Matching Methods

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March 20, 2023

Observational Studies

- Randomized Controlled Trial (RCT) is called the "gold standard" for causal inference
  - In a RCT, researcher can assign treatments randomly to the individuals
  - Therefore, treatment status is unrelated to any observed and unobserved confounders
  - Treatment and control group should be similar in all characteristics

Observational Studies

- But implementing a randomized experiment in social science is very expensive and sometimes has ethical issues
- In social science, many empirical studies use **non-experimental data**
  - It means researchers cannot assign treatment
- We call this type of empirical researches as **observational studies**

Observational Studies

- In contrast to RCT, in observational studies, **researchers can NOT control the assignment of treatment**
  - Thus, we need to directly control for the observed variables and use indirect methods to adjust for unobserved variables
  - Make "other thing equal" in observed and unobserved variables
- We want to **design** observational studies that **approximate experiments**:
  - "The planner of an observational study should always ask himself: How would the study be conducted if it were possible to do it by controlled experimentation" (Cochran 1965)

Matching Methods: Main Idea

Main Idea of Matching

- Assume all confounding factors are **observable** to researchers
- Matching is a way to eliminate selection bias
  - By constructing a control group with the same observable characteristics as the treatment group
- This can be accomplished by **matching** treated and untreated units with the same observable characteristics.

Main Idea of Matching

- **Example:**
  - We want to estimate the causal effect of job training program on worker's earnings
  - Suppose **age** is the only confounding factors that affect both earnings and job training decision

Matching: An Example

Trainees			Non-Trainees		
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3	29	14400	3	30	21000
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Avg:	28.5	16426	Avg:	33	20724

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Matching: An Example

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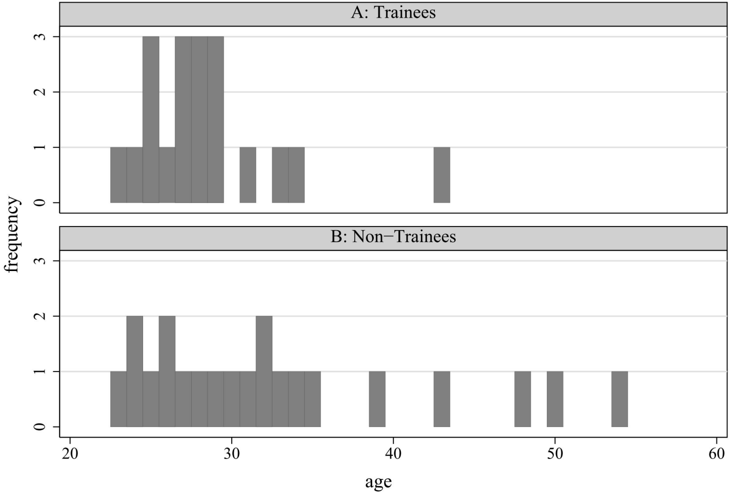
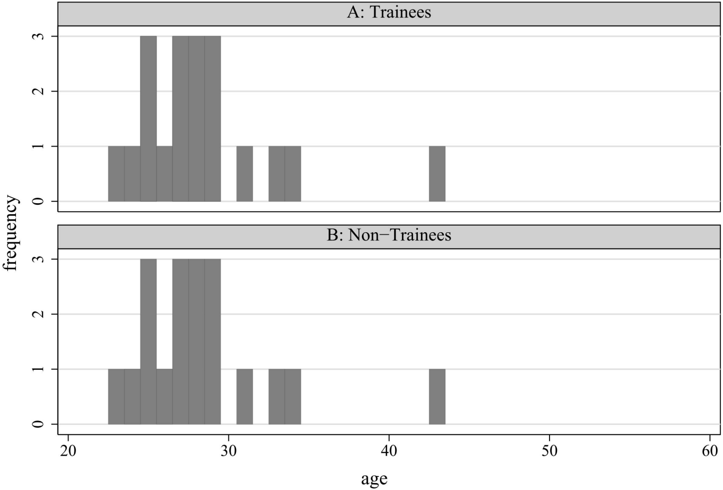
Avg:	28.5	16426	Avg:	33	20724	Avg:	28.5	13982

Age Distribution: Before Matching



Age Distribution: After Matching



Treatment Effect Estimates

Difference in average earnings between trainees and non-trainees:

- Before matching:

$$16426 - 20724 = -4298$$

- After matching:

$$16426 - 13982 = 2444$$

Matching Methods: Potential Outcomes Framework

Observed Outcome, Potential Outcomes, and Selection Bias

- If we find two individuals (groups) have different **observed outcomes** Y, it could be due to:

  1. They receive different treatment D:
     - $D_i \neq D_j$
     - **Causal effect of treatment**
  2. Given that they receive the same treatment, their value of potential outcomes $(\mathrm{Y}^1, \mathrm{Y}^0)$ are different:
     - Under the situation that both receive treatment $D = 1$ but $\mathrm{Y}_i^1 \neq \mathrm{Y}_j^1$
     - Under the situation that both do not receive treatment $D = 0$ but $\mathrm{Y}_i^0 \neq \mathrm{Y}_j^0$
     - **Selection bias**

Sources of Selection Bias: Self-selection

- For those getting treatment $D_i = 1$, they make this decision based on their value of potential outcomes
  - $\mathrm{Y}_i^1 \geq \mathrm{Y}_i^0 \Rightarrow D = 1$
- For those not getting treatment $D_i = 0$, they make this decision based on their value of potential outcomes
  - $\mathrm{Y}_i^0 \geq \mathrm{Y}_i^1 \Rightarrow D = 0$
- This self-selection behavior would result in selection bias:
  - $\mathrm{E}[\mathrm{Y}_i^0 | D_i = 1] \neq \mathrm{E}[\mathrm{Y}_i^0 | D_i = 0]$
  - $\mathrm{E}[\mathrm{Y}_i^1 | D_i = 1] \neq \mathrm{E}[\mathrm{Y}_i^1 | D_i = 0]$

