

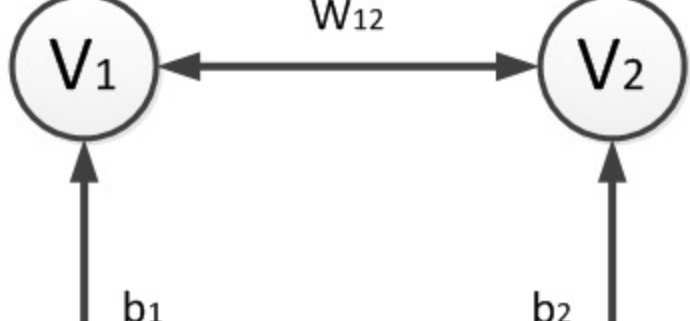
1. If $\Delta E = 3$, then:

- ☐
 $P(s = 1)$ increases when T increases.
- ☒
 $P(s = 1)$ decreases when T increases.

Correct

At $T = 1$, $P(s = 1)$ is 0.95, i.e. fairly close to 1. Increasing the temperature brings that probability closer to 0.5, i.e. it decreases it.

2. The Hopfield network shown below has two visible units: V_1 and V_2 . It has a connection between the two units, and each unit has a bias.



Let $W_{12} = -10$, $b_1 = 1$, and $b_2 = 1$ and the initial states of $V_1 = 0$ and $V_2 = 0$.

If the network always updates both units simultaneously, then what is the lowest energy value that it will encounter (given those initial states)?

If the network always updates the units one at a time, i.e. it alternates between updating V_1 and updating V_2 , then what is the lowest energy value that it will encounter (given those initial states)?

Write those two numbers with a comma between them. For example, if you think that the answer to that first question is 4, and that the answer to the second question is -7, then write this: **4, -7**

Please use a space after the comma in your response to be read/graded properly in the Coursera platform.

0, -1

Correct Response

From the initial state, both units will want to turn on.

If we update both of them at the same time, then both will turn on, leading to a configuration with energy 8. Next, both units will want to turn off,

bringing us back to the initial state, which has energy 0. We'll only ever alternate between those two states, so the lowest energy we'll see is

0.

If we update one unit, say V_1 , first, then it will turn on. Now we're in a state with energy -1. From that state, neither unit will want to

change its state, so we'll stay in that state forever.

3. This question is about Boltzmann Machines, a.k.a. a stochastic Hopfield networks. Recall from the lecture that when we pick a new state s_i for unit i , we do so in a stochastic way: $p(s_i = 1) = \frac{1}{1 + \exp(-\Delta E/T)}$, and $p(s_i = 0) = 1 - p(s_i = 1)$. Here, ΔE is the *energy gap*, i.e. the energy when the unit is off, minus the energy when the unit is on. T is the *temperature*. We can run our system with any temperature that we like, but the most commonly used temperatures are 0 and 1.

When we want to explore the configurations of a Boltzmann Machine, we initialize it in some starting configuration, and then repeatedly choose a unit at random, and pick a new state for it, using the probability formula described above.

Consider two small Boltzmann Machines with 10 units, with the same weights, but with different temperatures. One, the "cold" one, has temperature 0. The other, the "warm" one, has temperature 1. We run both of them for 1000 iterations (updates), as described above, and then we look at the configuration that we end up with after those 1000 updates.

Which of the following statements are true? (Note that an "energy minimum" is what could also reasonably be called a "local energy minimum")

- ☒
 If the weights are small, then over the course of those 1000 updates, the warm one is likely to have encountered more different configurations than the cold one.

Correct

The warm one will jump around between configurations, a lot. The cold one won't: it always reduces energy.

- ☒
 The warm one could end up in a configuration that's not an energy minimum.

Correct

The warm one could end up anywhere, because it's truly stochastic.

- ☒
 If the cold one is exponentially unfortunate, it could end up in a configuration that's not an energy minimum.

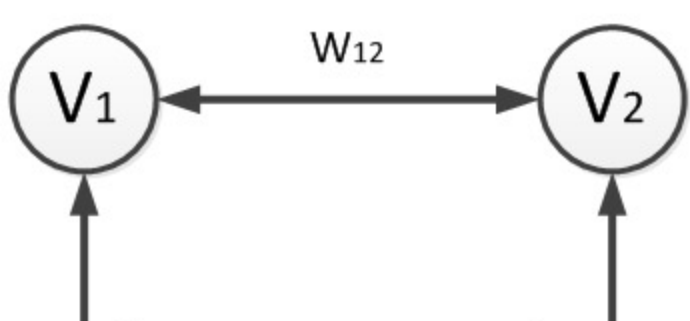
Correct

If the random choice of which unit to update is always the same, then there won't be much progress towards an energy minimum. However, that's very unlikely.

