Potential Outcomes Framework

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Causal Effect and Potential Outcomes

- Estimating causal effect of treatment is a challenging task
  - Because **we can NOT observe counterfactual outcomes** if one had chosen different treatments
- In order to obtain causal effect, we need to compare **observed outcomes** with **counterfactual outcomes**
- The **potential outcomes framework** provides a way to quantify causal effects

Potential Outcomes Framework

Potential Outcomes Framework

Treatment

- An intervention, whose effect(s) we wish to assess relative to some other (non-)intervention
- D_i: a dummy that indicate whether individual i receive treatment or not

$$D_i = \begin{cases} 1 & \text{if individual } i \text{ received the treatment} \\ 0 & \text{otherwise.} \end{cases}$$

- Examples:
  - Attend graduate school or not
  - Have health insurance or not
  - Win a lottery or not
  - Increase corporate tax rate or not
  - Democracy v.s. Dictatorship

Potential Outcomes Framework

Treatment

- D_i can be a multiple valued (continuous) variable

$$D_i = s$$

- Examples:
  - Schooling years
  - Number of children
  - Number of polices
  - Number of advertisements
  - Money supply
  - Income tax rate
- **In the following slides, we assume treatment variable D_i is a dummy**

Potential Outcomes Framework

Potential Outcomes

- A potential outcome is the outcome that would be realized if the individual received a specific value of the treatment
- Suppose there are two treatments for each individual:
  - $D_i = 1$
  - $D_i = 0$
- Thus, each individual i has two potential outcomes and one for each value of the treatment
  - Y_i^1: Potential outcome for an individual i if getting treatment
  - Y_i^0: Potential outcome for an individual i if not getting treatment

Potential Outcomes Framework

Potential Outcomes

- **Example:**
  - Annual earnings if attending graduate school
  - Annual earnings if not attending graduate school
- Again, potential outcome can be Y_i^s:
  - s can be continuous
  - More than two potential outcomes
- **How many treatments we have, how many potential outcomes will be**

Potential Outcomes Framework

Observed Outcomes

- For each particular individual, we only can observe one potential outcome
- Observed outcomes Y_i are realized as

$$\mathrm{Y}_i = \mathrm{Y}_i^1 D_i + \mathrm{Y}_i^0 (1 - D_i) \text{ or}$$

$$\mathrm{Y}_i = \begin{cases} \mathrm{Y}_i^1 & \text{if } D_i = 1 \\ \mathrm{Y}_i^0 & \text{if } D_i = 0 \end{cases}$$

- **Only one potential outcome can be realized**
- The unobserved outcome is called the "**counterfactual**" outcome

Casual Effects

Casual Effect

- **Causal effect:** the comparisons between the potential outcomes under each treatment
  - The differences between **observed (potential) outcome** and **counterfactual (potential) outcome**

Casual Effect for an Individual

Casual Effect for an Individual

- Individual Treatment Effect (ITE):

$$\tau_i = \mathrm{Y}_i^1 - \mathrm{Y}_i^0$$

  - Also call Individual Causal Effect
  - The difference between an individual i's outcome under treatment v.s. without treatment
  - **Example:**
    - The difference in individual i's earnings if he/she attends graduate school v.s. not attending graduate school
  - Almost always unidentified without strong assumptions

Individual Treatment Effect (ITE)

An Example

- Imagine a population with 4 people

i	Y_i^1	Y_i^0	Y_i	D_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?

- We want to evaluate the effect of attending graduate school on the annual earnings
  - D_i: Attending graduate school $D_i = 1$, otherwise $D_i = 0$
  - Y_i^1: (Potential) annual earnings if individual i attend graduate school
  - Y_i^0: (Potential) annual earnings if individual i do not attend graduate school
  - Y_i: Observed annual earnings for individual i

Individual Treatment Effect (ITE)

An Example

- What is Individual causal effect (ITE) of attending graduate school for David?
  - We only observe the annual earnings for David who attended graduate school
  - Only observe Y^1
- What is Individual causal effect (ITE) of attending graduate school for Bill?
  - We only observe the annual earnings for Bill who did not attend graduate school
  - Only observe Y^0

Individual Treatment Effect (ITE)

An Example

- Suppose **we can observe counterfactual outcomes**

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0

- The ITE for David: $\alpha_{David} = 1$
- The ITE for Bill: $\alpha_{Bill} = 0$

