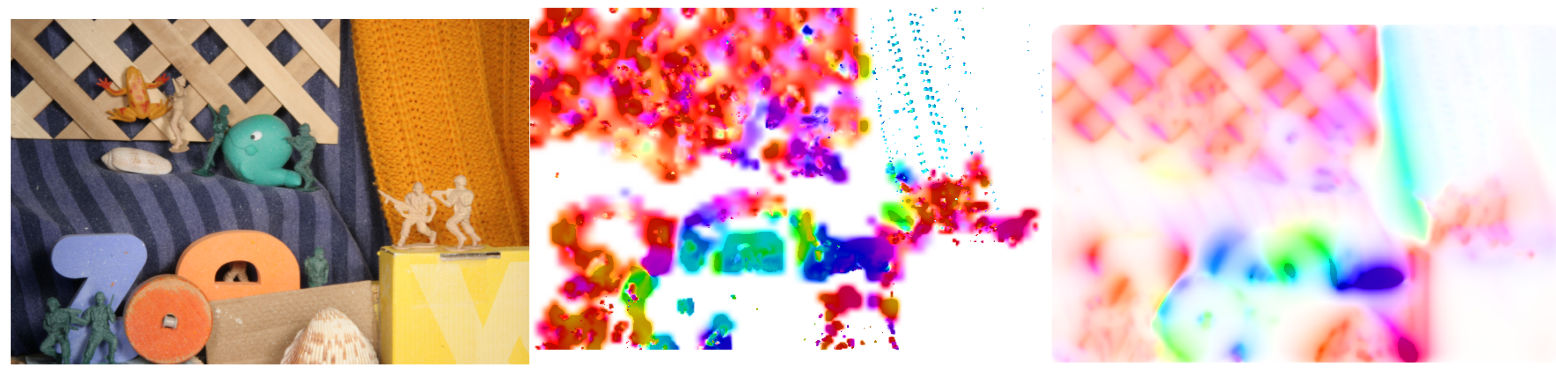

Mini-Project for Advanced Image Analysis: Optical Flow Estimation

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Introduction: Optical Flow

Small movements between two consecutive frames can be modeled as **optical flow**. The problem of **optical flow estimation** is investigated in this mini-project. The Lucas-Kanade [1] and the Horn-Shunck methods [2] are applied for optical flow estimation in this project. Estimation results on real images are presented for evaluating their performance.

Background

Brightness Constancy Constraint Optical flow is estimated by determining the local translation between two frames which makes the brightness constancy constraint fulfilled:

$$I(x,y,t) = I(x+\Delta x, y+\Delta y, t+\Delta t)$$

Small Movement Assumption the local movement is small, so the brightness constancy constraint is approximated as:

$$I_x u + I_y v = -I_t \Rightarrow \mathbf{a}^T\mathbf{u} = \mathbf{b}$$

Local Consistency Assumption neighbouring pixels undertake a same displacement $\mathbf{u}$.

Minimization Problem optimal flow is estimated by solving a minimization problem

$$\mathbf{u} = \underset{\mathbf{u}}{\operatorname{argmin}}\ E,\ \ E = \int\int ||\mathbf{a}^T\mathbf{u} - \mathbf{b}||^2 dxdy$$

Lucas-Kanade Method

Solve with least-square solution

$$\mathbf{u} = (\mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{b},\ \ \mathbf{A} = [\mathbf{a}_1, \cdots, \mathbf{a}_k]^T$$

Horn-Shunck Method

Add regularization on the smoothness of the flow and solve a joint minimization problem

$$E_{joint} = E + \int\int \alpha^2(||\bigtriangledown u||^2 + ||\bigtriangledown v||^2)dxdy$$

The optimization problem is solved iteratively

$$u^{k+1} = \bar{u}^k - \frac{I_x(I_x\bar{u}^k + I_y\bar{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

$$v^{k+1} = \bar{v}^k - \frac{I_y(I_x\bar{u}^k + I_y\bar{v}^k + I_t)}{\alpha^2 + I_x^2 + I_y^2}$$

Implementation

Conditioning $\mathbf{Au} = \mathbf{b}$ may be ill-posed. To deal with:

- check the harmonic mean: $\frac{\det(\mathbf{A})}{\operatorname{tr}(\mathbf{A})} > \tau$.
- check the minimum eigenvalue: $\lambda_{min} > \tau$.

