Selection Bias

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Causal Effects

- Individual Treatment Effect (ITE):

$$\alpha_{\mathsf{ITE}} = \mathrm{Y}_i^1 - \mathrm{Y}_i^0$$

- Average treatment effect (ATE):

$$\alpha_{\mathsf{ATE}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0] = \frac{1}{N} \sum_i [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

- Conditional average treatment effect (CATE):

$$\alpha_{\mathsf{CATE}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | X_i = f] = \frac{1}{N_f} \sum_{i:X_i=f} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

Causal Effects

- Average treatment effect on the treated (ATT):

$$\alpha_{\mathsf{ATT}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 1] = \frac{1}{N_1} \sum_{i:D_i=1} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

- Average treatment effect on the untreated (ATU):

$$\alpha_{\mathsf{ATU}} = \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | D_i = 0] = \frac{1}{N_0} \sum_{i:D_i=0} [\mathrm{Y}_i^1 - \mathrm{Y}_i^0]$$

Observed Outcome, Potential Outcomes, and Selection Bias

- **Causality is defined by potential outcomes**, not by realized (observed) outcomes
  - In fact, we can NOT observe all potential outcomes
- By using observed data, we can only establish association (correlation)
- That is, the observed difference in average outcome between those getting treatment and those not getting treatment

$$\alpha_{\text{corr}} = \underbrace{\mathrm{E}[\mathrm{Y}_i | D_i = 1] - \mathrm{E}[\mathrm{Y}_i | D_i = 0]}_{\text{Observed Difference in Average Outcome}}$$

Observed Outcome, Potential Outcomes, and Selection Bias

- Note that the values of **observed outcomes** Y depend on either treatment status D or the value of potential outcomes $(\mathrm{Y}^1,\mathrm{Y}^0)$

$$\mathrm{Y}_i = \mathrm{Y}_i^1 D_i + \mathrm{Y}_i^0 (1 - D_i) \text{ or}$$

$$\mathrm{Y}_i = \begin{cases} \mathrm{Y}_i^1 & \text{if } D_i = 1 \\ \mathrm{Y}_i^0 & \text{if } D_i = 0 \end{cases}$$

Observed Outcome, Potential Outcomes, and Selection Bias

- If we find two individuals (groups) have different **observed outcomes** Y, it could be due to:
  1. They receive different treatment D:
     - $D_i \neq D_j$
     - **Causal effect of treatment**
  2. Given that they receive the same treatment, their value of potential outcomes $(\mathrm{Y}^1, \mathrm{Y}^0)$ are different:
     - Under the situation that both receive treatment $D = 1$ but $\mathrm{Y}_i^1 \neq \mathrm{Y}_j^1$
     - Under the situation that both do not receive treatment $D = 0$ but $\mathrm{Y}_i^0 \neq \mathrm{Y}_j^0$
     - **Selection bias**

Observed Outcome, Potential Outcomes, and Selection Bias

- The observed association usually mix up causal effect (ATT) and selection bias

$$\alpha_{\text{corr}} = \underbrace{\mathrm{E}[\mathrm{Y}_i|D_i = 1] - \mathrm{E}[\mathrm{Y}_i|D_i = 0]}_{\text{Observed Difference in Average Outcome}}$$

$$= \mathrm{E}[\mathrm{Y}_i^1|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 1] + \mathrm{E}[\mathrm{Y}_i^0|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 0]$$

$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|D_i = 1]}_{\text{ATT}} + \underbrace{\mathrm{E}[\mathrm{Y}_i^0|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 0]}_{\text{Selection Bias}}$$

- **Selection Bias** implies:
  - The value of potential outcomes for treatment and control groups are different **even if both groups receive the same treatment** (e.g. Both are Y_i^0)
  - This means two groups could be quite different in other dimensions: **other things are NOT equal**