Conditional Independence Assumption (CIA)

Conditional Independence Assumption

$$(\mathrm{Y}_i^1, \mathrm{Y}_i^0) \perp\!\!\!\perp D_i | X_i$$

- X_i are observable characteristics (covariates) with value of k
- CIA asserts that conditional on observable characteristics X_i, potential outcomes are independent of treatment assigned
  - This assumption is also called **selection on observable**

Conditional Independence Assumption (CIA)

Conditional Independence Assumption

$$(\mathrm{Y}_i^1, \mathrm{Y}_i^0) \perp\!\!\!\perp D_i | X_i$$

- This implies X_i are key factors that determine the value of potential outcome for treatment group and control group
  - $\mathrm{Y}^0 = \mathrm{f}(X)$
  - $\mathrm{Y}^1 = \mathrm{f}(X)$
- Thus, people with the same value of covariates X_i, they should have similar value of potential outcomes
  - $\mathrm{E}[\mathrm{Y}_i^{\mathbf{0}} | X_i = k, D_i = 1] = \mathrm{E}[\mathrm{Y}_i^{\mathbf{0}} | X_i = k, D_i = 0]$
  - $\mathrm{E}[\mathrm{Y}_i^{\mathbf{1}} | X_i = k, D_i = 1] = \mathrm{E}[\mathrm{Y}_i^{\mathbf{1}} | X_i = k, D_i = 0]$

Conditional Independence Assumption (CIA)

- Job training example:
  - D_i: join job training
  - Y_i^1: potential earnings for joining job training
  - Y_i^0: potential earnings for not joining job training
  - X_i: age, education...etc.

Conditional Independence Assumption (CIA)

- Suppose only **age** can affect an individual's potential earnings
  - CIA suggests for those with the same age, their training decision are unrelated to the potential earnings
- That is, once controlling age, treatment and control group should be comparable (apple-to-apple comparison)
  - $\mathrm{E}[\mathrm{Y}_i^{\mathbf{0}} | X_i = 40, D_i = 1] = \mathrm{E}[\mathrm{Y}_i^{\mathbf{0}} | X_i = 40, D_i = 0]$
  - $\mathrm{E}[\mathrm{Y}_i^{\mathbf{1}} | X_i = 40, D_i = 1] = \mathrm{E}[\mathrm{Y}_i^{\mathbf{1}} | X_i = 40, D_i = 0]$

Common Support Assumption

Common Support Assumption

$$0 < \Pr(D_i = 1|X_i) < 1$$

- For each value of covariates X_i, there is a positive probability of being both treated and untreated
- In other words, it is NOT possible to perfectly predict one's treatment status by using specific value of X_i
  - For example, this exclude:
    - All individuals with age 40 are in treatment group: $Pr(D_i = 1|X_i = 40) = 1$
    - All individuals with age 40 are in control group: $Pr(D_i = 1|X_i = 40) = 0$
- It ensures that there is sufficient overlap in the characteristics of treated and untreated units to find adequate matched sample

Identification Results for Matching

$$\begin{aligned}
\alpha_{mat}(X) = \quad & \underbrace{\mathrm{E}[\mathrm{Y}_i|X_i, D_i = 1] - \mathrm{E}[\mathrm{Y}_i|X_i, D_i = 0]}_{\text{Observed Difference in Average Outcome at given } X_i} \\
& = \mathrm{E}[\mathrm{Y}_i^1|X_i, D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 0] \\
& = \mathrm{E}[\mathrm{Y}_i^1|X_i, D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 1] \\
& + \mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 0] \\
& = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|X_i, D_i = 1]}_{\text{Causal Effect (CATT)}} \\
& + \underbrace{\mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 0]}_{\text{Selection Bias}} \\
& = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|X_i, D_i = 1]}_{\text{Causal Effect (CATT)}} + \underbrace{0}_{\text{Selection Bias}}
\end{aligned}$$

- Remember CIA ensures $\mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 1] = \mathrm{E}[\mathrm{Y}_i^0|X_i, D_i = 0]$

Identification Results for Matching

$$
\begin{aligned}
\alpha_{mat}(X) = \quad & \underbrace{\mathrm{E}[\mathrm{Y}_i | X_i, D_i = 1] - \mathrm{E}[\mathrm{Y}_i | X_i, D_i = 0]}_{\text{Observed Difference in Average Outcome at given } X_i} \\
&= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i, D_i = 1]}_{\text{Causal Effect (CATT)}} + \underbrace{0}_{\text{Selection Bias}} \\
&= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i, D_i = 0]}_{\text{Causal Effect (CATU)}} \\
&= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i]}_{\text{Causal Effect (CATE)}}
\end{aligned}
$$

Identification Results for Matching

- Under CIA, matching estimator represent **conditional average treatment effect (CATE)**
  - Causal effect for the people with specific value of X_i
    - Causal effect of job training on annual earnings for people with age 40
    - Causal effect of job training on annual earnings for treated people with age 40
- How to obtain ATT, ATU, and ATE?
  - Take average of CATT, CATU, or CATE over all subgroups (all possible X-values)

Review: The Law of Iterated Expectations (LIE)

The Law of Iterated Expectations (LIE)

$$\mathrm{E}[\mathrm{Y}_i] = \mathrm{E}[\mathrm{E}[\mathrm{Y}_i|\mathrm{X}_i]]$$

- Intuitively, there are two ways to compute average outcome Y (e.g. earnings)

1. $\mathrm{E}[\mathrm{Y}_i] = \sum_{\mathrm{Y}_i} \mathrm{Y}_i \times Pr(\mathrm{Y}_i = y)$
   - Each value of outcome times its probability
2. $\mathrm{E}[\mathrm{E}[\mathrm{Y}_i|\mathrm{X}_i]] = \sum_{\mathrm{X}_i} \mathrm{E}[\mathrm{Y}_i|\mathrm{X}_i] \times Pr(\mathrm{X}_i = x) = \mathrm{E}[\mathrm{Y}_i]$
   - Average outcome of subgroup times the share of subgroup

