National Taiwan University **Xiang Jyun, Jhang**
ECON 5211 Labor Economics I **Homework 1**
Prof. Kuan Ming, Chen **March 10, 2024**

---

1 Identification

1.1 Heuristic Identification

1.1.1 *"We don't have enough sample size to identify the causal effects of the problem.*

The statement is false in most cases. Since the identification is to find the parameters underlying the data generating process, sample size has nothing to do with it. However, if the sample size is very small, then the model might not be able to sucessfully identify the casual effect even if it is identifiable.

1.1.2 *"We don't have a good identification strategy so I need to use a structural model."*

$\Rightarrow$ Not having a good identification strategy can be interpreted as "Have identification set but lack method to estimate it". In this case, turning to the structural model can be a good idea.

1.1.3 *"Because I have a structural model, I don't need to think about identification."*

$\Rightarrow$ The statement is false. When using structural model, identification is as crucial as reduced-form model since it helps us to understand the exact parameters underlying the data generating process

1.1.4

"Because I can use the maximum likelihood estimator, I can identify that."
$\Rightarrow$ The statement is false and it fails to distinguish the order between identification and estimation. Only after have a correct identification can we start to discuss estimation

1.2 Identification of OLS

Given the OLS regression model

$$y_i = \beta x_i + \epsilon_i$$

suppose the identification set is not a singleton, that is

$$\beta x_i = \beta^* x_i, \quad \text{for all } x_i$$

hence $\beta = \beta^*$, so the identification set is a singleton and thus β is identified

1.3 Identification of a Factor Model

1.3.1 T

o show that ρ is identified, we consider the model equation for $y_{i,t}$:

$$\begin{aligned} y_{i,t} &= \nu_{i,t} + \epsilon_{i,t} \\ &= \rho\nu_{i,t-1} + \xi_{i,t} + \epsilon_{i,t} \\ &= \rho(y_{i,t-1} - \epsilon_{i,t-1}) + \xi_{i,t} + \epsilon_{i,t} \\ &= \rho y_{i,t-1} + \underbrace{\xi_{i,t} + \epsilon_{i,t} - \rho\epsilon_{i,t-1}}_{\text{idiosyncratics}} \end{aligned}$$

Since we can observe $y_{i,t}$ and $y_{i,t-1}$, the model equation degenrate into the OLS example in question 1.2, so ρ can be identified

1.3.2 W

e first show that σ_ξ^2 is identified by considering the equation $\nu_{i,t} = \rho\nu_{i,t-1} + \xi_{i,t}$, since the process is stationary, we have

$$\mathrm{Var}(\nu_{i,t}) = \rho^2\nu_{i,t-1} + \mathrm{Var}(\xi_{i,t}) \quad \Leftrightarrow \quad \mathrm{Var}(\nu_{i,t}) = \sigma_\nu^2 = \frac{\sigma_\xi^2}{1-\rho^2}, \quad \text{for any } t \tag{1}$$

sicne we observe the data $y_{i,t}$, we can compute the covariance between $y_{i,t}$ and $y_{i,t-1}$

$$\mathrm{Cov}(y_{i,t}, y_{i,t-1}) = \mathrm{Cov}(\nu_{i,t} + \xi_{i,t}, \nu_{i,t-1} + \xi_{i,t-1}) \overset{\nu\perp\xi}{=} \sigma_\nu^2 \overset{(1)}{=} \frac{\sigma_\xi^2}{1-\rho^2}$$

given that ρ can be identified in question 1, we can conclude that σ_ξ^2 can also be identified.

1.3.3 N

ext we show σ_ϵ^2 is identified by considering the equation $y_{i,t} = \nu_{i,t} + \epsilon$, follow same procedure, we consider the variance of the terms

$$\begin{aligned} \mathrm{Var}(y_{i,t}) &= \sigma_\nu^2 + \sigma_\epsilon^2 \\ &\overset{(1)}{=} \frac{\sigma_\xi^2}{1-\rho^2} + \sigma_\epsilon^2 \end{aligned}$$

since $y_{i,t}$ is observed and we can compute $\mathrm{Var}(y_{i,t})$, also we have shown that σ_ξ^2 and ρ can be identified, we can conclude that σ_ϵ^2 can be identified.

1.3.4 h

ello

1.4 Simulation of MLE

2 Potential Outcome Framework

2.1 f

$$w_i = D_i Y_i(1) + (1 - D_i) Y_i(0)$$

2.2 f

$Y_i(1)$ represents the outcome (wage) if individual i choose to migrate; while $Y_i(0)$ represents the outcome if individual i choose not to migrate

$$\begin{cases} Y_i(1) = w_{i,1} = \mu_{i,1} + \epsilon_{i,1} \\ Y_i(0) = w_{i,0} = \mu_{i,0} + \epsilon_{i,0} \end{cases}$$

2.2.1 f

$D_i \in \{0, 1\}$ reflects on whether individual i choose to migrate

3 Control for Observables

3.1 Rosenbaum and Rubin

$$\begin{aligned} P[D = 1 | Y_0, Y_1, P] &= E[P[D = 1 | Y_0, Y_1, P, X] | Y_0, Y_1, P] \\ &= E[P[D = 1 | Y_0, Y_1, X] | Y_0, Y_1, P], \quad (\cdot \text{ CIA}) \\ &= E[P[D = 1 | X] | Y_0, Y_1, P] \\ &= E[P(X) | Y_0, Y_1, P] \\ &= E[P | Y_0, Y_1, P] \\ &= P \end{aligned}$$

3.2 Propensity Score

3.2.1 t

3.2.2 t

3.2.3 t

3.2.4 t

3.2.5 t

3.2.6 t

3.2.7 t

3.2.8 t

3.2.9 t

3.2.10 t