Difference-in-Differences Design

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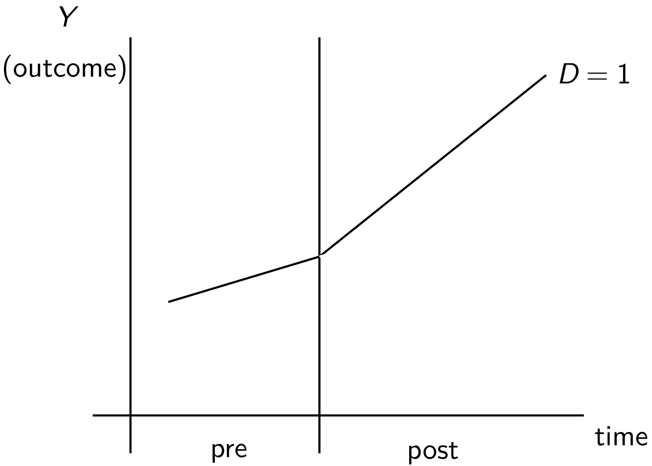
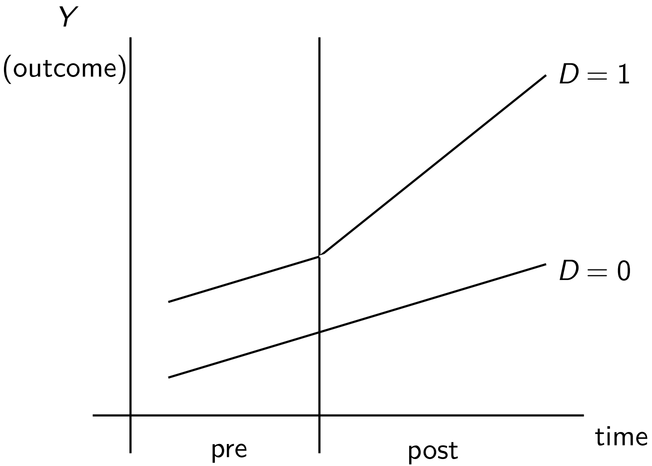
Difference-in-Differences Design: Main Idea

Main Idea of Difference-in-Differences (DID)

- If we can observe **group-level** outcomes several times
  - At least before and after treatment
- Assume **in the absence of treatment**, outcomes of treatment and control group **move in parallel way**
- Then, we can construct the **counterfactual trend in outcomes of treatment group** by using
  - **Trend in outcomes of control group**
- Comparing observed trend with counterfactual trend in outcome of treatment group, we can get causal effect of treatment

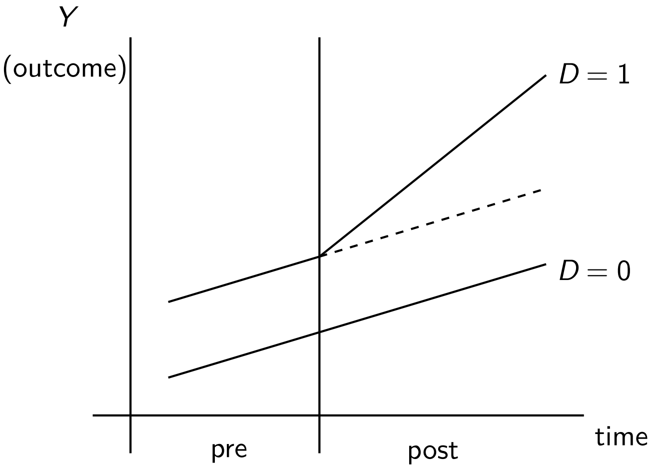


Y
(outcome)
$D = 1$
pre
post
time


Y
(outcome)
$D = 1$
$D = 0$
pre
post
time

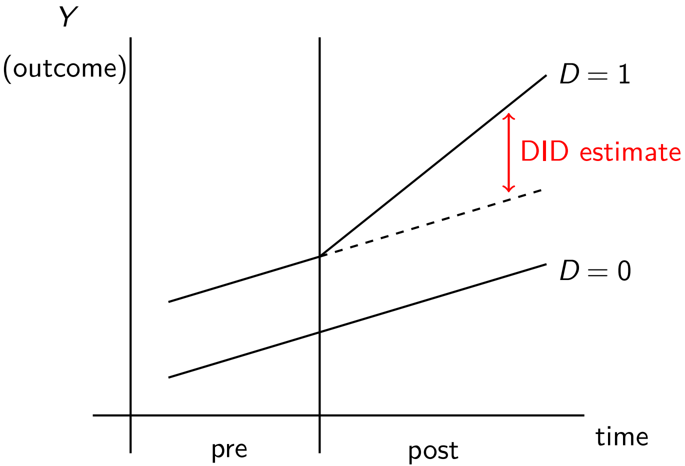
Main Idea of Difference-in-Differences

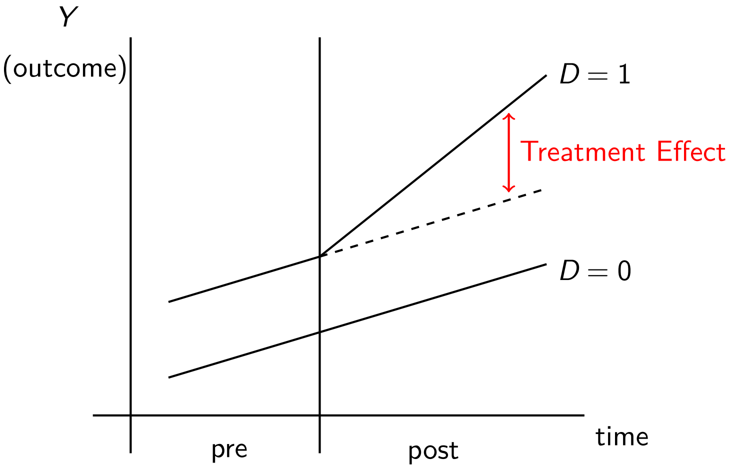
Graph



Main Idea of Difference-in-Differences

Graph





Y
(outcome)
D = 1
Treatment Effect
D = 0
pre
post
time

David Card and Alan B. Krueger (1994) "**Minimum Wages and Employment: A Case Study of the Fast-Food Industry in New Jersey and Pennsylvania**" AER

- They want to estimate the causal effect of raising minimum wage on employment of low-skilled workers

An Example of Difference-in-Differences

Card & Krueger (1994)

- What is the effect of increasing the minimum wage on employment?
- Minimum wage is effective only in certain jobs:
  - Low-skilled jobs
- How much does an increase in the minimum wage reduce demand for low-skilled workers?
  - In a competitive labour market, increases in the minimum wage would move up a downward-sloping labour demand curve.
  - Employment would fall.