Review: The Law of Iterated Expectations (LIE)

Example

- Suppose we want to compute average income in Taiwan

1. We know 40% of people in Taiwan earn 1 million NTD per year, 40% earn 2 million NTD, and 20% earn 3 million NTD

$$
\begin{aligned}
\mathrm{E}[\mathrm{Y}_i] &= \sum_{\mathrm{Y}_i} \mathrm{Y}_i \times Pr(\mathrm{Y}_i = y) \\
&= 1 \times 0.4 + 2 \times 0.4 + 3 \times 0.2 \\
&= 1.8
\end{aligned}
$$

Review: The Law of Iterated Expectations (LIE)

Example

2 Suppose we know the average annual income for male is 2 million and average annual income for female is 1.6 million

- The share of male to total population is 50%

$$
\begin{aligned}
\mathrm{E}[\mathrm{E}[\mathrm{Y}_i|\mathrm{X}_i]] &= \sum_{\mathrm{X}_i} \mathrm{E}[\mathrm{Y}_i|\mathrm{X}_i] \times Pr(\mathrm{X}_i = x) \\
&= 2 \times 0.5 + 1.6 \times 0.5 \\
&= 1.8 \\
&= \mathrm{E}[\mathrm{Y}_i]
\end{aligned}
$$

Identification Results for Matching

- Using a matching method, we can identify CATE

$$\alpha_{mat}(X) = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i]}_{\text{Causal Effect (CATE)}}$$

- Applying LIE, we can identify ATE by averaging all of the X-specific effects (CATE):

$$\mathrm{E}[\ \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i]}_{\text{Causal Effect (CATE)}}\ ] = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0]}_{\text{Causal Effect (ATE)}}$$

Identification Results for Matching

- **Example:**
  - Using matching methods, we get CATE for male and female

$$\alpha_{mat}(X = male) = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i = male]$$
$$\alpha_{mat}(X = female) = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i = female]$$

- Applying LIE, we can identify ATE by averaging CATE over all possible gender (male and female):

$$\mathrm{E}[\ \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i]}_{\text{Causal Effect (CATE)}}\ ] = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0]}_{\text{Causal Effect (ATE)}}$$

Identification Results for Matching

- Note that using a matching method, we can also identify CATT

$$\alpha_{mat}(X) = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i, D_i = 1]}_{\text{Causal Effect (CATT)}}$$

- Applying LIE, we can identify ATT by averaging all of the X-specific effects in treatment group (CATT):

$$\mathrm{E}[\underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i, D_i = 1 | D_i = 1]}_{\text{Causal Effect (CATT)}}] = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0] | D_i = 1]}_{\text{Causal Effect (ATT)}}$$

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Identification Results for Matching

- Note that using a matching method, we can also identify CATU

$$\alpha_{mat}(X) = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i, D_i = 0]}_{\text{Causal Effect (CATU)}}$$

- Applying LIE, we can identify ATU by averaging all of the X-specific effects in treatment group (CATU):

$$\mathrm{E}[\underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i, D_i = 0 | D_i = 0]}_{\text{Causal Effect (CATU)}}] = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0] | D_i = 0]}_{\text{Causal Effect (ATU)}}$$

Matching Methods: Estimation

Matching Estimator

- Again, we usually only have sample
  - Part of population data
- Suppose our sample is N individuals
- Treatment is job training and outcome is earning
  - N_1 individuals choose to join job training: treatment group
  - N_0 individuals choose not join it ($N_0 = N - N_1$): control group

Matching Estimator

Estimation for ATT

- Suppose we want to estimate ATT
  - Average treatment effect for treatment group
- In that case, a matching estimator of α_{ATT} can be constructed as:

$$\hat{\alpha}_{\text{ATT}} = \frac{1}{N_1} \sum_{D_i=1} (Y_i - Y_{j(i)})$$

  - We want to match **treated** individual i's outcome Y_i
    - We impute Y_i^0 using untreated units $Y_{j(i)}$ in control group
    - $Y_{j(i)}$: the outcome of an untreated observation j such that $X_{j(i)}$ is the **closest** value to X_i among the untreated observations.

Matching Estimator

Estimation for ATT

- We can also use the average:

$$\hat{\alpha}_{ATT} = \frac{1}{N_1} \sum_{D_i=1} \left\{ Y_i - \left(\frac{1}{M} \sum_{m=1}^{M} Y_{jm(i)} \right) \right\}$$

- Works well when we can find good matches for each treated unit, so M is usually small (typically, $M = 1$ or $M = 2$)

- Perfect matches are often not available

Matching Estimator
Estimation for ATU

- Suppose we want to estimate ATU
  - Average treatment effect for control group
- In that case, a matching estimator of α_{ATU} can be constructed as:

$$\hat{\alpha}_{\text{ATU}} = \frac{1}{N_0} \sum_{D_i=0} (Y_{j(i)} - Y_i)$$

  - We want to match **untreated** individual i's outcome Y_i
    - We impute Y_i^1 using treated units $Y_{j(i)}$ in treatment group
    - $Y_{j(i)}$: the outcome of an treated observation j such that $X_{j(i)}$ is the **closest** value to X_i among the treated observations.

