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RCV
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# Homework 1: Linear Algebra

3. $\hat{x} = (A^TA)^{-1}A^Tb$

| | |
|---|---|
| $n=1: (0,1)$ | $1 = m(0)+B$ |
| $n=2: (1,3.2)$ | $3.2 = m(1)+B$ |
| $n=3: (1.9,5)$ | $5 = m(1.9)+B$ |
| $n=4: (3,7.2)$ | $7.2 = m(3)+B$ |
| $n=5: (3.9,9.3)$ | $9.3 = m(3.9)+B$ |
| $n=6: (5,11.1)$ | $11.1 = m(5)+B$ |

$$A = \overset{6\times 2}{\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 1 & 1.9 \\ 1 & 3 \\ 1 & 3.9 \\ 1 & 5 \end{bmatrix}} \qquad x = \begin{bmatrix} B \\ m \end{bmatrix} \qquad b = \overset{6\times 1}{\begin{bmatrix} 1 \\ 3.2 \\ 5 \\ 7.2 \\ 9.3 \\ 11.1 \end{bmatrix}}$$

$$A^T = \overset{2\times 6}{\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1.9 & 3 & 3.9 & 5 \end{bmatrix}}$$

$$(A^TA) = \begin{bmatrix} 6(1) & 0+1+1.9+3+3.9+5 \\ 0+1+1.9+3+3.9+5 & 0+1+1.9^2+3^2+3.9^2+5^2 \end{bmatrix} = \begin{bmatrix} 6 & 14.8 \\ 14.8 & 53.82 \end{bmatrix}$$

$$(A^TA)^{-1} = \begin{bmatrix} 6 & 14.8 \\ 14.8 & 53.82 \end{bmatrix}^{-1} = \frac{1}{6(53.82)-(14.8^2)}\begin{bmatrix} 53.82 & -14.8 \\ -14.8 & 6 \end{bmatrix}$$

$$= \frac{1}{103.88}\begin{bmatrix} 53.82 & -14.8 \\ -14.8 & 6 \end{bmatrix} = \begin{bmatrix} 0.518 & -0.142 \\ -0.142 & 0.058 \end{bmatrix}$$

$$A^Tb = \begin{bmatrix} 1+3.2+5+7.2+9.3+11.1 \\ 0+3.2+(1.9)(5)+(3)(7.2)+(3.9)(9.3)+(5)(11.1) \end{bmatrix} = \begin{bmatrix} 36.8 \\ 126.07 \end{bmatrix}$$

$$(A^TA)^{-1}A^Tb = \overset{2\times 2}{\begin{bmatrix} 0.518 & -0.142 \\ -0.142 & 0.058 \end{bmatrix}}\overset{2\times 1}{\begin{bmatrix} 36.8 \\ 126.07 \end{bmatrix}} = \begin{bmatrix} 1.16046 \\ 2.08646 \end{bmatrix} = \hat{x}$$

4. Yes; these vectors are linearly independent and all possibly existing vectors in the vector space can be expressed as a linear combination of these vectors.

5. No; $\vec{t_1}$ and $\vec{t_2}$ are linearly dependent of one another. For example, $\vec{t_2}$ can be expressed as $\vec{t_2} = (1)\vec{t_1} + (0)\vec{t_3}$.

6. Yes; we know from question 4 that the first three vectors are capable basis vectors; the additional vector $\vec{w}^4$ is linearly dependent and doesn't present any changes to the dimensions & spans, so it remains spanning $\mathbb{R}^3$.