Received: 10 April 2017 | Revised: 25 December 2017
DOI: 10.1002/jae.2641

RESEARCH ARTICLE

WILEY Journal of APPLIED ECONOMETRICS

Should we use linearized models to calculate fiscal multipliers?

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Summary

We calculate the magnitude of the government consumption multiplier in linearized and nonlinear solutions of a New Keynesian model at the zero lower bound. Importantly, the model is amended with real rigidities to simultaneously account for the *macroeconomic evidence* of a low Phillips curve slope and the *microeconomic evidence* of frequent price changes. We show that the nonlinear solution is associated with a much smaller multiplier than the linearized solution in long-lived liquidity traps, and pin down the key features in the model which account for the difference. Our results caution against the common practice of using linearized models to calculate fiscal multipliers in long-lived liquidity traps.

1 | INTRODUCTION

The magnitude of the fiscal spending multiplier is a classic subject in macroeconomics. To calculate the magnitude of the multiplier, economists typically employ a linearized version of their actual nonlinear model. Does linearizing the nonlinear model matter for the conclusions about the multiplier? We document that this may be the case, especially in long-lived liquidity traps. When interest rates are expected to be constrained by the zero (or effective) lower bound for a protracted time period, the nonlinear solution suggests a much smaller multiplier than the linearized solution of the same model.

The financial crisis and "Great Recession" have revived interest in the magnitude of the fiscal spending multiplier. A quickly growing literature suggests that the fiscal spending multiplier can be very large when nominal interest rates are expected to be constrained by the zero (or effective) lower bound (ZLB henceforth) for a prolonged period (see, e.g., Christiano, Eichenbaum, & Rebelo, 2011; Coenen et al., 2012; Davig & Leeper, 2011; Eggertsson, 2010; Leeper, Traum, & Walker, 2015; Woodford, 2011). Erceg and Lindé (2014) show that in a long-lived liquidity trap fiscal stimulus can be self-financing. Conversely, the results of the above literature suggest that it is hard to reduce government debt in the short-run through aggressive government spending cuts in long-lived liquidity traps: fiscal consolidation can in fact be self-defeating in such a situation.

Importantly, the bulk of the existing literature analyzes fiscal multipliers in models where all equilibrium equations have been linearized around the steady state, except for the ZLB constraint on the monetary policy rule. Implicit in the linearization procedure is the assumption that the linearized solution is accurate even far away from the steady state. However, recent work by Boneva, Braun, and Waki (2016) suggests that linearization produces severely misleading results at the zero lower bound. Essentially, Boneva et al. argue that extrapolating decision rules far away from the steady state is invalid.

Our paper provides a positive analysis of the effect of spending-based fiscal stimulus on output and government debt using a fully nonlinear model. We compare the fiscal spending multipliers for output and government debt of the nonlinear and linearized solution as a function of the liquidity trap duration. Moreover, our framework allows us to pin down the key features that account for the difference between the multiplier schedule for the nonlinear and linearized solutions of the model.

The New Keynesian model employed in our analysis features monopolistic competition and Calvo sticky prices. The central bank follows a Taylor rule subject to the ZLB constraint on the nominal interest rate. The key difference to existing work is that we follow Lindé and Trabandt (2018) to introduce real rigidities into the model using the Kimball (1995) aggregator. The Kimball aggregator aggregates intermediate goods into a final good, and is commonly used in New Keynesian models (see, e.g., Smets & Wouters, 2007) as it allows us to simultaneously account for the *macroeconomic evidence* of a low Phillips curve slope and the *microeconomic evidence* of frequent price changes.

The key finding of our paper is that in a long-lived liquidity trap the fully nonlinear model implies a much smaller fiscal spending multiplier than the linearized version of the same model. More precisely, when the ZLB binds for 12 quarters, the nonlinear model implies a multiplier of about 0.7, whereas the linearized version of the same model implies a multiplier in excess of 2.[1] Importantly, our analysis suggests that the nonlinear model is incapable of producing a multiplier that is close to or exceeds unity when government spending follows a standard AR(1) process.