An Example of Difference-in-Differences

Card & Krueger (1994)

- Card & Krueger (1994) analyse the effect of a minimum wage increase in New Jersey (NJ) using a DID methodology
- In February 1992 NJ increased the state minimum wage from \$4.25 to \$5.05
- Pennsylvania (PA)'s minimum wage stayed at \$4.25.

- They surveyed about 400 fast food stores both in NJ and in PA both before and after the minimum wage increase in NJ.

An Example of Difference-in-Differences
Card & Krueger (1994)

- Two groups:
  - treatment group: NJ
  - control group: PA
- Two periods:
  - Pre-treatment period: February 1992
  - Post-treatment period: November 1992
- Let Y_{st} denote the average employment in state s at time t

An Example of Difference-in-Differences

Card & Krueger (1994)

- To estimate the effect of minimum wage on employment in NJ, we would like to know the following counterfactual:
  - **In absence of raising minimum wage to $5.05**, what the average employment level in NJ would be ?
- DID method suggests us construct the **counterfactual employment in NJ** by using:
  - Average employment level in NJ before reform +
  - The trend in average employment level in PA (control group)

$$Y_{NJ,Feb} + (Y_{PA,Nov} - Y_{PA,Feb})$$

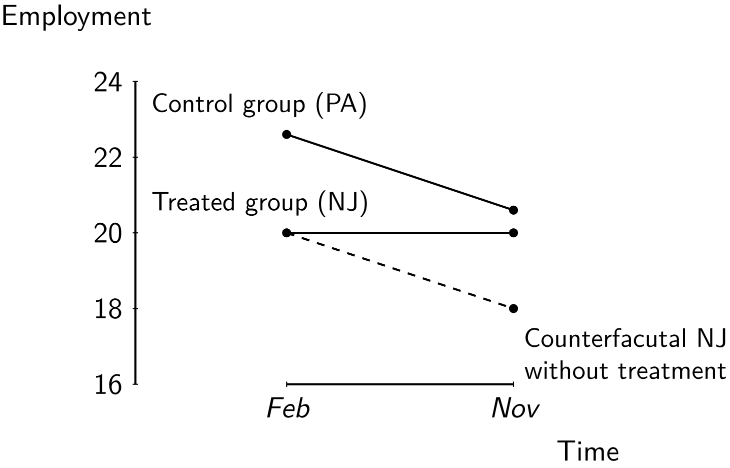
- We can identify the effect of minimum wage on employment in NJ by taking difference in **realized employment** and **counterfactual employment** in NJ:

$$\begin{aligned}\alpha_{DID} &= Y_{NJ,Nov} - [Y_{NJ,Feb} + (Y_{PA,Nov} - Y_{PA,Feb})] \\ &= (Y_{NJ,Nov} - Y_{NJ,Feb}) - (Y_{PA,Nov} - Y_{PA,Feb})\end{aligned}$$

- If PA is a good control group
- The trend in employment rate of PA should absorb any other changes in employment that are unrelated to increase minimum wage

An Example of Difference-in-Differences

Card & Krueger (1994)



An Example of Difference-in-Differences

Card & Krueger (1994)

		Stores by state		
Variable		PA (i)	NJ (ii)	Difference, NJ-PA (iii)
1.	Mean employment at February 1992	23.33 (1.35)	20.44 (0.51)	-2.89 (1.44)
2.	Mean employment at November 1992	21.17 (0.94)	21.03 (0.52)	-0.14 (1.07)
3.	Change in mean employment between Feb and Nov	-2.16 (1.25)	0.59 (0.54)	2.76 (1.44)

- Surprisingly, employment rose in NJ relative to PA after the minimum wage change.

$$\begin{aligned}\alpha_{DID} &= (Y_{NJ,Nov} - Y_{NJ,Feb}) - (Y_{PA,Nov} - Y_{PA,Feb}) \\ &= (21.03 - 20.44) - (21.17 - 23.33) \\ &= 0.59 - (-2.16) = 2.76\end{aligned}$$

- Instead of comparing the employment of NJ in February (before reform) and November (after reform)
- DID suggests we need to adjust for change (trend) in labor demand when there was no increase in minimum wage

Difference-in-Differences Design: Potential Outcomes Framework

DID and Potential Outcomes Framework

- Basic setup: two time periods, two groups
- Two periods
  - In period $t = 1$: one of the groups is treated
  - In period $t = 0$: neither group is treated
- Two groups
  - $D_i = 1$: those that are treated at $t = 1$ (treatment group)
  - $D_i = 0$: those that are always untreated (control group)

DID and Potential Outcomes Framework

- Potential Outcomes
  - Y^1_{it}: the potential outcome for unit i if he would receive treatment at time t
  - Y^0_{it}: the potential outcome for unit i if he would NOT receive treatment at time t

