Image Mosaicing

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Abstract

In this project, we will apply a Harris corner detector to find corners in two images, automatically find corresponding features, estimate a homography between the two images, and warp one image into the coordinate system of the second one to produce a mosaic containing the union of all pixels in the two images.

1. Introduction

Image Stitching is a widely used image processing technique. According matched feature points, we can combine many small perspective images into an image with a large viewing angle. This technique is also widely used in the synthesis of a wide-angle photo, satellite photo processing, medical image processing and other fields. The purpose of this project is to find corner features in multiple images and to align the images in a mosaic by estimating a homography between corresponding features.

2. Description of Algorithms

1. Read in two images and make them grayscale.

2. Apply Harris corner detector to both images: compute Harris R function over the image, and then do non-maximum suppression to get a sparse set of corner features.

3. Given two set of corners from the two images, compute normalized cross correlation (NCC) of image patches centered at each corner. Choose potential corner matches by finding pair of corners (one from each image) such that they have the highest NCC value. Then set a threshold to keep only matches that have a large NCC score to find correspondences between the two images.

4. Repeatedly sample minimal number of points needed to estimate a homography (4 pts in this case). Compute a homography from these four points. Map all points using the homagraphy and comparing distances between predicted and observed locations to determine the number of inliers. At the end, compute a least-squares homgraphy from all the inliers in the largest set of inliers.

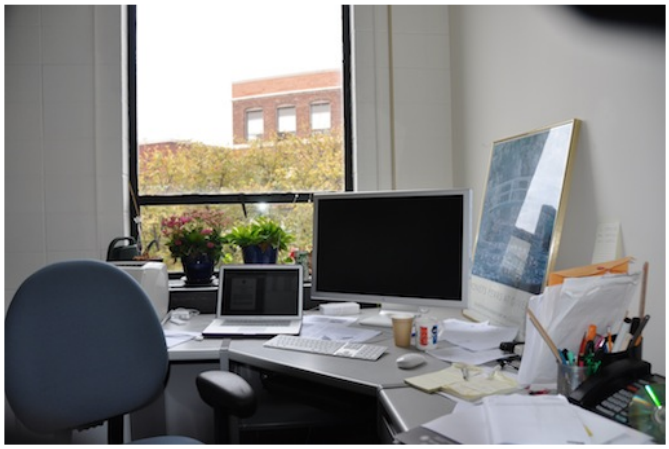
5. Determine how big to make the final output image so that it contains the union of all pixels in the two images. Copy the image that does not have to be warped into the appropriate location in the output. Warp the other image into the output image based on the estimated homography (or its inverse). Use blending schemes to blend pixels in the area of overlap between both images.

3. Experiments and values of parameters used

1) Read in two images and make them grayscale.

```
img1=imread('DSC_0308.JPG'); %read in image1
img2=imread('DSC_0309.JPG'); %read in image2
pic1=rgb2gray(img1); %gray scale image1
pic2=rgb2gray(img2); %gray scale image2
figure,imshow(img1);
figure,imshow(img2);
```

1. img1



2. img2

2) Apply Harris corner detector to both images: compute Harris R function over the image, and then do non-maximum suppression to get a sparse set of corner features.

```
%gaussian smoothing
der_sig=1;
x=-3:3;
Filter_der1D=exp(-(x).^2/(2*der_sig^2))/(der_sig*sqrt(2*pi));
pic1_smoothed=conv2(Filter_der1D,Filter_der1D',pic1,'same');
pic2_smoothed=conv2(Filter_der1D,Filter_der1D',pic2,'same');

%derivative filter
Filter_derx=[1 0 -1];
Filter_dery=Filter_derx';

%computer image 1 derivative by convolution in both x and y directions
%compute the derivative with respect to x
pic1_derx=conv2(pic1_smoothed,Filter_derx,'same');
%compute the derivative with respect to x at the border pixels
pic1_derx(:,1)=2*(pic1_smoothed(:,2) -pic1_smoothed(:,1));
pic1_derx(:,end)=2*(pic1_smoothed(:,end) -pic1_smoothed(:,end-1));
%compute the derivative with respect to y
pic1_dery=conv2(pic1_smoothed,Filter_dery,'same');
%compute the derivative with respect to y at the border pixels
pic1_dery(1,:)=2*(pic1_smoothed(2,:) -pic1_smoothed(1,:));
pic1_dery(end,:)=2*(pic1_smoothed(end,:) -pic1_smoothed(end-1,:));

%compute image 2 derivative by convolution in both x and y directions
%compute the derivative with respect to x
pic2_derx=conv2(pic2_smoothed,Filter_derx,'same');
%compute the derivative with respect to x at the border pixels
pic2_derx(:,1)=2*(pic2_smoothed(:,2) -pic2_smoothed(:,1));
pic2_derx(:,end)=2*(pic2_smoothed(:,end) -pic2_smoothed(:,end-1));
%compute the derivative with respect to y
pic2_dery=conv2(pic2_smoothed,Filter_dery,'same');
%compute the derivative with respect to y at the border pixels
pic2_dery(1,:)=2*(pic2_smoothed(2,:) -pic2_smoothed(1,:));
pic2_dery(end,:)=2*(pic2_smoothed(end,:) -pic2_smoothed(end-1,:));

%compute quadratic terms of the derivatives
pic1_der_x2=pic1_derx.^2;
pic1_der_y2=pic1_dery.^2;
pic1_der_xy=pic1_derx.*pic1_dery;
pic2_der_x2=pic2_derx.^2;
pic2_der_y2=pic2_dery.^2;
pic2_der_xy=pic2_derx.*pic2_dery;
%apply Gaussian filter to suppress noise and perform the smoothing in a
separable way
int_sig=2;
x=-6:6;
Filter_int1D=exp(-(x).^2/(2*int_sig^2))/(int_sig*sqrt(2*pi));
pic1_der_x2=conv2(Filter_int1D,Filter_int1D',pic1_der_x2,'same');
pic1_der_y2=conv2(Filter_int1D,Filter_int1D',pic1_der_y2,'same');
pic1_der_xy=conv2(Filter_int1D,Filter_int1D',pic1_der_xy,'same');
pic2_der_x2=conv2(Filter_int1D,Filter_int1D',pic2_der_x2,'same');
pic2_der_y2=conv2(Filter_int1D,Filter_int1D',pic2_der_y2,'same');
```

```
pic2_der_xy=conv2(Filter_int1D,Filter_int1D',pic2_der_xy,'same');

kappa=0.04;
%compute Harris R function over the images
r1=(pic1_der_x2.*pic1_der_y2-pic1_der_xy.^2)-
kappa*(pic1_der_x2+pic1_der_y2).^2;
r2=(pic2_der_x2.*pic2_der_y2-pic2_der_xy.^2)-
kappa*(pic2_der_x2+pic2_der_y2).^2;

%calculate the maximum of Harris R function
r1_max=max(r1(:));
r2_max=max(r2(:));
%set the threshold
threshold1=0.001*r1_max;
threshold2=0.001*r2_max;

height1=size(img1,1); %calculate the height of the image1
width1=size(img1,2);  %calculate the width of the image1
height2=size(img2,1); %calculate the height of the image2
width2=size(img2,2);  %calculate the width of the image2
result1 = zeros(height1, width1); %record the position of the corner of
image 1
result2 = zeros(height2, width2); %record the position of the corner of
image 2

%do non-maximum suppression of image 1
count1 = 0; %record the quantity of corners in image 1
for i = 2 : height1-1
    for j = 2 : width1-1
        % the size of window is 3*3
        if (r1(i, j) > threshold1 && r1(i, j) > r1(i-1, j-1) && r1(i,
j) > r1(i-1, j) && r1(i, j) > r1(i-1, j+1) && r1(i, j) > r1(i, j-1) &&
r1(i, j) > r1(i, j+1) && r1(i, j) > r1(i+1, j-1) && r1(i, j) > r1(i+1,
j) && r1(i, j) > r1(i+1, j+1))
            result1(i, j) = 1; % a sparse matrix
            count1 = count1 + 1;
        end;
    end;
end;

%do non-maximum suppression of image 2
count2 = 0; %record the quantity of corners in image 2
for i = 2 : height2-1
    for j = 2 : width2-1
        % the size of window is 3*3
        if (r2(i, j) > threshold2 && r2(i, j) > r2(i-1, j-1) && r2(i,
j) > r2(i-1, j) && r2(i, j) > r2(i-1, j+1) && r2(i, j) > r2(i, j-1)
&& ...
                r2(i, j) > r2(i, j+1) && r2(i, j) > r2(i+1, j-1) &&
r2(i, j) > r2(i+1, j) && r2(i, j) > r2(i+1, j+1));
            result2(i, j) = 1; % a sparse matrix
            count2 = count2 + 1;
        end;
    end;
end;

i = 1;
```

```
    for j = 1 : height1
        for k = 1 : width1
            if result1(j, k) == 1;
                % cor1 is a Nx2 matrix, stores the position of corners
in image1
                cor1(i, 1) = j;
                cor1(i, 2) = k;
                i = i + 1;
            end;
        end;
    end;

i = 1;
    for j = 1 : height2
        for k = 1 : width2
            if result2(j, k) == 1;
       % cor2 is a Nx2 matrix, stores the position of corners in image2
                cor2(i, 1) = j;
                cor2(i, 2) = k;
             i = i + 1;
            end;
        end;
    end;

%%display the Harris corners in image 1
[poscol1, posrow1] = find(result1 == 1);
figure,imshow(img1);
hold on;
plot(posrow1, poscol1, '*');

%%display the Harris corners in image 2
[poscol2, posrow2] = find(result2 == 1);
figure,imshow(img2);
hold on;
plot(posrow2, poscol2, '*');
```