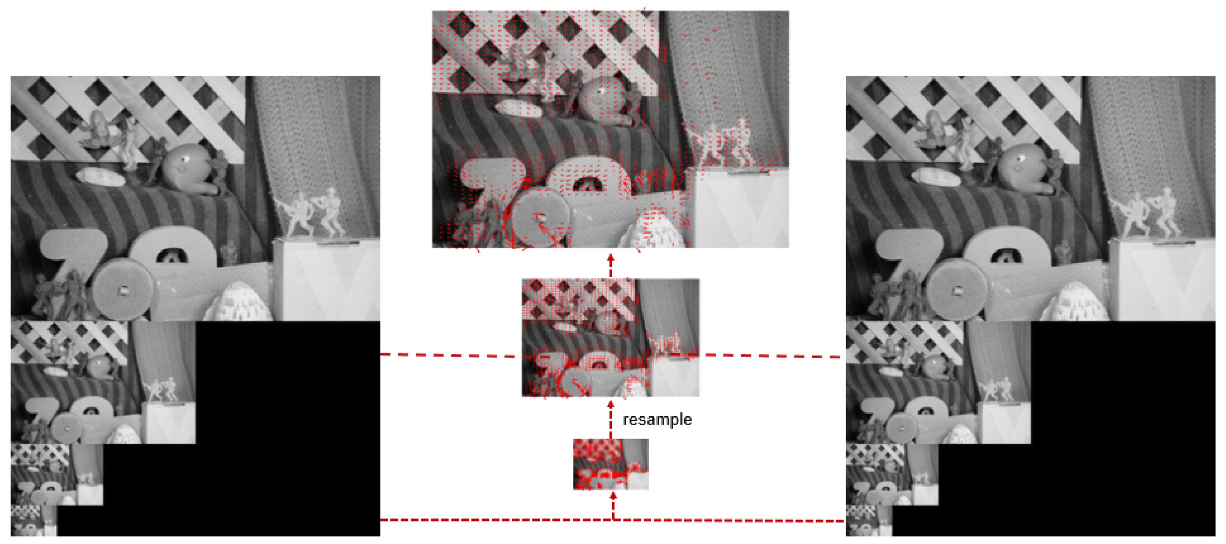
Gradient Kernel To compute I_x, I_y, different kernels are applied:

- $[-1, 0, 1]$.
- Derivative of Gaussian.

Weighting Kernel To sum up the contributions from neighbouring pixels, the following kernels are applied:

- Box filter.
- Gaussian Kernel.

Coarse-to-Fine To deal with large movement, image pyramid is used to establish a coarse-to-fine flow estimation.



Coarse-to-Fine Optical Flow Estimation.

References

[1] Lucas B D, Kanade T. An iterative image registration technique with an application to stereo vision[J]. 1981.

[2] Horn B K P, Schunck B G. Determining optical flow[J]. Artificial intelligence, 1981.

[3] Anders D B, Vedrana A D. Advanced Image Analysis. Lecture Note.

[4] Richard Szeliski. Computer vision: algorithms and applications. Springer Science & Business Media, 2010

