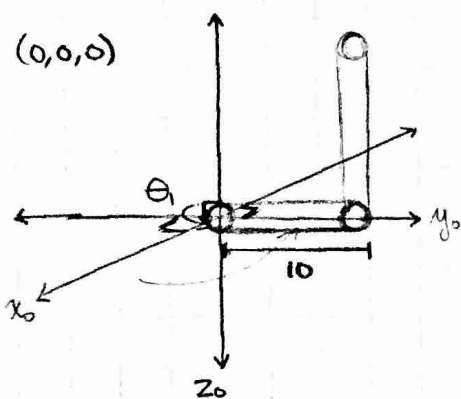


Kristina Poon



Θ_1 = Rotate about z_0 by Θ_0
 $\Theta_1 = 90^\circ$

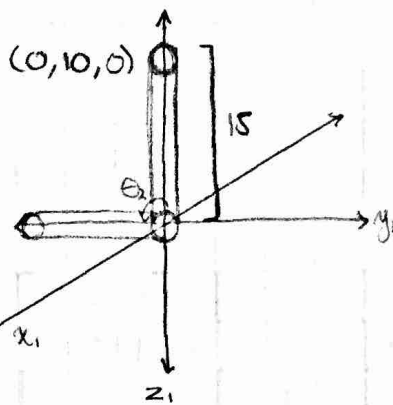
$$[{}^0R, {}^0t]$$

$$\text{Rot}(\hat{z}, \Theta_1) = \begin{bmatrix} \cos(\Theta_1) & -\sin(\Theta_1) & 0 \\ \sin(\Theta_1) & \cos(\Theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(\hat{y}, \beta_1) = \begin{bmatrix} \cos(\beta_1) & 0 & \sin(\beta_1) \\ 0 & 1 & 0 \\ \sin(\beta_1) & 0 & \cos(\beta_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(\hat{x}, \alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_1) & -\sin(\alpha_1) \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^0R, {}^0t] = \begin{bmatrix} 0 & -1 & 0 & | & 0 \\ 1 & 0 & 0 & | & 10 \\ 0 & 0 & 1 & | & 0 \end{bmatrix}$$



Θ_2 = Rotate about x_1 by α_1
 $\Theta_2 = 90^\circ$

$$\text{Rot}(\hat{z}, \Theta_1) \text{Rot}(\hat{y}, \beta_1) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(\hat{z}, \Theta_1) \text{Rot}(\hat{y}, \beta_1) \text{Rot}(\hat{x}, \alpha_1) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^2R, {}^2t] = \begin{bmatrix} 1 & 0 & 0 & | & 0 \\ 0 & 0 & -1 & | & 0 \\ 0 & 1 & 0 & | & 15 \end{bmatrix}$$

$$[{}^1R, {}^1t]$$

$$\text{Rot}(\hat{z}, \Theta) = \begin{bmatrix} \cos(\Theta) & -\sin(\Theta) & 0 \\ \sin(\Theta) & \cos(\Theta) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(\hat{y}, \beta_2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(\hat{x}, \alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\alpha_1) & -\sin(\alpha_1) \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\text{Rot}(\hat{z}, \Theta) \text{Rot}(\hat{y}, \beta_2) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{Rot}(\hat{z}, \Theta) \text{Rot}(\hat{y}, \beta_2) \text{Rot}(\hat{x}, \alpha_1) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$${}^1R_2 = {}^2R_1^{-1} = ({}^2R_1)^T$$

$${}^0R_1 {}^1R_2 = {}^0R_2$$

$${}^2R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \rightarrow ({}^2R_1)^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = {}^1R_2$$

$${}^0R_2 = {}^0R_1 {}^1R_2 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$${}^1R_0 = {}^0R_1^{-1} = ({}^0R_1)^T$$

$${}^0R_1 = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow ({}^0R_1)^T = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = {}^1R_0$$

$${}^2R_0 = {}^2R_1 {}^1R_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

$${}^0T_2 = \begin{bmatrix} 0 & 0 & -1 & | & 0 \\ 1 & 0 & 0 & | & 10 \\ 0 & -1 & 0 & | & 15 \end{bmatrix}$$

$${}^2T_0 = \begin{bmatrix} 0 & 1 & 0 & | & 0 \\ 0 & 0 & -1 & | & -10 \\ -1 & 0 & 0 & | & -15 \end{bmatrix}$$

$${}^0R_2 = {}^0R_1 {}^1R_2 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1) & 0 \\ \sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_2) & \sin(\theta_2) \\ 0 & -\sin(\theta_2) & \cos(\theta_2) \end{bmatrix}$$

$${}^0T_2 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1)\cos(\theta_2) & -\sin(\theta_1)\sin(\theta_2) & | & 0 \\ \sin(\theta_1) & \cos(\theta_1)\cos(\theta_2) & \cos(\theta_1)\sin(\theta_2) & | & 10 \\ 0 & -\sin(\theta_2) & \cos(\theta_2) & | & 15 \end{bmatrix}$$

$${}^2R_0 = {}^2R_1 {}^1R_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\theta_2) & -\sin(\theta_2) \\ 0 & \sin(\theta_2) & \cos(\theta_2) \end{bmatrix} \cdot \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 \\ -\sin(\theta_1) & \cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$${}^2T_0 = \begin{bmatrix} \cos(\theta_1) & \sin(\theta_1) & 0 & | & 0 \\ -\sin(\theta_1)\cos(\theta_2) & \cos(\theta_1)\cos(\theta_2) & -\sin(\theta_2) & | & -10 \\ -\sin(\theta_1)\sin(\theta_2) & \cos(\theta_1)\sin(\theta_2) & \cos(\theta_2) & | & -15 \end{bmatrix}$$