Technical University of Denmark

Xiao Hu

30550: Satellite Based Positioning

Report for assignment G&H

December 5, 2018

DTU Space
National Space Institute

30550: Satellite Based Positioning, Report for assignment G&H

Author(s):
Xiao Hu

Supervisor(s):
Dr. Daniel Olesen

National Space Institute
Technical University of Denmark
Elektrovej Building 328, room 007
2800 Kongens Lyngby
Denmark
www.space.dtu.dk

E-mail: xiahaa@space.dtu.dk

ABSTRACT

This report will present the theory and results accomplished for the assignment G&H which mainly includes the processing of real GPS-data and the simulation of the integration of the Galileo system and the GPS system.

Table of Contents

1 Assignment G: Processing of Real GPS-data

The main objective of the assignment is to estimate the receiver's position using real GPS data provided in the RINEX format file..

1.1 Theory

1.1.1 RINEX

Receiver Independent Exchange Format (RINEX) is a data interchange format for raw satellite navigation system data. The objective of defining this format is to provide users a standard for post-processing the received data to produce a more accurate result regardless the exact receiver used for data collection. The RINEX format used in this assignment is **the RINEX version 2.10**[1]. The file we will use is the **Observation file** which has a extension as **.05O**. Relevant lines which are useful in this assignment are:

- **line 13**: provides the approximate $x,\ y,\ z$ position of the receiver.
- **line 16**: number and type of stored observation.
- **line 25-EOF**: year, month, day, hour, minute, second (one block per 30 seconds), observed satellites and observations per each visible satellite. **It should be noted that some data may have lost**.

Among those observations, the third item corresponds to the pseudorange of the C/A code.

1.1.2 Tranmission time

To compensate the satellite motions during transmission, we will use the satellite positions at the transmit time instead of at the receive time. The transmission time for a satellite signal can be estimated as:

$$t_{trans_i} = t_{recv} - \frac{\rho_i}{c} \tag{1.1}$$

where ρ_i is the pseudorange between satellite i and the receiver, $c = 299792458m/s$ is the speed of light. The satellites move approximately $200 \sim 300$ meters during the 0.07 seconds of signal transmission.

1.1.3 Coordinate Movement

Another issue due to the transmission of the signal is the movement of the ECEF coordinate frame. Actually, the ECEF reference frame continues to rotate while the signal is being

[1] ftp://igs.org/pub/data/format/rinex210.txt

transmitted. As a result, after obtaining satellite positions at transmit time, they have to be transformed as follows:

$$\psi = t_{trans_i}\omega_{earth} \tag{1.2}$$

$$\mathbf{R} = \begin{bmatrix} cos(\psi) & sin(\psi) & 0 \\ -sin(\psi) & cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1.3}$$

$$\mathbf{p}_i' = \mathbf{R}\mathbf{p}_i \tag{1.4}$$

where $\mathbf{p}_i = [x,\ y,\ z]^T$ and $\mathbf{p}_i' = [x,\ y,\ z]^T$ are the postion of satellite i before and after compensating the rotation of the coordinate frame. The effect of this coordinate Movement would be around $20 \sim 30$ meters on final solved position.

1.1.4 Position Estimation

According to [1] and the lecture slides, the pseudorange R can be modeled as:

$$R = \rho + d\rho + c(dT - dt) + d_{ion} + d_{trop} + \epsilon \tag{1.5}$$

where R is the pseudorange, ρ is the geometric distance between the receiver and satellite, $d\rho$ is the orbit error (set to 0 in this experiment), c is the speed of light, dT is the receiver clock error, dt is the satellite clock error, d_{ion} and d_{trop} are the ionospheric delay and tropospheric delay, respectively, e is the error from the multipath effect and receiver noise. Now supposing all biased errors (e.g. the ionospheric effect, the tropospheric effect, the satellite clock error, the multipath error) have been compensated and only the gaussian random noise remained, the following pseudorange equation between the ith satellite and the receiver is given by

$$R_i = ||\mathbf{x} - \mathbf{x_i}|| + cdT + \epsilon_i \tag{1.6}$$

where $\mathbf{x} = [x,\ y,\ z]^T$, $\mathbf{x}_i = [x_{si},\ y_{si},\ z_{si}]^T$ are the $3D$ position vectors of the receiver and the i^{th} satellite, respectively, cdT is the receiver clock bias in unit of meter. The position estimation problem is nothing more than a unconstrained nonlinear least square problem defined as:

$$\min_{\mathbf{x}} \sum_{i=1}^{k} [R_i - (||\mathbf{x} - \mathbf{x_i}|| + cdT)]^2 \tag{1.7}$$

