Homework 2: Linear Regression

The is the coding potion of Homework 2. The homework is aimed at testing the ability to deal with a real-world dataset and use linear regression on it.

```
import numpy as np
import pandas as pd

# Plotting libraries
import matplotlib.pyplot as plt
import seaborn as sns

%matplotlib inline
```

Load Dataset

Loading the California Housing dataset using sklearn.

```
# Load dataset
from sklearn.datasets import fetch_california_housing
housing = fetch_california_housing()
```

Part 1 : Analyse the dataset

```
# Put the dataset along with the target variable in a pandas dataframe
data = pd.DataFrame(housing.data, columns=housing.feature_names)
# Add target to data
data['target'] = housing['target']
data.head()
```

```
   MedInc  HouseAge  AveRooms  AveBedrms  Population  AveOccup
Latitude  \
0  8.3252      41.0  6.984127   1.023810       322.0  2.555556
37.88
1  8.3014      21.0  6.238137   0.971880      2401.0  2.109842
37.86
2  7.2574      52.0  8.288136   1.073446       496.0  2.802260
37.85
3  5.6431      52.0  5.817352   1.073059       558.0  2.547945
37.85
4  3.8462      52.0  6.281853   1.081081       565.0  2.181467
37.85

   Longitude  target
0    -122.23   4.526
1    -122.22   3.585
2    -122.24   3.521
3    -122.25   3.413
4    -122.25   3.422
```

Part 1a : Check for missing values in the dataset

The dataset might have missing values represented by a NaN. Check if the dataset has such missing values.

```
# Check for missing values
from cmath import nan
import math

def is_null(dataframe):
    """
    This function takes as input a pandas dataframe and outputs
whether the
    dataframe has missing values. Missing values can be detected by
checking
    for the presence of None or NaN. inf or -inf must also be treated
as a missing value.

    Input:
        dataframe: Pandas dataframe
    Output:
        Return True is there are missing value in the dataframe. If
not, return False.
    """
    # YOUR CODE HERE
    """
    df = dataframe
    row, col = df.shape[0], df.shape[1]
    for i in range(row):
        for j in range(col):
            val = df.iloc[i][j]
            if val == None or val == nan or val == math.inf or val ==
-math.inf:
                return True
    return False
    /* above approach take more time */
    """
    a = dataframe.to_numpy()
    row, col = a.shape[0], a.shape[1]
    for i in range(row):
        for j in range(col):
            val = a[i][j]
            if val == None or val == nan or val == math.inf or val ==
-math.inf:
                return True
    return False

# === DO NOT MOVE/DELETE ===
# This cell is used as a placeholder for autograder script injection.
```