Flow Estimation

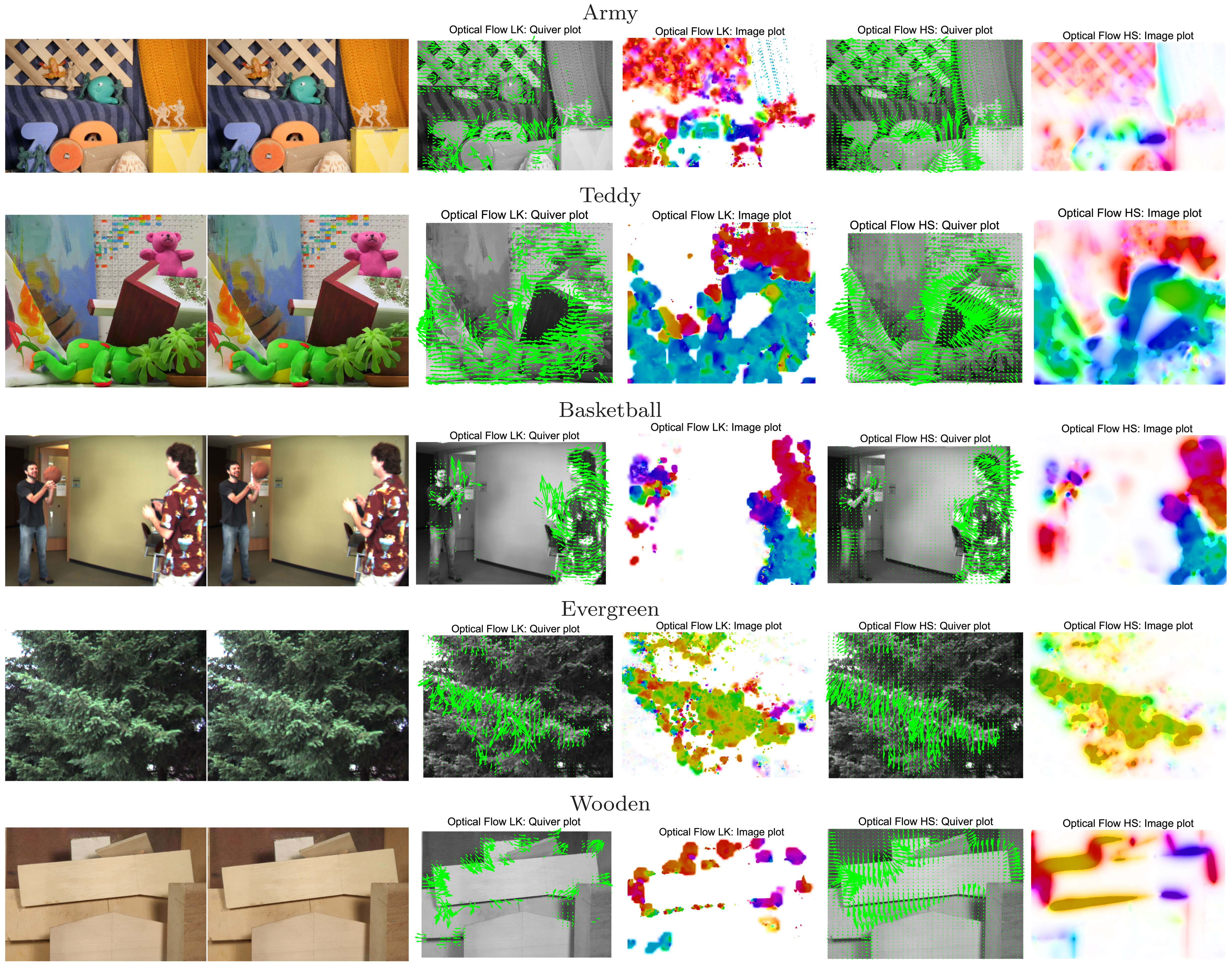
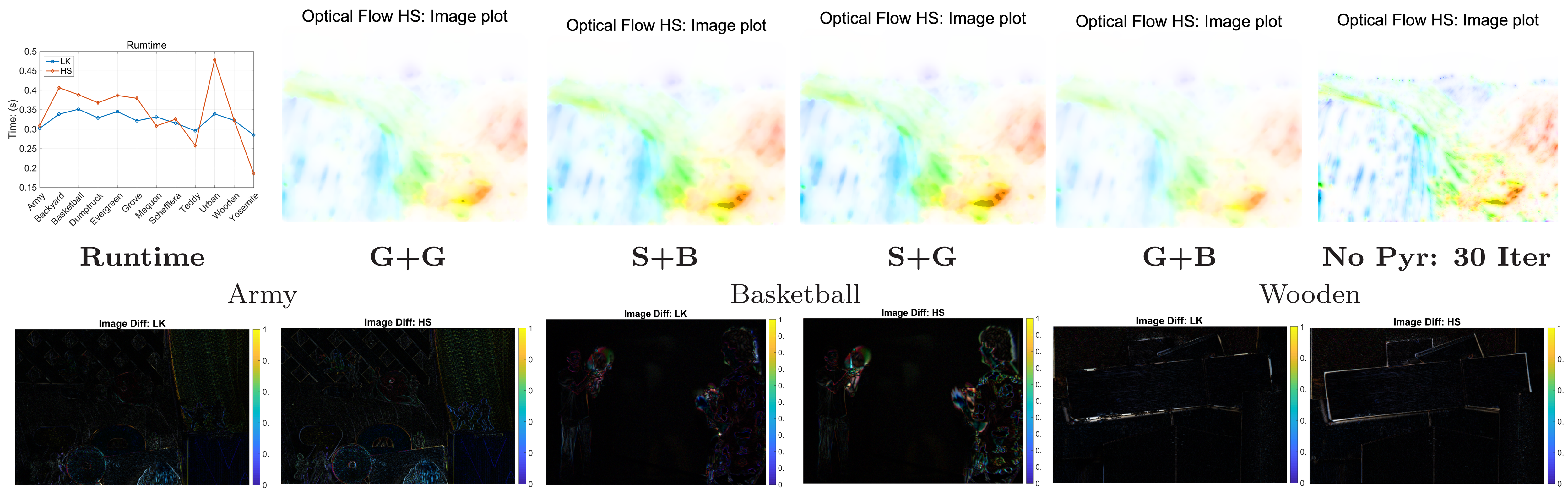


Image 1 **Image 2** **LK Quiver Plot** **LK Flow Image** **HS Quiver Plot** **HS Flow Image**

Analysis



Runtime **G+G** **S+B** **S+G** **G+B** **No Pyr: 30 Iter**

Diff LK **Diff HS** **Diff LK** **Diff HS** **Diff LK** **Diff HS**