Problem of Bad Controls

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Bad Control Problem: Main Idea

Bad Control Problem

- Controlling for additional covariates increases the likelihood that regression estimates have a causal interpretation
- **Bad control problem**: More controls are not always better
  - Bad controls are **variables that could themselves be outcomes, which are also affected by treatment**
- **The bad control problem is a version of selection bias**

Bad Control Problem

- We should **NOT include bad controls** into regression or matching process even if including them can change estimated coefficients of treatment effect
- Good controls are variables that is **pre-determined**
  - **The value of variables have been determined before getting treatment**
  - Whether the variables are pre-determined or not, depending on timing of treatment
  - **Examples:**
    - The effect of master degree
    - **Pre-determined variables:** Gender, age, birth place, father's education, mother's education

Bad Control Problem and Selection Bias

Example

- We are interested in the effect of a college degree on earnings.
- People can work in two occupations:
  - White collar ($W_i = 1$)
  - Blue collar ($W_i = 0$)
- Occupation is highly correlated with both education (treatment) and earnings (outcome)
  - Occupation is a potential omitted variable, should we include it into our regression ?
  - Should we look at the effect of college degree on earnings for those within an occupation (e.g. white collar) ?

Bad Control Problem and Selection Bias

Example

- Note that a college degree also increases the chance of getting a high-paying white collar job.
- That is, occupational choices are also affect by treatment (get college degree): **Bad Controls**

Bad Control Problem and Selection Bias

Example

- Comparisons of earnings by college degree status within an occupation are no longer apples-to-apples comparison
  - Those who have college degree and white color jobs
  - Those who do not have college degree but still have white color jobs
  - Two groups are different types of people (e.g. different ability)
- Even if college degree completion is randomly assigned

Bad Control Problem and Selection Bias

Intuition

- If our goal was to estimate the causal effect of college degree on earnings, it would be a bad idea to control for occupation
- The reason is that one of the main ways that education can affect one's earning is through changing occupation
- If our regression controls for occupation, we might shut down this channel and underestimate the effect of college degree
  - The causal effect of college degree on earnings given the occupation does not change

Good Controls

Example

$$\begin{tikzcd}
 & X \arrow[dl] \arrow[dr] & \\
D & & Y
\end{tikzcd}$$

- X is the confounding factor and good control variable
- If you want to estimate the (total) effect of treatment D, you should control for all confounding factors X

Bad Controls

Example

$$
\begin{array}{ccc}
 & W & \\
\nearrow & & \searrow \\
D & & Y
\end{array}
$$

- W is the mediator and bad control variable
- If you want to estimate the (total) effect of treatment D, you should NOT control for mediator W

Bad Control Problem: Formal Illustration

Bad Control Problem and Selection Bias

Formal Illustration

- The realization of earnings Y_i and occupations W_i is determined by college graduation status D_i

$$\mathrm{Y}_i = \mathrm{Y}_i^1 D_i + \mathrm{Y}_i^0 (1 - D_i)$$
$$\mathrm{W}_i = \mathrm{W}_i^1 D_i + \mathrm{W}_i^0 (1 - D_i)$$

- D_i: a dummy that indicate whether individual i gets college degree or not

$$D_i = \begin{cases} 1 & \text{if individual } i \text{ gets college degree} \\ 0 & \text{otherwise.} \end{cases}$$

Bad Control Problem and Selection Bias

Formal Illustration

- Potential outcomes for earnings:
  - Y_i^1: Potential earnings for an individual *i* getting college degree
  - Y_i^0: Potential earnings for an individual *i* not getting college degree

Bad Control Problem and Selection Bias

Formal Illustration

- Potential outcomes for occupation:
  - W_i^1: Potential occupation for an individual i getting college degree
  - W_i^0: Potential occupation for an individual i not getting college degree
- W_i^d: a dummy that indicate whether individual i have white collar job or not

$$W_i^1 = \begin{cases} 1 & \text{if individual } i \text{ with college degree becomes white collar} \\ 0 & \text{if individual } i \text{ with college degree becomes blue collar} \end{cases}$$

$$W_i^0 = \begin{cases} 1 & \text{if individual } i \text{ without college degree becomes white colla} \\ 0 & \text{if individual } i \text{ without college degree becomes blue collar} \end{cases}$$

Bad Control Problem and Selection Bias

Formal Illustration

- Assume that college degree completion D_i is randomly assigned
- So D_i is independent of all potential outcomes $(\mathrm{Y}_i^1, \mathrm{Y}_i^0, \mathrm{W}_i^1, \mathrm{W}_i^0)$

