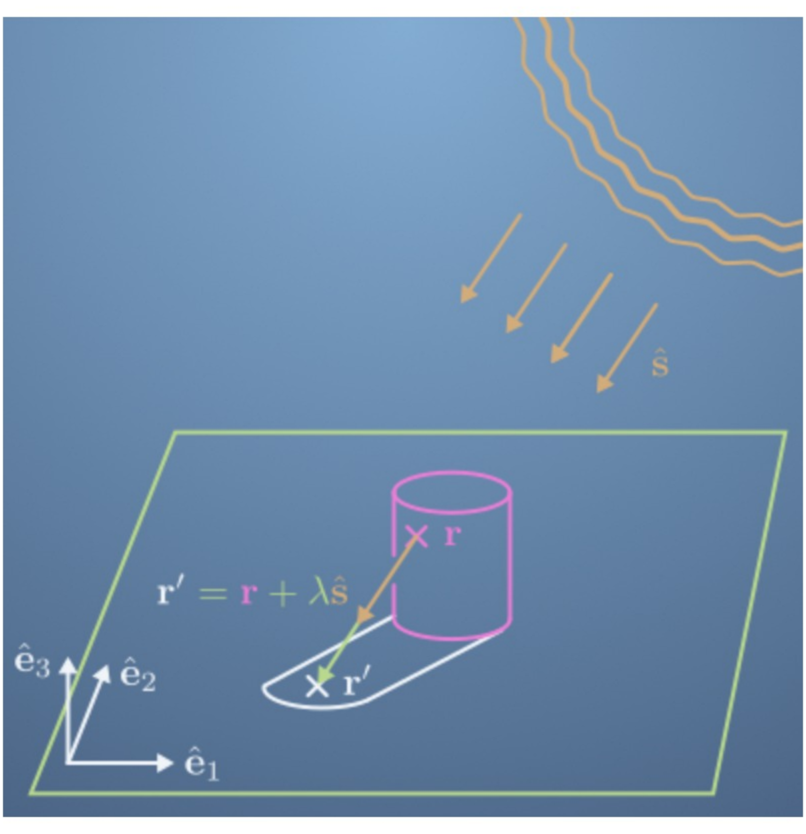
Congratulations! You passed!

Next Item

1. 1 / 1 points

Shadows are an example of a transformation that reduces the number of dimensions. For example, 3D objects in the world cast shadows on surfaces that are 2D.

We can consider a simple example for looking at shadows using linear algebra.



The sun is sufficiently far away that effectively all of its rays come in parallel to each other. We can describe their direction with the unit vector $\hat{\mathbf{s}}$.

We can describe the 3D coordinates of points on objects in our space with the vector $\mathbf{r}$. Objects will cast a shadow on the ground at the point $\mathbf{r}'$ along the path that light would have taken if it hadn't been blocked at $\mathbf{r}$, that is, $\mathbf{r}' = \mathbf{r} + \lambda\hat{\mathbf{s}}$.

The ground is at $\mathbf{r}'_3 = 0$; by using $\mathbf{r}'.\hat{\mathbf{e}}_3 = 0$, we can derive the expression, $\mathbf{r}.\hat{\mathbf{e}}_3 + \lambda s_3 = 0$, (where $s_3 = \hat{\mathbf{s}}.\hat{\mathbf{e}}_3$).

Rearrange this expression for λ and substitute it back into the expression for $\mathbf{r}'$ in order to get $\mathbf{r}'$ in terms of $\mathbf{r}$.

- (selected) $\mathbf{r}' = \mathbf{r} - \hat{\mathbf{s}}(\mathbf{r}.\hat{\mathbf{e}}_3)/s_3$

  Correct
  Well done!

- $\mathbf{r}' = \mathbf{r} - \hat{\mathbf{s}}$
- $\mathbf{r}' = \mathbf{r} + \hat{\mathbf{s}}$
- $\mathbf{r}' = \mathbf{r} + \hat{\mathbf{s}}(\mathbf{r}.\hat{\mathbf{e}}_3)/s_3$

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2. 2 / 2 points

From your answer above, you should see that $\mathbf{r}'$ can be written as a linear transformation of $\mathbf{r}$. This means we should be able to write $\mathbf{r}' = A\mathbf{r}$ for some matrix A.

To help us find an expression for A, we can re-write the expression above with Einstein summation convention.