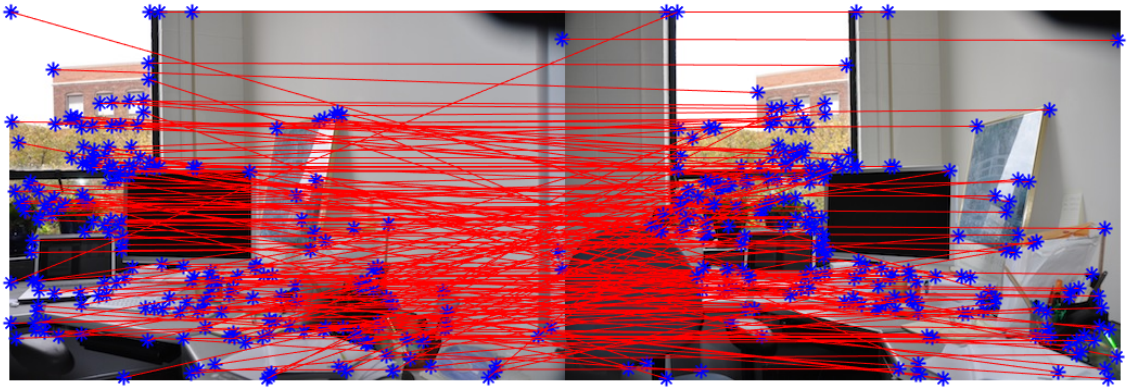
3. Harris corners in img1

4. Harris corners in img2

3) Given two set of corners from the two images, compute normalized cross correlation (NCC) of image patches centered at each corner. Choose potential corner matches by finding pair of corners (one from each image) such that they have the highest NCC value. Then set a threshold to keep only matches that have a large NCC score to find correspondences between the two images.

```
f=cell(1,count1);
g=cell(1,count2);

pic1_db=double(pic1);
pic2_db=double(pic2);

for i=1:count1
    patch1=pic1_db(cor1(i,1)-1:cor1(i,1)+1,cor1(i,2)-1:cor1(i,2)+1); %compute
the gray level of each corner and its 3x3 neighbours in image 1
    f{i}=patch1./(sqrt(sum(sum(patch1.^2))));
end

for i=1:count2
    patch2=pic2_db(cor2(i,1)-1:cor2(i,1)+1,cor2(i,2)-1:cor2(i,2)+1); %compute
the gray level of each corner and its 3x3 neighbours in image 2
    g{i}=patch2./(sqrt(sum(sum(patch2.^2))));
end

ncc1=zeros(count2,count1);
for i=1:count1
    for j=1:count2
        ncc1(j,i)=sum(sum(f{i}.*g{j})); % compute the NCC values
    end
end
%index1 records the exact corner in image2 which corresponds to image1
%max1_num records the highest NCC values of every corner in image1
```

```
corresponding to the corners in image2
[max1_num,index1]=max(ncc1);
tres=0.95; %set treshold be 0.95
% only the hightest NCC values bigger than treshold will be recorded
[max1pos_rol,max1pos_col]=find(max1_num>tres);
match1=zeros(length(max1pos_col),2); %records the corners of two images which
have the correspondence
for i=1:length(max1pos_col)
    match1(i,1)=max1pos_col(i);%records corners in image1 which correspond to
corners in image2
    match1(i,2)=index1(max1pos_col(i));%records corners in image2 which
correspond to corners in image1
end

ncc2=zeros(count1,count2);
for i=1:count2
    for j=1:count1
        ncc2(j,i)=sum(sum(f{j}.*g{i})); % compute the NCC values
    end
end
%index2 stores the exact corner in image1 which corresponds to image2
%max2_num stores the highest NCC values of every corner in image2
corresponding to the corners in image1
[max2_num,index2]=max(ncc2);
% only the hightest NCC values bigger than treshold will be recorded
[max2pos_rol,max2pos_col]=find(max2_num>tres);
match2=zeros(length(max2pos_col),2); %records the corners of two images which
have the correspondence
for i=1:length(max2pos_col)
    match2(i,2)=max2pos_col(i);%records corners in image2 which correspond to
corners in image1
    match2(i,1)=index2(max2pos_col(i));%records corners in image1 which
correspond to corners in image2
end
matchs=[];  %record the final feature points match relationship using two-way
match
loc1=[];    %record the location of the final matching points in image1
loc2=[];    %record the location of the final matching points in image2
for i=1:length(match1)
    for j=1:length(match2)
        if match1(i,:)==match2(j,:)
            matchs=[matchs;match1(i,:)];
            loc1=[loc1;cor1(match1(i,1),:)];
            loc2=[loc2;cor2(match1(i,2),:)];
        end
    end
end

%Draw lines between matching corners
X1=loc1(:,2);
Y1=loc1(:,1);
X2=loc2(:,2);
Y2=loc2(:,1);
dif=size(img1,2);
figure();
imshowpair(img1,img2,'montage');
hold on
for k=1:length(X1)
  plot(X1(k),Y1(k),'b*');
  plot(X2(k)+dif,Y2(k),'b*');
  line([X1(k),X2(k)+dif],[Y1(k),Y2(k)],'Color','r');
end
set(gcf,'Color','w');
```



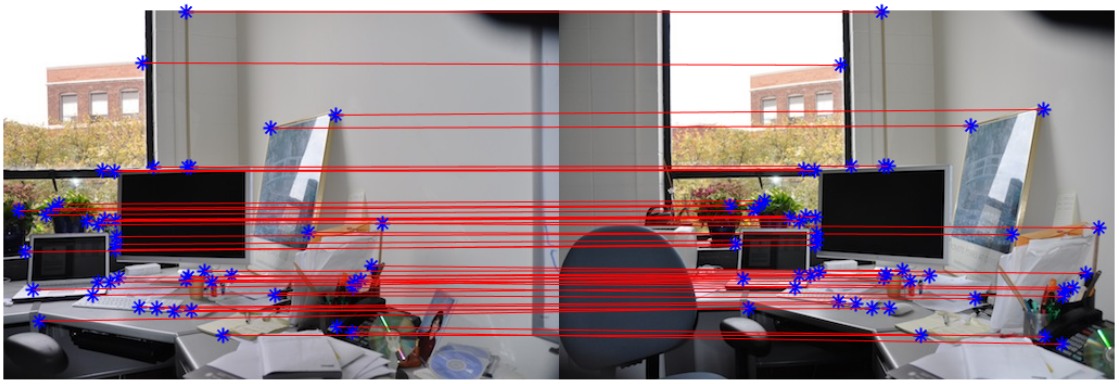
5. the correspondences between the two images after NCC