In order to determine the position of a receiver, pseudoranges from $n \geq 4$ satellites must be used at the same time. For positioning calculation, the first-order Taylor expansion is applied to (1.6) around the approximate position of the receiver $\hat{\mathbf{x}} = [\hat{x}, \hat{y}, \hat{z}]^T$:

$$R_i = ||\hat{\mathbf{x}} - \mathbf{x_i}|| + cdT + \mathbf{a}_i\Delta\mathbf{x} + c\Delta dT + \epsilon_i \tag{1.8}$$

$$\Leftrightarrow$$

$$\Delta R_i = R_i - (||\hat{\mathbf{x}} - x_i|| + cdT) = \mathbf{a}_i\Delta\mathbf{x} + c\Delta dT + \epsilon_i \tag{1.9}$$

$$\mathbf{a}_i = \left[\frac{\hat{x} - x_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||},\ \frac{\hat{y} - y_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||},\ \frac{\hat{z} - z_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||}\right] \tag{1.10}$$

The linear measurement equation can be obtained by stacking (2.8) together:

$$\underbrace{\begin{bmatrix} \Delta R_1 \\ \Delta R_2 \\ \vdots \\ \Delta R_k \end{bmatrix}}_{\mathbf{\Delta R}} = \underbrace{\begin{bmatrix} \mathbf{a}_1 & 1 \\ \mathbf{a}_2 & 1 \\ \vdots & \vdots \\ \mathbf{a}_k & 1 \end{bmatrix}}_{\mathbf{A}} \underbrace{\begin{bmatrix} \Delta \mathbf{x} & c\Delta dT \end{bmatrix}}_{\mathbf{\Delta x}} + \underbrace{\begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_k \end{bmatrix}}_{\epsilon} \tag{1.11}$$

where $\mathbf{A}$ is the design matrix which captures the receiver-satellite geometry. Usually (1.11) is solved by the Gauss-Newton method, which means the two steps are iterated until a given condition is met:

$$\text{solve: } \mathbf{A\Delta\hat{x}} = \mathbf{\Delta R} \tag{1.12}$$

$$\mathbf{\hat{x}} = \mathbf{\hat{x}} + \mathbf{\Delta\hat{x}} \tag{1.13}$$

1.1.5 DOP

Since $\epsilon_1,\ \epsilon_2,\ \cdots,\ \epsilon_k$ are independent identical random variables and $\epsilon_i \sim \mathcal{N}(0, \sigma_0^2)$, it can be derived that the covariance matrix $\mathbf{Q_{\hat{x}}}$ equals to $\sigma_0^2(\mathbf{A^T W A})^{-1}$ for weighted case. The DOP matrices is further defined as

$$\mathbf{Q}_{DOP} = \frac{\mathbf{Q_{\hat{x}}}}{\sigma_0^2\sigma_p^2} = \frac{(\mathbf{A^T W A})^{-1}}{\sigma_p^2} \tag{1.14}$$

$$\mathbf{Q}_{DOP_{ENU}} = \mathbf{R}_{ECEF}^{ENU}\mathbf{Q}_{DOP}\mathbf{R}_{ECEF}^{ENU\ T} \tag{1.15}$$

where σ_p^2 are the prior variances of measurements. Several relevant DOP values are defined as:

- $PDOP = \sqrt{trace(\mathbf{Q_{DOP}})}$ defines the geometric dilution of precision;
- $PDOP = \sqrt{trace(\mathbf{Q_{DOP}}(1:3, 1:3))}$ defines the position ($3D$) dilution of precision;
- $TDOP = \sqrt{\mathbf{Q_{DOP}}(4, 4)}$ defines the time dilution of precision;
- $HDOP = \sqrt{trace(\mathbf{Q_{DOP_{ENU}}}(1:2,\ 1:2))}$ defines the horizontal dilution of precision;
- $VDOP = \sqrt{trace(\mathbf{Q_{DOP_{ENU}}}(3,\ 3))}$ defines the vertical dilution of precision;