Matching

Estimation for ATE

- We can also use matching to estimate ATE
  - Average treatment effect for control group
- In that case, we match in both directions:
  1. If observation i is treated, we impute Y_i^0 using untreated units $Y_{j(i)}$ in control group
  2. If observation i is untreated, we impute Y_i^1 using treated units $Y_{j(i)}$ in treatment group
- The matching estimator for ATE is:

$$\hat{\alpha}_{\text{ATE}} = \frac{1}{N}\left\{\sum_{D_i=1}(Y_i - Y_{j(i)}) + \sum_{D_i=0}(Y_{j(i)} - Y_i)\right\}$$

Matching

Measure Closeness

- We usually use more than one characteristics to construct a matched sample
- When the vector of matching covariates has more than one variables (k>1)

$$X = \begin{pmatrix} X_1 \\ X_2 \\ \vdots \\ X_K \end{pmatrix}$$

- We need to define a **distance metric** to measure "closeness" to construct a matched sample

Matching

Measure Closeness

- The usual **Euclidean distance** is:

$$\begin{aligned} ||X_i - X_j|| &= \sqrt{(X_i - X_j)'(X_i - X_j)} \\ &= \sqrt{\sum_{k=1}^{K} (X_{ki} - X_{kj})^2}. \end{aligned}$$

  - Sum up the differences between treatment group and control group over k characteristics
    - **Drawback:** The Euclidean distance is NOT invariant to changes in the scale of the X's
    - For this reason, we often use alternative distances that are invariant to changes in scale

- A commonly used distance is the **normalized Euclidean distance**

$$||X_i - X_j|| = \sqrt{(X_i - X_j)' \widehat{V}^{-1} (X_i - X_j)}$$

where

$$\hat{V} = \begin{pmatrix} \hat{\sigma}_1^2 & 0 & \dots & 0 \\ 0 & \hat{\sigma}_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \hat{\sigma}_K^2 \end{pmatrix}.$$

- $\hat{\sigma}_k^2$ is the variance of variable k

Matching

Measure Closeness

- Notice that, the normalized Euclidean distance is equal to:

$$||X_i - X_j|| = \sqrt{\sum_{k=1}^{K} \frac{(X_{ki} - X_{kj})^2}{\hat{\sigma}_k^2}}.$$

$\Rightarrow$ Changes in the scale of X_{ki} affect also $\hat{\sigma}_k$, and the normalized Euclidean distance does not change

Matching and the "Curse of Dimensionality"

- Matching becomes unfeasible with many covariates
- This is also true even if we divided each of covariates into coarse categories (subclassification)

Matching and the "Curse of Dimensionality"

- Assume we have k covariates and divided each of them into 3 coarse categories
  - age could be "young", "middle age" or "old"
  - income could be "low", "medium" or "high"
- The number of subclassification cells is 3^k.
  - For $k = 10$, we obtain $3^{10} = 59049$
- Many cells may contain only treated or untreated observations
  - We may not be able to construct matched sample
  - Violate common support assumption

Matching and the "Curse of Dimensionality"

- Matching discrepancies $||X_i - X_{j(i)}||$ tend to increase with k, the dimension of X
- It is difficult to find good matches in large dimensions: you need many observations if k is large

Propensity Score Matching: Main Idea

Propensity Score Matching

- Instead of matching over k dimensions, the method of **propensity score matching (PSM)** allows the matching problem to be reduced to a single dimension
  - The **propensity score** is defined as the treatment probability conditional on a set of observed variables X_i:

$$p(X_i) = E[D_i|X_i] = Pr(D_i = 1|X_i)$$

  - Intuitively, propensity score $p(X_i)$ summarized all information of a set of covariates X_i into a single value
  - Then, we can just control (match) $p(X_i)$ to eliminate selection bias

Propensity Score Matching

- Rosenbaum and Rubin (1983) proved that CIA (selection on observables) implies:

$$(Y_i^1, Y_i^0) \perp\!\!\!\perp D_i | p(X_i)$$

  - Conditioning on the propensity score $p(X_i)$ is enough to make treatment status be independent of the potential outcomes
  - Substantial dimension reduction in the matching variables!

Propensity Score Matching

Propensity Score Theorem

Suppose the CIA holds, such that $(\mathrm{Y}_i^1, \mathrm{Y}_i^0) \perp\!\!\!\perp D_i | X_i$. Then $(\mathrm{Y}_i^1, \mathrm{Y}_i^0) \perp\!\!\!\perp D_i | p(X_i)$

- If potential outcomes are independent of treatment status conditional on a set of covariates X_i
- Then, potential outcomes are independent of treatment status D_i conditional on the propensity score $p(X_i)$

Propensity Score Matching

- Goal of Proof:
  - Assume that $(\mathrm{Y}_i^1, \mathrm{Y}_i^0) \perp\!\!\!\perp D_i | X_i$. Then:

$$\Rightarrow \; Pr(D_i = 1 | \mathrm{Y}_i^1, \mathrm{Y}_i^0, p(X_i)) = p(X_i) = Pr(D_i = 1 | p(X_i))$$

$$\Rightarrow \; (\mathrm{Y}_i^1, \mathrm{Y}_i^0) \perp\!\!\!\perp D_i | p(X_i)$$

Propensity Score Matching

Proof: Assume that $(Y_i^1, Y_i^0) \perp\!\!\!\perp D_i | X_i$. Then:

$$
\begin{aligned}
Pr(D_i = 1 | Y_i^1, Y_i^0, p(X_i)) &= E[D_i | Y_i^1, Y_i^0, p(X_i)] \\
&= E[E[D_i | Y_i^1, Y_i^0, p(X_i), X_i] | Y_i^1, Y_i^0, p(X_i)] \\
&= E[E[D_i | Y_i^1, Y_i^0, X_i] | Y_i^1, Y_i^0, p(X_i)] \\
&= E[E[D_i | X_i] | Y_i^1, Y_i^0, p(X_i)] \\
&= E[p(X_i) | Y_i^1, Y_i^0, p(X_i)] \\
&= p(X_i)
\end{aligned}
$$

Propensity Score Matching

Using a similar argument, we obtain

$$\begin{aligned} Pr(D_i = 1 | p(X_i)) &= E[D_i | p(X_i)] \\ &= E[E[D_i | p(X_i), X_i] | p(X_i)] \\ &= E[E[D_i | X_i] | p(X_i)] \\ &= E[p(X_i) | p(X_i)] \\ &= p(X_i) \end{aligned}$$