Causal Effect for General Population

Causal Effect for General Population

- People might be more interested in the **causal effect for general population**
- ITE might not represent causal effect for general population
  - We usually cannot rule out that the ITE differs across individuals ("effect heterogeneity")
- Understand the treatment effect (causal effect) for general population:
  - Estimate the **population average of the individual treatment effects**

Review: Expectation

- We usually use $\mathrm{E}[Y_i]$ (the expectation of a variable Y_i) to denote **population average** of Y_i
- Suppose we have a population with N individuals

$$\mathrm{E}[\mathrm{Y}_i] = \frac{1}{N}\sum_{i=1}^{N}\mathrm{Y}_i$$

- For example: average income for whole population in Taiwan

Causal Effect for General Population

- Assume we have a population with N individuals
- Average Treatment Effect (ATE):

$$\alpha_{\mathsf{ATE}} = \mathrm{E}[\tau_i] = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0] = \frac{1}{N}\sum_{i=1}^{N}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

  - Average of ITEs over the population
    - Average effect of attending graduate school on annual earnings for whole population
  - **Example:**
    - Average difference between the earnings of the same individuals if they attend graduate schools v.s. if not attending graduate schools
  - We'll spend a lot time trying to identify/estimate ATE

Average Treatment Effect (ATE)

An Example

- Missing data problem also arises when we estimate ATE

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?
$E[Y_i^1]$	?				
$E[Y_i^0]$		?			
$E[Y_i^1 - Y_i^0]$					?

- What is the effect of attending graduate school on average annual earnings of whole population (ATE)?
- $\alpha_{\text{ATE}} = E[Y_i^1 - Y_i^0] = ?$

Average Treatment Effect (ATE)

An Example

- Suppose **we can observe counterfactual outcomes**

i	Y_i^1	Y_i^0	Y_i	D_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[\mathrm{Y}_i^1]$	1.75				
$E[\mathrm{Y}_i^0]$		1.25			
$E[\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$					0.5

- What is the effect of attending graduate school on average annual earnings of whole population (ATE)?

- $\alpha_{\text{ATE}} = \dfrac{1+1+0+0}{4} = 0.5$

Causal Effect for a Specific Sub-population

Review: Conditional Expectation

- We usually use $\mathrm{E}[Y_i|X_i = 1]$ to denote the average of Y_i in the population that has $X_i = 1$
- Suppose the population has N_1 individuals with $X = 1$

$$\mathrm{E}[Y_i|X_i = 1] = \frac{1}{N_1} \sum_{i:X=1} \mathrm{Y}_i$$

- For example: average income for those who have master degree in Taiwan
- The above quantities are unique for specific population

Causal Effect for a Specific Sub-population

- Conditional average treatment effect (CATE) for a subpopulation:

$$\alpha_{\mathsf{CATE}} = \mathrm{E}[\tau_i | X_i = f] = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i = f] = \frac{1}{N_f} \sum_{i:X_i=f} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

  - N_f is the number of units in the subpopulation
  - Average of ITEs over the female population
    - Average effect of attending graduate school on annual earnings for **female**
  - **Example:**
    - Average difference between the earnings of **female** if they attend graduate schools v.s. if not attending graduate schools

Causal Effect for Treatment Group

- Average treatment effect on the treated (ATT):

$$\alpha_{\mathsf{ATT}} = \mathrm{E}[\tau_i | D_i = 1] = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 1] = \frac{1}{N_1} \sum_{i:D_i=1} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

where $N_1 = \sum_i 1(D_i = 1)$

  - Note that ATT is a special case of CATE
  - Average of ITEs over the treated population
    - Average effect of attending graduate school on annual earnings for those attending graduate school ($D_i = 1$)
  - **Example:**
    - Average difference between the earnings of those attending graduate schools v.s. earnings if they had not attended graduate schools
  - We'll also spend a lot time trying to identify/estimate ATT

Average Treatment Effect on Treated (ATT)

- Missing data problem also arises when we estimate ATT

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?
$E[Y_i^1 \mid D_i = 1]$	2.5				
$E[Y_i^0 \mid D_i = 1]$		?			
$E[Y_i^1 - Y_i^0 \mid D_i = 1]$					?

- What is the effect of attending graduate school on average annual earnings for those who choose to attend graduate school (ATT)?