DID and Potential Outcomes Framework

- Observed Outcomes
  - Y_{it} is the observed outcome for unit i at time t
  - Observed outcomes Y_{it} are realized as

$$Y_{it} = \mathrm{Y}^0_{it}(1 - D_i) + \mathrm{Y}^1_{it} D_i$$

  - Observed outcomes at $t = 0$:

$$Y_{i0} = \mathrm{Y}^0_{i0}$$

  - Observed outcomes at $t = 1$:

$$Y_{i1} = \mathrm{Y}^0_{i1}(1 - D_i) + \mathrm{Y}^1_{i1} D_i$$

Identification Results for DID

- Our main interest is average treatment effect on treated (ATT):
- DID can help us identify ATT

$$\alpha_{\text{ATT}} = E[\mathrm{Y}_{i1}^1 - \mathrm{Y}_{i1}^0 | D_i = 1]$$

- Missing data problem: $E[\mathrm{Y}_{i1}^0 | D_i = 1]$ is unknown

Common Trend Assumption

$$
\begin{aligned}
E[\mathrm{Y}^0_{i1} - \mathrm{Y}^0_{i0} | D_i = 1] &= E[\mathrm{Y}^0_{i1} - \mathrm{Y}^0_{i0} | D_i = 0] \\
&= E[\mathrm{Y}_{i1} - \mathrm{Y}_{i0} | D_i = 0]
\end{aligned}
$$

- The treatment group and control group would have exhibited the same trend in the absence of the treatment
- We can use common trend assumption to construct a counterfactual for treatment group at $t = 1$

$$
\begin{aligned}
\mathrm{E}[\mathrm{Y}^0_{i1} | D_i = 1] &= \mathrm{E}[\mathrm{Y}^0_{i0} | D_i = 1] + \mathrm{E}[\mathrm{Y}^0_{i1} - \mathrm{Y}^0_{i0} | D_i = 0] \\
&= \mathrm{E}[\mathrm{Y}_{i0} | D_i = 1] + \mathrm{E}[\mathrm{Y}_{i1} - \mathrm{Y}_{i0} | D_i = 0]
\end{aligned}
$$

- We can use **observed outcomes** to represent **unobserved** $\mathrm{E}[\mathrm{Y}^0_{i1} | D_i = 1]$

Identification Results for DID

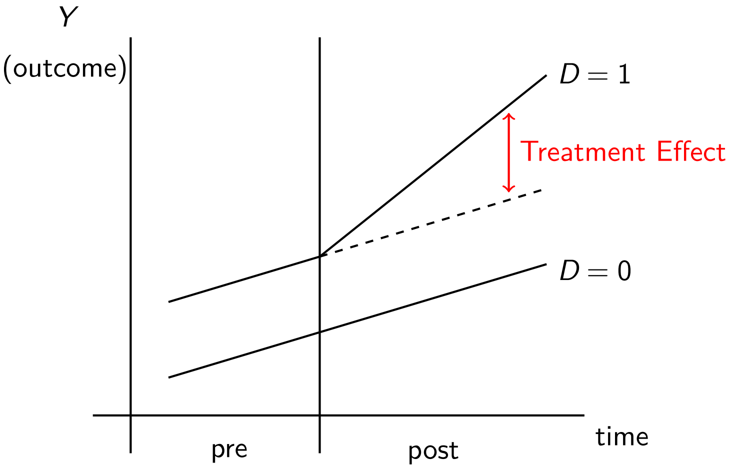
- Apply common trend assumption:

$$\begin{aligned}
\alpha_{\mathsf{ATT}} &= \mathrm{E}[\mathrm{Y}_{i1}^{1} - \mathrm{Y}_{i1}^{0} | D_i = 1] \\
&= \mathrm{E}[\mathrm{Y}_{i1}^{1} | D_i = 1] - \mathrm{E}[\mathrm{Y}_{i1}^{0} | D_i = 1] \\
&= \mathrm{E}[\mathrm{Y}_{i1}^{1} | D_i = 1] - \mathrm{E}[\mathrm{Y}_{i0}^{0} | D_i = 1] - \mathrm{E}[\mathrm{Y}_{i1}^{0} - \mathrm{Y}_{i0}^{0} | D_i = 0] \\
&= \mathrm{E}[\mathrm{Y}_{i1}^{1} - \mathrm{Y}_{i0}^{0} | D_i = 1] - \mathrm{E}[\mathrm{Y}_{i1}^{0} - \mathrm{Y}_{i0}^{0} | D_i = 0] \\
&= \mathrm{E}[\mathrm{Y}_{i1} - \mathrm{Y}_{i0} | D_i = 1] - \mathrm{E}[\mathrm{Y}_{i1} - \mathrm{Y}_{i0} | D_i = 0] = \alpha_{\mathsf{DID}}
\end{aligned}$$

- The **average treatment effect on treated (ATT)** can be identified by difference in trend of outcome between treatment and control groups

Identification Results for DID

Graphical Interpretation



Difference-in-Differences Design: Estimation

- We can estimate the DID estimator in a regression framework
- Advantages:
  - It is easy to calculate standard errors
  - We can control for other variables which may reduce the selection bias further
  - It is easy to include multiple periods
  - We can study treatments with different **treatment intensity** (e.g. varying increases in the minimum wage for different states): **continous DID**

DID Estimation

Basic Two Periods/Groups

- Basic case: two groups and two periods
- We can estimate the DID estimator in a regression framework
- To implement DID method in a regression framework, we estimate:

$$Y_{ist} = \mu + \gamma D + \delta Post + \alpha(D \cdot Post) + \varepsilon_{ist},$$

  - D is a dummy indicating treatment group
  - $Post$ is a dummy indicating post-treatment period
  - γ captures differences across groups that are constant over time
  - δ captures differences over time that are common to all groups

$$Y_{ist} = \mu + \gamma D + \delta Post + \alpha(D \cdot Post) + \varepsilon_{ist},$$