Bad Control Problem and Selection Bias

Formal Illustration

- We have no trouble estimating the causal effect of D_i on Y_i since independence gives us ATE:

$$\mathrm{E}[\mathrm{Y}_i|D_i = 1] - \mathrm{E}[\mathrm{Y}_i|D_i = 0] = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

  - In practice, we can estimate these ATE of getting college degree on earning by regressing Y_i on D_i

$$\mathrm{Y}_i = \delta + \alpha D_i + \epsilon_i$$

Bad Control Problem and Selection Bias

Formal Illustration

- We have no trouble estimating the causal effect of D_i on W_i since independence gives us ATE:

$$\mathrm{E}[\mathrm{W}_i|D_i = 1] - \mathrm{E}[\mathrm{W}_i|D_i = 0] = \mathrm{E}[\mathrm{W}_i^1 - \mathrm{W}_i^0]$$

  - Similarly, we can estimate these ATE of getting college degree on having white color job by regressing W_i on D_i

$$\mathrm{W}_i = \delta + \alpha D_i + \epsilon_i$$

Bad Control Problem and Selection Bias

Formal Illustration

- Bad controls means that a comparison of earnings Y_i **conditional on** W_i does NOT have a causal interpretation

$$\mathrm{Y}_i = \delta + \alpha D_i + \beta W_i + \epsilon_i$$

- Consider the difference in mean earnings between college graduates and others conditional on working in a white collar job.

- We can compute this in a regression including W_i or by regressing Y_i on D_i in the sample where $W_i = 1$

$$\begin{aligned} \alpha &= \mathrm{E}[\mathrm{Y}_i | W_i = 1, D_i = 1] - \mathrm{E}[\mathrm{Y}_i | W_i = 1, D_i = 0] \\ &= \mathrm{E}[\mathrm{Y}_i^1 | W_i^1 = 1, D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1, D_i = 0] \end{aligned}$$

Bad Control Problem and Selection Bias

Formal Illustration

- By independence of D_i and all potential outcomes $(\mathrm{Y}_i^1, \mathrm{Y}_i^0, \mathrm{W}_i^1, \mathrm{W}_i^0)$

$$\begin{aligned}\mathrm{E}[\mathrm{Y}_i^1 | W_i^1 = 1, D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1, D_i = 0] \\ = \mathrm{E}[\mathrm{Y}_i^1 | W_i^1 = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1]\end{aligned}$$

- **Including bad controls (i.e. occupation) leads to selection bias**:

$$\begin{aligned}&\mathrm{E}[\mathrm{Y}_i^1 | W_i^1 = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1] \\ &= \mathrm{E}[\mathrm{Y}_i^1 | W_i^1 = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^1 = 1] + \mathrm{E}[\mathrm{Y}_i^0 | W_i^1 = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1] \\ &= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | W_i^1 = 1]}_{\text{Causal Effect}} + \underbrace{\mathrm{E}[\mathrm{Y}_i^0 | W_i^1 = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1]}_{\text{Selection Bias}}\end{aligned}$$

Bad Control Problem and Selection Bias

Formal Illustration

$$\mathrm{E}[\mathrm{Y}_i^1 | W_i^1 = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1]$$
$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | W_i^1 = 1]}_{\text{Causal Effect}} + \underbrace{\mathrm{E}[\mathrm{Y}_i^0 | W_i^1 = 1] - \mathrm{E}[\mathrm{Y}_i^0 | W_i^0 = 1]}_{\text{Selection Bias}}$$

- Selection bias implies the potential outcome (earnings) is different for:
    - Those who have college degree and work at white-color jobs
    - Those who do not have college degree but work at white-color jobs
- Selection bias reflects the fact that college changes the composition of the pool of white collar workers

Control-Based Causal Inference v.s. Design-Based Causal Inference

Control-Based Causal Inference

- So far, we have learned several control-based causal inference methods
  - Matching, regression, or machine learning
- These methods are all based on CIA (**selection on observables**)
  - Assumed all confounding factors can be observed
  - Thus, we can eliminate selection bias by comparing the treated and untreated units with the similar observed characteristics

Unobservable Omitted Variable

- Even if we can control all observed variable, selection bias might still exist due to **unobservable omitted variables**
  - That is, the treated and untreated units may be very different in some characteristics that we can NOT observe
  - In other words, this means CIA (**selection on observables**) is not valid
  - Thus, we can NOT eliminate selection bias by including more covariates into regression or matching process

Design-Based Causal Inference

- Next four weeks, we will learn several methods to deal with unobservable omitted variables
  - Difference-in-differences design
  - Synthetic control method
  - Regression discontinuity design
- The above methods utilize an exogenous factor that drives change in treatment status to estimate the causal effect