4) Use RANSAC to robustly estimate the homography from the noisy correspondences. Repeatedly sample minimal number of points needed to estimate a homography (4 pts in this case). Compute a homography from these four points. Map all points using the homagraphy and comparing distances between predicted and observed locations to determine the number of inliers. At the end, compute a least-squares homgraphy from all the inliers in the largest set of inliers.

```
inlier_nummax=4;
inlier_loc=[];
for k=1:20000
  a=ceil(length(loc1)*rand(1,4)); %randomly select corresponding four
points
  while
a(1)==a(2)||a(1)==a(3)||a(1)==a(4)||a(2)==a(3)||a(2)==a(4)||a(3)==a(4)
      a=ceil(length(loc1)*rand(1,4));
  end
  x11=loc1(a(1),1);
  y11=loc1(a(1),2);
  x12=loc2(a(1),1);
  y12=loc2(a(1),2);
  x21=loc1(a(2),1);
  y21=loc1(a(2),2);
  x22=loc2(a(2),1);
  y22=loc2(a(2),2);
  x31=loc1(a(3),1);
  y31=loc1(a(3),2);
  x32=loc2(a(3),1);
  y32=loc2(a(3),2);
  x41=loc1(a(4),1);
  y41=loc1(a(4),2);
  x42=loc2(a(4),1);
```

```
  y42=loc2(a(4),2);

  A=[x11,y11,1,0,0,0,-x11*x12,-y11*x12;
     0,0,0,x11,y11,1,-x11*y12,-y11*y12;
     x21,y21,1,0,0,0,-x21*x22,-y21*x22;
     0,0,0,x21,y21,1,-x21*y22,-y21*y22;
     x31,y31,1,0,0,0,-x31*x32,-y31*x32;
     0,0,0,x31,y31,1,-x31*y32,-y31*y32;
     x41,y41,1,0,0,0,-x41*x42,-y41*x42;
     0,0,0,x41,y41,1,-x41*y42,-y41*y42];

  b=[x12,y12,x22,y22,x32,y32,x42,y42]';
  h=A\b; %Compute a homography from these corresponding four points

  inlier_num=0;
  inlier_loc=0;
  for i=1:length(loc1)
    %Map all points using the homagraphy

loc2_predicted(i,1)=(h(1)*loc1(i,1)+h(2)*loc1(i,2)+h(3))/(h(7)*loc1(i,1
)+h(8)*loc1(i,2)+1);

loc2_predicted(i,2)=(h(4)*loc1(i,1)+h(5)*loc1(i,2)+h(6))/(h(7)*loc1(i,1
)+h(8)*loc1(i,2)+1);
    %Compare distances between predicted and observed locations
    distance=sqrt((loc2_predicted(i,1)-
loc2(i,1))^2+(loc2_predicted(i,2)-loc2(i,2))^2);
    if distance<1 %the inliers will be recorded only when the distance
less than 1
      inlier_num=inlier_num+1;
      inlier_loc=[inlier_loc;i];
    end
  end
  if inlier_num>inlier_nummax
      inlier_nummax=inlier_num;
      hom=h; %compute a least-squares homgraphy from all the inliers in
the largest set of inliers
      inlier_maxloc=inlier_loc; %record the exact corners that are
inliers
  end
end

%record the position of inliers in image1 and image2
for i=1:length(inlier_maxloc)-1
    inlier1_loc(i,1)=loc1(inlier_maxloc(i+1),1);
    inlier1_loc(i,2)=loc1(inlier_maxloc(i+1),2);
    inlier2_loc(i,1)=loc2(inlier_maxloc(i+1),1);
    inlier2_loc(i,2)=loc2(inlier_maxloc(i+1),2);
end

%Draw lines between matching inliers corners
X1=inlier1_loc(:,2);
Y1=inlier1_loc(:,1);
X2=inlier2_loc(:,2);
Y2=inlier2_loc(:,1);
dif=size(img1,2);
figure();
```

```
imshowpair(img1,img2,'montage');
hold on
for k=1:length(X1)
  plot(X1(k),Y1(k),'b*');
  plot(X2(k)+dif,Y2(k),'b*');
  line([X1(k),X2(k)+dif],[Y1(k),Y2(k)],'Color','r');
end
set(gcf,'Color','w');
```



6. a least-squares homgraphy after using RANSAC

5) Determine how big to make the final output image so that it contains the union of all pixels in the two images. Copy the image that does not have to be warped into the appropriate location in the output. Warp the other image into the output image based on the estimated homography (or its inverse). Use blending schemes to blend pixels in the area of overlap between both images.

```
% create meshgrid for the warped image2
[xi,yi] = meshgrid(1:width2,1:height2);
img2_grid = ones(3,height2*width2);
img2_grid(1,:) = reshape(xi,1,height2*width2);
img2_grid(2,:) = reshape(yi,1,height2*width2);

% use the inverse of the homography
new_hom=[hom(1),hom(2),hom(3);hom(4),hom(5),hom(6);hom(7),hom(8),1];
img2_newgrid = new_hom\img2_grid;
%the left side of the new image
left = fix(min([1,min(img2_newgrid(1,:)./img2_newgrid(3,:))]));
%the right side of the new image
right = fix(max([width1,max(img2_newgrid(1,:)./img2_newgrid(3,:))]));
%the top side of the new image
top = fix(min([1,min(img2_newgrid(2,:)./img2_newgrid(3,:))]));
%the bottom side of the new image
bottom = fix(max([height1,max(img2_newgrid(2,:)./img2_newgrid(3,:))]));
%the height of the new image
new_height = bottom - top + 1;
%the width of the new image
new_width = right - left + 1;
```

```
newimg_left = left; % x coordate of lefttop corner of the new image
newimg_top  = top;  % y coordate of lefttop corner of the new image
unwarped_img1_left = 2 - left; % x coordate of lefttop corner of
unwarped image
unwarped_img1_top  = 2 - top;  % y coordate of lefttop corner of
unwarped image

% create meshgrid for the new image
[XI,YI] = meshgrid(newimg_left:newimg_left+new_width-1,
newimg_top:newimg_top+new_height-1);
newimg_grid = ones(3,new_height*new_width);
newimg_grid(1,:) = reshape(XI,1,new_height*new_width);
newimg_grid(2,:) = reshape(YI,1,new_height*new_width);

newimg_grid = new_hom*newimg_grid;
XI = reshape(newimg_grid(1,:)./newimg_grid(3,:), new_height,
new_width);
YI = reshape(newimg_grid(2,:)./newimg_grid(3,:), new_height,
new_width);

% Separation of the three primary colors
R=double(img2(:,:,1));
G=double(img2(:,:,2));
B=double(img2(:,:,3));
%Interpolation
finalImage(:,:,1) = interp2(R, XI, YI);
finalImage(:,:,2) = interp2(G, XI, YI);
finalImage(:,:,3) = interp2(B, XI, YI);
%display the warped image
figure,imshow(uint8(finalImage));

[finalImage] = blend(unwarped_img1_left, unwarped_img1_top, finalImage,
img1);
% display the final image
figure,imshow(uint8(finalImage));


function [finalImage] = blend(unwarped_x, unwarped_y, img_warp, img_unwarp)
% Blend two image
% Input:
% img_warp - original image to be warped
% img_unwarp - the other image not to be warped
% unwarped_x -  coordinate of the lefttop corner of unwarped image
% unwarped_y - y coordinate of the lefttop corner of unwarped image
% Output: finalImage

% set the grayscale in the final image to be 0 where exists NAN
img_warp(isnan(img_warp))=0;
% Set a maskA, in the region of warped image, the value is 1, otherwise, is
0.
maskA = (img_warp(:,:,1)>0 |img_warp(:,:,2)>0 | img_warp(:,:,3)>0);
finalImage = zeros(size(img_warp));
%copy the unwarped image to the final image
finalImage(unwarped_y:unwarped_y+size(img_unwarp,1)-1, unwarped_x:
unwarped_x+size(img_unwarp,2)-1,:) = img_unwarp;
% In the region of unwarped image, the value is 1, otherwise is 0.
mask = (finalImage(:,:,1)>0 | finalImage(:,:,2)>0 | finalImage(:,:,3)>0);
% In the overlaid region, the value is 1,otherwise, is 0.
```