$\Rightarrow$ $Pr(D_i = 1 | \mathrm{Y}_i^1, \mathrm{Y}_i^0, p(X_i)) = p(X_i) = Pr(D_i = 1 | p(X_i))$

$\Rightarrow$ $(\mathrm{Y}_i^1, \mathrm{Y}_i^0) \perp\!\!\!\perp D_i | p(X_i)$

Propensity Score Matching

- From CIA, to get causal effect, we need only control for covariates that affect the probability of treatment
- The propensity score theorem says something more:
  - **The only covariate you really need to control for is the probability of treatment itself** $p(X_i) = Pr(D_i = 1|X_i)$

Identification Results or Propensity Score Matching

- Similar to identification results of matching estimator, the only difference is that we control $p(X_i)$ instead of a set of covariates X_i

- Remember CIA and propensity score theorem ensures $\mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 1] = \mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 0]$

Identification Results or Propensity Score Matching

$$\alpha_{psm}(X) = \underbrace{\mathrm{E}[\mathrm{Y}_i|p(X_i), D_i = 1] - \mathrm{E}[\mathrm{Y}_i|p(X_i), D_i = 0]}_{\text{Observed Difference in Average Outcome at given } X_i}$$

$$= \mathrm{E}[\mathrm{Y}_i^1|p(X_i), D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 0]$$

$$= \mathrm{E}[\mathrm{Y}_i^1|p(X_i), D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 1]$$

$$+ \mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 0]$$

$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|p(X_i), D_i = 1]}_{\text{Causal Effect (CATT)}}$$

$$+ \underbrace{\mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|p(X_i), D_i = 0]}_{\text{Selection Bias}}$$

$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|p(X_i), D_i = 1]}_{\text{Causal Effect (CATT)}} + \underbrace{0}_{\text{Selection Bias}}$$

$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|p(X_i), D_i = 0]}_{\text{Causal Effect (CATU)}} = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|p(X_i)]}_{\text{Causal Effect (CATE)}}$$

Identification Results for Matching

- Using a matching method, we can identify CATE

$$\alpha_{psm}(X) = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | p(X_i)]}_{\text{Causal Effect (CATE)}}$$

- Applying LIE, we can identify ATE by averaging all of the $p(X)$-specific effects (CATE):

$$\mathrm{E}[\underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | p(X_i)]}_{\text{Causal Effect (CATE)}}] = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0]}_{\text{Causal Effect (ATE)}}$$

Identification Results for Propensity Score Matching

- Note that using a matching method, we can also identify CATT

$$\alpha_{psm}(X) = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | p(X_i), D_i = 1]}_{\text{Causal Effect (CATT)}}$$

- Applying LIE, we can identify ATT by averaging all of the $p(X)$-specific effects for treatment group (CATT):

$$\mathrm{E}[\underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | p(X_i), D_i = 1 | D_i = 1]]}_{\text{Causal Effect (CATT)}} = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 1]}_{\text{Causal Effect (ATT)}}$$

Identification Results for Propensity Score Matching

- Note that using a matching method, we can also identify CATU

$$\alpha_{psm}(X) = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | p(X_i), D_i = 1]}_{\text{Causal Effect (CATU)}}$$

- Applying LIE, we can identify ATU by averaging all of the $p(X)$-specific effects for treatment group (CATU):

$$\mathrm{E}[\underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | p(X_i), D_i = 0 | D_i = 0]}_{\text{Causal Effect (CATU)}}] = \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 0]}_{\text{Causal Effect (ATU)}}$$

Propensity Score Matching: Estimation – Nearest Neighbor

Propensity Score Matching

Estimation

- There are two ways to estimate causal effect of treatment using PSM
  1. Nearest Neighbor:
     - By matching each treated observation to the untreated observation with the same or similar values of the propensity score
  2. Weighting Approach
     - Skip the cumbersome matching procedure and re-weight sample

Propensity Score Matching

Estimation: Nearest Neighbor

- There are two steps to estimate causal effect of treatment using PSM with nearest neighbor

  1. Estimate the propensity score: $\hat{p}(X) = \hat{P}r(D_i = 1|X_i)$ using logit or porbit regression

$$D_i = \beta_0 + \beta_1 X_i^1 + \beta_2 X_i^2 + ... + \beta_k X_i^k + \epsilon_i$$

  2. By matching each treated observation to the observation (control group) with the same or similar values of the propensity score $\hat{P}r(D_i = 1|X_i)$

Propensity Score Matching: An Example

Estimation: Nearest Neighbor

Trainees			Non-Trainees		
unit	pro-score	earnings	unit	pro-score	earnings
1	0.28	17700	1	0.43	20900
2	0.34	10200	2	0.50	31000
3	0.29	14400	3	0.30	21000
4	0.25	20800	4	0.27	9300
5	0.29	6100	5	0.54	41100
6	0.23	28600	6	0.48	29800
7	0.33	21900	7	0.39	42000
8	0.27	28800	8	0.28	8800
9	0.31	20300	9	0.24	25500
10	0.26	28100	10	0.33	15500
11	0.25	9400	11	0.26	400
12	0.27	14300	12	0.31	26600
13	0.29	12500	13	0.26	16500
14	0.24	19700	14	0.34	24200
15	0.25	10100	15	0.25	23300
16	0.43	10700	16	0.24	9700
17	0.28	11500	17	0.29	6200
18	0.27	10700	18	0.35	30200
19	0.28	16300	19	0.32	17800
			20	23	9500
			21	32	25900
Avg:		16426	Avg:		20724