- ☐
 For the cold one, $P(s_i = 1)$ can be any value between 0 and 1, depending on the weights.

Un-selected is correct

4. The Boltzmann Machine shown below has two visible units V_1 and V_2 . There is a connection between the two, and both units have a bias.



Let $W_{12} = -2$, $b_1 = 1$, and $b_2 = 1$.

What is $P(V_1 = 1, V_2 = 0)$? Write your answer with at least 3 digits after the decimal point.

0.3655

Correct Response

There are four configurations. Each has an energy, and from the energies you can calculate the probabilities. $E(V_1 = 0, V_2 = 0) = 0$, because nothing is on, there.

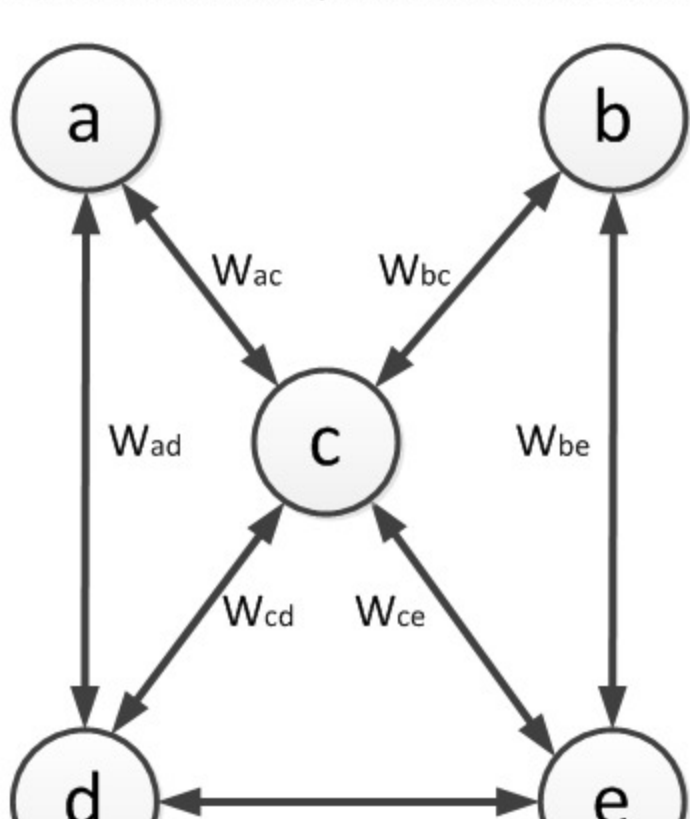
$E(V_1 = 0, V_2 = 1) = -1$, because only one bias comes into play.

$E(V_1 = 1, V_2 = 0) = -1$, likewise. $E(V_1 = 1, V_2 = 1) = 0$, because the two biases contribute energy -2 (-1 each), but the connection contributes energy 2,

now that both units are on; that makes a total of 0. Thus, $\sum_s \exp(-E(s)) = \exp(0) + \exp(1) + \exp(1) + \exp(0) \approx 7.43656$ (where the sum is over the four different states s). Therefore,

$E(V_1 = 1, V_2 = 0) \approx \frac{\exp(1)}{7.43656} \approx 0.3655$, and that's the answer.

5. The figure below shows a Hopfield network with five binary threshold units: a, b, c, d, and e. The network has many connections, but no biases.



Let $W_{ac}=W_{bc}=1$, $W_{ce}=W_{cd}=2$, $W_{be}=-3$, $W_{ad}=-2$, and $W_{de}=3$.

What is the energy of the configuration with the lowest energy? What is the energy of the configuration with the second lowest energy (considering all configurations, not just those that are energy minima)?

Write your answer with a comma between the two numbers. For example, if you think that the energy of the lowest energy configuration is -17, and that

the energy of the second lowest energy configuration is -13, then you should write this: -17, -13

-7, -6

Correct Response

You can simply try all 32 configurations to find the answer, or you can do some clever elimination, such as "if we want low energy, then unit c will definitely be on, because it only has positive connections."