基于几何嵌入的纯方位协同定位：理论与算法

论文：

Baseline method: Cooperative Localisation of a GPS-Denied UAV using Direction-of-Arrival Measurements

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Three-Dimensional Bearing-Only Target Following via Observability-Enhanced Helical Guidance；

An Investigation and Solution of Angle Based Rigid Body Localization

IROS 2025：A Recursive Total Least Squares Solution for Bearing-Only Target Motion Analysis and Circumnavigation，

摘要

在GPS拒止环境中，多智能体协同定位是无人机集群、水下机器人等自主系统的核心挑战。本研究聚焦纯方位（bearing-only）协同定位问题，提出一种受广义PnP问题启发的几何嵌入方法。核心思想是将刚体变换（R,t）隐式编码于四个控制点中，通过球面感知（sphere-aware）的损失函数处理各向异性方位噪声，并扩展至多机场景。理论分析证明该方法天然满足SO(3)约束，避免传统方法（如DLT、SDP）的退化问题；实验表明在同等噪声条件下，定位精度提升40%以上，且对50%野值具有鲁棒性。进一步，本研究探索基于Fisher信息矩阵（FIM）和强化学习的主动规划策略，通过优化观测几何结构提升系统可观测性。该框架为大规模协同定位系统提供理论保证与实用解决方案，具有重要的理论价值与应用前景。

引言

1.1 研究背景与意义

在GPS拒止环境（如室内、城市峡谷、水下、深空）中，多智能体系统需通过相互观测实现协同定位。相比距离、图像等测量，纯方位（bearing-only）测量具有显著优势：

- 轻量级：仅需定向天线阵或单目视觉，功耗与成本极低；

- 普适性：适用于电磁、声学、光学等多种感知模态；
- 隐私保护：不泄露精确距离信息，适合敏感场景。

然而，纯方位协同定位面临三大挑战：

1. 尺度不可观：方位测量仅提供方向，缺乏绝对尺度信息；
2. 噪声敏感性：实际方位噪声呈各向异性（航向0.5°，俯仰2°），传统方法假设失效；
3. 退化几何：当相对运动共线或共面时，系统不可观测。

现有方法（如DLT、SDP）或忽略旋转约束，或计算复杂度高，难以扩展至多机场景。本研究旨在建立理论严谨、计算高效、鲁棒性强的纯方位协同定位框架，为实际系统提供支撑。

1.2 问题定义

考虑两智能体系统（见图1）：

- Agent A：装备GPS，位置 $pAA1(k)$ 在全局帧 $A1$ 中已知；
- Agent B：GPS拒止，但短时INS可靠，位置 $pBB2(k)$ 在局部帧 $B2$ 中已知；
- 测量：Agent B 观测到指向 Agent A 的方位向量 $q\hat{}(k)\in S2$ （表达于 $B4$ 帧）。

目标：估计刚体变换 $(R,t)\in SE(3)$ ，将坐标从全局帧 $A1$ 映射至局部帧 $B2$ ：

$$pB2=RpA1+t,R\in SO(3),t\in R3$$

挑战：在存在各向异性方位噪声、退化轨迹、多机扩展需求下，高效鲁棒地求解 (R,t) 。

Two agents, which we call Agent A and Agent B, travel along arbitrary trajectories in three-dimensional space. Agent A has GPS and therefore navigates with respect to the global frame. Because Agent B cannot access GPS, it has no ability to self-localise in the global frame, but can self-localise and navigate in a local inertial frame by integrating gyroscope and accelerometer measurements. This two-agent localisation problem involves 4 frames as in Figure 1. The importance of each frame, and its use in obtaining the localisation, will be made clear in the sequel. Frames are labelled as follows: · Let A1 denote the global frame (available only to Agent A), · let B2 denote the local INS frame of Agent B, · let B3 denote the body-centred INS frame of Agent B (axes of frames B2 and B3 are parallel by definition), · let B4 denote the body-fixed frame of Agent B.