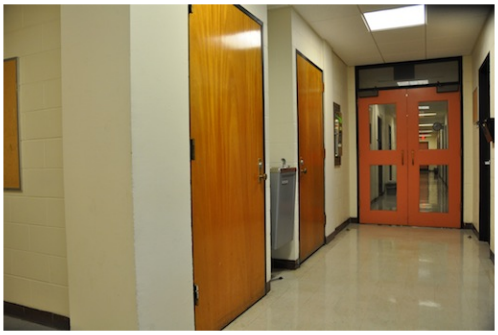
```
mask = and(maskA, mask);

% calculate the overlaid region of the two images
[~,col_mask] = find(mask);
left = min(col_mask);% find the left boundary of the overlaid region
right = max(col_mask);% find the right boundary of the overlaid region
mask = ones(size(mask));
if( unwarped_x<2) %if the warped image is on the right of the unwarped image
  mask(:,left:right) = repmat(linspace(0,1,right-left+1),size(mask,1),1);
else
  mask(:,left:right) = repmat(linspace(1,0,right-left+1),size(mask,1),1);
end

% blend each channel
img_warp(:,:,1) = img_warp(:,:,1).*mask;
img_warp(:,:,2) = img_warp(:,:,2).*mask;
img_warp(:,:,3) = img_warp(:,:,3).*mask;

if( unwarped_x<2) %if the warped image is on the right of the unwarped image
  mask(:,left:right) = repmat(linspace(1,0,right-left+1),size(mask,1),1);
else
  mask(:,left:right) = repmat(linspace(0,1,right-left+1),size(mask,1),1);
end

finalImage(:,:,1) = finalImage(:,:,1).*mask;
finalImage(:,:,2) = finalImage(:,:,2).*mask;
finalImage(:,:,3) = finalImage(:,:,3).*mask;

finalImage(:,:,1) = img_warp(:,:,1) + finalImage(:,:,1);
finalImage(:,:,2) = img_warp(:,:,2) + finalImage(:,:,2);
finalImage(:,:,3) = img_warp(:,:,3) + finalImage(:,:,3);
```

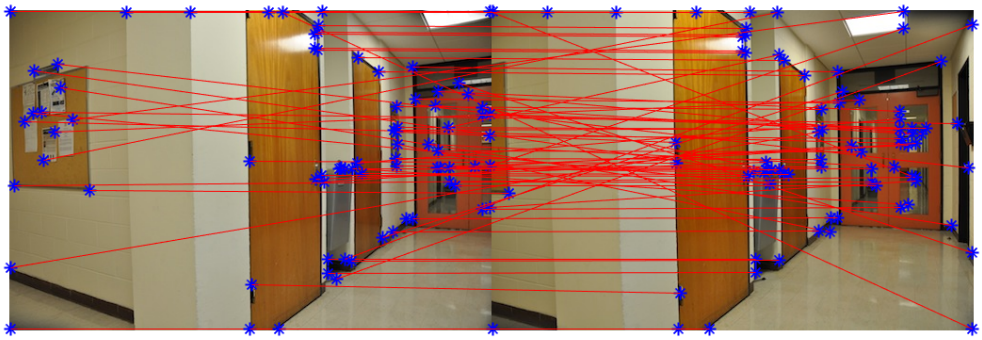
7. a warped image

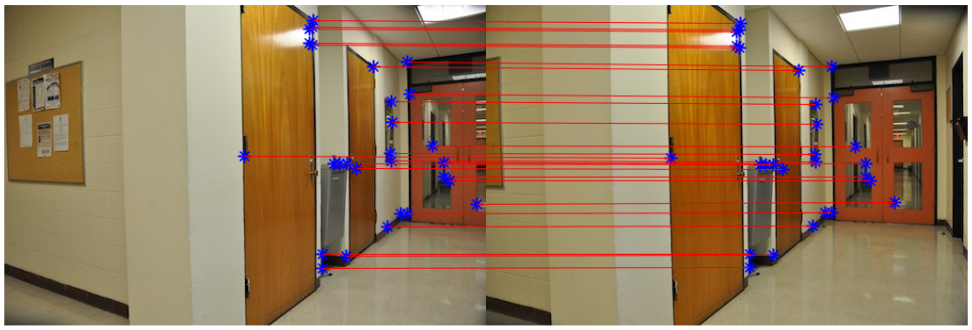
8. the final mosaicing image

4. Observations

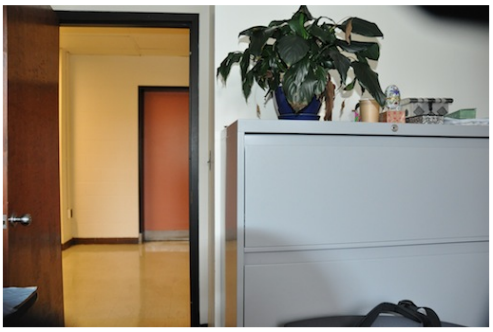
The program also uses other images as the samples. We can see the final mosaic image is good. For example: 1).

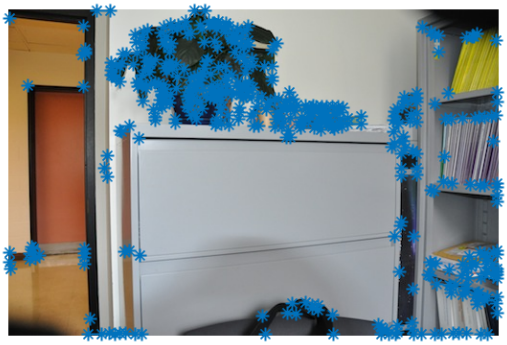


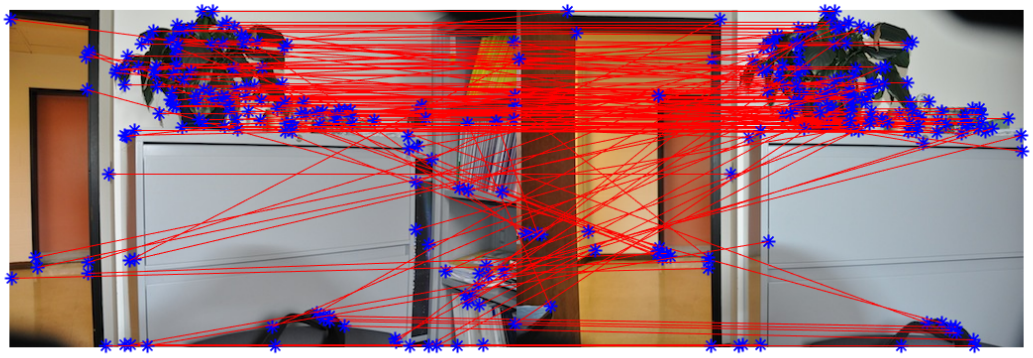


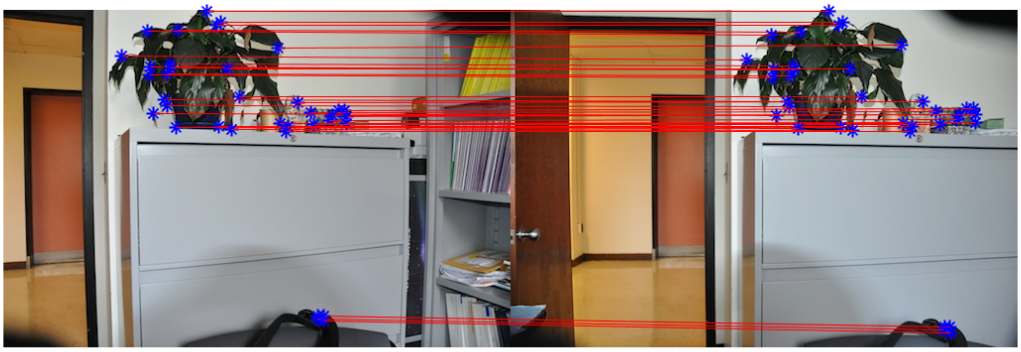


2).