Which of the answers below correspond to the answer to Question 1? (Select all that apply)

- [x] $r'_i = r_i - s_i r_3/s_3$

  Correct
  This answer is correct and concise, but more difficult to see it as a matrix multiplication on r.

- [ ] None of the other options.

  Un-selected is correct

- [x] $r'_i = r_i - s_i[\hat{\mathbf{e}}_3]_j r_j/s_3$

  Correct
  This form probably flows most naturally from the previous question.

- [x] $r'_i = (I_{ij} - s_i I_{3j}/s_3) r_j$

  Correct
  Another way to write the unit vectors is in terms of the identity matrix $[\hat{\mathbf{e}}_a]_j = I_{aj}$. Think about why this is true.

- [x] $r'_i = (I_{ij} - s_i[\hat{\mathbf{e}}_3]_j/s_3) r_j$

  Correct
  In this form, it's easier to see this as a matrix multiplication. The term in brackets has free indices i and j. Compare this to $[Ar]_i = A_{ij} r_j$.

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3. 1 / 1 points

Based on your answer to the previous question, or otherwise, you should now be able to give an expression for A in its component form by evaluating the components A_{ij} for each row i and column j.

Since A will take a 3D vector, $\mathbf{r}$, and transform it into a 2D vector, $\mathbf{r}'$, we only need to write the first two rows of A. That is, A will be a 2×3 matrix. Remember, the columns of a matrix are the vectors in the new space that the unit vectors of the old space transform to - and in our new space, our vectors will be 2D.

What is the value of A?

- (selected) $\begin{bmatrix} 1 & 0 & -s_1/s_3 \\ 0 & 1 & -s_2/s_3 \end{bmatrix}$

  Correct
  Well done!

- $\begin{bmatrix} -s_1/s_3 & 0 & 0 \\ 0 & -s_2/s_3 & 0 \end{bmatrix}$
- $\begin{bmatrix} r_1 & 0 & -s_1/s_3 \\ 0 & r_2 & -s_2/s_3 \end{bmatrix}$
- $\begin{bmatrix} s_1/s_3 & 0 & -s_1/s_3 \\ 0 & s_2/s_3 & -s_2/s_3 \end{bmatrix}$

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4. 1 / 1 points

A is a 2x3 matrix, but if you were to evaluate its *third* row, what would it's components be?

```
# Fill in the components of the 'third' row of A,
A3 = [0, 0, 0]

```

Run

Reset

Correct Response
Correct! the matrix has all zeros in its final row, as r' never has any value in the third direction.

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5. 1 / 1 points

Assume the Sun's rays come in at a direction $\hat{\mathbf{s}} = \begin{bmatrix} 4/13 \\ -3/13 \\ -12/13 \end{bmatrix}$.

Construct the matrix, A, and apply it to a point, $\mathbf{r} = \begin{bmatrix} 6 \\ 2 \\ 3 \end{bmatrix}$, on an object in our space to find the coordinates of that point's shadow.

Give the coordinates of $\mathbf{r}'$.

```
# Give the coordinates for r'
rp = [7, 5/4]

```

Run

Reset

Correct Response
Well done!

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6. 1 / 1 points

Another use of non-square matrices is applying a matrix to a list of vectors.

Given our transformation $\mathbf{r}' = A\mathbf{r}$, this can be generalized to a matrix equation, $R' = AR$, where R' and R are matrices where each column are corresponding r' and r vectors, i.e.,

$$\begin{bmatrix} r'_1 & s'_1 & t'_1 & u'_1 & \dots \\ r'_2 & s'_2 & t'_2 & u'_2 & \dots \end{bmatrix} = A \begin{bmatrix} r_1 & s_1 & t_1 & u_1 & \dots \\ r_2 & s_2 & t_2 & u_2 & \dots \\ r_3 & s_3 & t_3 & u_3 & \dots \end{bmatrix}.$$

In Einstein notation, $r'_i = A_{ij} r_j$ becomes $R'_{ia} = A_{ij} R_{ja}$.

For the same $\hat{\mathbf{s}}$ as in the previous question, apply A to the matrix

$$R = \begin{bmatrix} 5 & -1 & -3 & 7 \\ 4 & -4 & 1 & -2 \\ 9 & 3 & 0 & 12 \end{bmatrix}.$$

Observe that it's the same result as treating the columns as separate vectors and calculating them individually.

```
Rp = [[8,   0,   -3,   11],
      [7/4,  -19/4,  1,   -5]]

```

Run

Reset

Correct Response
Well done!