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30550: Satellite Based Positioning

Report for assignment D-E

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30550: Satellite Based Positioning, Report for assignment D-E

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ABSTRACT

This report will present the procedures, results accomplished for assignments D-E which mainly includes the interpolation of satellite's position using precise ephemeris data and the computation of pseudo ranges by considering the delay of ionosphere, troposphere, the error caused by satellites' clock error as well as the clock error of the receiver.

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1 Assignment D: Interpolation of Satellite Positions

The main objective of the assignment is to obtain satellite positions at any epoch in time. Since the precise satellite positions are given in the SP3 file at every 15 minute, an interpolation routine is necessary to estimate satellite positions at any time.

1.1 Theory

In this experiment, two interpolation methods are carried out:

1. Cubic spline interpolation;
2. Lagrange interpolation;

1.1.1 Lagrange Interpolation

Given a set of K points: (x_1, y_1), $\cdots$, (x_K, y_K), where no two x_i are the same, the lagrange interpolation is defined with a linear combination as:

$$L(x) = \sum_{j=0}^{K} y_j \ell_j(x) \tag{1.1}$$

$$\ell_j(x) = \prod_{0 \le m \le K, m \ne j} \frac{x - x_m}{x_j - x_m} \cdots \frac{x - x_K}{x_j - x_K}. \tag{1.2}$$

The Runge-Kutta numerical integration is used to estimate the movements of satellites during a given interval. Let an initial value problem be specified as follows:

$$\dot{y} = f(t, y), \quad y(t_0) = y_0. \tag{1.3}$$

$$k_1 = h\ f(t_n, y_n), \tag{1.4}$$

$$k_2 = h\ f\left(t_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right), \tag{1.5}$$

$$k_3 = h\ f\left(t_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right), \tag{1.6}$$

$$k_4 = h\ f\left(t_n + h, y_n + k_3\right). \tag{1.7}$$

$$y_{n+1} = y_n + \tfrac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right), \tag{1.8}$$

$$t_{n+1} = t_n + h \tag{1.9}$$

where $\dot{y}$ denoting the rate at which y changes, y_0 is the initial value. Here y_{n+1} is the RK4 approximation of $y(t_{n+1})$. The RK4 method is a fourth-order method, meaning that the local truncation error is on the order of $O(h^5)$.

1.2 Tasks

- Implement a MATLAB function for interpolation of satellite positions.
- Evaluation of correctness, which can be done in 2 ways:
    1. 1. interpolate satellite positions for one satellite e.g. for every minute during an hour, then plot the positions together with the positions given in the sp3 file. Finally, check visually whether the satellite orbit is continuous.
    2. 2. remove one of the positions in the input file, estimate the position using the code, and compare with the position given in the sp3 file.
- Provide a list of satellite positions at 12:05 for the given day.
- Provide an evaluation of how much the satellites move, in general, during 5 minutes.

1.3 Code

List of relevant functions in the attachment:

- **ex4_interpolation_sat_position.m**: main entry;
- **sp3fileParser.m**: parsing the sp3 file;
- **find_data.m**: extract relevant data of given epoch from the whole sp3 data;
- **find_neighbor_ids.m**: find K nearest neighbors for interpolation;
- **interp_sat_pos.m**: interpolation;
- **move_estimation_via_RK.m**: Runge-Kutta numerical integration;

1.4 Experiments

1.4.1 Interpolation of satellite positions: 1 hour trace

In this experiment, the cubic and Lagrange interpolation (interpolation sample number is 5) are used to interpolate satellite positions for one satellite for 1 hour with 1 minute as the step. The result is shown Fig 1.1. The 5^{th} satellite is chosen for this interpolation. Blue square cubes are positions given in the sp3 file. From this figure, it can be seen the trace is continuous. Since there are no significant differences between the Lagrange interpolation and the cubic spline interpolation. For further tests, only the cubic spline interpolation will be carried out.