- α is the coefficient of interest
  - Capture the different trends in outcome between treatment and control group
- We will show that α can represent the DID estimator:

$$\begin{aligned} \alpha = & \{\mathrm{E}[\mathrm{Y}_{ist}|D=1, Post=1] - \mathrm{E}[\mathrm{Y}_{ist}|D=1, Post=0]\} \\ & - \{\mathrm{E}[\mathrm{Y}_{ist}|D=0, Post=1] - \mathrm{E}[\mathrm{Y}_{ist}|D=0, Post=0]\} \end{aligned}$$

$$Y_{ist} = \mu + \gamma D + \delta Post + \alpha(D \cdot Post) + \varepsilon_{ist},$$

- If $E[\varepsilon_{st}|D, Post] = 0$ Then, we can show that
  - **Pre-treatment mean of outcome for control group:** $\mathrm{E}[\mathrm{Y}_{ist}|D = 0, Post = 0] = \mu$
  - **Post-treatment mean of outcome for control group:** $\mathrm{E}[\mathrm{Y}_{ist}|D = 0, Post = 1] = \mu + \delta$
  - **Pre-treatment mean of outcome for treatment group:** $\mathrm{E}[\mathrm{Y}_{ist}|D = 1, Post = 0] = \mu + \gamma$
  - **Post-treatment mean of outcome for treatment group:** $\mathrm{E}[\mathrm{Y}_{ist}|D = 1, Post = 1] = \mu + \gamma + \delta + \alpha$

- α can represent treatment effect identified by DID design α_{DID}:

$$\begin{aligned}\alpha_{DID} &= \{\mathrm{E}[\mathrm{Y}_{ist}|D=1, Post=1] - \mathrm{E}[\mathrm{Y}_{ist}|D=1, Post=0]\} \\ &- \{\mathrm{E}[\mathrm{Y}_{ist}|D=0, Post=1] - \mathrm{E}[\mathrm{Y}_{ist}|D=0, Post=0]\} \\ &= \{(\mu+\gamma+\delta+\alpha) - (\mu+\gamma)\} - \{(\mu+\delta) - \mu\} \\ &= \alpha\end{aligned}$$

	Pre	Post	Pre/Post difference
Control Group	μ	$\mu + \delta$	δ
Treatment Group	$\mu + \gamma$	$\mu + \gamma + \delta + \alpha$	$\delta + \alpha$
DID			α

Difference-in-Differences Design: STATA Example

Empirical Example 1: Eissa and Jeffrey (1996)

Eissa, Nada, and Jeffrey B. Liebman. (1996) "**Labor Supply Responses to the Earned Income Tax Credit**" QJE

- They want to look at the effect of tax credit on labor supply

Empirical Example 1: Eissa and Jeffrey (1996)

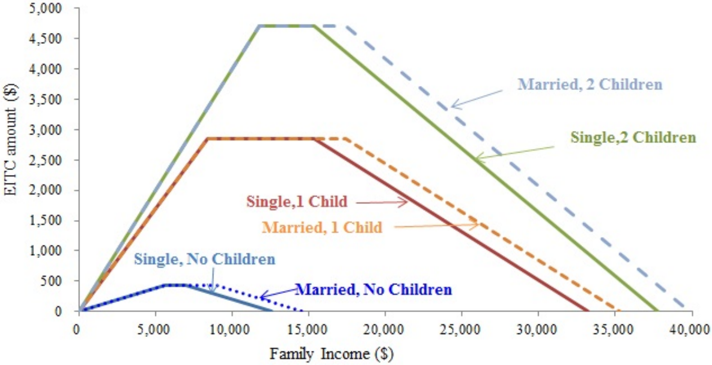
STATA Implementation

- See DID.do
- Use eitc.dta

Empirical Example 1: Eissa and Jeffrey (1996)

- Earned Income Tax Credit (EITC) is a refundable tax credit that subsidizes earnings of working poor in US
- The amount of cash transfer depends on the number of children and previous year earnings
- In 1994, the amount of EITC had large increase for those who have children
- The author examined how did labor supply respond to this change in tax credit using DID design

EITC benefit rule



- The first step of DID analysis is to define treatment and control groups
  - Treatment group: those who have at least one children
    - They receive much more tax credit after 1994
  - Control group: those who do not have children
    - Their tax credit did not increase after 1994

- D: a dummy that indicate whether individual i had children or not

$$D = \begin{cases} 1 & \text{if individual } i \text{ had at least one children} \\ 0 & \text{if individual } i \text{ did not have children} \end{cases}$$

- *Post*: a dummy that indicate whether individual i was observed after 1994 (Post-treatment period)

$$Post = \begin{cases} 1 & \text{if individual } i \text{ was observed after 1994} \\ 0 & \text{if individual } i \text{ was observed before 1994} \end{cases}$$

- $D \times Post$: a treatment dummy that indicate whether individual i was affected by 1994 EITC expansion

STATA Command

Step 1: Define treatment and control groups

- Example:

```
** a dummy for treatment group
gen anykids = (children >= 1)
** a dummy for post-treatment period
gen post93 = (year >= 1994)
** treatment variable (DID key variable)
gen eitc = post93*anykids
```

Empirical Example 1: Eissa and Jeffrey (1996)

Step 2: Graphical Analysis

- Plot the time trend of outcomes for treatment and control groups
    - Check whether there is a **common trend** in outcomes of treatment and control groups **before reform**
    - Examine whether the outcomes of treatment group exhibits different trend **after reform**

STATA Command

Step 2: Graphical Analysis

- Example:

```
1 ** generate time trend in outcome ("work") for treatment and control group ("anykids")
2 collapse (mean) work, by(year anykids)
```

- **collapse:** This command converts the data into a dataset of summary statistics, such as sums, means, medians, and so on
- **Line 2:** converts the data into mean of “work” (Labor Force Participation Rates) by group and year - group and year mean

- Example:

```
** for control group
gen work0 = work if anykids==0
label var work0 "Single women, no children"
** for treatment group
gen work1 = work if anykids==1
label var work1 "Single women, children"
```

