Definitions and Formulae for Inventory Management

Calculating Total Cost of Managing Inventory

- D = Annual demand (in units of product).

  Generally, upper-case D is used for annual demand, and lower-case d is used to indicate demand for a shorter period such as days, weeks, months, and quarters.

- Q = Order quantity (in units of product). Quantity for which an order is placed.
  - Number of orders:

    Using order quantity Q, number of orders in a year is (D ÷ Q) orders
  - Time between orders:

    Using order quantity Q, time between orders is (Q ÷ D) year (this will usually give a fraction of a year that can be converted to months or weeks or days)
  - Average inventory:

    Average of inventory levels at the start and the end of a period

    = (Starting Inventory + Ending Inventory) ÷ 2
  - Optimal order quantity

    Order quantity, when calculated using economic order quantity (EOQ), is referred to as Q*.

- S = Cost of ordering from a vendor (in dollars or other currency).

  OR

  Cost of setting up a process when switching from one product to another.

  Generally, includes costs that are incurred (as opposed to allocated) for ordering or setting up, such as:

  Transportation costs

  Receiving costs

  Costs related to the transaction

  Cost of cleaning machines

  Cost of changing tooling and dies

  Cost of unusable product when restarting after switching product

- H = Annual inventory holding cost per unit per year (in dollars or other currency)

  Generally, includes costs that are incurred for keeping items in inventory such as:

  Opportunity cost of foregoing other investment opportunities

  Storage costs, including rent paid for warehouse (or rent foregone), handling, electricity for temperature control, and insurance and taxes

  Deterioration costs, including items getting spoilt, breaking, getting stolen, expiring, becoming obsolete or going out of style

  - H is commonly represented as a product of a holding (or carrying) rate and the cost per unit of the product.

    For example, H is 25% of cost per unit, which is $18

    So, H = (0.25 * $18) = $ 4.50.

    Interpretation:

    It costs $ 4.50 to hold (or carry) a unit of the item in inventory for a year.

- TC = Total annual cost of managing inventory (in dollars or other currency).

  Calculated as sum of annual holding costs and annual ordering costs.

  - Annual Cost of Ordering when quantity is Q and annual demand is D:

    = Number of orders in a year * Cost of placing an order

    OR Number of setups in a year * Cost of making a setup

    = (D ÷ Q) * S

  - Annual Cost of Holding when quantity Q:

    = Average inventory * Cost of holding a unit of the item in inventory for a year

    When Q is assumed to be depleting at a constant rate until it reaches 0, average inventory is calculated as (Q + 0) ÷ 2, so Q ÷ 2.

    So, Annual cost of holding

    = (Q ÷ 2) * H

  - TC = (D ÷ Q) * S + (Q ÷ 2) * H

Economic Order Quantity (EOQ)

- EOQ is the optimal order quantity that minimizes the total annual cost of managing inventory, consisting of the two components of annual ordering (or setup) cost and annual holding cost.
- The base model EOQ has the following conditions and assumptions
    - Demand is known and is evenly spread (there is no variation or change).
    - Lead time, from order placement to delivery, is unchanging and known.
    - Because demand and lead time are known and unchanging, no stock outs occur.
    - Items are ordered or produced in a batch, and the batch arrives or is completed all at one time.
    - The purchase price or cost of the item is constant, and there are no discounts for larger quantities.
    - Ordering cost per order remains constant regardless of order quantity.
    - The EOQ is calculated for only one product without considering interactions with other items in inventory (that is, it does not account for conditions such as sharing transportation costs when ordering multiple items from a supplier)

- Q* = Order quantity, when calculated using economic order quantity (EOQ).

$$Q^* = \sqrt{\frac{2\,D\,S}{H}}$$

Can also be written as: EOQ = Square Root ((2 * D * S) ÷ H)

- Total Annual Cost of managing inventory when using EOQ

$$TC\,(Q^*) = \sqrt{2\,D\,S\,H}$$

Can also be written as: TC = Square Root (2 * D * S * H)

This will give the same total cost as when using Q* in the more generally applicable formula for all values of Q: (D ÷ Q) * S + (Q ÷ 2) * H.

Continuous Review System

- An inventory management system in which the inventory position is monitored continuously (commonly after every transaction of sale or withdrawal of an item from inventory), and when the inventory position reaches a predetermined threshold (or lower) called the reorder point, a fixed quantity order is placed.
    - Inventory position = Inventory on hand + Order that is on the way from the supplier
    - Continuous review system is also commonly referred to as the Fixed Order Quantity System or Q system
    - The base model of this system uses EOQ for the quantity and the ROP for reorder point.
- Lead Time

  Time from placing an order to receiving it.
    - Assumed to be known and unchanging in the base model
- Service Level

  Extent to which orders are filled from stock, allowing for uncertainty in demand
    - Until an order placed with a supplier arrives, there is a probability of stocking out
    - Service level probability = (1 – Stock out probability)
        - For example, with Stock out Probability of 10%, Service Level is

          = (1 – 0.10) = 0.90 = 90%
    - Accounting for uncertainty in demand requires:
        - Standard deviation (σ) of demand (lower-case d) for lead time (LT) duration
        - Need to use managerial discretion or generally accepted company or industry standard to select service level probability
- Reorder Point

  ROP or R is the threshold level of inventory that triggers an order

  The idea is that when inventory reaches this level (ROP), place an order of quantity EOQ.

  ROP = Demand during lead time + Safety stock
    - Demand during lead time (ddlt) = d * LT

      E.g. for 3 weeks LT, ddlt = weekly demand * 3 weeks
    - Safety stock = $z * \sigma_{LT}$

      (Safety stock = z score * standard deviation of ddlt)

- z = Safety factor from normal distribution for desired service level
- σ_{LT} = Standard deviation of demand during lead time

- ROP = $(d * LT) + (z * \sigma_{\text{per period}} * \sqrt{LT})$
  - Lower-case d signifies that it is not annual Demand. It is demand for the period of a day or week or month – the same unit in which lead time is stated.
  - The z score for a desired service level is selected using a z table

    OR, in Microsoft Excel, using =NORM.S.INV(Probability).

    Use the probability in its decimal form, i.e., 0.90 for 90%, and round the z score to one or two places after the decimal.

    For example, z score for 90% service level:

    =NORM.S.INV(0.90) = 1.28
  - On a side note (i.e., you do not have to know the information in this bullet point): the standard deviation per period is multiplied by square root of lead time (and not just by the lead time) because, effectively, it is adding the variances (square of standard deviation) of multiple periods and then taking the square root to convert back to standard deviation.

    For example, when standard deviation of daily demand is 3,

    variance of daily demand = $(\text{std. dev})^2 = 9$

    variance of 2-day demand is $2 * 9 = 18$

    standard deviation of 2-day demand is square root of 18 = 4.24

    This standard deviation of 2-day demand can also be calculated as

    3 * square root of 2 = 4.24

Periodic Review System

- An inventory management system in which the inventory position is monitored at fixed periodic intervals P, and an amount equal to target inventory T minus the inventory position is ordered.
  - Also referred to as the Fixed Order Period System or P system.