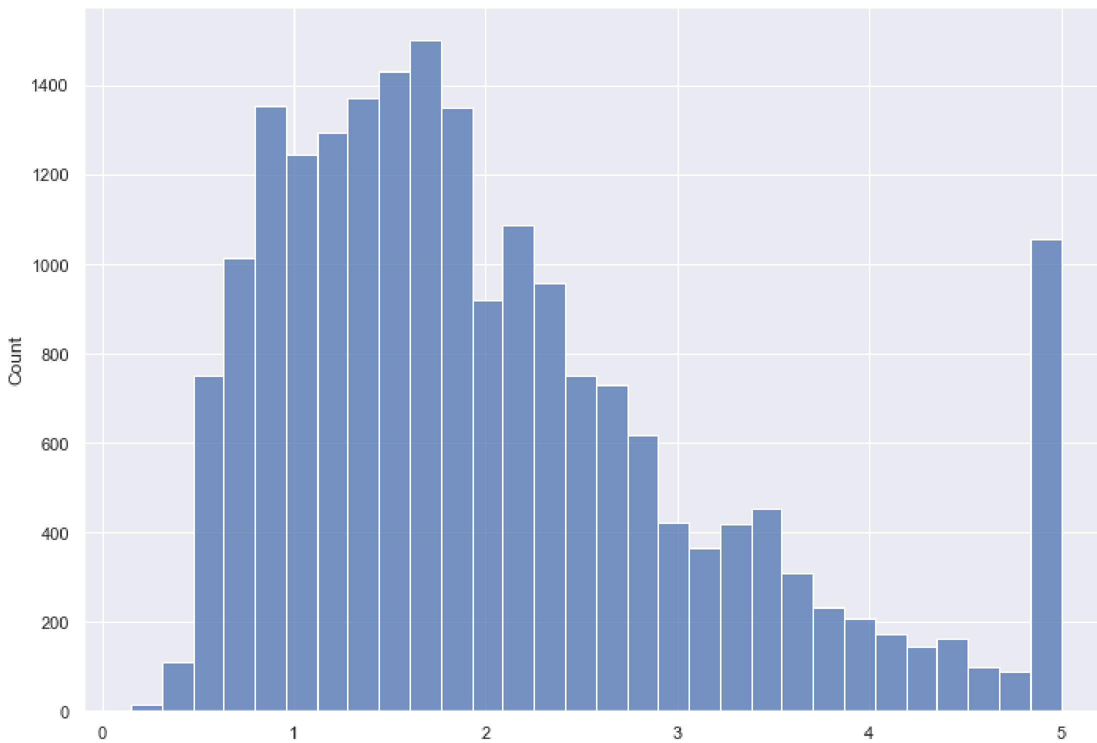
```
# This dataset has no null values; you can run this cell as a sanity check.
print(f"The data has{'' if is_null(data) else ' no'} missing values.")
assert not is_null(data)
```

```
The data has no missing values.
```

Part 1b: Studying the distribution of the target variable

Plot the histogram of the target variable over a fixed number of bins (say, 30).

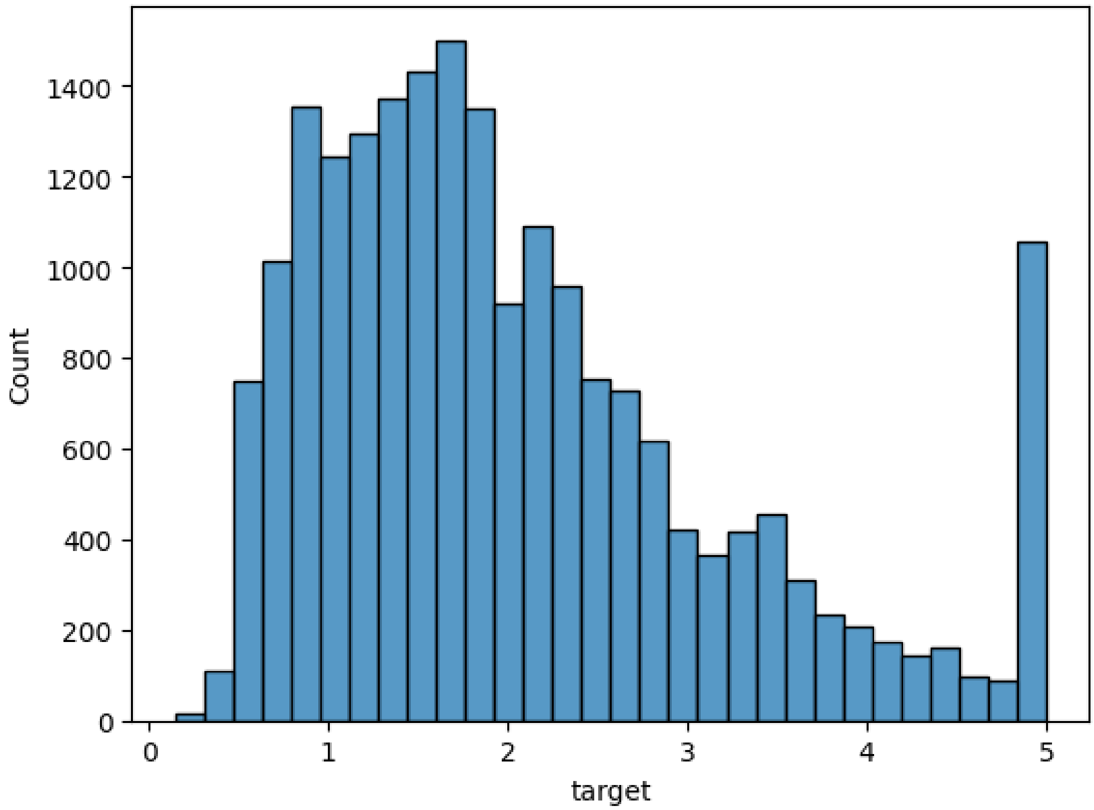
Example histogram output:



Hint: Use the histogram plotting function available in Seaborn in Matplotlib.

```
# Plot histogram of target variable

# YOUR CODE HERE
sns.histplot(data = data, x = 'target', bins = 30)
plt.show()
```



Part 1c: Plotting the correlation matrix

Given the dataset stored in the `data` variable, plot the correlation matrix for the dataset. The dataset has 9 variables (8 features and one target variable) and thus, the correlation matrix must have a size of 9x9.

Hint: You may use the correlation matrix computation of a dataset provided by the `pandas` library.

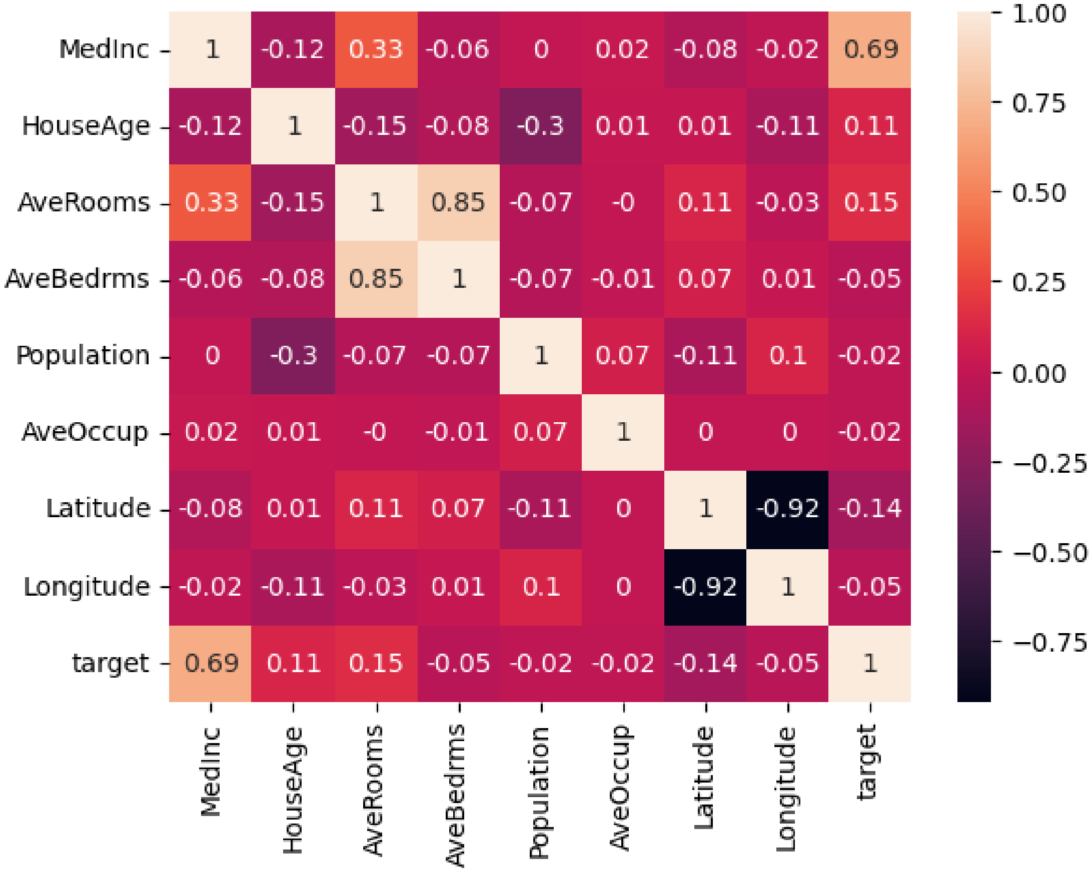
Link: What is a correlation matrix?

```
# Correlation matrix
def get_correlation_matrix(dataframe):
    """
    Given a pandas dataframe, obtain the correlation matrix
    computing the correlation between the entities in the dataset.

    Input:
        dataframe: Pandas dataframe
    Output:
        Return the correlation matrix as a pandas dataframe, rounded
off to 2 decimal places.
    """
    # YOUR CODE HERE
    return dataframe.corr().round(2)
```

```python
# Plot the correlation matrix
correlation_matrix = get_correlation_matrix(data)
# annot = True to print the values inside the square
sns.heatmap(data=correlation_matrix, annot=True)
```

```
<AxesSubplot: >
```




```python
# === DO NOT MOVE/DELETE ===
# This cell is used as a placeholder for autograder script injection.

# You can check your output against the expected correlation matrix
below:
ground_truth = np.array([
    [1.0, -0.12, 0.33, -0.06, 0.0, 0.02, -0.08, -0.02, 0.69],
    [-0.12, 1.0, -0.15, -0.08, -0.3, 0.01, 0.01, -0.11, 0.11],
    [0.33, -0.15, 1.0, 0.85, -0.07, 0.0, 0.11, -0.03, 0.15],
    [-0.06, -0.08, 0.85, 1.0, -0.07, -0.01, 0.07, 0.01, -0.05],
    [0.0, -0.3, -0.07, -0.07, 1.0, 0.07, -0.11, 0.1, -0.02],
    [0.02, 0.01, 0.0, -0.01, 0.07, 1.0, 0.0, 0.0, -0.02],
    [-0.08, 0.01, 0.11, 0.07, -0.11, 0.0, 1.0, -0.92, -0.14],
    [-0.02, -0.11, -0.03, 0.01, 0.1, 0.0, -0.92, 1.0, -0.05],