STATA Command

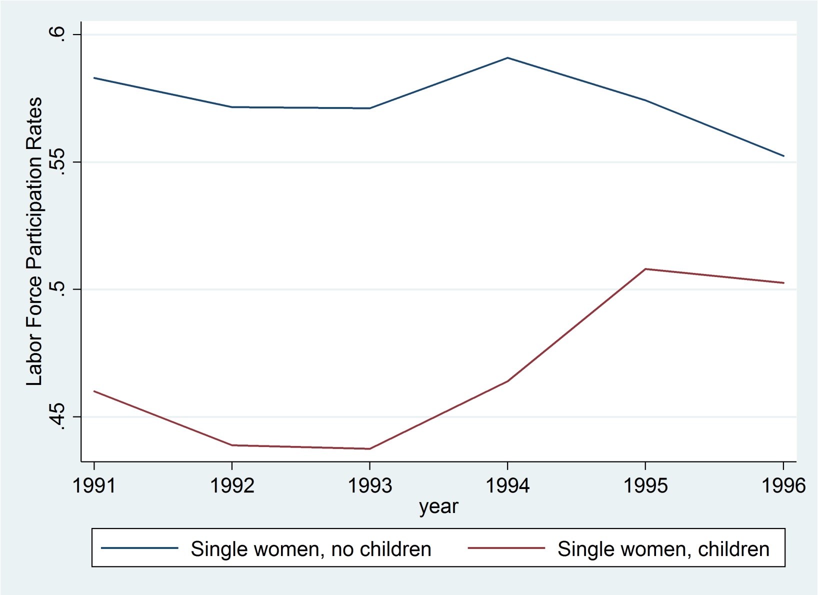
Step 2: Graphical Analysis

- Example:

```
** graph outcome by group
twoway (line work0 year, sort) (line work1 year, sort), ytitle(Labor Force Participation Rates)
graph export "$pic\eitc_DID.png", replace width(3000)
```

- Create a twoway gragh ("work" by "year") for treatment and control groups

DID graph



Labor Force Participation Rates
.45
.5
.55
.6
1991
1992
1993
1994
1995
1996
year
Single women, no children
Single women, children

STATA Command

Step 3: Show the group means in the pre/post-treatment period

- Example:

```
** pre-treatment
mean work if post93==0 & anykids==0
mean work if post93==0 & anykids==1
** post-treatment
mean work if post93==1 & anykids==0
mean work if post93==1 & anykids==1
```

- We can estimate the following DID regression:

$$Y_i = \mu + \gamma D_i + \delta Post_i + \alpha (D_i \cdot Post_i) + X_i' \beta + \varepsilon_i,$$

- Simpe DID regression:

```
** Simpe DID regression
reg work post93 anykids eitc,r
```

STATA Command

Step 4: DID regression

- Control more covariates :

```
** add more variables
**Create age-squared variable
gen age2 = age^2

** Create Non-labor income
gen nonlaborinc = finc - earn
** Control more covariates^^I
reg work post93 anykids eitc nonwhite age age2 ed
    finc nonlaborinc,r
```

DID Results

```
. reg work post93 anykids eitc nonwhite age age2 ed finc nonlaborinc,r

Linear regression                               Number of obs     =     13,746
                                                F(9, 13736)       =     122.54
                                                Prob > F          =     0.0000
                                                R-squared         =     0.1993
                                                Root MSE          =     .44741
```

work	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
post93	-.008003	.011667	-0.69	0.493	-.030872	.014866
anykids	-.072796	.0118336	-6.15	0.000	-.0959914	-.0496006
eitc	.0429062	.0156294	2.75	0.006	.0122704	.0735421
nonwhite	-.0636661	.0081387	-7.82	0.000	-.0796191	-.0477131
age	.0333505	.003121	10.69	0.000	.0272328	.0394681
age2	-.0004231	.0000427	-9.92	0.000	-.0005067	-.0003395
ed	.0144307	.001531	9.43	0.000	.0114297	.0174317
finc	9.02e-06	7.44e-07	12.13	0.000	7.56e-06	.0000105
nonlaborinc	-.000027	1.18e-06	-22.83	0.000	-.0000293	-.0000247
_cons	-.1554345	.0584772	-2.66	0.008	-.2700577	-.0408112

- Show the treatment effect by treatment intensity:

```
** treatment intensity
gen onekid = (children==1)
gen twokid = (children>=2)
gen eitc_one = post93*onekid
gen eitc_two = post93*twokid
reg work post93 onekid twokid eitc_one eitc_two
    nonwhite age age2 ed finc nonlaborinc,r
```

DID Results

```
. reg work post93 onekid twokid eitc_one eitc_two nonwhite age age2 ed finc nonlaborinc,r
```

Linear regression		
Number of obs	=	13,746
F(11, 13734)	=	111.21
Prob > F	=	0.0000
R-squared	=	0.2018
Root MSE	=	.44676

work	Coef.	Robust Std. Err.	t	P>|t|	[95% Conf.	Interval]
post93	-.0080963	.0116636	-0.69	0.488	-.0309584	.0147659
onekid	-.0257933	.0140935	-1.83	0.067	-.0534185	.0018319
twokid	-.1084904	.0134775	-8.05	0.000	-.1349081	-.0820726
eitc_one	.0194807	.0202555	0.96	0.336	-.0202229	.0591842
eitc_two	.0576754	.0175247	3.29	0.001	.0233244	.0920263
nonwhite	-.0589521	.0081535	-7.23	0.000	-.0749341	-.0429702
age	.0353075	.0031283	11.29	0.000	.0291756	.0414394
age2	-.0004527	.0000428	-10.57	0.000	-.0005366	-.0003688
ed	.0145814	.0015276	9.55	0.000	.0115872	.0175756
finc	8.95e-06	7.41e-07	12.08	0.000	7.50e-06	.0000104
nonlaborinc	-.0000266	1.18e-06	-22.59	0.000	-.000029	-.0000243
_cons	-.1873079	.0584447	-3.20	0.001	-.3018675	-.0727482