1.2 Tasks

- Extract pseudoranges from RINEX observation file.
- Read satellite positions from the corresponding sp3 file, correct for transmission time and interpolate satellite positions.
- Correct for rotation of reference frame during transmission.
- Estimate elevation and azimuth, correct satellite clock error, ionosphere, and troposphere delay.
- Estimate position and receiver clock error using least squares, determine DOP values.
- Compare position solution with the known position given in data file.

1.3 Code

List of relevant functions in the attachment:

- **ex8.m**: main entry.
- **prepare__RINEX__data.m** entry for preparing data from RINEX file.
- **prepare__SP3__data.m** entry for preparing data from sp3 file.
- **utils\RINEX__file__parser.m** entry for parsing a RINEX observation file.
- **utils\interp__sat__clk__err.m** entry for interpolating the satellite clock error.

Some of the previously made functions are also used for this assignment:

- **utils\sp3fileParser.m**: parsing the sp3 file.
- **utils\find__data.m**: extract relevant data of given epoch from the whole sp3 data.
- **utils\find__neighbor__ids.m**: find K nearest neighbors for interpolation.
- **utils\interp__sat__pos.m**: interpolation;
- **utils\iono__delay__first__order__group.m**: estimate the ionospheric delay.
- **utils\tropo__delay__via__saastamoinen__model.m**: estimate the tropospheric delay.
- **utils\navSolver.m**: position estimation using corrected pseudoranges.

1.4 Experiments

The time epoch at $03:28$ is used in the following experiments. A TEC value of $10e^{16}$ for estimation of the ionospheric delay.

Position Estimation& Comparison

The position estimation results are shown in Table 1.1. It can be seen from Table 1.1

Table 1.1. Position Estimation Results.

	x: (m)	**y**: (m)	**z**: (m)	**dT**: (ms)
RINEX	3513638.5600	778956.1840	5248216.2480	N/A
Initial	3513838.5600	779156.1840	5248416.2480	0
Estimated	3513635.6260	778954.7644	5248224.2905	0.6071

Table 1.2. Position Estimation Errors.

	err_x: (m)	err_y: (m)	err_z: (m)	err_{norm}: (m)
Estimated	-2.934	-1.4196	8.0425	8.6779

and Table 1.2 that the estimated position hit the known position provided in the RINEX file with an error smaller than 10 meter. The results of latitude, longitude and height are shown in Table 1.3. The DOP values for this experiment are given in Table 1.4. From[2], we can see that the GDOP is less than 2, which means the geometric distribution of visible satellites is quite good for positioning. The corresponding skyplot of all visible satellites is shown in Fig 1.1. From Fig 1.1, we can clearly see that visible satellites are placed well with several satellites along the zenith direction but also several satellites with different elevation angles, which will be beneficial for vertical position estimation.

Table 1.3. Latitude, Longitude and Height Estimation Results.

	Latitude: (deg)	**Longitude**: (deg)	**z**: (m)
RINEX	55.739017	12.500020	94.014546
Estimated	55.739081	12.500008	98.875931

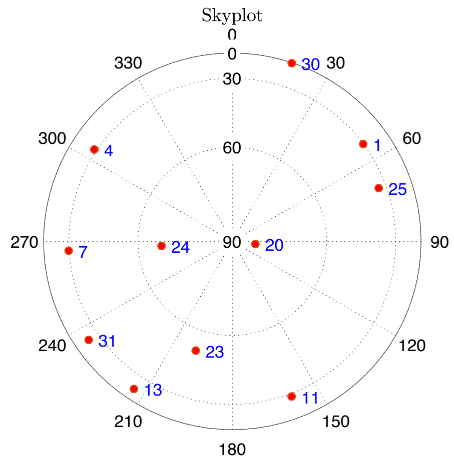

Figure 1.1. The corresponding skyplot of all visible satellites.

Table 1.4. DOP Values.