1.4.2 Interpolation of satellite positions: 1 truth removial

In this experiment, 5 consecutive satellite positions from the sp3 file will be used for interpolation. However, each time, I will remove 1 position, but use other 4 samples to interpolate the deleted position. Finally, the interpolated position will be compared with the position given in the sp3 file. Here, the Euclidean norm is used to represent the error: $err = \sqrt{ex^2 + ey^2 + ez^2}$. Results are shown in Fig 1.2. It can be anticipated than id 1, 5 will have the biggest error since, for those two cases, the minimum interpolation interval is 15 minutes. From this figure, it can be concluded that for interpolation, it should at least use one sample before the interpolated time and one sample after the interpolated time.

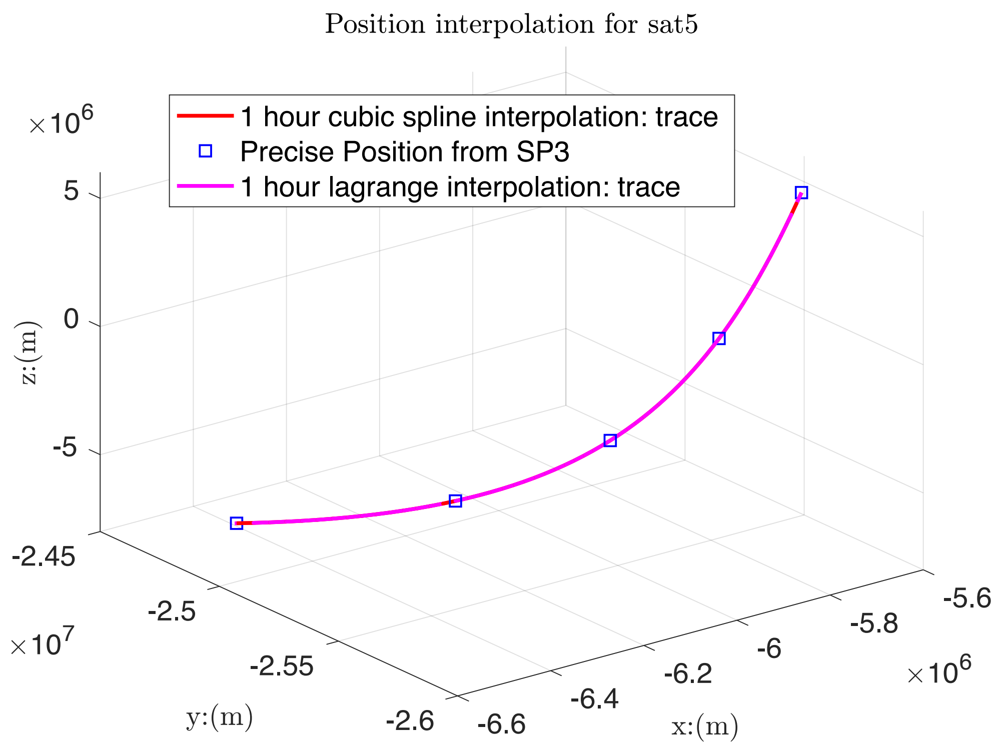

Figure 1.1. 1 hour interpolation of satellites positions using the cubic spline (red) and the Lagrange interpolation (magenta).

1.4.3 Interpolation of satellite positions: lists of satellite positions at 12:05

Satellite positions at 12:05 is given in Table 1.1 and plotted in Fig 1.3.

1.4.4 Interpolation of satellite positions: movements

In order to estimate the movements of satellites from 12:00 to 12:05, two methods are used.

- Divide 5 minutes equally to N samples (N=500 in this report), interpolate positions at these samples, numerical integration using the Euclidean distance between two points.
- Runge-Kutta numerical integration using satellite orbital equations. The satellite's orbital velocity is used for numerical integration, $s = s + v * d_t$.