- $\alpha_{\text{ATT}} = E[Y_i^1 - Y_i^0 \mid D_i = 1] = ?$

Average Treatment Effect on Treated (ATT)

- Suppose **we can observe counterfactual outcomes**

i	Y_i^1	Y_i^0	Y_i	D_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[\mathrm{Y}_i^1 \mid D_i = 1]$	2.5				
$E[\mathrm{Y}_i^0 \mid D_i = 1]$		1.5			
$E[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 \mid D_i = 1]$					1

- What is the effect of attending graduate school on average annual earnings of those who choose to attend graduate school (ATT)?

- $\alpha_{\text{ATT}} = \dfrac{1+1}{2} = 1$

Causal Effect for a Control Group

- Average treatment effect on the untreated (ATU):

$$\alpha_{\mathsf{ATU}} = \mathrm{E}[\tau_i | D_i = 1] = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 0] = \frac{1}{N_0} \sum_{i:D_i=0} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

where $N_0 = \sum_i 1(D_i = 0)$

  - Note that ATU is a special case of CATE
  - Average of ITEs over the untreated population
    - Average effect of attending graduate school on annual earnings for those NOT attending graduate school ($D_i = 0$)
  - **Example:**
    - Average difference between the earnings of those NOT attending graduate schools v.s. earnings if they had attended graduate schools

Average Treatment Effect on Untreated (ATU)

- Missing data problem also arises when we estimate ATU

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?
$E[Y_i^1 \mid D_i = 0]$	?				
$E[Y_i^0 \mid D_i = 0]$		1			
$E[Y_i^1 - Y_i^0 \mid D_i = 0]$					?

- What is the effect of attending graduate school on average annual earnings for those who choose NOT to attend graduate school (ATU)?

- $\alpha_{\text{ATU}} = E[Y_i^1 - Y_i^0 \mid D_i = 0] = ?$

Average Treatment Effect on Untreated (ATU)

- Suppose **we can observe counterfactual outcomes**

i	Y_i^1	Y_i^0	Y_i	D_i	$Y_i^1 - Y_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[Y_i^1 \mid D_i = 0]$	1				
$E[Y_i^0 \mid D_i = 0]$		1			
$E[Y_i^1 - Y_i^0 \mid D_i = 0]$					0

- What is the effect of attending graduate school on average annual earnings of those who choose NOT to attend graduate school (ATU)?

- $\alpha_{\text{ATU}} = \dfrac{0+0}{2} = 0$

Summary

- Individual Treatment Effect (ITE):

$$\alpha_{\mathsf{ITE}} = \mathrm{Y}_i^1 - \mathrm{Y}_i^0$$

- Average treatment effect (ATE):

$$\alpha_{\mathsf{ATE}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0] = \frac{1}{N} \sum_i [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

- Conditional average treatment effect (CATE):

$$\alpha_{\mathsf{CATE}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i = f] = \frac{1}{N_f} \sum_{i:X_i=f} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

Summary

- Average treatment effect on the treated (ATT):

$$\alpha_{\text{ATT}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 1] = \frac{1}{N_1} \sum_{i:D_i=1} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

- Average treatment effect on the untreated (ATU):

$$\alpha_{\text{ATU}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 0] = \frac{1}{N_0} \sum_{i:D_i=0} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

Fundamental Problem of Causal Inference

Fundamental Problem of Causal Inference

- We can never **directly observe** causal effects (ITE, ATE, CATE, ATT or ATU)
- Because we can never observe both potential outcomes (Y_i^1, Y_i^0) for any individual
- For someone receiving the treatment $(D_i = 1)$
  - Y_i^1 is observed
  - But Y_i^0 is the **unobserved** counterfactual outcome
    - It represents what would have happened to an individual i if assigned to control
- We need to compare **potential outcomes**, but we only have **observed outcomes**
- Causal inference is a **missing data problem**