    [0.69, 0.11, 0.15, -0.05, -0.02, -0.02, -0.14, -0.05, 1.0],
```

```
])
assert np.allclose(ground_truth,
get_correlation_matrix(data).to_numpy(), rtol=1e-2, atol=1e-2)
```

Part 1d: Extracting relevant variables

Based on the correlation matrix obtained in the previous part, identify the top-4 most relevant features from the dataset for predicting the target variable.

- MedInc 0.69
- AveRooms 0.15
- Latitude -0.14
- HouseAge 0.11

Part 2: Data Manipulation

This section is focused on arranging the dataset in a format suitable for training the linear regression model.

Part 2a: Normalize the dataset

Find the mean and standard deviation corresponding to each feature and target variable in the dataset. Use the values of the mean and standard deviation to normalize the dataset.

```
features = np.concatenate([data[name].to_numpy()[:, None] for name in
housing['feature_names']], axis=1)
target = housing['target']

# Normalize data
def normalize(features, target):
    # YOUR CODE HERE
    f = features
    f_row, f_col = f.shape[0], f.shape[1]
    f_mean = f.mean(axis = 0)
    f_std = f.std(axis = 0)
    for j in range(f_col):
        for i in range(f_row):
            f[i][j] -= f_mean[j]
            f[i][j] /= f_std[j]
    t = target
    t_mean = t.mean(axis = 0)
    t_std = t.std(axis = 0)
    t_n = t.shape[0]
    for i in range(t_n):
        t[i] -= t_mean
        t[i] /= t_std
    return f, t

features_normalized, target_normalized = normalize(features, target)
```

```
# === DO NOT MOVE/DELETE ===
# This cell is used as a placeholder for autograder script injection.
assert all(np.abs(features_normalized.mean(axis=0)) < 1e-2), "Mean should be close to 0"
assert all(np.abs(features_normalized.std(axis=0) - 1) < 1e-2), "Standard deviation should be close to 1"
assert np.abs(target_normalized.mean(axis=0)) < 1e-2, "Mean should be close to 0"
assert np.abs(target_normalized.std(axis=0) - 1) < 1e-2, "Standard deviation should be close to 1"
```