The results using the first method is shown in Fig 1.4, while the result of Runge-Kutta integration is given as $1.160377384143887e + 06$. It can be seen results using the two methods are on the same level, which means the result from the first method is reasonable. Moreover, we should expect some differences for the movements of different satellites since some satellites are close to the earth (near the perigee) while some are near the apogee. Because they should sweep the same area in the same interval, some satellites will move faster than others, which can be shown in the figure.

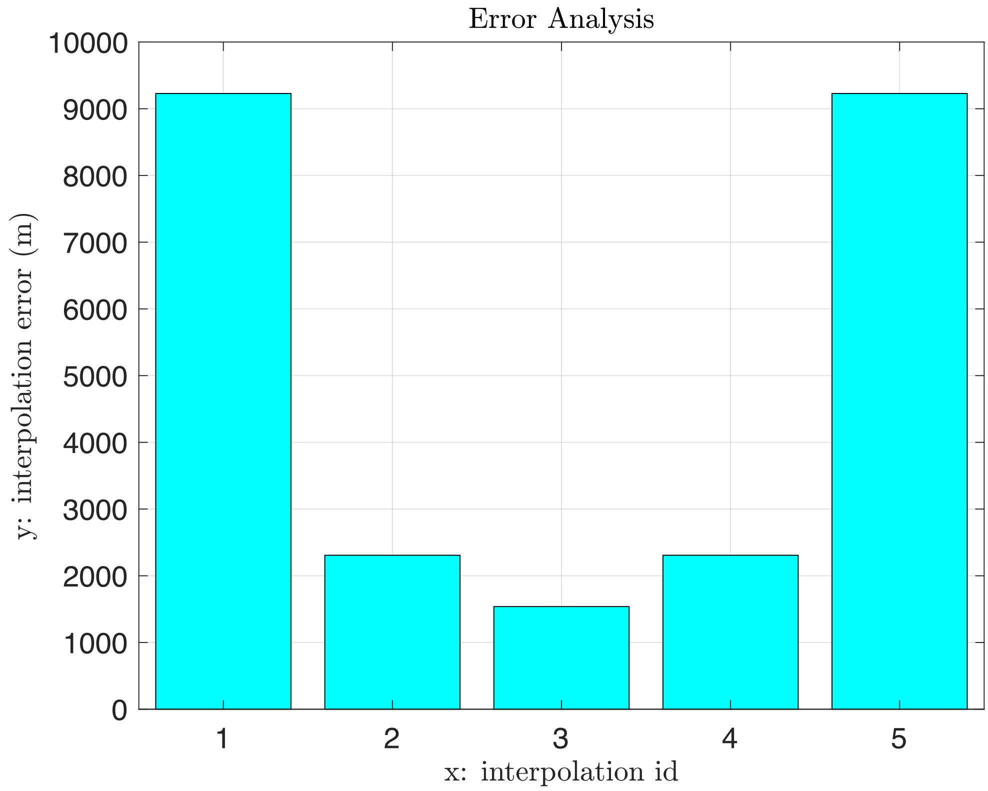

Figure 1.2. Error of interpolation, e.g. for id = 1 (t_0), I will use samples $2(t_0 + 15)$, $3(t_0 + 30)$, $4(t_0 + 45)$, $5(t_0 + 60)$ to interpolate the position at t_0.

1.4.5 Conclusion

Satellite positions at any time can be estimated with the interpolation of neighboring positions from the sp3 file. The interpolation accuracy depends on the interpolated interval. The big interval will encounter a big error.