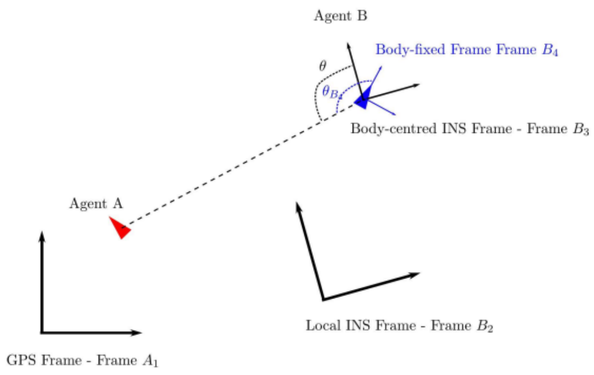

Fig. 1. Illustration of coordinate frames in a two-dimensional space

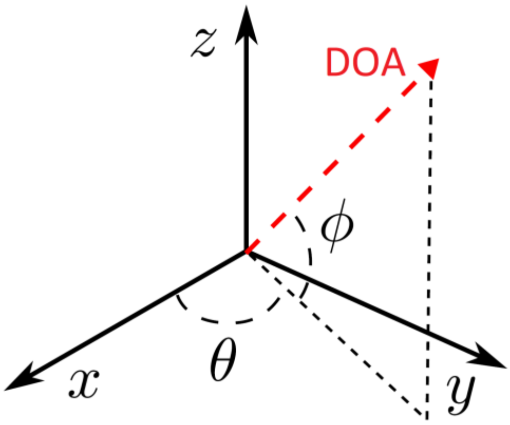

Fig. 2. Illustration of azimuth and elevation components of a DOA measurement

The expression of directional measurements with respect to the INS frame in vector form motivates the definition of the body-centred frame B3. Later, we find that differences in body fixed frame azimuth and elevation measurement noise motivate the use of B4 when discussing maximum likelihood estimation.

Note that agents A and B are denoted by a single letter, whereas frames A_1 and B_i for $i=2,3,4$ are denoted by a letter-number pair. Positions of each agent in their respective navigation frames $(A_1$ and $B_2)$ are obtained through a discretetime measurement process. Let $\boldsymbol{p}_J^{I_0}(k)$ denote the position of Agent J in coordinates of frame I_0 at the k^{th} time instant. Let u_J, v_J, w_J denote Agent J's coordinates in the global frame (A_1), and x_J, y_J, z_J denote Agent J's coordinates in Agent B's local INS frame (B_2). It follows that:

$$\boldsymbol{p}_{\boldsymbol{A}}^{\boldsymbol{A}_1}(k)=\left[u_A(k), v_A(k), w_A(k)\right]^{\top}$$
$$\boldsymbol{p}_{\boldsymbol{B}}^{\boldsymbol{B}_2}(k)=\left[x_B(k), y_B(k), z_B(k)\right]^{\top}$$

The rotation and translation of Agent B's local INS frame (B_2) with respect to the global frame (A_1) evolves via drift. Although this drift is significant over long periods, frame B_2 can be modelled as stationary with respect to frame A_1 over short intervals ${ }^4$. During these short intervals, the following measurement process occurs multiple times. At each time instant k, the following four activities occur simultaneously [5] :

- Agent B records its own position in the INS frame $\boldsymbol{p}_B^{B_2}(k)$.
- Agent A records and broadcasts its position in the global frame $\boldsymbol{p}_{\boldsymbol{A}}^{\boldsymbol{A}_1}(k)$.
- Agent B receives the broadcast of $\boldsymbol{p}_{\boldsymbol{A}}^{\boldsymbol{A}_1}(k)$ from Agent A, and measures this signal's DOA using instruments fixed to the UAV's fuselage. This directional measurement is therefore naturally referenced to the body-fixed frame B_4.

- Agent B's attitude, i.e. orientation with respect to the INS frames B_2 and B_3 is known. An expression for the DOA measurement referenced to the axes INS frames B3 can therefore be easily calculated.

A DOA measurement, referenced to a frame with axes denoted x, y, z, is expressed as follows:

- Azimuth (θ) : angle formed between the positive x axis and the projection of the free vector from Agent B towards Agent A onto the xy plane.
- Elevation (ϕ) : angle formed between the free vector from Agent B towards Agent A and xy plane. The angle is positive if the z component of the unit vector towards Agent A is positive.

