IV and Regression Discontinuity Design

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Fuzzy Regression Discontinuity Design: Main Idea

Fuzzy RDD

Overview

- Fuzzy RDD:
  - The probability of getting the treatment is no longer a deterministic function of the assignment variable X_i
  - But there is still a discontinuity in the probability of getting treatment at the threshold

Fuzzy RDD

Overview

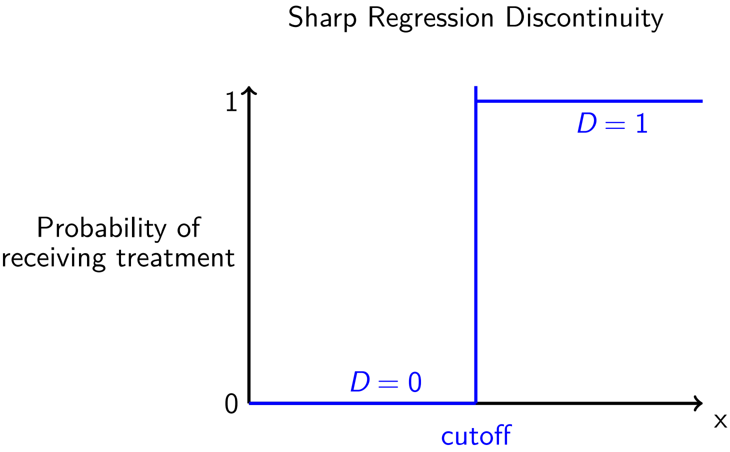
Assumption FRD

$$\lim_{\varepsilon \to 0} \Pr[D_i = 1 | X_i = c + \varepsilon] \neq \lim_{\varepsilon \to 0} \Pr[D_i = 1 | X_i = c - \varepsilon]$$

- **Sharp RDD**:
  - This is also true, but it further required the jump in probability to be from 0 to 1
  - Nobody below the cutoff gets the "treatment", everybody above the cutoff gets it

Treatment Probability and assignment variable

Sharp RDD



Fuzzy RDD

Overview

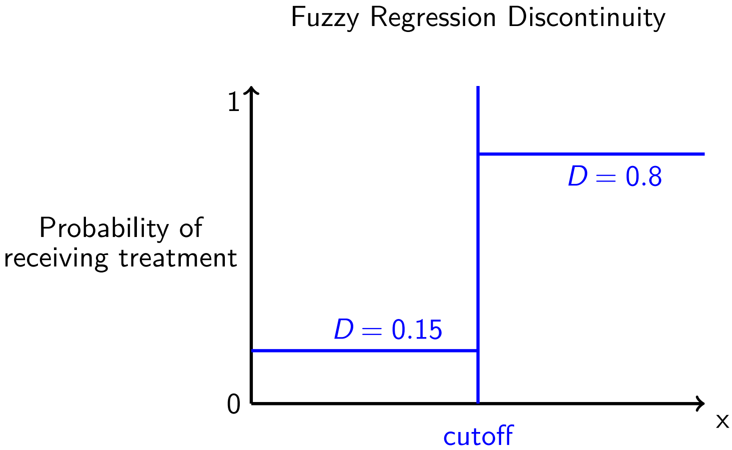
Assumption FRD

$$\lim_{\varepsilon \to 0} \Pr[D_i = 1 | X_i = c + \varepsilon] \neq \lim_{\varepsilon \to 0} \Pr[D_i = 1 | X_i = c - \varepsilon]$$

- **Fuzzy RDD**:
  - The probability of getting the treatment jumps discontinuously at the cutoff (NOT jump from 0 to 1)
  - Some individuals above cutoff do NOT get treatment and some individuals below cutoff do receive treatment

Treatment Probability and assignment variable

Fuzzy RDD



Fuzzy Regression Discontinuity Design: Potential Outcomes Framework

Fuzzy RDD and Potential Outcomes

- Treatment Eligibility

$$Z_i = \begin{cases} 1 & \text{if } X_i \geq c \text{, eligible for a treatment} \\ 0 & \text{if } X_i < c \text{, not eligible for a treatment} \end{cases}$$

Fuzzy RDD and Potential Outcomes

- Potential Treatments
  - D_i^z: treatment status given the value of Z
  - D_i^1: treatment status if eligible for a treatment (above cutoff c)

$$\mathrm{D}_i^1 = \begin{cases} 1 & \text{if getting a treatment} \\ 0 & \text{if not getting a treatment} \end{cases}$$