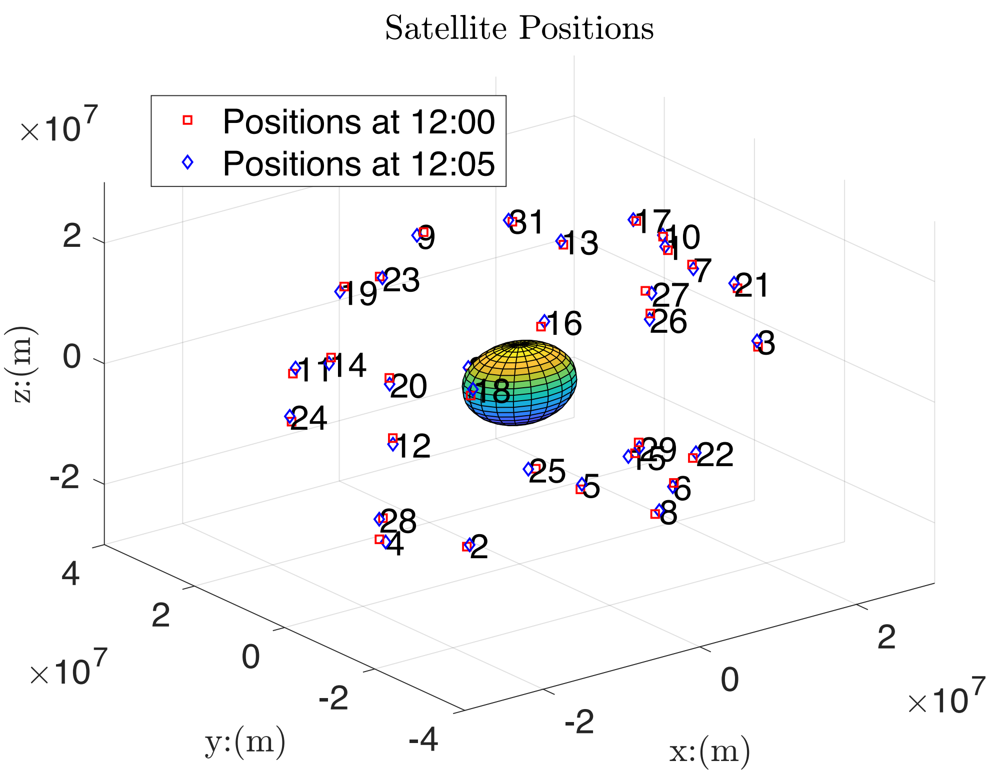

Figure 1.3. Satellite positions at 12:05.