- use i.yeari.anykids to generate a set of dummy variables representing the interaction term between year and treatment group

```
reg work i.year##i.anykids  nonwhite age age2 ed
    finc nonlaborinc,r
```

DID Results

```
. reg work i.year##i.anykids  nonwhite age age2 ed finc nonlaborinc,r

Linear regression                               Number of obs     =     13,746
                                                F(17, 13728)      =      66.34
                                                Prob > F          =     0.0000
                                                R-squared         =     0.1998
                                                Root MSE          =     .44742

------------------------------------------------------------------------------
             |               Robust
        work |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        year |
       1992  |   .0024691   .0189465     0.13   0.896    -.0346686    .0396068
       1993  |    .004421   .0192407     0.23   0.818    -.0332934    .0421354
       1994  |   .0106316   .0192887     0.55   0.582     -.027177    .0484401
       1995  |  -.0139931    .020325    -0.69   0.491    -.0538328    .0258466
       1996  |  -.0158603   .0212575    -0.75   0.456    -.0575279    .0258074
             |
   1.anykids |  -.0581311   .0187154    -3.11   0.002    -.0948158   -.0214463
             |
year#anykids |
     1992 1  |  -.0216739   .0250962    -0.86   0.388    -.0708658     .027518
     1993 1  |    -.02384   .0253685    -0.94   0.347    -.0735658    .0258858
     1994 1  |   .0000791   .0256784     0.00   0.998     -.050254    .0504122
     1995 1  |   .0558004     .02687     2.08   0.038     .0031315    .1084693
     1996 1  |   .0314123   .0281061     1.12   0.264    -.0236796    .0865041
```

- Creating a placebo DID model is when you arbitrarily choose a treatment time before your actual treatment time
- Test to see if you get a "significant" treatment effect (Hope not)

```
gen placebo = (year >= 1992)
gen placeboXany = anykids*placebo
reg work anykids placebo placeboXany if year<1994
```

DID Results

```
. reg work anykids placebo placeboXany if year<1994

      Source |       SS           df       MS      Number of obs   =     7,401
-------------+----------------------------------   F(3, 7397)      =     41.89
       Model |  30.910802         3  10.3036007   Prob > F        =    0.0000
    Residual |  1819.32944     7,397  .245955041   R-squared       =    0.0167
-------------+----------------------------------   Adj R-squared   =    0.0163
       Total |  1850.24024     7,400  .250032465   Root MSE        =    .49594

------------------------------------------------------------------------------
        work |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
     anykids |  -.1229792   .0196401    -6.26   0.000    -.1614794   -.0844791
     placebo |  -.0116737   .0184985    -0.63   0.528     -.047936    .0245885
 placeboXany |  -.0101282   .0244038    -0.42   0.678    -.0579666    .0377103
       _cons |   .5830325    .014899    39.13   0.000     .5538262    .6122388
------------------------------------------------------------------------------
```

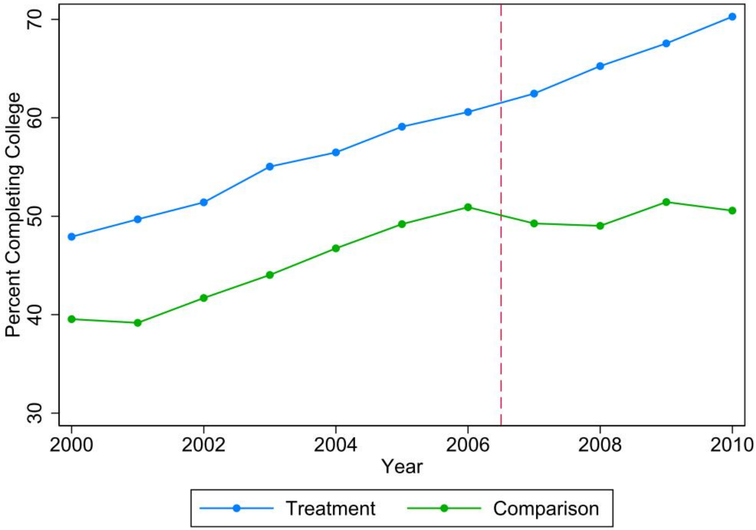
Difference-in-Differences Design: Examine Common Trend Assumption

Test Common Trend Assumption

- The key assumption for any DID strategy is **common trend assumption**
- The outcome in treatment and control group would follow **the same time trend in the absence of the treatment**.
    - This does not mean that they have to have the same mean of the outcome!
    - Common trend assumption is difficult to verify.
    - We can use **pre-treatment data** to show that the trends are the same:
        - Graphical evidence
        - DID event-study design

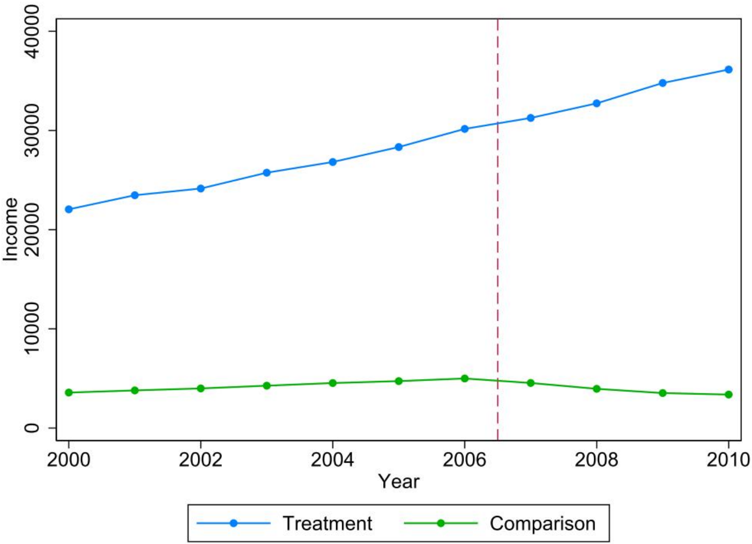
Test Common Trend Assumption

Graphical Evidence



Test Common Trend Assumption

Graphical Evidence



- We can include leads and lags into the DID design:
    1. Examine common trend assumption.
    2. Analyze whether the treatment effect changes over time after treatment happens
- It is so-called DID event-study design.