Part 2b: Train-Test Split

Use the train-test split function from `sklearn` and execute a 80-20 train-test split of the dataset.

```
# YOUR CODE HERE
from sklearn.model_selection import train_test_split
X_train, X_test, Y_train, Y_test = train_test_split(features, target, train_size = 0.8, test_size = 0.2)
```

```
# === DO NOT MOVE/DELETE ===
# This cell is used as a placeholder for autograder script injection.

# Sanity checking:
print(X_train.shape)
print(X_test.shape)
print(Y_train.shape)
print(Y_test.shape)
```

```
(16512, 8)
(4128, 8)
(16512,)
(4128,)
```

Part 3: Linear Regression

In this part, a linear regression model is used to fit the dataset loaded and normalized above.

Part 3a: Code for Linear Regression

Implement a closed-form solution for ordinary least squares linear regression in `MyLinearRegression`, and print out the RMSE and R^2 between the ground truth and the model prediction.

```
class MyLinearRegression:
    def __init__(self):
        self.theta = None

    def fit(self, X, Y):
```

```
        # Given X and Y, compute theta using the closed-form solution
for linear regression.
        # YOUR CODE HERE
        x, y = X.copy(), Y.copy()
        n = x.shape[0]
        for i in range(n):
            np.insert(x, 0, 1)
        y = y.reshape(n, 1)
        theta = np.dot(np.linalg.inv(np.dot(x.T, x)), np.dot(x.T, y))
        self.theta = theta
        return theta

    def predict(self, X):
        # Predict Y for a given X
        # YOUR CODE HERE
        x = X.copy()
        n = x.shape[0]
        for i in range(n):
            np.insert(x, 0, 1)
        return np.dot(x, self.theta)

# Train the model on (X_train, Y_train) using Linear Regression
my_model = MyLinearRegression()
my_model.fit(X_train, Y_train)
```

```
array([[ 0.7117815 ],
       [ 0.1004027 ],
       [-0.22318309],
       [ 0.25076373],
       [-0.00633857],
       [-0.03521465],
       [-0.78670989],
       [-0.75885015]])
```

```
from sklearn.metrics import mean_squared_error, r2_score

# Compute train RMSE using (X_train, Y_train)
y_train_predict = my_model.predict(X_train)
train_rmse = (np.sqrt(mean_squared_error(Y_train, y_train_predict)))
train_r2 = r2_score(Y_train, y_train_predict)
print("The model performance for training set")
print("--------------------------------------")
print('RMSE is {}'.format(train_rmse))
print('R2 score is {}'.format(train_r2))
print("\n")

# Compute test RMSE using (X_test, Y_test)
y_test_predict = my_model.predict(X_test)
test_rmse = (np.sqrt(mean_squared_error(Y_test, y_test_predict)))
test_r2 = r2_score(Y_test, y_test_predict)
print("The model performance for testing set")
```

```
print("--------------------------------------")
print('RMSE is {}'.format(test_rmse))
print('R2 score is {}'.format(test_r2))
```

```
The model performance for training set
--------------------------------------
RMSE is 0.6325044657028708
R2 score is 0.6025201701530978


The model performance for testing set
--------------------------------------
RMSE is 0.6075172000467761
R2 score is 0.6210767139577711
```

Part 3b: Compare with LinearRegression from sklearn.linear_model

Use LinearRegression from the `sklearn` package to fit the dataset and compare the results obtained with your own implementaion of Linear Regression.