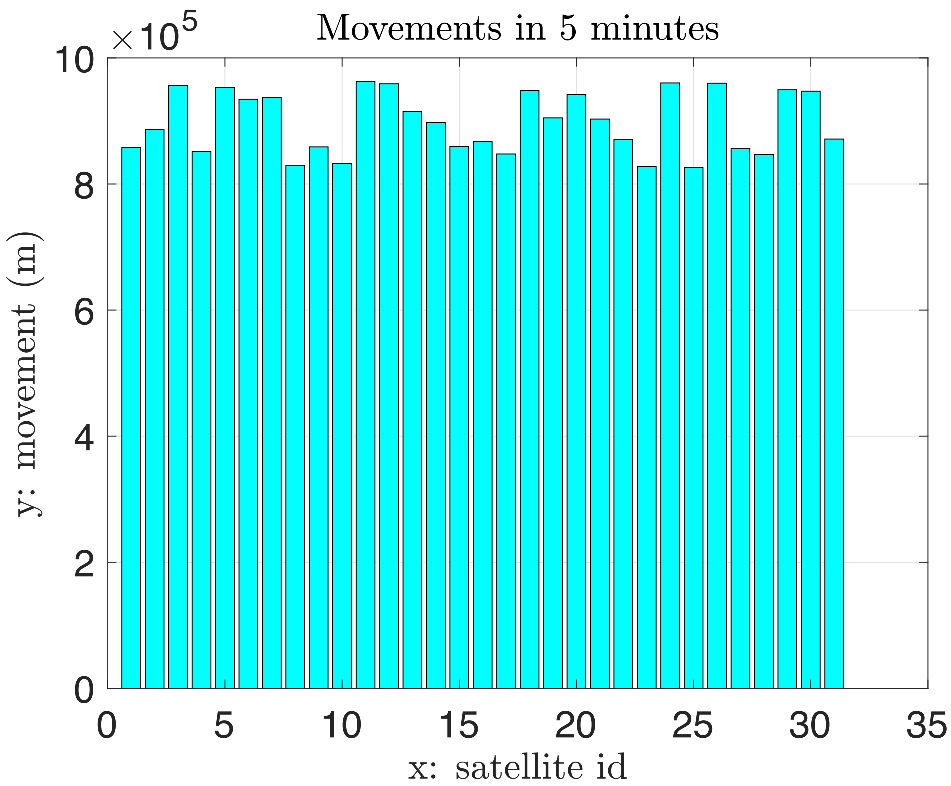

Figure 1.4. Satellite movements for 31 satellites.

Table 1.1. Test cases

ID	x: · x 1.0e+07 m	y: · x 1.0e+07 m	z: · x 1.0e+07 m
1	1.352275032487963	-0.878809847323457	2.087610849817902
2	-1.437693324821914	-1.391716787680864	-1.700311212213889
3	2.278033299450926	-1.312518790408642	0.347465621006790
4	-1.874589722163271	-0.287491825262037	-1.879538575727778
5	-0.645297886408642	-2.506036461925000	-0.590949023093518
6	0.663989172338889	-2.247232429460185	-1.180835507769444
7	2.479907440194444	0.452062120471296	0.865999828226543
8	1.123530648196914	-1.148451948476852	-2.118240865507408
9	-0.260765008742901	1.822258602618210	1.912366834147222
10	1.599183109164506	-0.399532648460494	2.024800011246914
11	-2.374441160230556	0.844283180228086	0.791095146870370
12	-2.333718341039815	-1.246492233753704	0.233368790093210
13	1.575683818774383	1.818078591017901	1.174826106816049
14	-2.402990711784568	0.051592075679321	1.148973836728086
15	1.698258643957407	0.536337840034877	-2.000086494548457
16	-0.707951370467284	-1.783790148741667	1.880740308229321
17	1.542611490504321	0.156099993267284	2.110349378693210
18	-1.619835102425309	-1.781284963483024	1.076742904667593
19	-1.072757189849691	2.120701954924691	1.162881803342901
20	-0.220957856325000	2.498488708658025	-0.800678691275309
21	2.293612391333951	-0.775056021610494	1.112435400433333
22	1.984220046017901	-0.467085079438889	-1.694158860153703
23	-1.511764039328395	0.415714282056482	2.130005106953087
24	-1.926576246199692	1.750498232476852	-0.477810309570370
25	0.849657017333333	1.280070572010185	-2.166874058700000
26	2.365694501997222	1.224199805620679	-0.198119115875000
27	0.638074638526852	-1.822718815569136	1.888229314681481
28	-1.244578594919444	0.950211598745679	-2.147860085833333
29	0.009795386511728	-2.640899917435186	-0.171773051395062
30	0.723936071380247	2.394859482751543	-0.821747491325309
31	0.879830663560185	1.771993636657716	1.780695066435185

2 Assignment E: Error Sources and Pseudoranges

The main objective of the assignment is to simulate real code pseudoranges from a GPS receiver by adding atmospheric errors and clock errors. Four kinds of error will be considered in this experiment:

- Ionospheric delay;
- Tropospheric delay;
- Satellite clock error;
- Receiver clock error;

2.1 Theory

According to [1] and the lecture slides, the pseudorange R can be modeled as:

$$R = \rho + d\rho + c(dT - dt) + d_{ion} + d_{trop} + e \tag{2.1}$$

where R is the pseudorange, ρ is the geometric distance between the receiver and satellite, $d\rho$ is the orbit error (set to 0 in this experiment), c is the speed of light (299792458 m/s), dT is the receiver clock error (set to 0.1 ms), dt is the satellite clock error (read from the sp3 file), d_{ion} and d_{trop} are the ionospheric delay and tropospheric delay, respectively, e is the error from the multipath effect and receiver noise (0 in this experiment).