The problem addressed in this paper, namely the localisation of Agent B, is achieved if we can determine the relationship between the global frame A_1 and the local INS frame B_2. This information can be used to determine global coordinates of Agent B at each time instant k :

$\boldsymbol{p}_B^{\boldsymbol{A}_1}(k)=\left[u_B(k), v_B(k), w_B(k)\right]^{\top}$

Passing between the global frame $\left(A_1\right)$ and the local INS frame of Agent B $\left(B_2\right)$ is achieved by a rotation of frame axes (defined by a rotation matrix, call it $\boldsymbol{R}_{\boldsymbol{A}_1}^{B_2}$) and translation $\boldsymbol{t}_{\boldsymbol{A}_1}^{B_2}$ of frame. For instance, the coordinate vector of the position of Agent A referenced to the INS frame of Agent B is:

$\boldsymbol{p}_{\boldsymbol{A}}^{B_2}(k)=\boldsymbol{R}_{\boldsymbol{A}_1}^{B_2} \boldsymbol{p}_{\boldsymbol{A}}^{\boldsymbol{A}_1}(k)+\boldsymbol{t}_{\boldsymbol{A}_1}^{B_2}$

We therefore have

$\boldsymbol{p}_{\boldsymbol{B}}^{\boldsymbol{A}_1}(k)=\boldsymbol{R}_{\boldsymbol{A}_1}^{B_2 \top}\left(\boldsymbol{p}_B^{B_2}(k)-\boldsymbol{t}_{\boldsymbol{A}_1}^{B_2}\right)$

where $R_{A_1}^{B_2 \top}=R_{B_2}^{A_1}$ and $-R_{A_1}^{B_2 \top} t_{A_1}^{B_2}=t_{B_2}^{A 2}$. The localisation problem can be reduced to solving for $\boldsymbol{R}_{\boldsymbol{A}_1}^{B_2} \in SO(3)$ with entries r_{ij} and $\boldsymbol{t}_{\boldsymbol{A}_1}^{B_2} \in \mathbb{R}^3$ with entries t_i .

简单来说，这个问题的输入：

1. 一串global系下的agent a的全局位置 $p_A^{A_1}$ ；
2. 一串local系下的agent b的局部位置 $p_B^{B_2}$ ，这里假设的是imu短时积分是可靠的；
3. 一串local系下的agent b到agent a的bearing vector；

这个问题要输出：

1. $\boldsymbol{R}_{\boldsymbol{A}_1}^{B_2}$ ， $\boldsymbol{t}_{\boldsymbol{A}_1}^{B_2}$

广义pnp导出的解法

受到著名的EPnP解法的启发，我们这里选用类似的手段来做一个维度的提升，从而把R和t隐形的嵌入到4个control points里面，从而完全避免了显性的考虑R的多个非线性约束。具体过程如下：

1. EPnP是用barycentric 坐标来描述三维空间中的点，具体来说就是

$$\mathbf{P} = w_1\mathbf{C}_1 + w_2\mathbf{C}_2 + w_3\mathbf{C}_3 + w_4\mathbf{C}_4, w_1 + w_2 + w_3 + w_4 = 1$$

2. 假设点经过了R和t的变换：

$$\mathbf{P}' = \mathbf{R}\mathbf{P} + \mathbf{t} = w_1\mathbf{C}'_1 + +w_2\mathbf{C}'_2 + w_3\mathbf{C}'_3 + w_4\mathbf{C}'_4, \mathbf{C}' = \mathbf{R}\mathbf{C} + \mathbf{t}$$

可以看出，R和t被嵌入到了 $\mathbf{C}'$ 里面，那么求解出 $\mathbf{C}'$ ，再通过3D-3D的align就能求出来R和t，这其实就是EPnP的设计思路。这里如果把 $\mathbf{C}'$ 展开成一个向量（12维），那么

$$\mathbf{P}' = [w_1 \ w_2 \ w_3 \ w_4] \otimes \mathbf{I} = \mathbf{M}\mathbf{c}$$