	$HDOP$	$VDOP$	$PDOP$	$GDOP$
Estimated	0.8257	1.2323	1.4834	1.6581

1.4.1 Conclusion & Analysis

In this experiment, the position estimation is carried out with the real GPS data. By comparing the position estimation results, it can be shown that the implemented routines for pseudoranges correction and position estimation can correctly estimate the receiver's position.

The real accuracy cannot be obtained unless there is a more accurate "ground truth". However, from this coarse comparison, it can be seen that the horizontal position obtained is more accurate than the vertical position. The major error comes from the vertical position estimation even if the geometric distribution of visible satellites is relatively good in terms of GDOP. This is easy to understand that compared with horizontal position, there are not satellites below the horizontal plane tangent at the receiver position because of the visibility constraint. So satellites cannot form an equal distributed geometry for vertical position.

The potential application of this kind of GPS-data processing could be:

- Estimate the atmospheric delay by reversing this process. This is possible since if we know accurately the receiver's position, we can model the position error to some atmospheric parameters. This can be used for example: predict solar activities, etc.
- Benchmark the performance of different models of troposphere and ionosphere.

2 Assignment H: Galileo Simulator

The main objective of the assignment is to implement a Galileo simulator and demonstrate the improvement of using multi-constellation.

2.1 Theory

According to [1], the orbital position of one satellite can be computed with

$$n = \sqrt{\frac{GM}{a^3}} \tag{2.1}$$

$$M = n(t - t_p) \tag{2.2}$$

$$M = E - esinE \tag{2.3}$$

$$\mathbf{r} = \begin{bmatrix} acosE - ae \\ a\sqrt{1-e^2}sinE \\ 0 \end{bmatrix} \tag{2.4}$$

where, n is the mean motion, GM is the earth's gravitational constant $3986004.418 * 10^8\ m/s^2$, a is the length of the semi-major axis of ellipsoid (6378137.0 m), e is the eccentricity of the ellipsoid calculated by $e^2 = 2f - f^2$ with f being the flattening $(\frac{1}{298.257223563})$.
The transformation from the orbital coordinate frame to the inertial coordinate frame can be computed using the inclination i, the right ascension of the ascending node Ω and the argument of perigee ω.

$$\mathbf{r}_I = \mathbf{R}_3(-\Omega)\mathbf{R}_1(-i)\mathbf{R}_3(-\omega)\mathbf{r}_{orbit} \tag{2.5}$$

And then to the ECEF coordinate frame (ignoring the polar motion, precession, and nutation):

$$\mathbf{r}_T = \mathbf{R}_3(\theta)\mathbf{r}_I \tag{2.6}$$

θ is the Greenwich apparent sidereal time (GAST). GAST is the angle between the Mean Greenwich Meridian and the direction of the vernal equinox. 1 hour equals 15 degrees of rotation.

2.1.1 Position Estimation

Single Constellation

A is the design matrix which captures the receiver–satellite geometry is defined as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & 1 \\ \mathbf{a}_2 & 1 \\ \vdots & \vdots \\ \mathbf{a}_k & 1 \end{bmatrix} \tag{2.7}$$

$$\mathbf{a}_i = \left[\frac{\hat{x} - x_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||}, \frac{\hat{y} - y_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||}, \frac{\hat{z} - z_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||} \right] \tag{2.8}$$

Then the PDOP can be computed as:

$$\mathbf{M} = \mathbf{A}^T \mathbf{A} \tag{2.9}$$

$$PDOP = \sqrt{trace(\mathbf{M}^{-1}(1:3, 1:3))} \tag{2.10}$$

Multi Constellations

Suppose there are two navigation systems, then there are two independent clock errors. Similarly, the design matrix **A** is given as:

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}_1 & 1 & 0 \\ \mathbf{a}_2 & 1 & 0 \\ \vdots & \vdots & \vdots \\ \mathbf{a}_{k_1} & 1 & 0 \\ \mathbf{a}'_1 & 0 & 1 \\ \mathbf{a}'_2 & 0 & 1 \\ \vdots & \vdots & \vdots \\ \mathbf{a}'_{k_2} & 0 & 1 \end{bmatrix} \tag{2.11}$$

$$\mathbf{a}_i = \left[\frac{\hat{x} - x_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||}, \frac{\hat{y} - y_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||}, \frac{\hat{z} - z_i}{||\hat{\mathbf{x}} - \mathbf{x_i}||} \right], \tag{2.12}$$

$$i = 1, \cdots, k_1 \; and \; 1, \cdots, k_2$$

Then the PDOP for multi Constellations can be computed as:

$$\mathbf{M} = \mathbf{A}^T \mathbf{A} \tag{2.13}$$

$$PDOP = \sqrt{trace(\mathbf{M}^{-1}(1:3, 1:3))} \tag{2.14}$$

2.2 Tasks

- Determine Galileo satellite positions over 24 hours, and verify the code by plots in the inertial coordinate system.
- Convert the satellite positions from the inertial to the terrestrial reference system (WGS84).

- Integrate the Galileo simulator together with a GPS simulator, determine the number of visible satellites, and the PDOP for a given location on the ground in GPS-only case, Galileo-only case, and GPS/Galileo case.

2.3 Code

List of relevant functions in the attachment:

- **ex9_galileo_display.m**: main entry for displaying Galileo satellites.
- **ex_9_galileo_gps_display.m** entry for displaying GPS/Galileo satellites.
- **ex_9_gdop.m** entry for analyzing 24 hour variations of PDOP at given point.
- **calc_sat_pos_with_Kepler.m** entry for compute satellite positions using the Kepler parameters.
- **GalileoKepParams.m** load Galileo Kepler parameters.
- **ExampleHelperSat.m** drawing.
- **assign3.m**: GUI entry;

2.4 Experiments

2.4.1 Galileo at one instance

Fig 2.1 shows the 27 Galileo satellite positions at one instance in time, and the positions over 12 hours. The inclination of satellites of the Galileo system is 1 degree higher than that of the GPS system, which means it would provide better coverage of polar latitudes. Compared with GPS which has 6 orbital planes, the Galileo system has 3 orbital planes. Fig 2.2 shows the Galileo satellite positions and the GPS satellite positions. Obviously, with two systems operating simultaneously, there would be a denser network of satellites.

PDOP Comparison

The antenna is assumed to be placed near the DTU 101 with the latitude, longitude, and height being $(55.78575300466123, 12.525384183973078, 40)$.

The DOP values for this experiment are given in Table 2.1. From Table 2.1 and Fig. 2.3, it is clear that the integration of the Galileo and GPS system would have a lower PDOP value, which means a better satellite distribution. It can also be observed that the PDOP values from the Galileo system are lower than those from the GPS system, which corresponds to the design intention of the Galileo system[1].

2.4.2 GUI

A Matlab GUI is designed and shown in Fig 2.4. Latitude, longitude, and altitude can be arbitrarily selected simply by sliding corresponding sliders. By clicking the **compute** button, the PDOP values will be automatically computed and displayed in corresponding text boxes. The 24 **Hour Analysis** button will display the 24-hour PDOP values.