DID Event-Study Design

- The estimated regression would be:

$$Y_{it} = \alpha + \beta D_i + \sum_{k=-m}^{q} \delta_t \mathbf{I}[t - E_i = k] \\ + \sum_{k=-m}^{q} \gamma_t D_i \cdot \mathbf{I}[t - E_i = k] + X_{it}'\theta + \varepsilon_{it},$$

  - E_i represents the timing when treatment happens.
  - $\mathbf{I}[t - E_i = k]$ is an indicator for being k years from the treatment event
  - t is the calendar year
    - Treatment occurs in $k = 0$ ($t = E_i$)
    - For example, $\mathbf{I}[t - E_i = -1]$ is a dummy variable indicating one year before treatment occurs
    - We usually use time $k = -1$ as baseline year

DID Event-Study Design

- The estimated regression would be:

$$Y_{it} = \alpha + \beta D_i + \sum_{k=-m}^{q} \delta_t \mathbf{I}[t - E_i = k] + \sum_{k=-m}^{q} \gamma_t D_i \cdot \mathbf{I}[t - E_i = k] + X_{it}'\theta + \varepsilon_{it},$$

  - $\gamma_{-2}, \gamma_{-3}, ..., \gamma_{-m}$ represent pre-trend
    - These coefficients should be zero if common trend assumption holds
  - $\gamma_0, \gamma_1, ..., \gamma_q$ represent post-treatment effects

Hsing-Wen Han, Kuang-Ta Lo, Yung-Yu Tsai, and Tzu-Ting Yang (2023), "**The Effect of Financial Resources on Fertility: Evidence from Administrative Data on Lottery Winners**", Working Paper

- During the past fifty years, fertility rates in developed countries have declined dramatically
- Low fertility rate leads to the growth of an aging population, workforce shortages, and reductions in tax revenue.
- Many countries initiated child-related cash transfer policies to encourage childbearing.
  - On average, the public spending of child-related cash benefits accounts for 1.1% of GDP in OECD countries.
- The rationale behind these policies is that people do not have enough income to afford the expense of raising children, so the government needs to subsidize them.

- Empirically, there is an endogenous problem between income and fertility.
    - Reverse Causality
    - Income effect confounds with substitution effect
        - Both working and raising children are time-consuming activities
        - A sudden increase in wage income can increase the relative price of having children
        - Higher wage income would make people work more and reduce demand for children

Empirical Example: Han et al. (2023)

DID Event-Study Design

- This paper examines the fertility impact of the large and permanent income shock generated by winning lottery prizes.
- We implement an DID event-study design to examine the causal effect of large income shock on fertility.
- Compare the trend in fertility before and after receiving a windfall gain between:
  - Households winning 1,000,000 NT$ from lottery prizes.
  - Households winning less than 10,000 NT$.

- We estimate the following regression:

$$
\begin{aligned}
Y_{it} = \alpha + \beta D_i &+ \sum_{k=-3}^{6} \delta_t \mathbf{I}[t - E_i = k] \\
&+ \sum_{k=-3}^{6} \gamma_t D_i \cdot \mathbf{I}[t - E_i = k] + X'_{it}\theta + \varepsilon_{it},
\end{aligned}
$$

  - D_i represents treatment group dummy.
  - Treatment Group:
    - Households who earn more than 1,000,000 NT$ by winning lotteries in a given year
  - Control group:
    - Households who earn less than 10,000 NT$ from winning lotteries during sample period

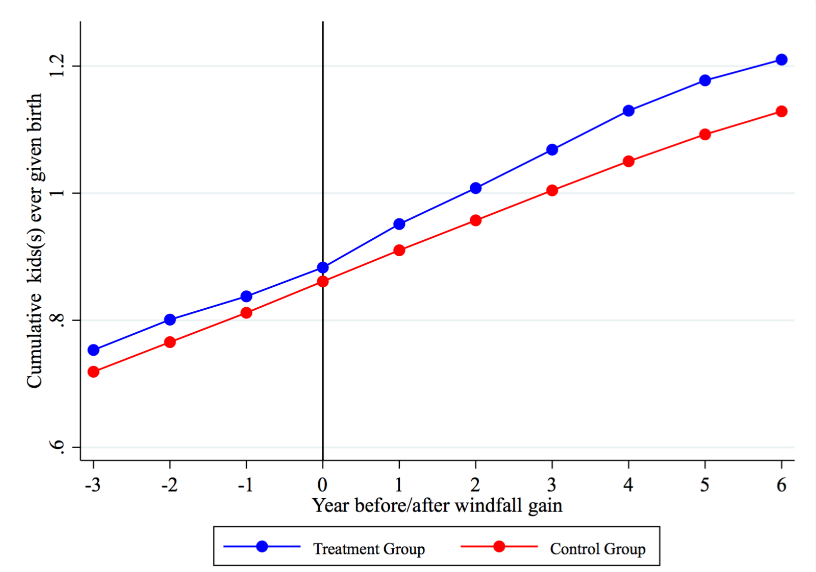
- We estimate the following regression:

$$Y_{it} = \alpha + \beta D_i + \sum_{k=-3}^{6} \delta_t \mathbf{I}[t - E_i = k]$$
$$+ \sum_{k=-3}^{6} \gamma_t D_i \cdot \mathbf{I}[t - E_i = k] + X_{it}'\theta + \varepsilon_{it},$$

  - Outcome variable Y_{it}:
    - Cumulative number of children for household i in the year t
  - E_i is the lottery-winning year
  - We use $\mathbf{I}[t - E_i = k]$, where $k = -3, -2, 0, 1, 2, 3, 4, 5, 6$, to denote dummy variables for the year before and after winning lottery.
  - For example, $\mathbf{I}[t - E_i = 1]$ represents a dummy for the first year after winning lottery.