2.1.1 Ionosheric delay

The first order group delay for L1 frequency in the zenith direction can be modeled as

$$d_g = \frac{40.3}{f^2} TEC \tag{2.2}$$

where the TEC (Total Electron Content) is measured in $TECU$, 1 $TECU$ = 10^{16} $electrons/m^2$, f is the frequency of L1 signal (1575.42 MHz). The typical size of first order ionosheric delay in zenith direction is $5 - 15$ m in the afternoon and $1 - 3$ m at night. To compensate for the effect generated by the elevation angle, the obliquity factor, given as the following equation, is used.

$$OF_\zeta = \left[1 - \left(\frac{R_E sin(\zeta)}{R_E + h_I}\right)^2\right]^{(-\frac{1}{2})} \tag{2.3}$$

where h_I is the mean ionospheric height (350 km), R_E is the Earth mean radius (6371 km), ζ is the zenith angle. The overall estimation of the ionospheric effect is given as

$$d_{ion} = OF_\zeta d_g \tag{2.4}$$

2.1.2 Tropospheric delay

The tropospheric delay is estimated with the Saastamoinen troposphere model[1]:

$$d_{trop} = 0.002277D\left(P_s + \left(\frac{1255}{T_s} + 0.05\right)e_s\right) \tag{2.5}$$

$$D = 1 + 0.0026cos(2\phi_{ant}) + 0.00028h_{ant} \tag{2.6}$$

where P_s, T_s, e_s are the pressure (hPa), temperature (K) and water vapor pressure (hPa) at surface respectively, h_{ant} and ϕ_{ant} are the mean-sea-level height and latitude of the antenna. Here, standard sea-level meteorological values for temperature, total pressure and relative humidity are used: $T_s = 18°\ Celsius$, $P_s = 1013\ hPa$, $RH = 50\%$. The partial pressure of water vapor e_s (in hPa) is estimated with the relative humidity RH

$$e_s = 6.106RHe^{\left(\frac{17.15T-4684}{T-38.45}\right)} \tag{2.7}$$

where T is temperature, given in Kelvin. The delay after considering the elevation angle is given by

$$d_{trop}(elv) = d_{trop}(zenith)/sin(elv) \tag{2.8}$$

The size of zenith tropospheric delay is approximately 2.4 m at sea level and 1 m at the Mount Everest. The slant tropospheric delay is approximately 24 m at 5° elevation.

2.1.3 Satellite clock error

The satellite clock error for each satellite is given in the sp3-file in microseconds. However, they are given at every 15 minute. In order to estimate the satellite clock error at given epoch, a linear interpolation is used.

2.1.4 Receiver clock error

In this report, the receiver clock error is assumed to be a constant, 0.1 milliseconds.

2.2 Tasks

- Implement a MATLAB function for estimating the ionospheric delay for the L1 frequency.
- Implement a MATLAB function for estimating the tropospheric delay for the visible satellites.
- Compose pseudoranges using the geometric distances and the four errors.

2.3 Code

List of relevant functions in the attachment:

- **ex5_modeling_error_sources.m**: main entry for single point test;

[1]The Saastamoinen troposphere model are given differently in the lecture slide, [1] and the ESA Navipedia: Gelileo Tropospheric Correction Model.

- **sp3fileParser.m**: parsing the sp3 file;
- **find_data.m**: extract relevant data of given epoch from the whole sp3 data;
- **find_neighbor_ids.m**: find K nearest neighbors for interpolation;
- **interp_sat_pos.m**: interpolation;
- **calc_pseudorange.m**: pseudorange composition;
- **iono_delay_first_order_group.m**: estimation of ionospheric delay;
- **tropo_delay_via_saastamoinen_model.m**: estimation of tropospheric delay;
- **sat_clock_error.m**: estimation of satellite clock error;
- **rec_clock_error.m**: estimation of receiver clock error;
- **assign3.m**: GUI entry;

2.4 Experiments

Setup of TECU

The vertical total electron content (VTEC)-value is visually and manully selected from the "seSolstorm" service[2]. In this experiment, TEC is assumed to be 10 (in $TECU$).