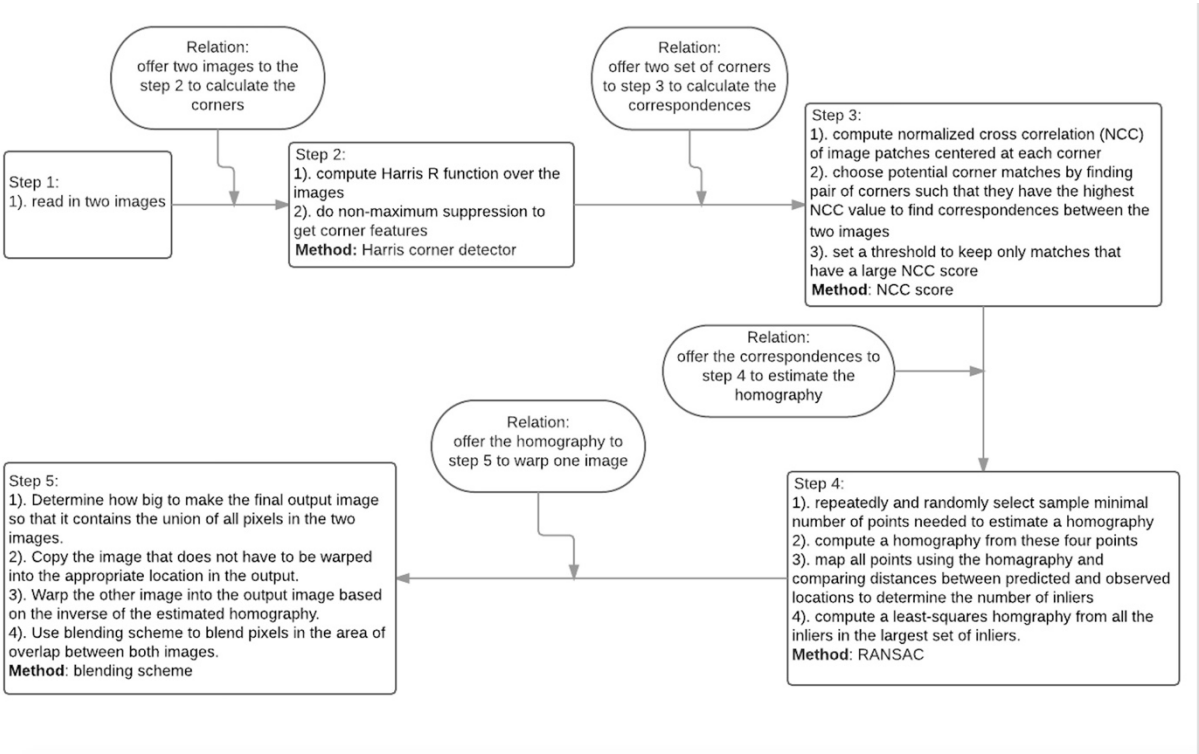




5. Conclusion

Using Harris corner detector can detect a large number of corners. After using NCC to find correspondence between the two images, the correspondences are likely to have many errors. Using RANSAC can robustly estimate the homography from the noisy correspondences. Warping the image into the output image based on the inverse of the estimated homography can have no gaps. It is a good way to produce an image mosaic.

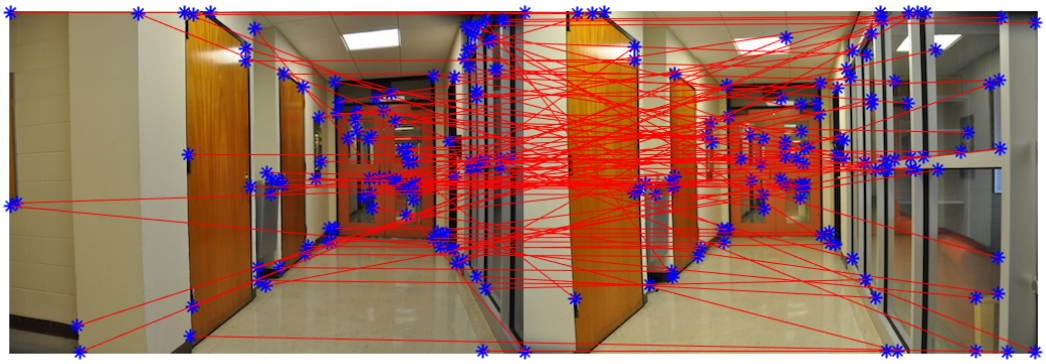
6. Flowchart



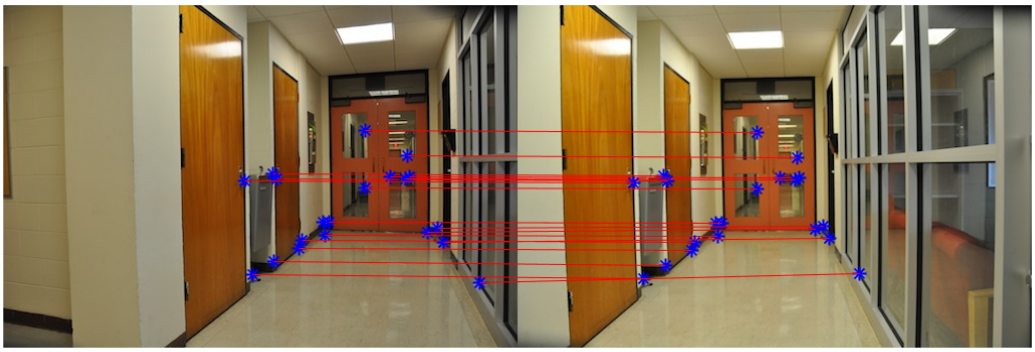
Step 1:

step 2:


step 3:



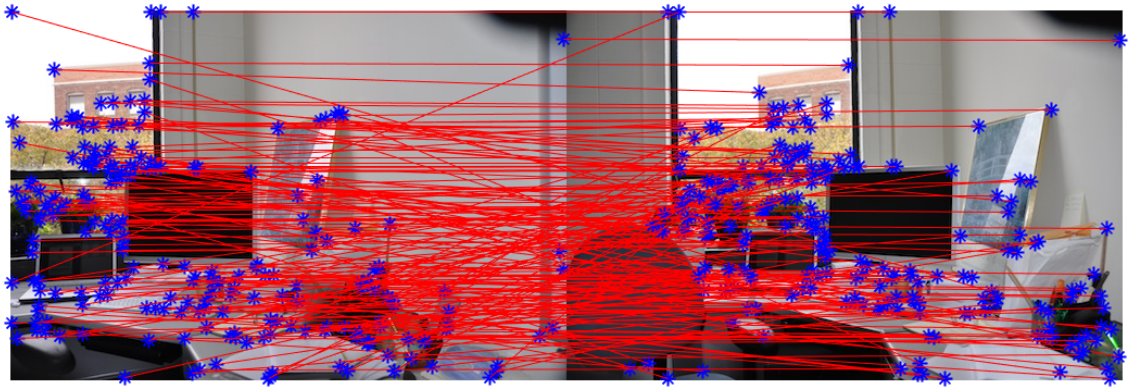
step 4:

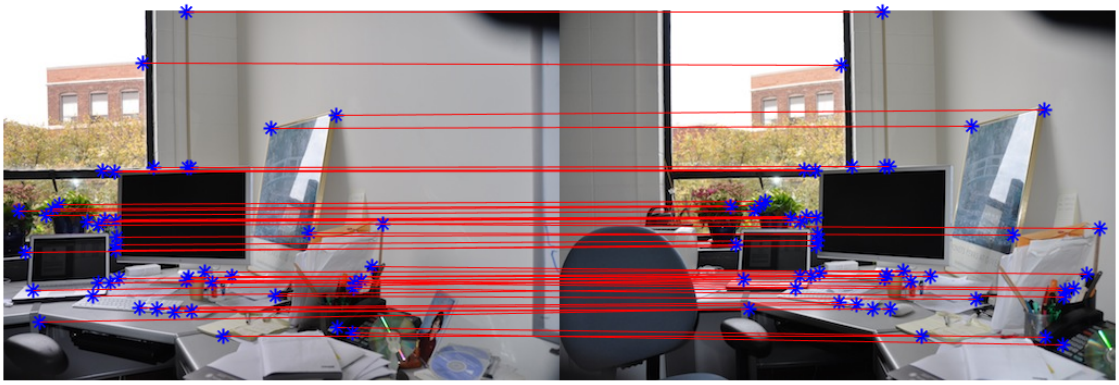


step 5:

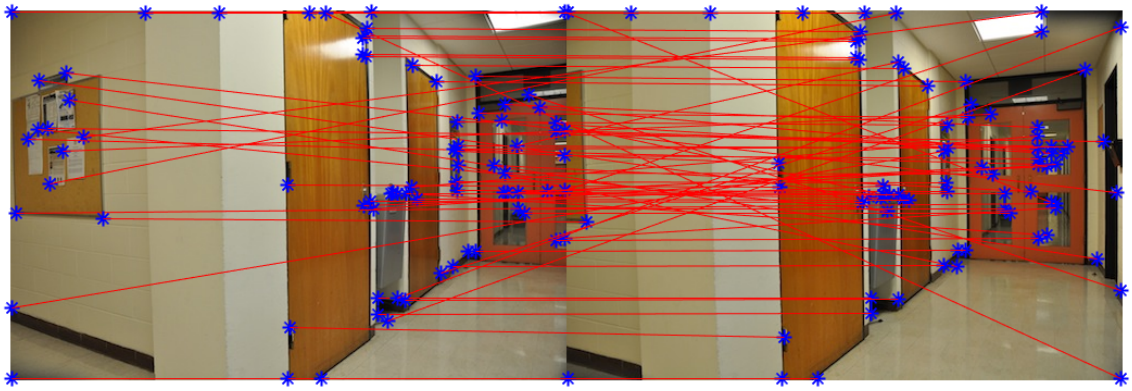

7. Image showing

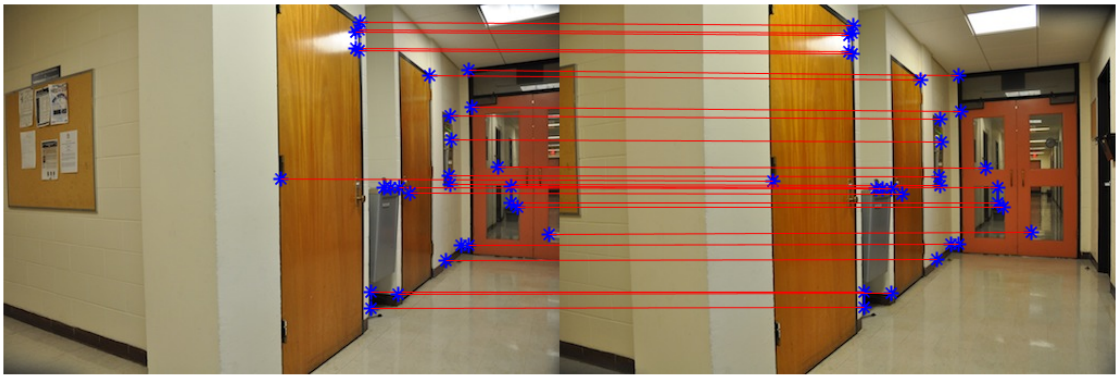
1).



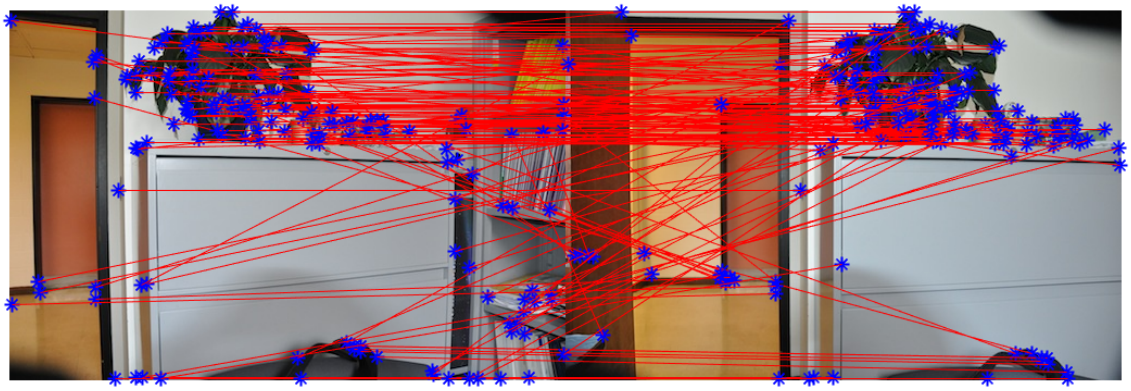


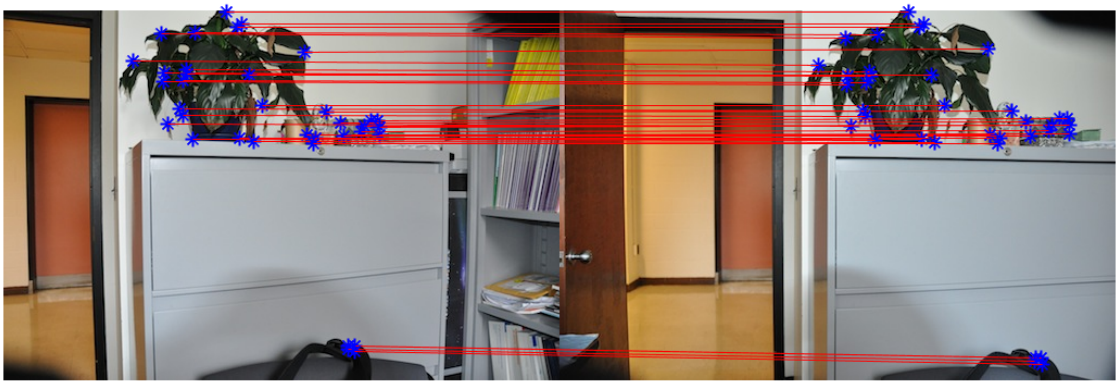
2).



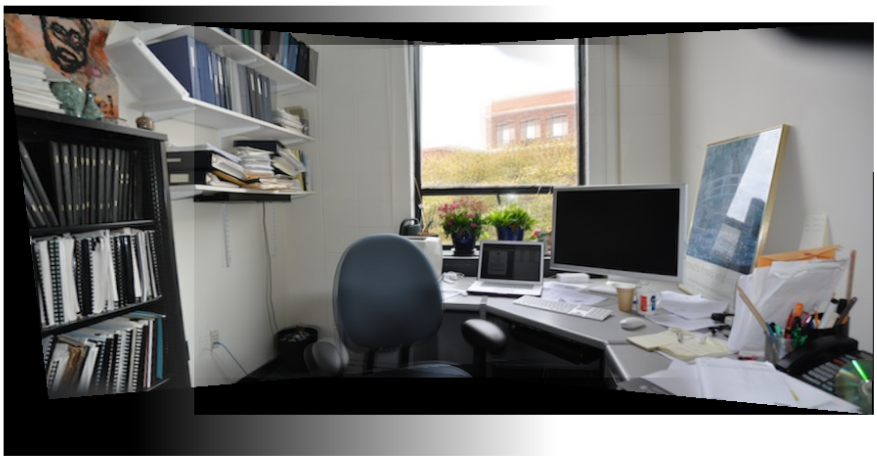


3).