  - D_i^0: treatment status if not eligible for a treatment (below cutoff c)

$$\mathrm{D}_i^0 = \begin{cases} 1 & \text{if getting a treatment} \\ 0 & \text{if not getting a treatment} \end{cases}$$

Fuzzy RDD and Potential Outcomes

- Observed Treatment

$$D_i = \begin{cases} \mathrm{D}_i^1 & \text{if } Z_i = 1, X_i \geq c \\ \mathrm{D}_i^0 & \text{if } Z_i = 0, X_i < c \end{cases}$$

- or, in a more compact notation: $\mathrm{D}_i = Z_i \mathrm{D}_i^1 + (1 - Z_i)\mathrm{D}_i^0$

Fuzzy RDD and Potential Outcomes

- In sharp RDD, the **eligible for a treatment** Z_i is the same as **getting a treatment** D_i
  - $Z_i = D_i$
- In fuzzy RDD, the **eligible for a treatment** Z_i does NOT represent the **getting a treatment** D_i
  - $Z_i \neq D_i$

Fuzzy RDD and Potential Outcomes

- The variation in getting a treatment D_i (enroll in NTU) was not entirely from the treatment eligibility Z_i (passing test score cutoff) but also from individual choice
- Thus, $\mathrm{D}_i^1 = 1$ or $\mathrm{D}_i^1 = 0$
  - Students whose test score is above threshold can enroll in NTU or not enroll in NTU
- Similarly, $\mathrm{D}_i^0 = 1$ or $\mathrm{D}_i^0 = 0$
  - Students whose test score is below threshold can enroll in NTU or not enroll in NTU

Fuzzy RDD and Potential Outcomes

- Potential Outcomes
  - Y_i^1: potential outcome if getting a treatment (either $\mathrm{D}_i^1 = 1$ or $\mathrm{D}_i^0 = 1$)
  - Y_i^0: potential outcome if not getting a treatment (either $\mathrm{D}_i^0 = 0$ or $\mathrm{D}_i^1 = 0$)
- Observed Outcomes

$$Y_i = \begin{cases} \mathrm{Y}_i^1 & \text{if } \mathrm{D}_i^1 = 1 \text{ or } \mathrm{D}_i^0 = 1 \\ \mathrm{Y}_i^0 & \text{if } \mathrm{D}_i^0 = 0 \text{ or } \mathrm{D}_i^1 = 0 \end{cases}$$

- or, in a more compact notation: $\mathrm{Y}_i = D_i^z \mathrm{Y}_i^1 + (1 - D_i^z)\mathrm{Y}_i^0$

Fuzzy RDD is IV

- The discontinuity in outcome is actually the **average causal effect** of **treatment eligibility** $Z_i = 1(X_i \geq c)$ at cutoff c

$$\lim_{\varepsilon \to 0} \mathrm{E}[Y_i | X_i = c + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[Y_i | X_i = c - \varepsilon] \tag{1}$$

- To recover the causal effect of **receiving treatment** D_i
- Divide (3) by the jump in the treatment probability at cutoff

$$\lim_{\varepsilon \to 0} \mathrm{E}[D_i | X_i = c + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[D_i | X_i = c - \varepsilon]$$

Fuzzy RDD is IV

- The the average causal effect of **receiving treatment** defined in fuzzy RDD :

$$\alpha_{FRD} = \frac{\lim_{\varepsilon \to 0} \mathrm{E}[Y_i | X_i = c + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[Y_i | X_i = c - \varepsilon]}{\lim_{\varepsilon \to 0} \mathrm{E}[D_i | X_i = c + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[D_i | X_i = c - \varepsilon]}$$

- This is a **Wald estimate** at cutoff c
- So we can consider fuzzy RDD as an IV estimate
- Use treatment eligibility $Z_i = 1(X_i \geq c)$ as an instrument for treatment received D_i
    - Use an indicator for the test score above threshold as an instrument for attending NTU

Fuzzy RDD is IV

Assumptions

- **First-Stage Relationship**: $Z_i = 1(X_i \geq c)$ affects treatment probability
- **Local Independent Assumption**: In a neighborhood of cutoff c

$$(\mathrm{Y}_i^1, \mathrm{Y}_i^0, \mathrm{D}_i^1, \mathrm{D}_i^0) \perp\!\!\!\perp Z_i$$