Propensity Score Matching: An Example

Estimation: Nearest Neighbor

Trainees			Non-Trainees			Matched Sample		
unit	pro-score	earnings	unit	pro-score	earnings	unit	pro-score	earnings
1	0.28	17700	1	0.43	20900			
2	0.34	10200	2	0.50	31000			
3	0.29	14400	3	0.30	21000			
4	0.25	20800	4	0.27	9300			
5	0.29	6100	5	0.54	41100			
7	0.33	21900	7	0.39	42000			
8	0.27	28800	8	0.28	8800			
9	0.31	20300	9	0.24	25500			
10	0.26	28100	10	0.33	15500			
11	0.25	9400	11	0.26	400			
12	0.27	14300	12	0.31	26600			
13	0.29	12500	13	0.26	16500			
14	0.24	19700	14	0.34	24200			
15	0.25	10100	15	0.25	23300			
16	0.43	10700	16	0.24	9700			
17	0.28	11500	17	0.29	6200			
18	0.27	10700	18	0.35	30200			
19	0.28	16300	19	0.32	17800			
			20	23	9500			
			21	32	25900			
Avg:		16426	Avg:		20724	Avg:		

Propensity Score Matching: An Example

Estimation: Nearest Neighbor

Trainees			Non-Trainees			Matched Sample		
unit	pro-score	earnings	unit	pro-score	earnings	unit	pro-score	earnings
1	0.28	17700	1	0.43	20900	8	0.28	8800
2	0.34	10200	2	0.50	31000	14	0.34	24200
3	0.29	14400	3	0.30	21000	17	0.29	6200
4	0.25	20800	4	0.27	9300	15	0.25	23300
5	0.29	6100	5	0.54	41100	17	0.29	6200
6	0.23	28600	6	0.48	29800	20	0.23	9500
7	0.33	21900	7	0.39	42000	10	0.33	15500
8	0.27	28800	8	0.28	8800	4	0.27	9300
9	0.31	20300	9	0.24	25500	12	0.31	26600
10	0.26	28100	10	0.33	15500	11,13	0.26	8450
11	0.25	9400	11	0.26	400	15	0.25	23300
12	0.27	14300	12	0.31	26600	4	0.27	9300
13	0.29	12500	13	0.26	16500	17	0.29	6200
14	0.24	19700	14	0.34	24200	9,16	0.24	17700
15	0.25	10100	15	0.25	23300	15	0.25	23300
16	0.43	10700	16	0.24	9700	1	0.43	20900
17	0.28	11500	17	0.29	6200	8	0.28	8800
18	0.27	10700	18	0.35	30200	4	0.27	9300
19	0.28	16300	19	0.32	17800	8	0.28	8800
			20	23	9500			
			21	32	25900			
Avg:		16426	Avg:		20724	Avg:		13982

Propensity Score Matching

Statistical Inference

- A valid method to calculate standard errors has not been known until very recently (see, Abadie and Imbens, 2016)
- Abadie, Alberto, and Guido W. Imbens. "**Matching on the Estimated Propensity Score**." Econometrica 84.2 (2016): 781-807.
  - Need to take into account that propensity scores are estimated
  - The adjustment for ATE is always negative: smaller standard errors
  - The adjustment for ATT can be positive or negative: smaller or larger standard errors

Propensity Score Matching – STATA Example

STATA Example

Dehejia et al. (1999)

Rajeev H. Dehejia; Sadek Wahba (1999) "**Causal Effects in Nonexperimental Studies: Reevaluating the Evaluation of Training Programs**" Journal of the American Statistical Association

- The authors wants to examine the effect of job training on workers' earnings
- We use this example to go through the procedure of implementing PSM
- See **matching.do**

STATA Example

Dehejia et al. (1999)

- See **matching.do**
- Use lalonde.dta
- Install the following ado files:
  - psmatch2.ado

STATA Example

Step 1: Test Differences in Outcomes in Pre-matching Data

```
ttest re78, by(treat)
reg re78 treat,r
```

- Test differences in outcome for treatment group and control group

STATA Example

Step 1: Test Differences in Outcomes in Pre-matching Data

```
. ** Step 1: Test Differences in Outcomes in Pre-matching Data
. ttest re78, by(treat)

Two-sample t test with equal variances
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |     429     6984.17    352.1654    7294.162    6291.981    7676.359
       1 |     185    6349.144    578.4229    7867.402     5207.95    7490.338
---------+--------------------------------------------------------------------
combined |     614    6792.834    301.4942    7470.731    6200.748    7384.921
---------+--------------------------------------------------------------------
    diff |            635.0262    657.1374               -655.4917    1925.544
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      t =   0.9664
Ho: diff = 0                                     degrees of freedom =      612

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.8329         Pr(|T| > |t|) = 0.3342          Pr(T > t) = 0.1671
```

STATA Example

Step 2: Test Differences in Covariates in Pre-matching Data

```
ttest age, by(treat)
ttest educ, by(treat)
reg age treat,r
reg educ treat,r
```

- Test differences in sample characteristics for treatment group and control group

STATA Example

Step 2: Test Differences in Covariates in Pre-matching Data

```
. ttest age, by(treat)

Two-sample t test with equal variances
```

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf.	Interval]
0	429	28.0303	.5207845	10.78665	27.00669	29.05392
1	185	25.81622	.5260475	7.155019	24.77836	26.85408
combined	614	27.36319	.3987723	9.881187	26.58007	28.14632
diff		2.214087	.8652112		.5149437	3.91323

```
    diff = mean(0) - mean(1)                                      t =   2.5590
Ho: diff = 0                                     degrees of freedom =      612

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.9946         Pr(|T| > |t|) = 0.0107          Pr(T > t) = 0.0054
```

STATA Example

Step 3: PSM Estimation – teffects psmatch

Syntax:

```
teffects psmatch (outcome) (treatment covariates,
    logit), nn(#) ate
```