What accounts for the large difference between the nonlinear and linearized solutions in a prolonged liquidity trap? We document that the difference can almost entirely be accounted for by the nonlinearities in the price setting block of the model—the Phillips curve. Key here is the nonlinearity implied by the Kimball aggregator. The Kimball aggregator implies that the demand elasticity for intermediate goods is state dependent; that is, the firm's demand elasticity is an increasing function of its relative price. In short, the demand curve is quasi-kinked. While the fully nonlinear model takes this state-dependency explicitly into account, a linear approximation replaces that nonlinearity with a linear function. Put differently, linearization replaces the quasi-kinked demand curve with a linear function.[2] Intuitively, in a deep recession that triggers the ZLB to bind for a long time, the Kimball aggregator implies that firms do not find it attractive to cut their prices much since that reduces the demand elasticity and thereby does not crowd in substantially more demand. With more fiscal spending in such a situation, firms also find it less attractive to increase their prices. Thus—with policy rates stuck at zero—aggregate inflation increases only little and therefore the real interest rate falls by little: The multiplier does not increase much with the duration of the ZLB. When the model is linearized, the response of aggregate inflation is notably stronger due to the nature of a linear approximation of a quasi-kinked demand curve at the steady state with no price dispersion. Hence the drop in the real interest rate is larger following a spending hike and the multiplier is magnified. The bottom line: The linearized version of the model exaggerates the rise in expected and actual inflation due to a sizable approximation error and thereby exagerates the magnitude of the fiscal multiplier in long-lived liquidity traps.

We perform several robustness checks. In particular, we compare our benchmark results based on the Kimball (1995) aggregator to those when a Dixit and Stiglitz (1977) aggregator is used instead. We also examine whether it is important which type of shock takes the model economy to the zero lower bound. In addition, we study the effects of the price indexation for the resulting multiplier. Moreover, we investigate the sensitivity of our results with respect to the government spending process. Finally, and perhaps most importantly, we compare the sensitivity of our results with respect to the solution method of the nonlinear model. Our benchmark solution method is based on Fair and Taylor (1983). That solution method solves the model by imposing certainty equivalence. As a robustness check, we also solve the model using global methods—that is, solving the model without certainty equivalence. In other words, we compare the deterministic solution of the linear and nonlinear model with the fully stochastic linear and nonlinear model solution in which agent's decision rules are affected by the variance of shocks hitting the economy.[3] Importantly, we document that the fiscal multiplier in the nonlinear model is little affected by shock uncertainty. The nonlinearity of the Kimball aggregator and the low slope of the Phillips curve based on macro- and micoeconomic evidence are responsible for that result. By contrast, the linear model is affected in a dramatic way by shock uncertainty: The fiscal multiplier is even more elevated due to the approximation error. Hence our basic finding of a significant difference between the linearized and nonlinear solutions in long-lived liquidity traps is even further strengthened when allowing future shock uncertainty to affect the model solution.

We argue that the results based on the Kimball specification appear to dominate those based on the Dixit–Stiglitz specification for at least two reasons. First, in contrast to Dixit–Stiglitz, the Kimball specification does not produce a "missing deflation" puzzle at the onset of the Great Recession, see Lindé and Trabandt (2018). In other words, inflation does not fall much in response to an adverse Great Recession type shock. Second, the small rise in inflation expectations in response to

[1] Note that both the linearized and nonlinar model imply a multiplier of 1/3 in normal times when monetary policy is unconstrained.
[2] It is well known that in a linearized model the Kimball (1995) aggregator and Dixit and Stiglitz (1977) aggregator—the latter featuring a constant demand elasticity—are observationally equivalent up to a factor of proportionality.
[3] In the stochastic economy, the probability of being at the ZLB is 10%.

fiscal stimulus with the Kimball specification is consistent with evidence provided by Dupor and Li (2015). These authors argue that expected inflation reacted little to spending shocks in the USA during the Great Recession. By contrast, inflation expectations react much more under the Dixit–Stiglitz specification.

Our results have potentially important implications for the scope of fiscal stimulus to be self-financing, and the extent to which fiscal consolidations can be self-defeating. In the nonlinear model, fiscal stimulus is never a "free lunch" and, conversely, fiscal consolidations are never self-defeating. The linearized model arrives at the opposite conclusions: Fiscal stimulus can be self-financing in a sufficiently long-lived liquidity trap and fiscal consolidations can be self-defeating. These findings cast doubt on the existing literature on the fiscal implications of fiscal stimulus. It should be noted, however, that we study a model environment in which the fiscal output multiplier is relatively small in normal times. Had we considered a medium-sized model with Keynesian accelerator effects in which the multiplier is in the mid-range of the empirical evidence when monetary policy is unconstrained, the multiplier might be magnified sufficiently in a long-lived liquidity trap to obtain a "fiscal free lunch" for a transient spending hike.[4] We elaborate more on this in the conclusions.

