30540 Mapping from Aerial and Satellite Images
F19 - CCA and MAD

Danjal Berg - s143277

February 2019

1 Theory

1.1 CCA

In Canonical Correlation Analysis (CCA) the relationship between two or more groups of variables. As data preparation should the expectation value be zero for the data sets $E\{X\} = E\{Y\} = 0$. In CCA is the correlation maximised i.e

$$max \quad \rho(\boldsymbol{a^T X, b^T Y}) = \frac{\boldsymbol{a^T \Sigma_{12} b}}{\sqrt{\boldsymbol{a^T \Sigma_{11} a b^T \Sigma_{22} b}}} \tag{1}$$

Where $\boldsymbol{a^T \Sigma_{12} b}$ is maximised subject to $\boldsymbol{a^T \Sigma_{11} a} = \boldsymbol{b^T \Sigma_{22} b} = 1$. The constrain is solved by Lagrange multipliers.

$$L(\boldsymbol{a}, \boldsymbol{b}, \lambda_1, \lambda 2) = \boldsymbol{a^T \Sigma_{12} b} - \frac{1}{2}\lambda_1(\boldsymbol{a^T \Sigma_{11} a^T} - 1) - \frac{1}{2}\lambda_1(\boldsymbol{b^T \Sigma_{22} b^T} - 1) \tag{2}$$

When the partial derivative has been taken with respect to $\boldsymbol{a}$, $\boldsymbol{b}$ λ_1 and λ_2 the following equations should be solved.

$$\boldsymbol{a} = \frac{1}{\lambda_1}\boldsymbol{\Sigma_{11}^{-1} \Sigma_{12} b} \tag{3}$$

and

$$\boldsymbol{\Sigma_{21} \Sigma_{11}^{-1} \Sigma_{12} b} = \rho^2 \boldsymbol{\Sigma_{22} b} \tag{4}$$

Where $\lambda_1 = \lambda_2 = \rho$. $\boldsymbol{b}$ is computed by calculating the generalized eigenvalues and corresponding eigenvectors of equation 4. $\boldsymbol{a}$ could also be solved for but if the data sets are sorted so $\boldsymbol{Y}$ has lower dimensions than $\boldsymbol{X}$ then is it more computational efficient to solve for $\boldsymbol{b}$.

1.2 MAD

The multivariate alteration detection (MAD) is defined as:

$$\begin{bmatrix} X \\ Y \end{bmatrix} \rightarrow \begin{bmatrix} a_p^T X & - & b_p^T Y \\ & \vdots & \\ a_1^T X & - & b_1^T Y \end{bmatrix} \tag{5}$$

We want to maximise the variance $V\{\boldsymbol{a^T X} - \boldsymbol{b^T Y}\} = 2(1 - Corr\{\boldsymbol{a^T X}, \boldsymbol{b^T Y}\}$ and therefore minimise ρ.

$$\sigma^2_{MAD,i} = 2(1 - \rho_{p-i+1}) \tag{6}$$

2 Example

In this example are a multi spectral image over Copenhagen used. There are 6 spectral bands and two images. The first image is from 1996 and the second are from 2005.

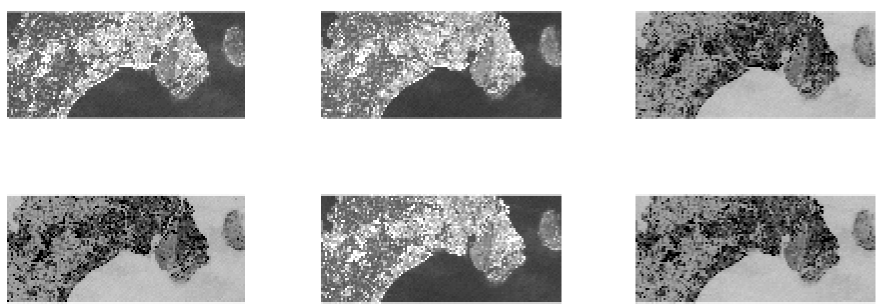

Figure 1: Image over Copenhagen with CCA used from 1986.

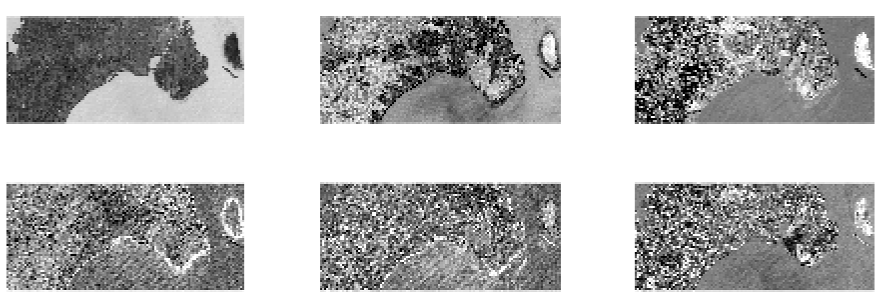

Figure 2: Image over Copenhagen with CCA used from 2005.

3 MATLAB code

```
clc; clear;

B_86 = double(imread('udsnit_1986_06_27_band453.tiff'));
F_86 = double(imread('udsnit_1986_06_27_farve.tiff'));
B_05 = double(imread('udsnit_2005_08_18_band453.tiff'));
F_05 = double(imread('udsnit_2005_08_18_farve.tiff'));

D86 = cat(3,F_86,B_86);
D05 = cat(3,F_05,B_05);

X = reshape(D86,1050*2350,[]);
Y = reshape(D05,1050*2350,[]);

X = X-mean(X);
Y = Y-mean(Y);

nrows = 1050;
ncols = 2350;
nvars =  6;

XY = cat(2,X,Y);

S = cov(XY);

S11 = S(1:6,1:6);
S22 = S(7:12,7:12);
S12 = S(1:6,7:12);
S21 = S(7:12,1:6);

S_cal = S21*inv(S11)*S12;


[b,D,W] = eig(S_cal,S22);
[lambda,ind] = sort(diag(D),'descend');
rho2 = diag(lambda);
rho = sqrt(diag(rho2));
R = diag(1./rho);

b = b(:,ind);
W = W(:,ind);

a = R*inv(S11)*S12*b;

```

```
img_CCA = X*a;
img_CCA = reshape(img_CCA, nrows, ncols, nvars);

img_CCA2 = Y*b;
img_CCA2 = reshape(img_CCA2, nrows, ncols, nvars);

figure
for i = 1:6
    subplot(2,3,i)
    imshowrgb(img_CCA,[i i i],2)
end

figure
for i = 1:6
    subplot(2,3,i)
    imshowrgb(img_CCA2,[i i i],2)
end

%% MAD

MAD = diag(sort(2*(1-diag(rho)),'descend'));

MAD_XY = X*a-Y*b;
MAD_XY = reshape(MAD_XY, nrows, ncols, nvars);
figure
for i = 1:6
    subplot(2,3,i)
    imshowrgb(MAD_XY,[i i i],2)
end
```