  - In a neighborhood of cutoff c, the assignment to treatment is random

Fuzzy RDD is IV

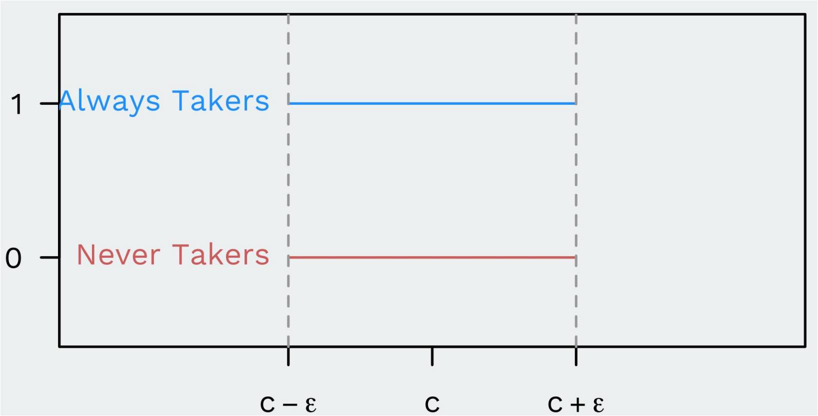
Assumptions

- **Exclusion Restriction**: $Z_i = 1(X_i \geq c)$ affects outcome Y_i only through changing treatment status D_i
- **Monotonicity Assumption**: $\mathrm{D}_i^1 \geq \mathrm{D}_i^0$
  - No one is discouraged from taking the treatment by crossing the threshold

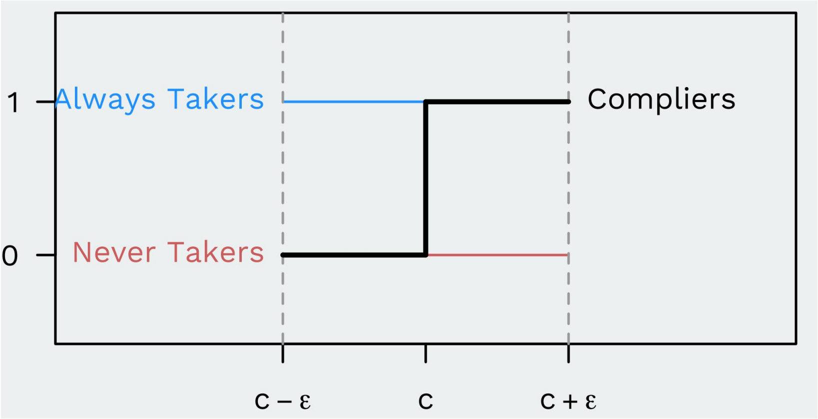
Fuzzy RDD and Compliers

- We can define four types of individuals based on whether they fellow the treatment assignment:
  - **Compliers**: $\mathrm{D}_i^1 > \mathrm{D}_i^0$ ($\mathrm{D}_i^0 = 0$ and $\mathrm{D}_i^1 = 1$)
    - David's test score is above NTU cutoff and enrolled in NTU
    - David's test score is below NTU cutoff and did not enroll in NTU
  - **Always-takers**: $\mathrm{D}_i^1 = \mathrm{D}_i^0 = 1$
    - John always can enroll in NTU (whether or not his test score is above NTU cutoff)
  - **Never-takers**: $\mathrm{D}_i^1 = \mathrm{D}_i^0 = 0$
    - Hank never enroll in NTU (whether or not his test score is above NTU cutoff)
  - **Defiers**: $\mathrm{D}_i^1 < \mathrm{D}_i^0$ ($\mathrm{D}_i^0 = 1$ and $\mathrm{D}_i^1 = 0$)
    - Jimmy's test score is above NTU cutoff and did NOT enroll in NTU
    - Jimmy's test score is below NTU cutoff but enrolled in NTU

Fuzzy RDD and Compliers



Fuzzy RDD and Compliers



Identification Results for Fuzzy RDD

Fuzzy RDD Identify LATE at cutoff c

$$\alpha_{FRD} = \frac{\lim_{\varepsilon \to 0} \mathrm{E}[Y_i | X_i = x + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[Y_i | X_i = x - \varepsilon]}{\lim_{\varepsilon \to 0} \mathrm{E}[D_i | X_i = x + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[D_i | X_i = x - \varepsilon]}$$
$$= \mathrm{E}[\mathrm{Y}_i^1 - \mathrm{Y}_i^0 | \mathrm{D}_i^1 > \mathrm{D}_i^0, X_i = c]$$