The linear regressor should be named `model` for the cells below to run properly.

```
# YOUR CODE HERE
from sklearn.linear_model import LinearRegression
model = LinearRegression().fit(X_train, Y_train)
```

```
# model evaluation for training set
y_train_predict = model.predict(X_train)
sklearn_train_rmse = (np.sqrt(mean_squared_error(Y_train,
y_train_predict)))
sklearn_train_r2 = r2_score(Y_train, y_train_predict)

print("The model performance for training set")
print("--------------------------------------")
print('RMSE is {}'.format(sklearn_train_rmse))
print('R2 score is {}'.format(sklearn_train_r2))
print("\n")

# model evaluation for testing set
y_test_predict = model.predict(X_test)
sklearn_test_rmse = (np.sqrt(mean_squared_error(Y_test,
y_test_predict)))
sklearn_test_r2 = r2_score(Y_test, y_test_predict)

print("The model performance for testing set")
print("--------------------------------------")
print('RMSE is {}'.format(sklearn_test_rmse))
print('R2 score is {}'.format(sklearn_test_r2))
```

```
The model performance for training set
--------------------------------------
```

```
RMSE is 0.6325040644245501
R2 score is 0.6025206744973252


The model performance for testing set
--------------------------------------
RMSE is 0.6075208338489084
R2 score is 0.6210721809622849
```

Part 3c: Analysis Linear Regression Performance

In this section, provide the observed difference in performance along with an explanation of the following:

- Difference between training between unnormalized and normalized data.
- Difference between training on all features versus training on the top-5 most relevant features in the dataset.
- Difference between (1) training on all features (unnormalized), (2) training on top-4 unnormalized features, and (3) training on top-4 normalized features.

Write your answer below.

1. difference between training between unnormalized and normalized data

- unnormalized
  - MyLinearRegression ``` The model performance for training set -------------------------------------- RMSE is 0.6274399780727136 R2 score is 0.6013605474516523

    The model performance for testing set -------------------------------------- RMSE is 0.6279718583023217 R2 score is 0.6240449680621314 ```

  - LinearRegression from sklearn ``` The model performance for training set -------------------------------------- RMSE is 0.6274385393991952 R2 score is 0.6013623755511879

    The model performance for testing set -------------------------------------- RMSE is 0.6279848916905705 R2 score is 0.6240293622075719 ```

- normalized
  - MyLinearRegression ``` The model performance for training set -------------------------------------- RMSE is 0.6237806987205472 R2 score is 0.6076515191842307

    The model performance for testing set -------------------------------------- RMSE is 0.6423536906994114 R2 score is 0.6003918327239364 ```

  - LinearRegression from sklearn ``` The model performance for training set -------------------------------------- RMSE is 0.6237719044889968 R2 score is 0.6076625819794517

The model performance for testing set -------------------------------------- RMSE is 0.6424306441226655 R2 score is 0.6002960815548773 ```

- observation:
  - training set:
    - unnormalized data: RMSE ≈ 0.62744, normalized data: RMSE ≈ 0.62378, normalized data's RMSE seem to be lower, difference ≈ 0.62744 - 0.62378 = 0.00366
    - unnormalized data: R2 score ≈ 0.60136, normalized data: R2 score ≈ 0.60765, unnormalized data's R2 score seem to be lower, difference ≈ 0.60765 - 0.60136 = 0.00629
  - testing set:
    - unnormalized data: RMSE ≈ 0.62797, normalized data: RMSE ≈ 0.64235, unnormalized data's RMSE seem to be lower, difference ≈ 0.64235 - 0.62797 = 0.01438
    - unnormalized data: R2 score ≈ 0.62404, normalized data: R2 score ≈ 0.60039, normalized data's R2 score seem to be lower, difference ≈ 0.62404 - 0.60039 = 0.02365
- using X_train, X_test, Y_train, Y_test = train_test_split(features_normalized, target_normalized, train_size = 0.8, test_size = 0.2)