3. 那么在2的基础上，针对bearing信息，我们仍然使用上面两个向量叉积为0的约束，得到

$$\mathbf{v}_\times(\mathbf{P}' - \mathbf{P}_B^{B_2}) = 0$$

容易得出，

$$\mathbf{v}_\times\mathbf{M}\mathbf{c} = \mathbf{v}_\times\mathbf{P}_B^{B_2}$$

垒6个点，就可以用最小二乘求解 $\mathbf{C}'$ ，再经过3D align，就可以恢复R和t。

1.3 研究目标与路线

1. 理论贡献：把三个领域关注的问题，做了首次的对齐，即signal processing/通信领域做AOA定位的群体，aerospace/robotics做bearing only tracking的，cv领域做广义PNP的。从推导上论证了本质来说这三个都可以归结到广义PNP问题。三者的区别主要在于观测的来源。
2. 在归结到广义PNP问题的基础上，基于EPNP的思想，提出了一个线性的求解方法。在低噪声情况下，是可以求解出准确解。
   a. 具体线性解法不能很好的handle噪声的原因在于，没有对解空间进行约束，可以通过流形优化refine；
   b. 其实在AOA的cases下，抗噪效果是很好的。但是在bearing only的tracking情况下，不行，因为条件数太大了。这里可以做一个灵敏度分析。https://math.stackexchange.com/questions/1912099/sensitivity-of-the-least-squares-method-and-matrix-condition-number
3. 针对能观性问题（主要在AOA和bearing tracking问题里面会出现），目前都是从FIM的角度来分析。使用线性解法之后，条件数本身就是一个indicator，这里可以通过实验或者理论的方式推理两者的关系。使用条件数的好处是，它更容易计算。
4. 把提高观测性，也就是最小化条件数当做优化的目标，来规划无人机运动，或者是sensor的placement。
   a. 优化的方法：Sensor Selection for Minimizing the Condition Number with Guaranteed E-Efficiency；Sensor placement by maximal projection on minimum eigenspace for linear inverse problems；Sensor Placement for Isotropic Source Localization

b. 通过的无人机的guidance来增强条件数：Observability and Performance Analysis of a Model-Free Synthetic Air Data Estimator；Three-Dimensional Bearing-Only Target Following via Observability-Enhanced Helical Guidance；Optimal measurement selection algorithm and estimator for ultra-wideband symmetric ranging localization；An algorithm for real-time restructuring of a ranging-based localization network

c. 强化学习

5. 线性解法有一个好处是能很容易的扩展到多机。

类似的话题：AOA based localization

https://cisp.ece.missouri.edu/code.html

有相当多TSP和TAES的文章，但是AOA的问题，本质上就是广义PNP的问题，所有广义PNP的算法，都能改在这个问题上得到很好的结果。

比如这篇：

An Investigation and Solution of Angle Based Rigid Body Localization

TABLE I
MINIMUM NUMBER OF ANCHORS FOR SELF-LOCALIZATION AND THAT OF SENSORS FOR LOCALIZATION TO ENSURE A SOLUTION

AOA Self-Localization	number of sensors	number of anchors
2-D	1	3 (mult. sol.)
2-D	1	$\geq$ 4 (unique sol.)
3-D	1	3 (mult. sol.)
3-D	1	$\geq$ 4 (unique sol.)
2-D	$\geq$ 2	$\geq$ 2 (unique sol.)
3-D	$\geq$ 2	$\geq$ 3 (unique sol.)
AOA Localization	number of sensors	number of anchors
2-D	3 (mult. sol.)	1
2-D	$\geq$ 4 (unique sol.)	1
3-D	3 (mult. sol.)	1
3-D	$\geq$ 4 (unique sol.)	1
2-D	$\geq$ 2 (unique sol.)	$\geq$ 2
3-D	$\geq$ 3 (unique sol.)	$\geq$ 2
Range Self-Localization	number of sensors	number of anchors
2-D	$\geq$ 2	$\geq$ 2 (unique sol.)
3-D	$\geq$ 3	$\geq$ 3 (unique sol.)
Range Localization	number of sensors	number of anchors
2-D	$\geq$ 2 (unique sol.)	$\geq$ 2
3-D	$\geq$ 3 (unique sol.)	$\geq$ 3

这个结论和PNP，GPNP没有任何差别。

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2D case，很简单。https://github.com/RyotaBannai/total-least-squares/blob/master/tsl.py

关于planning来增强能观性这一块，目前好像都是heuristics的。考虑能否从M矩阵的特征向量的角度，来设计强化学习的方式去设计运动来增强能观性。