- The estimate in fuzzy RDD represents the **causal effect for compliers** (local average treatment effect, LATE) at cutoff c
- **Compliers** are those who receive the treatment when they follow treatment eligibility rule ($X_i \geq c$), but would not otherwise receive it ($X_i < c$)

Fuzzy RDD

Estimation

- Fuzzy RDD implies the discontinuity in treatment assignment Z_i can act as an instrument for actual receipt of treatment D_i
- Therefore, we can estimate treatment effect in fuzzy RDD by using two-stage least squares (TSLS)

$$D_i = \alpha_1 + \rho_1 Z_i + \mathrm{f}_1(X_i) + \upsilon_i$$
$$Y_i = \alpha_2 + \rho_2 D_i + \mathrm{f}_2(X_i) + \eta_i$$

- As in the sharp RDD setting, we can use either parametric or nonparametric approaches to estimate treatment effect

Empirical Example: Marie (2009)

Motivation

Olivier Marie (2009) "**The Best Ones Come Out First! Early Release from Prison and Recidivism: A Regression Discontinuity Approach**", Working Paper

- Examine the causal effect of early release from prison on recidivism rate

Empirical Example: Marie (2009)

Motivation

- Prison systems in many countries suffer from overcrowding and high recidivism rates after release
- Some countries use early discharge of prisoners on electronic monitoring
- Difficult to estimate impact of early release program on future criminal behavior:
  - Best behaved prisoners are usually the ones to be released early
- Marie (2008) considers the Home Detention Curfew (HDC) program in England and Wales

Empirical Example: Marie (2009)

Identification Strategy

- **Fuzzy RDD**: Only offenders sentenced to more than three months (88 days) in prison are eligible for HDC, but not all of those are offered HDC

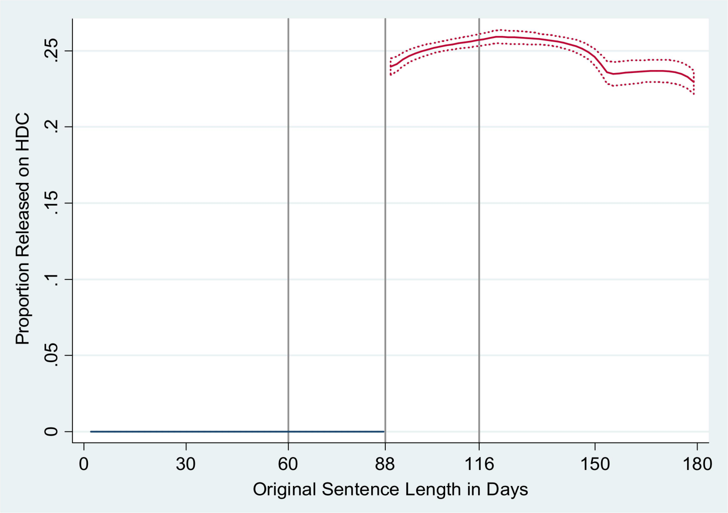
$$\alpha_{FRD} = \frac{\lim_{\varepsilon \to 0} \mathrm{E}[Rec_i | X_i = c + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[Rec_i | X_i = c - \varepsilon]}{\lim_{\varepsilon \to 0} \mathrm{E}[HDC_i | X_i = c + \varepsilon] - \lim_{\varepsilon \to 0} \mathrm{E}[HDC_i | X_i = c - \varepsilon]}$$

- Rec_i (outcome): one year recidivism rate
- X_i (assignment variable): original sentence length
- HDC_i (treatment variable): a dummy indicating offered HDC

Empirical Example: Marie (2009)