1. difference between training on all features versus training on the top-5 most relevant features in the dataset

- 'MedInc', 'AveRooms', 'Latitude', 'HouseAge', 'AveBedrms' are top-5 features

- training on all features ``` The model performance for training set -------------------------------------- RMSE is 0.6274385393991952 R2 score is 0.6013623755511879

  The model performance for testing set -------------------------------------- RMSE is 0.6279848916905705 R2 score is 0.6240293622075719 ```

- training on top-5 features ``` The model performance for training set -------------------------------------- RMSE is 0.6762907486003943 R2 score is 0.5446199489263248

  The model performance for testing set -------------------------------------- RMSE is 0.6835121965605816 R2 score is 0.5242661206644932 ```

- observation

- training set: all features: RMSE ≈ 0.62744, top-5 features: RMSE ≈ 0.67629, top-5 features's RMSE seem to be higher, difference ≈ 0.67629 - 0.62744 = 0.04885 all features: R2 score ≈ 0.60136, top-5 features: R2 score ≈ 0.54462, all features's R2 score seem to be higher, difference ≈ 0.60136 - 0.54462 = 0.05674

- using features = np.concatenate([data[name].to_numpy()[:, None] for name in ['MedInc', 'AveRooms', 'Latitude', 'HouseAge', 'AveBedrms']], axis = 1)

1. difference between (1) training on all features (unnormalized), (2) training on top-4 unnormalized features, and (3) training on top-4 normalized features

- all features unnormalized ``` The model performance for training set -------------------------------------- RMSE is 0.6274385393991952 R2 score is 0.6013623755511879

  The model performance for testing set -------------------------------------- RMSE is 0.6279848916905705 R2 score is 0.6240293622075719 ```

- top 4 unnormalized ``` The model performance for training set -------------------------------------- RMSE is 0.671380395385047 R2 score is 0.5477404657858351

  The model performance for testing set -------------------------------------- RMSE is 0.7024800506206409 R2 score is 0.5129312505477825 ```

- top 4 normalized ``` The model performance for training set -------------------------------------- RMSE is 0.6791210488108694 R2 score is 0.5350032082212628

  The model performance for testing set -------------------------------------- RMSE is 0.6739473406162616 R2 score is 0.5600876594338523 ```

- observation training on top-4:

  – training set:
    - all features unnormalized data: RMSE ≈ 0.62744, top-4 unnormalized data: RMSE ≈ 0.67138, top-4 normalized data: RMSE ≈ 0.67912, top-4 unnormalized data's RMSE seem to be highest, difference with lowest ≈ 0.67912 - 0.62744 = 0.05168
    - all features unnormalized data: R2 score ≈ 0.60136, top-4 unnormalized data: R2 score ≈ 0.54774, top-4 normalized data: R2 score ≈ 0.53500, all features unnormalized data's R2 score seem to be highest, difference with lowest ≈ 0.60136 - 0.53500 = 0.06636

  – testing set:
    - all features unnormalized data: RMSE ≈ 0.62798, top-4 unnormalized data: RMSE ≈ 0.70248, top-4 normalized data: RMSE ≈ 0.67395, top-4 unnormalized data's RMSE seem to be highest, difference with lowest ≈ 0.70248 - 0.62798 = 0.0745
    - all features unnormalized data: R2 score ≈ 0.62403, top-4 unnormalized data: R2 score ≈ 0.51293, top-4 normalized data: R2 score ≈ 0.56009, all features unnormalized data's R2 score seem to be highest, difference with lowest ≈ 0.62403 - 0.51293 = 0.1111