Potential Outcome Framework in Economics

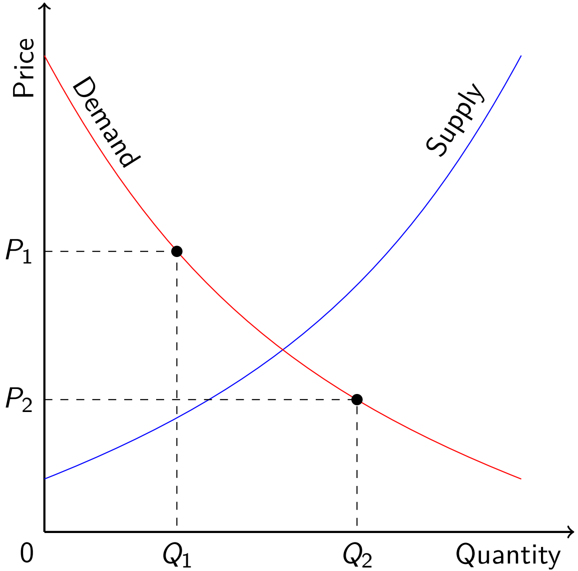
Potential Outcome Framework in Economics

Demand and Supply

- The concepts of potential and observed outcomes are deeply ingrained in economics
  - A demand function represents the potential quantity demanded as a function of price
  - Only the quantity under equilibrium price is realized
  - Other quantities along demand curve are counterfactual

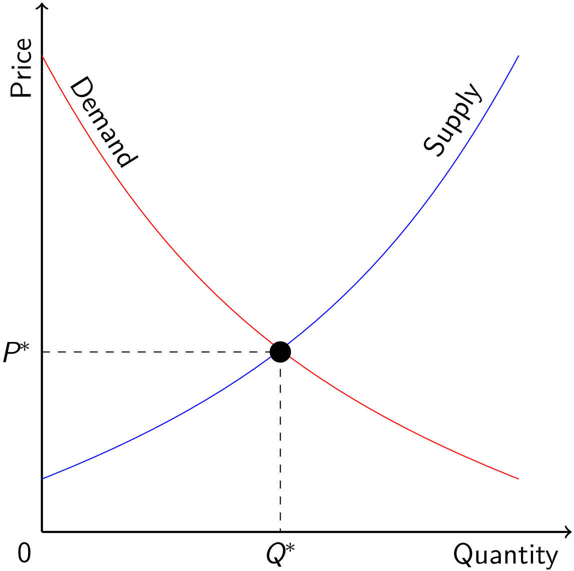
Potential Outcome Framework in Economics

Demand and Supply



Potential Outcome Framework in Economics

Demand and Supply



Stable Unit Treatment Value Assumption

Stable Unit Treatment Value Assumption (SUTVA)

Assumption

Observed outcomes are realized as

$$\mathrm{Y}_i = \mathrm{Y}_i^1 D_i + \mathrm{Y}_i^0 (1 - D_i)$$

- Implies that observed outcomes for an individual i are **unaffected** by the treatment status of other individual j
- Individual i's observed outcomes are only affected by his/her own treatment
- Rules out possible treatment effect from other individuals (spillover effect/externality)

Stable Unit Treatment Value Assumption (SUTVA)

- Could write out potential outcomes in a more extensive fashion, taking into account both one's own treatment status and the treatment status of others

$$\begin{cases} \mathrm{Y}_i^{11} & \text{if } D_i = 1 \text{ and } D_j = 1 \\ \mathrm{Y}_i^{10} & \text{if } D_i = 1 \text{ and } D_j = 0 \\ \mathrm{Y}_i^{01} & \text{if } D_i = 0 \text{ and } D_j = 1 \\ \mathrm{Y}_i^{00} & \text{if } D_i = 0 \text{ and } D_j = 0 \end{cases}$$

- **Example:**
  - Your health status depends on whether you smoke and your father/mother smoke

Stable Unit Treatment Value Assumption (SUTVA)

Examples for Spillover Effect

- **Contagion**:
  - The effect of being vaccinated on one's probability of contracting a disease depends on whether others have been vaccinated
- **Displacement**:
  - Police interventions designed to suppress crime in one location may displace criminal activity to nearby locations.
- **Communication**:
  - Interventions that convey information about commercial products, entertainment, or political causes may spread from individuals who receive the treatment to others who are nominally untreated

Stable Unit Treatment Value Assumption (SUTVA)

- SUTVA may be problematic, so we should choose the units of analysis to minimize interference across units.
- Recent literatures on causal inference are trying to deal with this assumption

Suggested Readings

- Chapter 1, Mastering Metrics: The Path from Cause to Effect
- Chapter 2, Mostly Harmless Econometrics
- Chapter 4, Causal Inference: The Mixtape