- **nn(#)**: specify number of matches per observation; default is nn(1)
  - The number of variables generated may be more than nn(#) because of tied distances
- **logit**: use logit to predict propensity score (the default)
- **ate**: estimate average treatment effect in population (the default)
- **atet**: estimate average treatment effect on the treated

STATA Example

Step 3: PSM Estimation – teffects psmatch

Example:

```
teffects psmatch (re78) (treat age educ  black
    hispan  nodegree  married  re74  re75, logit), nn
    (1) atet
teffects psmatch (re78) (treat age educ  black
    hispan  nodegree  married  re74  re75, logit), nn
    (1) ate
```

- Outcome: re78 (earnings in 1978)
- Treatment: treat (get job training or not)

STATA Example

Step 3: PSM Estimation – teffects psmatch

```
. teffects psmatch (re78) (treat age educ  black  hispan  nodegree  married  re74  re75, logit), nn(1) ate

Treatment-effects estimation                   Number of obs      =        614
Estimator      : propensity-score matching     Matches: requested =          1
Outcome model  : matching                                     min =          1
Treatment model: logit                                        max =          4
------------------------------------------------------------------------------
             |              AI Robust
        re78 |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ATE          |
       treat |
   (1 vs 0)  |  -304.6074   1076.527    -0.28   0.777    -2414.562    1805.347
------------------------------------------------------------------------------
```

STATA Example

Step 3: PSM Estimation – teffects psmatch

```
. teffects psmatch (re78) (treat age educ  black  hispan  nodegree  married  re74  re75, logit), nn(1) atet

Treatment-effects estimation                    Number of obs      =        614
Estimator      : propensity-score matching     Matches: requested =          1
Outcome model  : matching                                     min =          1
Treatment model: logit                                        max =          4
```

re78	Coef.	AI Robust Std. Err.	z	P>|z|	[95% Conf.	Interval]
ATET						
treat (1 vs 0)	**1968.8**	**1126.321**	**1.75**	**0.080**	**-238.7493**	**4176.349**

STATA Example

Step 3: PSM Estimation – teffects psmatch

Understanding the matching process:

```
teffects psmatch (re78) (treat age educ  black
    hispan  nodegree  married  re74  re75), nn(1)
    atet gen(matchnum)
```

- **gen(matchnum)**: specifies that the observation numbers of the nearest neighbors be stored in the new variables matchnum1, matchnum2, ...
- This option is required if you wish to perform postestimation based on the matching results

STATA Example

Step 3: PSM Estimation – teffects psmatch

Understanding the matching process:

```
predict ps1, ps
predict y0 y1, po
predict te
```

- **predict ps1, ps**: predict propensity score (i.e. probability of getting treatment)
- **predict y0 y1, po**: generate the potential outcome with or without treatment
- **predict te**: get treatment effect for each observation

STATA Example

Step 3: PSM Estimation – teffects psmatch

ps1	y0	y1	te
.3612301	14421.13	9930.046	-4491.084
.7753658	1525.014	3595.894	2070.88
.3217561	2158.959	24909.45	22750.49
.2236759	701.9201	7506.146	6804.226
.2983612	14344.29	289.7899	-14054.5
.3009301	8900.347	4056.494	-4843.853

STATA Example

Step 3: PSM Estimation – teffects psmatch

	id	matchnum1	treat	re78	ps1	y0	y1	te
1	1	254	1	9930.046	.3612301	14421.13	9930.046	-4491.084
2	254	1	0	14421.13	.3614458	14421.13	9930.046	-4491.084

STATA Example

Step 3: PSM Estimation – psmatch2

Syntax:

```
psmatch2 treatment covariates, out(outcome) n(#)
    logit ate
```

- **n(#)**: specify number of matches per observation; default is nn(1)
  - The number of variables generated may be more than n(#) because of tied distances
- **out(var)**: specify an outcome variable
- **ate**: display ATT, ATU, ATE

STATA Example

Step 3: PSM Estimation – psmatch2

Example:

```
psmatch2 treat age educ  black  hispan  nodegree
    married  re74  re75, out(re78) logit n(1) ate
```

- The PSM estimate is similar to the one using teffects

STATA Example

Compare teffects psmatch and psmatch2

- The **teffects psmatch** command has one very important advantage over **psmatch2**
  - **teffects psmatch** takes into account the fact that propensity scores are estimated rather than known when calculating standard errors.
  - **teffects psmatch** calculates standard errors based on this paper:
    - Abadie, Alberto, and Guido W. Imbens. "**Matching on the Estimated Propensity Score**." Econometrica 84.2 (2016): 781-807.

STATA Example

Compare teffects psmatch and psmatch2

- But **psmatch2** can allow matching without replacement, which is quite useful.

STATA Example

Step 3: PSM Estimation – psmatch2

Example:

```
psmatch2 treat age educ  black  hispan  nodegree
    married  re74  re75, out(re78) logit n(1)
    noreplace
```

- **noreplace**: STATA will perform PSM without replacement so that each untreated observation can be used only once.

STATA Example

Step 4: Post Matching Analysis – teffects psmatch

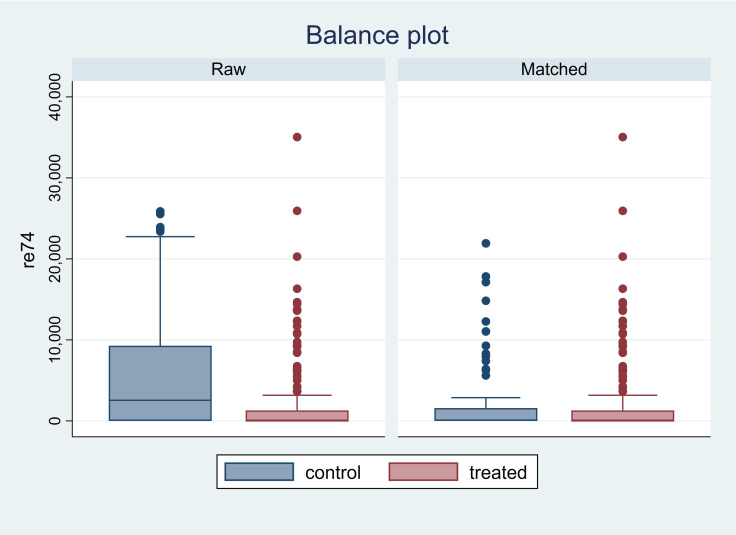
Example:

```
tebalance box re74
tebalance density educ
tebalance density
```