2.4.1 Point test

The antenna is assumed to be placed near the DTU 101 with the latitude, longitude and height being $(55.78575300466123, 12.525384183973078, 40)$. The epoch time is $2018-09-16:12:00:05$.

The ionospheric and tropospheric errors for all visible satellites at this point are given in Table 2.1. It can be seen from Sec. 2.1.1 and Sec. 2.1.2 that once the position of the

Table 2.1. The ionospheric and tropospheric errors for all visible satellites.

elevation angle: $^\circ$	$\mathbf{d_{ino}}$: (m)	$\mathbf{d_{trop}}$: (m)
51.563749991650930	2.009720619511114	3.073028976092045
14.377804376731538	4.099939118704490	9.693767694010592
47.430752748569226	2.116071149569006	3.268478426261661
30.070702126071616	2.839302226284274	4.803945178630212
65.083397953012266	1.771088768092310	2.654147342287291
36.633224255005878	2.501407964189823	4.034093612023026
80.487136298691340	1.644024582075247	2.440667339360591
37.099669748228656	2.480821181963063	3.990535517815306
8.303707879699919	4.683762366000505	16.667363207747151
19.442835418683210	3.621717560541795	7.231447590618250
27.108009625758054	3.025571233726567	5.282568504628498
44.050696556931513	2.218180897092655	3.461992373575292

antenna and the TEC are fixed, the ionospheric and tropospheric errors will only depend on the elevation angles. It can be easily derived that the ionospheric and tropospheric

[2] visit `http://sesolstorm.kartverket.no/moreplots.xhtml`, click "Plasmainnhold i ionosfæren (VTEC)" to get the VTEC map, then estimate a value based on the map

errors are inversely proportional to elevation angles. Consequently, satellite with large elevation angle will have small ionospheric and tropospheric errors and vice versa, which can be verified with the delays in Table 2.1.

For ionospheric errors, since $TEC = 10\,TECU$ indicates a relatively low level of ionospheric activity, the estimated errors are in the size of $1 - 5$ m, which is reasonable according to the typical size given in the lecture slides.

For tropospheric errors, the estimated errors are in the size of $2 - 17$ m with the maximum error corresponding to the satellite with an elevation angle of 8.3°, which is also reasonable according to the analysis in Sec. 2.1.2.

The satellite and receiver clock errors for all visible satellites at this point are given in Table 2.2. Since 1 microsec roughly means 300 m error, the satellite errors in the Table 2.2

Table 2.2. The satellite and receiver clock errors for all visible satellites.

clock error: $\cdot 1e^2$ **microsec**	**cdt**: $\cdot 1e^5$ (m)	**clock error: millisec**	**cdT**: $\cdot 1e^4$ (m)
-0.887968547388889	-0.266206273448404	0.1	2.99792458
1.369666929944445	0.410615815569359	0.1	2.99792458
-1.153204196611111	-0.345721920677960	0.1	2.99792458
1.815711888777778	0.544336730156513	0.1	2.99792458
-7.175379044944446	-2.151124520965587	0.1	2.99792458
-0.972670799833333	-0.291599369906861	0.1	2.99792458
0.578928733055556	0.173558467889551	0.1	2.99792458
-5.207432259833333	-1.561148917043930	0.1	2.99792458
-0.554504475777778	-0.166236259765421	0.1	2.99792458
3.443967622777778	1.032475518904967	0.1	2.99792458
7.447308556333335	2.232646937587602	0.1	2.99792458
-4.220427770500000	-1.265252415129655	0.1	2.99792458

are reasonable. And from the first and their columns, we can see that the satellite and receiver clock errors are on a similar level ≈ 100 *microsec*, so their errors should be on a similar scale, which is verified in the Table 2.2.