[1] https://gssc.esa.int/navipedia/index.php/Galileo_Space_Segment

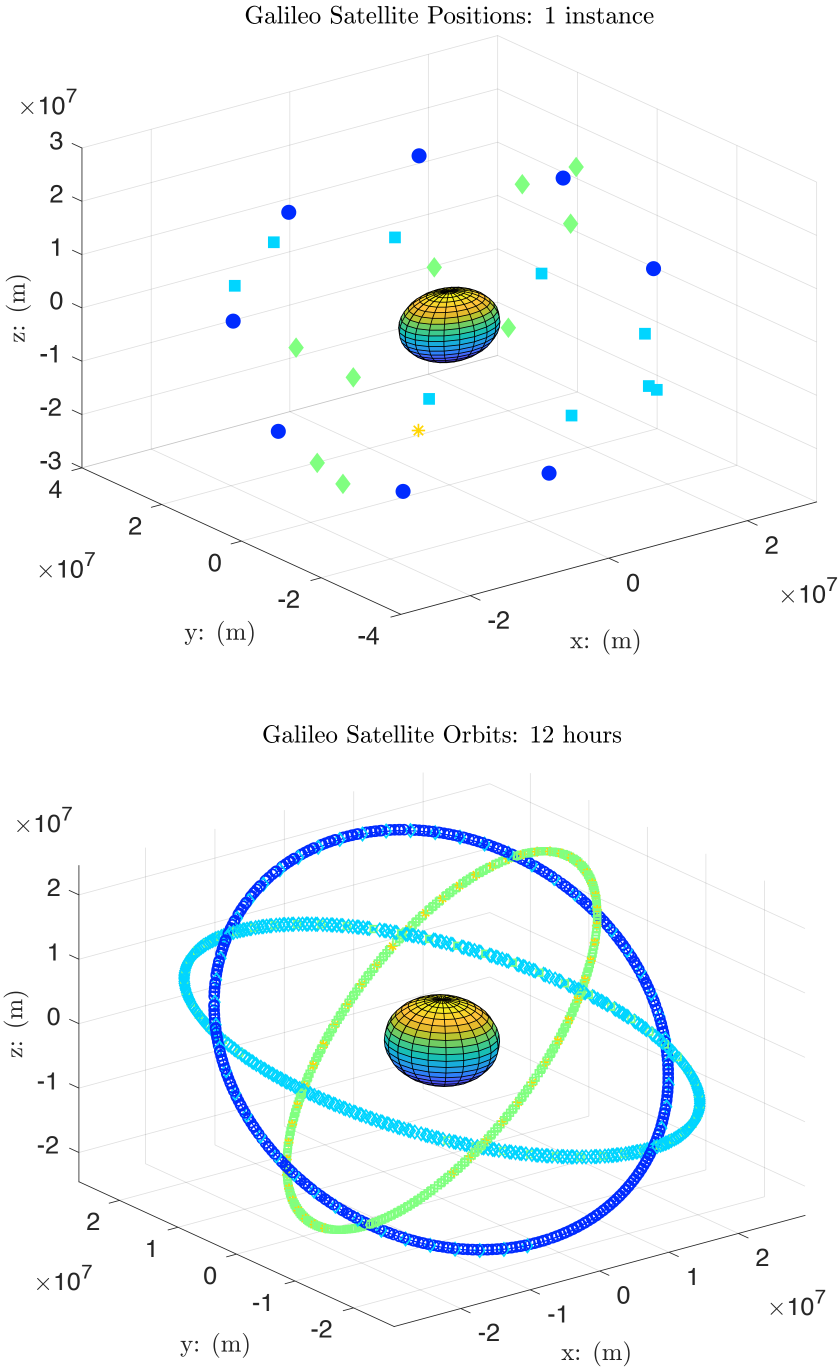

Figure 2.1. Galileo Satellite Orbits Plots.

