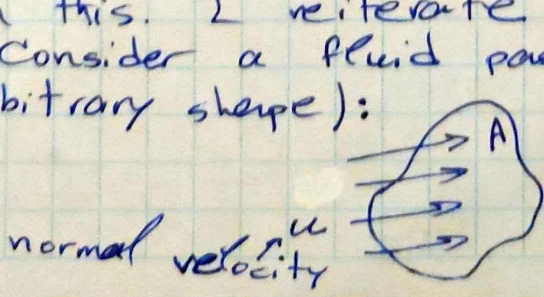
We can generalize this relationship to other objects as well:

$$C_D = \psi\left(\frac{\rho V D}{\mu}\right) = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

projected area $= \frac{\pi D^2}{4}$ for sphere

2) the flowrate: $\dfrac{\text{volume of fluid displaced}}{\text{Time}}$

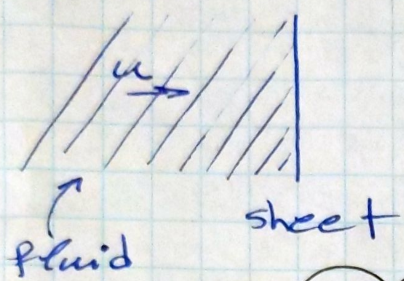
There are better but more complicated methods to explain this. I reiterate what we discussed in class. Consider a fluid passing through an area A (with arbitrary shape):



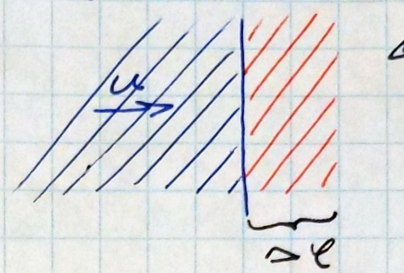
← A sheet of area "A".

Looking at the sheet from a side:

time = t



time = t + Δt



$\Delta V = A\Delta \ell$
displaced volume of the fluid (volume of fluid passed through area A)

$$\frac{\text{Volume passed}}{\text{Time}} = \frac{A\Delta \ell}{\Delta t}$$

$$\text{flowrate} = \frac{\Delta V}{\Delta t}$$

$$\Rightarrow \quad \frac{\Delta V}{\Delta t} = \frac{A\Delta \ell}{\Delta t}$$

take limit $\Delta t \to 0$

$$\Rightarrow \underbrace{\lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t}}_{\text{flowrate}} = A \underbrace{\lim_{\Delta t \to 0} \frac{\Delta \ell}{\Delta t}}_{\text{velocity}} \Rightarrow \boxed{Q = Au}$$

$\rightarrow$ mass flowrate $= \rho Q = \rho A u$

mass flux $= \dfrac{\text{mass flowrate}}{\text{unit area}} = \rho u$