Test Common Trend Assumption

Raw Data: Cumulative Number of Children



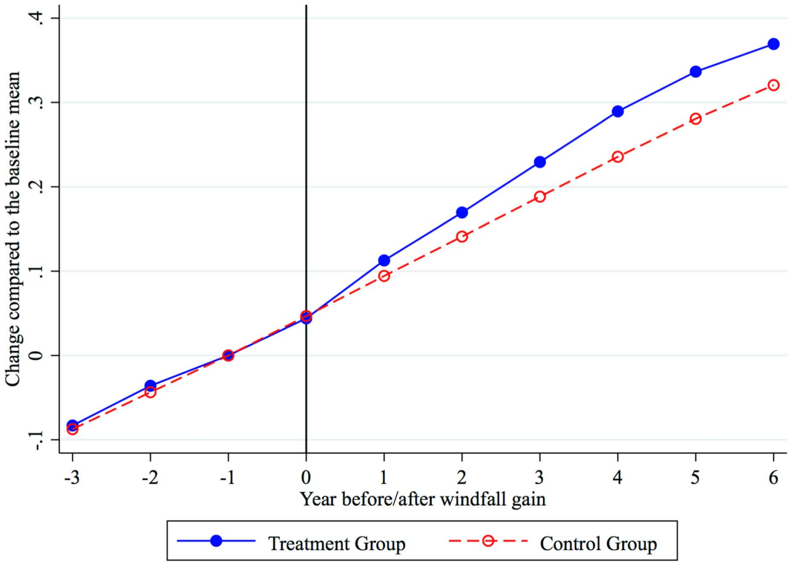
Test Common Trend Assumption

Raw Data: Cumulative Number of Children

- Since we focus on trend rather than level, we sometimes subtract the baseline mean ($k = -1$) from the outcome at each time period

Test Common Trend Assumption

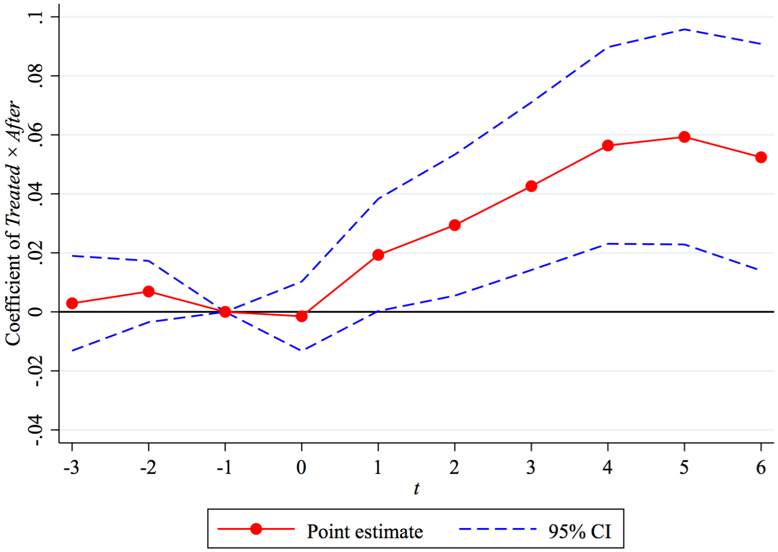
Subtract the Baseline Mean: Cumulative Number of Children



- We can formally examine common trend assumption by showing the estimated coefficients $\gamma_{-2}, \gamma_{-3}, ..., \gamma_6$
- If common trend assumption is valid γ_{-2}, γ_{-3} should be close to zero
- $\gamma_0, \gamma_1, ..., \gamma_6$ represent the treatment effects of winning lotteries

Test Common Trend Assumption

DID Event-Study Design: Cumulative Number of Children



Another Way to Test Common Trend Assumption

- Conduct a DID estimation using pre-treatment data
- Arbitrarily choose a "treatment timing" in the pre-treatment period

$$Y_i = \mu + \gamma D_i + \delta Placebo_i + \alpha(D_i \cdot Placebo_i) + X_i'\beta + \varepsilon_i,$$

- *Placebo* is a dummy indicating fake "post-treatment" period
- If common trend assumption is valid, we would expect $\alpha = 0$

Difference-in-Differences Design: Other Issues

Change in Sample Composition

- In repeated cross-sectional data, it is possible the treatment might result in composition change of treatment and control groups
  - Example: Policy reform could induce migration
- This could bias the estimates of causal effect
- **Valid DID estimate requires:** The distribution of covariates X in treatment and control groups should be similar for the pre-treatment and post-treatment periods
- **Conduct a DID estimation using covariates as "outcomes"**
  - Hope to see there is NO effect of DID estimator

Statistical Inference in DID Estimation

- Many papers using a DD strategy use data from many years (not only 1 pre and 1 post period)
- The variables of interest in many of these setups only vary at a group level (say state) and outcome variables are often serially correlated
- As Bertrand, Duflo, Mullainathan (2004) point out, conventional standard errors often severely **understate** the standard deviation of the estimators

Statistical Inference in DID Estimation

- Simple solution:
  - Clustering standard errors at the group level
  - In STATA one would simply add cl(state) to the regression equation if one analyzes state level variation
- Other solutions:
  - Block bootstrapping standard errors
  - Wild bootstrap clustering standard errors

Suggested Readings

- Chapter 5, Mastering Metrics: The Path from Cause to Effect
- Chapter 5, Mostly Harmless Econometrics
- Chapter 9, Causal Inference: The Mixtape