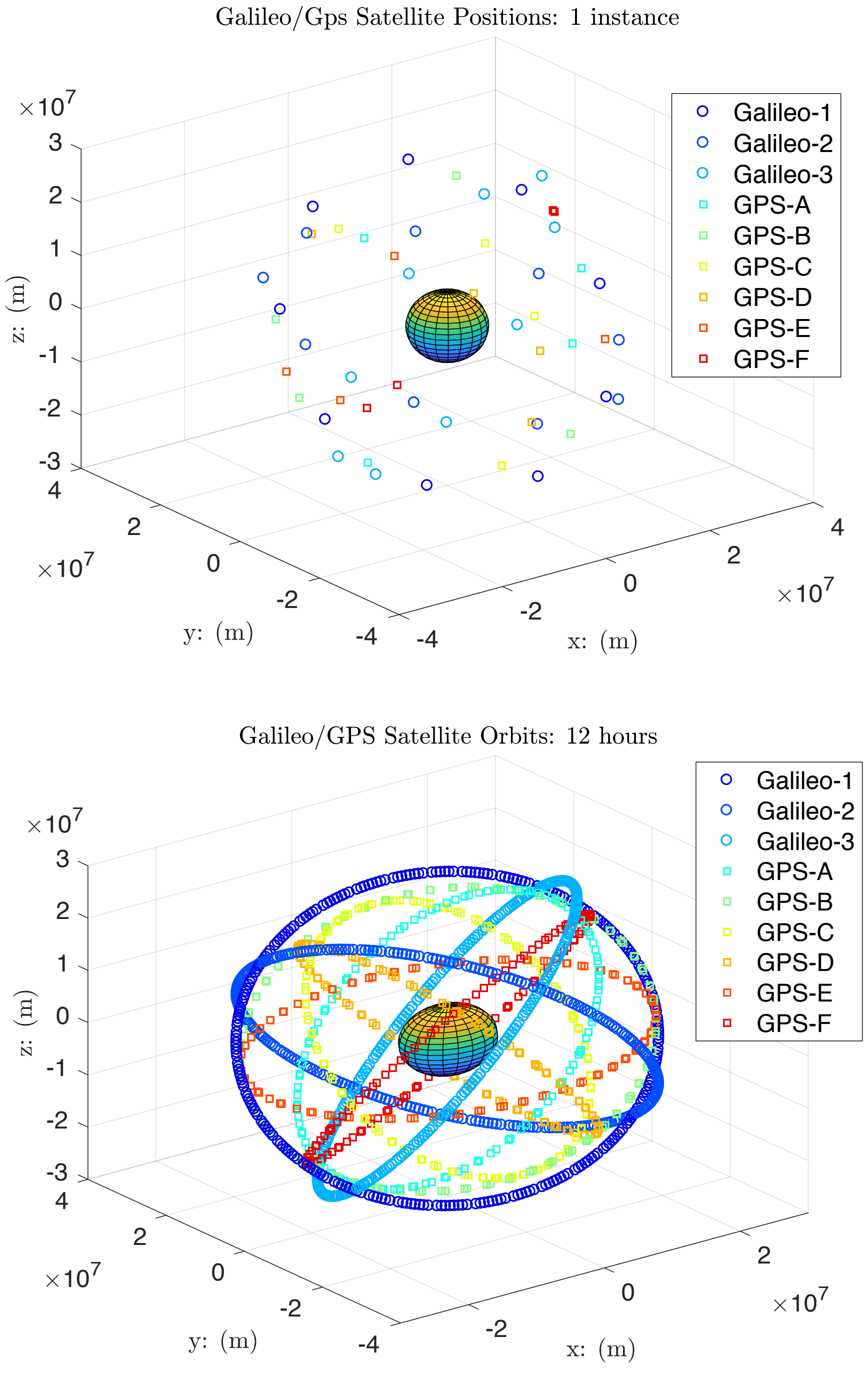

Figure 2.2. Galileo/GPS Satellite Orbits Plots.

Table 2.1. PDOP obtained with Galileo used, GPS used and Galileo/GPS used.

timestamp: (s)	*Galileo*: (PDOP)	*GPS*: (PDOP)	*Galileo/GPS*: (PDOP)
0	1.5107	1.4635	1.0249
900	1.5126	2.0010	1.1871
1800	1.3754	1.5914	1.0320
2700	1.3767	1.6583	1.0162
3600	1.3252	2.1504	1.0849
4500	1.3547	1.6625	1.0442
5400	1.5256	2.0056	1.1314
6300	1.6796	1.7321	1.1106
7200	1.7524	1.7726	1.2304
8100	1.6412	1.6435	1.1574