8. Multi images mosaic



9. Appendix

```
img1=imread('DSC_0308.JPG'); %read in image1
img2=imread('DSC_0309.JPG'); %read in image2
pic1=rgb2gray(img1); %gray scale image1
pic2=rgb2gray(img2); %gray scale image2
figure,imshow(img1);
figure,imshow(img2);


%gaussian smoothing
der_sig=1;
x=-3:3;
Filter_der1D=exp(-(x).^2/(2*der_sig^2))/(der_sig*sqrt(2*pi));
pic1_smoothed=conv2(Filter_der1D,Filter_der1D',pic1,'same');
pic2_smoothed=conv2(Filter_der1D,Filter_der1D',pic2,'same');

%derivative filter
Filter_derx=[1 0 -1];
Filter_dery=Filter_derx';

%computer image 1 derivative by convolution in both x and y directions
%compute the derivative with respect to x
pic1_derx=conv2(pic1_smoothed,Filter_derx,'same');
%compute the derivative with respect to x at the border pixels
pic1_derx(:,1)=2*(pic1_smoothed(:,2) -pic1_smoothed(:,1));
pic1_derx(:,end)=2*(pic1_smoothed(:,end) -pic1_smoothed(:,end-1));
%compute the derivative with respect to y
pic1_dery=conv2(pic1_smoothed,Filter_dery,'same');
%compute the derivative with respect to y at the border pixels
pic1_dery(1,:)=2*(pic1_smoothed(2,:) -pic1_smoothed(1,:));
pic1_dery(end,:)=2*(pic1_smoothed(end,:) -pic1_smoothed(end-1,:));

%compute image 2 derivative by convolution in both x and y directions
%compute the derivative with respect to x
pic2_derx=conv2(pic2_smoothed,Filter_derx,'same');
%compute the derivative with respect to x at the border pixels
pic2_derx(:,1)=2*(pic2_smoothed(:,2) -pic2_smoothed(:,1));
pic2_derx(:,end)=2*(pic2_smoothed(:,end) -pic2_smoothed(:,end-1));
%compute the derivative with respect to y
pic2_dery=conv2(pic2_smoothed,Filter_dery,'same');
%compute the derivative with respect to y at the border pixels
pic2_dery(1,:)=2*(pic2_smoothed(2,:) -pic2_smoothed(1,:));
pic2_dery(end,:)=2*(pic2_smoothed(end,:) -pic2_smoothed(end-1,:));

%compute quadratic terms of the derivatives
pic1_der_x2=pic1_derx.^2;
pic1_der_y2=pic1_dery.^2;
pic1_der_xy=pic1_derx.*pic1_dery;
pic2_der_x2=pic2_derx.^2;
pic2_der_y2=pic2_dery.^2;
pic2_der_xy=pic2_derx.*pic2_dery;
```

```
%apply Gaussian filter to suppress noise and perform the smoothing in a
separable way
int_sig=2;
x=-6:6;
Filter_int1D=exp(-(x).^2/(2*int_sig^2))/(int_sig*sqrt(2*pi));
pic1_der_x2=conv2(Filter_int1D,Filter_int1D',pic1_der_x2,'same');
pic1_der_y2=conv2(Filter_int1D,Filter_int1D',pic1_der_y2,'same');
pic1_der_xy=conv2(Filter_int1D,Filter_int1D',pic1_der_xy,'same');
pic2_der_x2=conv2(Filter_int1D,Filter_int1D',pic2_der_x2,'same');
pic2_der_y2=conv2(Filter_int1D,Filter_int1D',pic2_der_y2,'same');


pic2_der_xy=conv2(Filter_int1D,Filter_int1D',pic2_der_xy,'same');

kappa=0.04;
%compute Harris R function over the images
r1=(pic1_der_x2.*pic1_der_y2-pic1_der_xy.^2)-
kappa*(pic1_der_x2+pic1_der_y2).^2;
r2=(pic2_der_x2.*pic2_der_y2-pic2_der_xy.^2)-
kappa*(pic2_der_x2+pic2_der_y2).^2;

%calculate the maximum of Harris R function
r1_max=max(r1(:));
r2_max=max(r2(:));
%set the threshold
threshold1=0.001*r1_max;
threshold2=0.001*r2_max;

height1=size(img1,1); %calculate the height of the image1
width1=size(img1,2);  %calculate the width of the image1
height2=size(img2,1); %calculate the height of the image2
width2=size(img2,2);  %calculate the width of the image2
result1 = zeros(height1, width1); %record the position of the corner of
image 1
result2 = zeros(height2, width2); %record the position of the corner of
image 2

%do non-maximum suppression of image 1
count1 = 0; %record the quantity of corners in image 1
for i = 2 : height1-1
    for j = 2 : width1-1
        % the size of window is 3*3
        if (r1(i, j) > threshold1 && r1(i, j) > r1(i-1, j-1) && r1(i,
j) > r1(i-1, j) && r1(i, j) > r1(i-1, j+1) && r1(i, j) > r1(i, j-1) &&
r1(i, j) > r1(i, j+1) && r1(i, j) > r1(i+1, j-1) && r1(i, j) > r1(i+1,
j) && r1(i, j) > r1(i+1, j+1))
            result1(i, j) = 1; % a sparse matrix
            count1 = count1 + 1;
        end;
    end;
end;
```

```
%do non-maximum suppression of image 2
count2 = 0; %record the quantity of corners in image 2
for i = 2 : height2-1
    for j = 2 : width2-1
        % the size of window is 3*3
        if (r2(i, j) > threshold2 && r2(i, j) > r2(i-1, j-1) && r2(i,
j) > r2(i-1, j) && r2(i, j) > r2(i-1, j+1) && r2(i, j) > r2(i, j-1)
&& ...
                r2(i, j) > r2(i, j+1) && r2(i, j) > r2(i+1, j-1) &&
r2(i, j) > r2(i+1, j) && r2(i, j) > r2(i+1, j+1));
            result2(i, j) = 1; % a sparse matrix
            count2 = count2 + 1;
        end;
    end;
end;

i = 1;


    for j = 1 : height1
        for k = 1 : width1
            if result1(j, k) == 1;
                % cor1 is a Nx2 matrix, stores the position of corners
in image1
                cor1(i, 1) = j;
                cor1(i, 2) = k;
                i = i + 1;
            end;
        end;
    end;

i = 1;
    for j = 1 : height2
        for k = 1 : width2
            if result2(j, k) == 1;
       % cor2 is a Nx2 matrix, stores the position of corners in image2
                cor2(i, 1) = j;
                cor2(i, 2) = k;
             i = i + 1;
            end;
        end;
    end;

%%display the Harris corners in image 1
[poscol1, posrow1] = find(result1 == 1);
figure,imshow(img1);
hold on;
plot(posrow1, poscol1, '*');

%%display the Harris corners in image 2
[poscol2, posrow2] = find(result2 == 1);
figure,imshow(img2);
hold on;
plot(posrow2, poscol2, '*');
```

```
f=cell(1,count1);
g=cell(1,count2);

pic1_db=double(pic1);
pic2_db=double(pic2);

for i=1:count1
    patch1=pic1_db(cor1(i,1)-1:cor1(i,1)+1,cor1(i,2)-1:cor1(i,2)+1); %compute
the gray level of each corner and its 3x3 neighbours in image 1
    f{i}=patch1./(sqrt(sum(sum(patch1.^2))));
end

for i=1:count2
    patch2=pic2_db(cor2(i,1)-1:cor2(i,1)+1,cor2(i,2)-1:cor2(i,2)+1); %compute
the gray level of each corner and its 3x3 neighbours in image 2
    g{i}=patch2./(sqrt(sum(sum(patch2.^2))));
end

ncc1=zeros(count2,count1);
for i=1:count1
    for j=1:count2
        ncc1(j,i)=sum(sum(f{i}.*g{j})); % compute the NCC values
    end
end
%index1 records the exact corner in image2 which corresponds to image1
%max1_num records the highest NCC values of every corner in image1


corresponding to the corners in image2
[max1_num,index1]=max(ncc1);
tres=0.95; %set treshold be 0.95
% only the hightest NCC values bigger than treshold will be recorded
[max1pos_rol,max1pos_col]=find(max1_num>tres);
match1=zeros(length(max1pos_col),2); %records the corners of two images which
have the correspondence
for i=1:length(max1pos_col)
    match1(i,1)=max1pos_col(i);%records corners in image1 which correspond to
corners in image2
    match1(i,2)=index1(max1pos_col(i));%records corners in image2 which
correspond to corners in image1
end

ncc2=zeros(count1,count2);
for i=1:count2
    for j=1:count1
        ncc2(j,i)=sum(sum(f{j}.*g{i})); % compute the NCC values
    end
end
%index2 stores the exact corner in image1 which corresponds to image2
%max2_num stores the highest NCC values of every corner in image2
corresponding to the corners in image1
[max2_num,index2]=max(ncc2);
% only the hightest NCC values bigger than treshold will be recorded
[max2pos_rol,max2pos_col]=find(max2_num>tres);
match2=zeros(length(max2pos_col),2); %records the corners of two images which
have the correspondence
for i=1:length(max2pos_col)
    match2(i,2)=max2pos_col(i);%records corners in image2 which correspond to
corners in image1
    match2(i,1)=index2(max2pos_col(i));%records corners in image1 which
correspond to corners in image2
end
```

```