- **tebalance box**: Produces box plots that are used to check for balance in matched samples after **teffects**
- **tebalance density**: Produces density plots that are used to check for covariate balance after estimation by a **teffects**
- If you do not specify variable, it will plot the density of propensity score

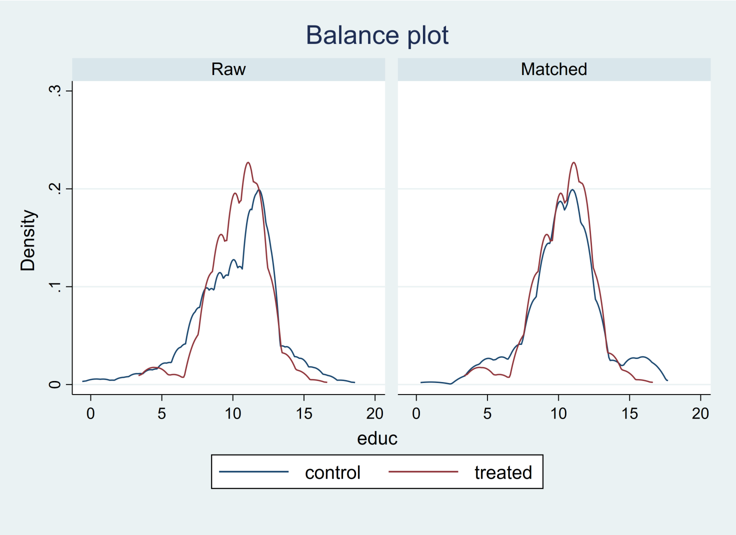
STATA Example

Step 4: Post Matching Analysis – teffects psmatch



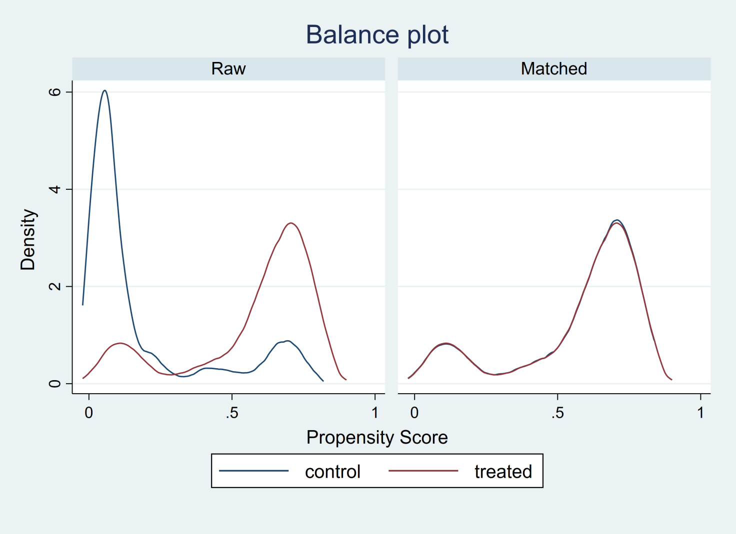
STATA Example

Step 4: Post Matching Analysis – teffects psmatch



STATA Example

Step 4: Post Matching Analysis – teffects psmatch



STATA Example

Step 4: Post Matching Analysis – psmatch2

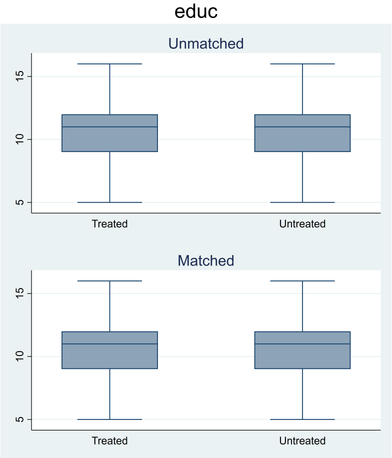
Example:

```
pstest age educ  black  hispan  nodegree  married  re74  re75, both
pstest educ, box both
pstest _pscore, density both
```

- command **pstest**: calculates and optionally graphs several measures of the extent of balancing of the variables between two groups.
- option **both**: compares the extent of balancing between the two samples before and after having performed matching.
- option **box**: draw box plot to compare two groups
- option **density**: draw density plot to compare two groups

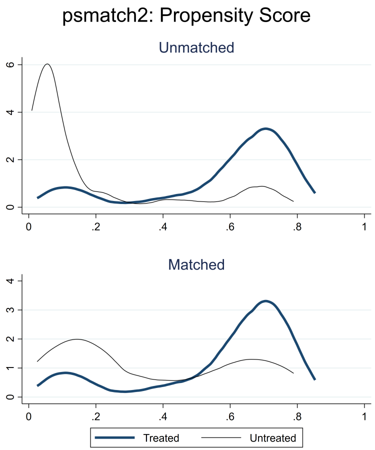
STATA Example

Step 4: Post Matching Analysis – psmatch2



STATA Example

Step 4: Post Matching Analysis – psmatch2



Propensity Score Matching

Drawback

- PSM is hugely popular method to estimate treatment effects even if it relies on **unconvincing assumption**:
  - Selection on observables (CIA)

Suggested Readings

- Chapter 2, Mastering Metrics: The Path from Cause to Effect
- Chapter 3, Mostly Harmless Econometrics
- Chapter 5, Causal Inference: The Mixtape