Sources of Selection Bias: Self-selection

- For those getting treatment $D_i = 1$, they make this decision based on their value of potential outcomes
  - $\mathrm{Y}_i^1 \geq \mathrm{Y}_i^0 \Rightarrow D = 1$
- For those not getting treatment $D_i = 0$, they make this decision based on their value of potential outcomes
  - $\mathrm{Y}_i^0 \geq \mathrm{Y}_i^1 \Rightarrow D = 0$
- This self-selection behavior would result in selection bias:
  - $\mathrm{E}[\mathrm{Y}_i^0 | D_i = 1] \neq \mathrm{E}[\mathrm{Y}_i^0 | D_i = 0]$
  - $\mathrm{E}[\mathrm{Y}_i^1 | D_i = 1] \neq \mathrm{E}[\mathrm{Y}_i^1 | D_i = 0]$

Observed Association

- Observed association is neither necessary nor sufficient for causality
- **Example:**
  - The observed difference in average earnings between those attending graduate school v.s. those not attending graduate school

$$\alpha_{\text{corr}} = \mathrm{E}[\mathrm{Y}_i | D_i = 1] - \mathrm{E}[\mathrm{Y}_i | D_i = 0] = 1.5$$

i	Y_i^1	Y_i^0	Y_i	D_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	3	?	3	1	?
Tina	2	?	2	1	?
Mary	?	1	1	0	?
Bill	?	1	1	0	?
$E[\mathrm{Y}_i \| D_i = 1]$			2.5		
$E[\mathrm{Y}_i \| D_i = 0]$			1		

Average Treatment Effect on Treated (ATT)

- But we are interested in causal effect (ATT):

$$\alpha_{\text{ATT}} = \mathrm{E}[\mathrm{Y}_i^1 | D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0 | D_i = 1] = 1$$

- Suppose we can observe counterfactual outcomes

i	Y_i^1	Y_i^0	Y_i	D_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[\mathrm{Y}_i^1 \| D_i = 1]$	2.5				
$E[\mathrm{Y}_i^0 \| D_i = 1]$		1.5			

Observed Association and Selection Bias

$$\underbrace{\mathrm{E}[\mathrm{Y}_i|D_i = 1] - \mathrm{E}[\mathrm{Y}_i|D_i = 0]}_{\text{Observed Difference in Average Outcome (1.5)}}$$

$$= \underbrace{\mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0|D_i = 1]}_{\text{ATT (1)}} + \underbrace{\mathrm{E}[\mathrm{Y}_i^0|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 0]}_{\text{Selection Bias}}$$

Selection Bias

- $\alpha_{\text{corr}} \neq \alpha_{\text{ATT}}$

$$\text{Selection Bias} = \mathrm{E}[\mathrm{Y}_i^0|D_i = 1] - \mathrm{E}[\mathrm{Y}_i^0|D_i = 0] = 0.5$$

i	Y_i^1	Y_i^0	Y_i	D_i	$\mathrm{Y}_i^1 - \mathrm{Y}_i^0$
David	3	2	3	1	1
Tina	2	1	2	1	1
Mary	1	1	1	0	0
Bill	1	1	1	0	0
$E[\mathrm{Y}_i^0\|D_i = 1]$		1.5			
$E[\mathrm{Y}_i^0\|D_i = 0]$		1			

- Here, selection bias is positive (0.5 million NT$)
- Those who attend graduate school could be more intelligent so they can earn more even if they did not attend graduate school

Causal Effect and Identification Strategy

- Identification strategy tells us what we can learn about a **causal effect** from the **observed data**
  - The main goal of identification strategy is to eliminate the **selection bias**
- Identification depends on **assumptions**, not on estimation strategies
  - Estimation strategies: OLS, MLE, GMM
  - If an effect is not identified, no estimation method will recover it
- "**What's your identification strategy?**" =
  - What are the assumptions that allow you to claim you've estimated a causal effect?

Suggested Readings

- Chapter 1 and 2, Mastering Metrics: The Path from Cause to Effect
- Chapter 2, Mostly Harmless Econometrics
- Chapter 4, Causal Inference: The Mixtape