matchs=[];  %record the final feature points match relationship using two-way
match
loc1=[];    %record the location of the final matching points in image1
loc2=[];    %record the location of the final matching points in image2
for i=1:length(match1)
    for j=1:length(match2)
        if match1(i,:)==match2(j,:)
            matchs=[matchs;match1(i,:)];
            loc1=[loc1;cor1(match1(i,1),:)];
            loc2=[loc2;cor2(match1(i,2),:)];
        end
    end
end

%Draw lines between matching corners
X1=loc1(:,2);
Y1=loc1(:,1);
X2=loc2(:,2);
Y2=loc2(:,1);
dif=size(img1,2);
figure();
imshowpair(img1,img2,'montage');
hold on


for k=1:length(X1)
  plot(X1(k),Y1(k),'b*');
  plot(X2(k)+dif,Y2(k),'b*');
  line([X1(k),X2(k)+dif],[Y1(k),Y2(k)],'Color','r');
end
set(gcf,'Color','w');


inlier_nummax=4;
inlier_loc=[];
for k=1:20000
  a=ceil(length(loc1)*rand(1,4)); %randomly select corresponding four
points
  while
a(1)==a(2)||a(1)==a(3)||a(1)==a(4)||a(2)==a(3)||a(2)==a(4)||a(3)==a(4)
      a=ceil(length(loc1)*rand(1,4));
  end
  x11=loc1(a(1),1);
  y11=loc1(a(1),2);
  x12=loc2(a(1),1);
  y12=loc2(a(1),2);
  x21=loc1(a(2),1);
  y21=loc1(a(2),2);
  x22=loc2(a(2),1);
  y22=loc2(a(2),2);
  x31=loc1(a(3),1);
  y31=loc1(a(3),2);
  x32=loc2(a(3),1);
  y32=loc2(a(3),2);
  x41=loc1(a(4),1);
  y41=loc1(a(4),2);
  x42=loc2(a(4),1);
```

```
  y42=loc2(a(4),2);

  A=[x11,y11,1,0,0,0,-x11*x12,-y11*x12;
     0,0,0,x11,y11,1,-x11*y12,-y11*y12;
     x21,y21,1,0,0,0,-x21*x22,-y21*x22;
     0,0,0,x21,y21,1,-x21*y22,-y21*y22;
     x31,y31,1,0,0,0,-x31*x32,-y31*x32;
     0,0,0,x31,y31,1,-x31*y32,-y31*y32;
     x41,y41,1,0,0,0,-x41*x42,-y41*x42;
     0,0,0,x41,y41,1,-x41*y42,-y41*y42];

  b=[x12,y12,x22,y22,x32,y32,x42,y42]';
  h=A\b; %Compute a homography from these corresponding four points

  inlier_num=0;
  inlier_loc=0;
  for i=1:length(loc1)
    %Map all points using the homagraphy

loc2_predicted(i,1)=(h(1)*loc1(i,1)+h(2)*loc1(i,2)+h(3))/(h(7)*loc1(i,1
)+h(8)*loc1(i,2)+1);

loc2_predicted(i,2)=(h(4)*loc1(i,1)+h(5)*loc1(i,2)+h(6))/(h(7)*loc1(i,1
)+h(8)*loc1(i,2)+1);
    %Compare distances between predicted and observed locations
    distance=sqrt((loc2_predicted(i,1)-
loc2(i,1))^2+(loc2_predicted(i,2)-loc2(i,2))^2);
    if distance<1 %the inliers will be recorded only when the distance
less than 1
      inlier_num=inlier_num+1;
      inlier_loc=[inlier_loc;i];
    end
  end
  if inlier_num>inlier_nummax
      inlier_nummax=inlier_num;
      hom=h; %compute a least-squares homgraphy from all the inliers in
the largest set of inliers
      inlier_maxloc=inlier_loc; %record the exact corners that are
inliers
  end
end

%record the position of inliers in image1 and image2
for i=1:length(inlier_maxloc)-1
    inlier1_loc(i,1)=loc1(inlier_maxloc(i+1),1);
    inlier1_loc(i,2)=loc1(inlier_maxloc(i+1),2);
    inlier2_loc(i,1)=loc2(inlier_maxloc(i+1),1);
    inlier2_loc(i,2)=loc2(inlier_maxloc(i+1),2);
end

%Draw lines between matching inliers corners
X1=inlier1_loc(:,2);
Y1=inlier1_loc(:,1);
X2=inlier2_loc(:,2);
Y2=inlier2_loc(:,1);
dif=size(img1,2);
figure();
```

```
imshowpair(img1,img2,'montage');
hold on
for k=1:length(X1)
  plot(X1(k),Y1(k),'b*');
  plot(X2(k)+dif,Y2(k),'b*');
  line([X1(k),X2(k)+dif],[Y1(k),Y2(k)],'Color','r');
end
set(gcf,'Color','w');

% create meshgrid for the warped image2
[xi,yi] = meshgrid(1:width2,1:height2);
img2_grid = ones(3,height2*width2);
img2_grid(1,:) = reshape(xi,1,height2*width2);
img2_grid(2,:) = reshape(yi,1,height2*width2);

% use the inverse of the homography
new_hom=[hom(1),hom(2),hom(3);hom(4),hom(5),hom(6);hom(7),hom(8),1];
img2_newgrid = new_hom\img2_grid;
%the left side of the new image
left = fix(min([1,min(img2_newgrid(1,:)./img2_newgrid(3,:))]));
%the right side of the new image
right = fix(max([width1,max(img2_newgrid(1,:)./img2_newgrid(3,:))]));
%the top side of the new image
top = fix(min([1,min(img2_newgrid(2,:)./img2_newgrid(3,:))]));
%the bottom side of the new image
bottom = fix(max([height1,max(img2_newgrid(2,:)./img2_newgrid(3,:))]));
%the height of the new image
new_height = bottom - top + 1;
%the width of the new image
new_width = right - left + 1;


newimg_left = left; % x coordate of lefttop corner of the new image
newimg_top  = top;  % y coordate of lefttop corner of the new image
unwarped_img1_left = 2 - left; % x coordate of lefttop corner of
unwarped image
unwarped_img1_top  = 2 - top;  % y coordate of lefttop corner of
unwarped image

% create meshgrid for the new image
[XI,YI] = meshgrid(newimg_left:newimg_left+new_width-1,
newimg_top:newimg_top+new_height-1);
newimg_grid = ones(3,new_height*new_width);
newimg_grid(1,:) = reshape(XI,1,new_height*new_width);
newimg_grid(2,:) = reshape(YI,1,new_height*new_width);

newimg_grid = new_hom*newimg_grid;
XI = reshape(newimg_grid(1,:)./newimg_grid(3,:), new_height,
new_width);
YI = reshape(newimg_grid(2,:)./newimg_grid(3,:), new_height,
new_width);

% Separation of the three primary colors
R=double(img2(:,:,1));
G=double(img2(:,:,2));
B=double(img2(:,:,3));
%Interpolation
finalImage(:,:,1) = interp2(R, XI, YI);
finalImage(:,:,2) = interp2(G, XI, YI);
finalImage(:,:,3) = interp2(B, XI, YI);
%display the warped image
figure,imshow(uint8(finalImage));

[finalImage] = blend(unwarped_img1_left, unwarped_img1_top, finalImage,
img1);
% display the final image
figure,imshow(uint8(finalImage));
```

```
function [finalImage] = blend(unwarped_x, unwarped_y, img_warp, img_unwarp)
% Blend two image
% Input:
% img_warp - original image to be warped
% img_unwarp - the other image not to be warped
% unwarped_x -  coordinate of the lefttop corner of unwarped image
% unwarped_y - y coordinate of the lefttop corner of unwarped image
% Output: finalImage

% set the grayscale in the final image to be 0 where exists NAN
img_warp(isnan(img_warp))=0;
% Set a maskA, in the region of warped image, the value is 1, otherwise, is
0.
maskA = (img_warp(:,:,1)>0 |img_warp(:,:,2)>0 | img_warp(:,:,3)>0);
finalImage = zeros(size(img_warp));
%copy the unwarped image to the final image
finalImage(unwarped_y:unwarped_y+size(img_unwarp,1)-1, unwarped_x:
unwarped_x+size(img_unwarp,2)-1,:) = img_unwarp;
% In the region of unwarped image, the value is 1, otherwise is 0.
mask = (finalImage(:,:,1)>0 | finalImage(:,:,2)>0 | finalImage(:,:,3)>0);
% In the overlaid region, the value is 1,otherwise, is 0.


mask = and(maskA, mask);

% calculate the overlaid region of the two images
[~,col_mask] = find(mask);
left = min(col_mask);% find the left boundary of the overlaid region
right = max(col_mask);% find the right boundary of the overlaid region
mask = ones(size(mask));
if( unwarped_x<2) %if the warped image is on the right of the unwarped image
  mask(:,left:right) = repmat(linspace(0,1,right-left+1),size(mask,1),1);
else
  mask(:,left:right) = repmat(linspace(1,0,right-left+1),size(mask,1),1);
end

% blend each channel
img_warp(:,:,1) = img_warp(:,:,1).*mask;
img_warp(:,:,2) = img_warp(:,:,2).*mask;
img_warp(:,:,3) = img_warp(:,:,3).*mask;

if( unwarped_x<2) %if the warped image is on the right of the unwarped image
  mask(:,left:right) = repmat(linspace(1,0,right-left+1),size(mask,1),1);
else
  mask(:,left:right) = repmat(linspace(0,1,right-left+1),size(mask,1),1);
end

finalImage(:,:,1) = finalImage(:,:,1).*mask;
finalImage(:,:,2) = finalImage(:,:,2).*mask;
finalImage(:,:,3) = finalImage(:,:,3).*mask;

finalImage(:,:,1) = img_warp(:,:,1) + finalImage(:,:,1);
finalImage(:,:,2) = img_warp(:,:,2) + finalImage(:,:,2);
finalImage(:,:,3) = img_warp(:,:,3) + finalImage(:,:,3);
```