It can be seen from Table 2.1 and Table 2.2, the satellite and receiver clock errors are much bigger than the ionospheric and tropospheric delay. Consequently, it is very important to eliminate the clock errors from the pseudoranges.

2.4.2 GUI

A Matlab GUI is designed and shown in Fig 2.1. Latitude, longitude, and altitude can be arbitrarily selected simply by sliding corresponding sliders. TEC value and receiver clock error can be easily changed. Epoch value can be either typed or selected via the corresponding sliders.

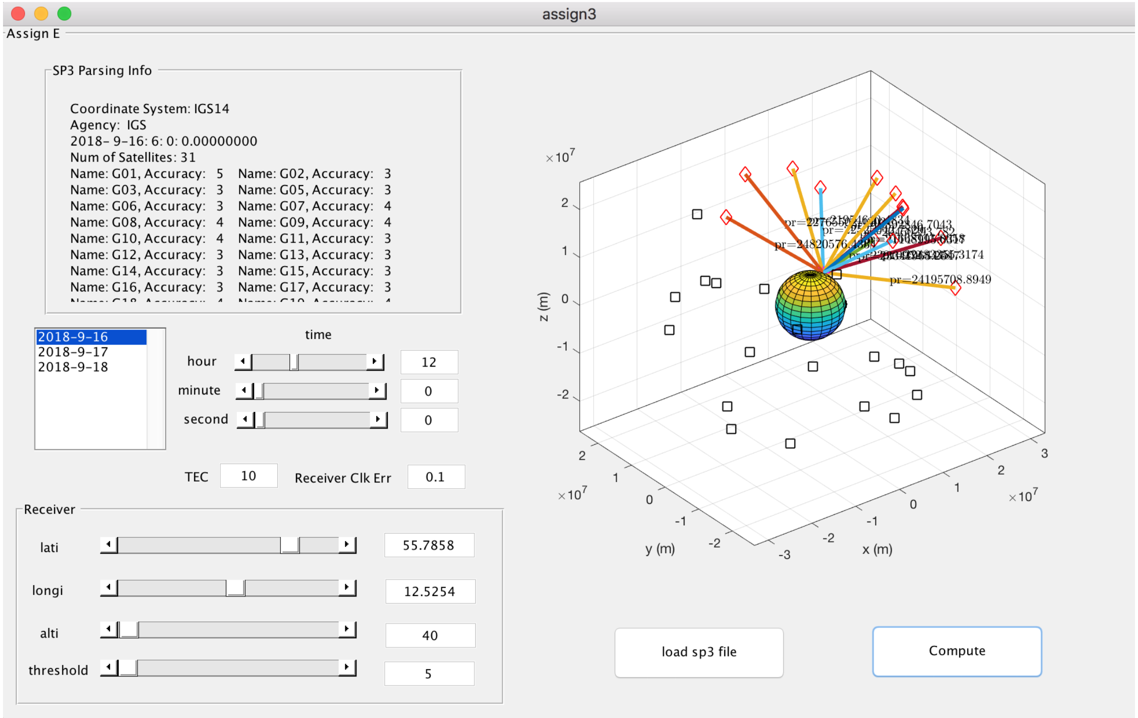

Figure 2.1. Snapshot of the designed GUI.

2.4.3 Conclusion

In this experiment, the ionospheric delay, the tropospheric delay, the satellite clock error, the receiver clock error are estimated using corresponding models or relevant data. The sizes of the four errors are analyzed. It can be concluded that the satellite clock error and the receiver clock error are two biggest error sources that should be eliminated correctly.

References

[1] Pratap Misra and Per Enge. Global positioning system: signals, measurements and performance second edition.

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