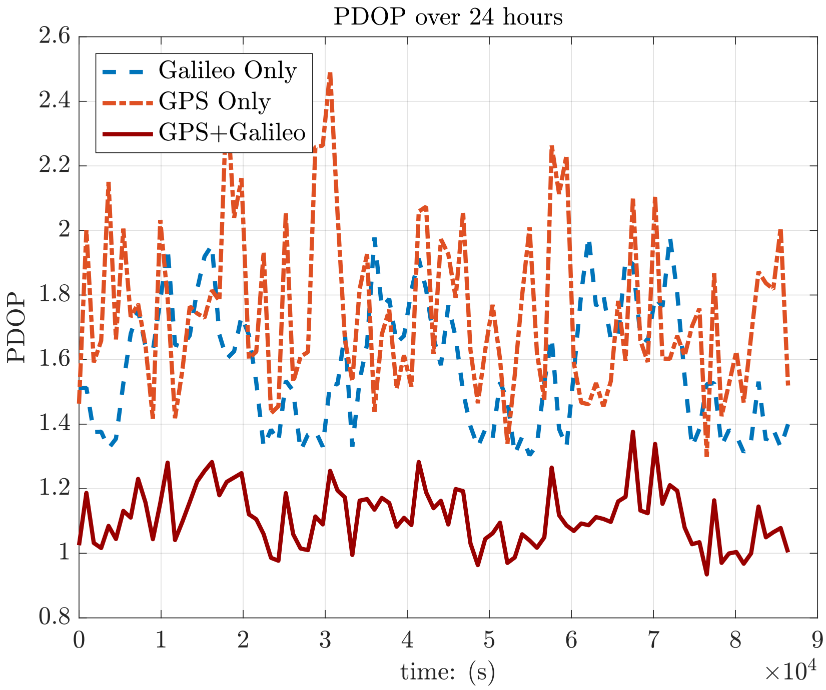

Figure 2.3. Comparison of PDOP with Galileo used, GPS used and Galileo/GPS used for 24 hours.

2.4.3 Conclusion

In this experiment, a Galileo simulator is developed. By comparing the PDOP values with GPS satellites only, Galileo satellites only, and combined GPS/Galileo satellites, it can be concluded that the integration of multi-constellation will significantly improve the availability and the geometric distribution of satellites, which will improve the position accuracy in the end.

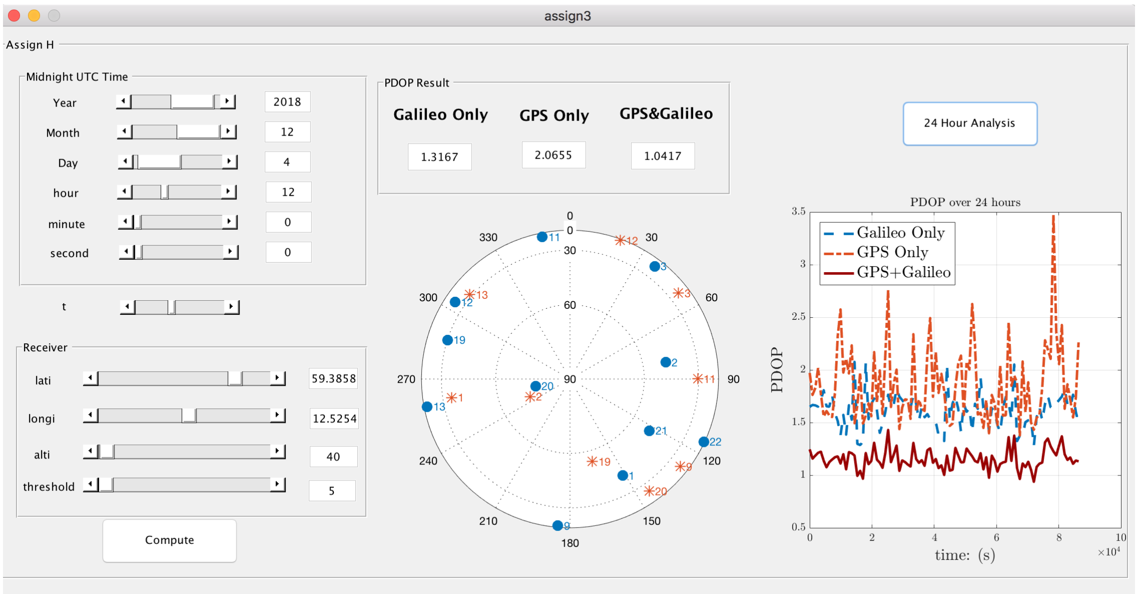

Figure 2.4. Snapshot of the designed GUI.

References

[1] Pratap Misra and Per Enge. Global positioning system: signals, measurements and performance second edition.

[2] Allan Aasbjerg Nielsen. *Least Squares Adjustment: Linear and Nonlinear Weighted Regression Analysis*. 2013.

National Space Institute
Technical University of Denmark
Elektrovej Building 328, room 007
2800 Kongens Lyngby
Denmark www.space.dtu.dk

E-mail: xiahaa@space.dtu.dk