Graphical Analysis

Figure 2 : Proportion Released on HDC by Original Sentence Length

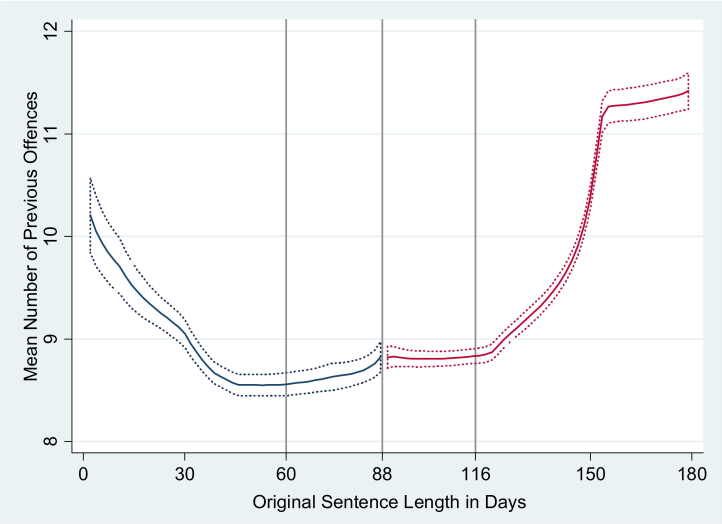


Note: Dotted lines show the confidence intervals. Line smoothed with 14 days local averages.

Empirical Example: Marie (2009)

Graphical Analysis

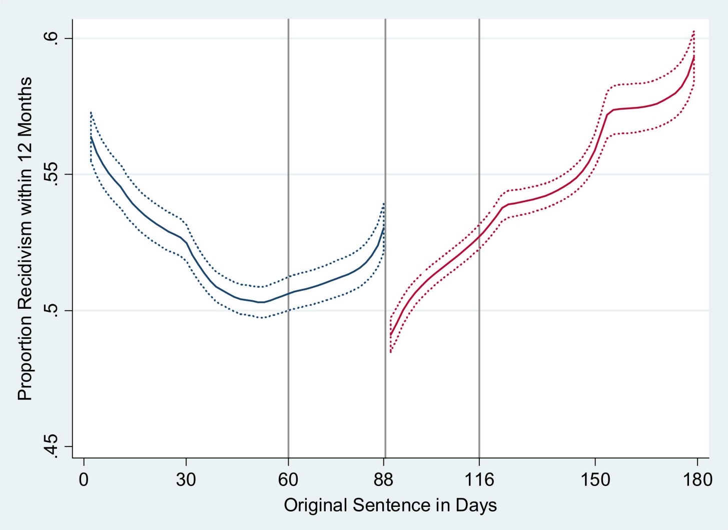
Figure 3 : Number of Previous Offences by Original Sentence Length



Empirical Example: Marie (2009)

Graphical Analysis

Figure 4: One Year Recidivism Rate by Original Sentence Length



Empirical Example: Marie (2009)

Graphical Analysis

Table 5: RD Estimates of HDC Impact on Recidivism

Panel A: Recidivism Within 12 Months of Release	**Estimation on Individuals Sentenced to Between 58 and 118 Days: +/- 4 Weeks**		
	(1)	**(2)**	**(3)**
Discontinuity of HDC Participation Around Threshold ($HDC^{+} - HDC^{-}$)	.242 (.003)	.243 (.003)	.237 (.003)
Difference in Recidivism Around Threshold ($Rec^{+} - Rec^{-}$)	-.023 (.005)	-.022 (.005)	-.016 (.005)
Estimated Effect of HDC on Recidivism Participation ($Rec^{+} - Rec^{-}$)/ ($HDC^{+} - HDC^{-}$)	*-.094* *(.020)*	*-.090* *(.018)*	*-.066* *(.018)*
Controls	No	Yes	Yes
Prison Fixed Effects	No	No	Yes
Sample Size	41,761	41,761	41,761

Panel B: Recidivism Within 24 Months of Release	**Estimation on Individuals Sentenced to Between 58 and 118 Days: +/- 4 Weeks**		
	(1)	**(2)**	**(3)**
Discontinuity of HDC Participation Around Threshold ($HDC^{+} - HDC^{-}$)	.242 (.003)	.243 (.003)	.237 (.003)
Difference in Recidivism Around Threshold ($Rec^{+} - Rec^{-}$)	-.019 (.005)	-.019 (.004)	-.013 (.005)
Estimated Effect of HDC on Recidivism Participation ($Rec^{+} - Rec^{-}$)/ ($HDC^{+} - HDC^{-}$)	*-.079* *(.020)*	*-.077* *(.018)*	*-.053* *(.019)*
Controls	No	Yes	Yes
Prison Fixed Effects	No	No	Yes
Sample Size	41,761	41,761	41,761

Note: Robust standard errors in parenthesis. The estimation is based on individuals sentenced to between 59 and 118 days. The controls included in column (2) are: gender, age, ethnic minority, breached in the past