Our paper is related to Boneva et al. (2016), Christiano and Eichenbaum (2012), Christiano, Eichenbaum, and Johannsen (2016), Fernandez-Villaverde, Grey, Guerrón-Quintana, and Rubio-Ramírez (2015), Eggertsson and Singh (2016), and Nakata (2016). Importantly, none of the above papers considers the case of a Kimball (1995) aggregator. Boneva et al. report that the multiplier is smaller in a fully nonlinear model compared to the linearized model version. Their model features a Dixit–Stiglitz aggregator. Eggertsson and Singh report that the multipliers of the nonlinear and linearized model differ only very little. Their model features a Dixit–Stiglitz aggregator and assumes firms-specific labor markets, implying that price dispersion is irrelevant for the nonlinear model dynamics. By contrast, our analysis shows how important these assumptions are: Moving to the frequently used Kimball aggregator and allowing for price dispersion alters the conclusions about the multiplier substantially. Nakata and Fernández-Villaverde et al. show that future shock uncertainty may have potentially important implications for the equilibrium dynamics of the model. As mentioned above, our robustness analysis shows that allowing for future shock uncertainty has a quantitatively small impact on our results for the nonlinear model. Christiano and Eichenbaum and Christiano et al. analyze multiplicity of equilibria in a nonlinear New Keynesian model. They document that there is a unique stable-under-learning rational expectations equilibrium in their model and that all other equilibria are not stable under learning.

The remainder of the paper is organized as follows. Section 2 presents the New Keynesian model and Section 3 the results. Section 4 provides an in-depth robustness analysis. Section 5 discusses potential implications of our work for future empirical work. Finally, Section 6 concludes.

2 | NEW KEYNESIAN MODEL

The model that we study is very similar to the one developed Erceg and Lindé (2014). We deviate from Erceg and Lindé in so far as we allow for a Kimball (1995) aggregator which aggregates intermediate goods into a final good. The specification of the Kimball aggregator nests the standard Dixit and Stiglitz (1977) specification as a special case. Below, we outline the model environment. All linearized and nonlinear equilibrium equations are available in the Appendix.[5]

2.1 | Households

The utility functional for the representative household is

$$\max_{\{C_t,N_t,B_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left(\ln\left(C_t - C\nu_t\right) - \frac{N_t^{1+\chi}}{1+\chi} \right), \tag{1}$$

[4] A large empirical literature has examined the effects of government spending shocks, mainly focusing on the post-WWII prefinancial crisis period when monetary policy had latitude to adjust interest rates. The bulk of this research suggests a government spending multiplier in the range of 0.5 to somewhat above unity (see, e.g., Blanchard, Erceg, & Lindé, 2016; Hall, 2009; Ramey, 2011; and references therein).

[5] A Technical Appendix with all derivations is available here: https://sites.google.com/site/mathiastrabandt/home/downloads/LindeTrabandt_Multiplier_TechApp.pdf. (See also Supporting Information).

where the discount factor β satisfies $0 < \beta < 1$. As in Erceg and Lindé (2014), the utility function depends on the household's current consumption C_t in deviation from a "reference level" $C\nu_t$, where C denotes steady-state consumption and ν_t is an exogenous consumption preference shock.[6] The utility function also depends negatively on hours worked N_t.

The household's budget constraint in period t states that its expenditure on goods and net purchases of (zero-coupon) government bonds B_t must equal its disposable income:

$$P_t C_t + B_t = (1 - \tau_N) W_t N_t + (1 + i_{t-1}) B_{t-1} - T_t + \Gamma_t. \tag{2}$$

Thus the household purchases the final consumption good at price P_t. The household is subject to a constant distortionary labor income tax τ_N and earns after-tax labor income $(1 - \tau_N) W_t N_t$. The household pays lump-sum taxes net of transfers T_t and receives a proportional share of the profits Γ_t of all intermediate firms.

Utility maximization yields the standard consumption Euler equation:

$$1 = \beta E_t \left(\frac{1 + i_t}{1 + \pi_{t+1}} \frac{C_t - C\nu_t}{C_{t+1} - C\nu_{t+1}} \right), \tag{3}$$

where $1 + \pi_{t+1} = P_{t+1}/P_t$.

We also have the following labor supply schedule:

$$N_t^{\chi} = \frac{1 - \tau_N}{C_t - C\nu_t} \frac{W_t}{P_t}. \tag{4}$$

Equations 3 and 4 are the key equations for the household side of the model.

2.2 | Firms and price setting

2.2.1 | Final goods production

The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G\left(\frac{Y_t(f)}{Y_t} \right) \mathrm{d}f = 1. \tag{5}$$

As in Dotsey and King (2005) and Levin, Lopez-Salido, and Yun (2007), we assume that $G(\cdot)$ is given by the following strictly concave and increasing function:

$$G\left(\frac{Y_t(f)}{Y_t} \right) = \frac{\omega}{1 + \psi} \left((1 + \psi) \frac{Y_t(f)}{Y_t} - \psi \right)^{\frac{1}{\omega}} - \left(\frac{\omega}{1 + \psi} - 1 \right), \tag{6}$$

where $\omega = \frac{\phi(1+\psi)}{1+\phi\psi}$. Here $\phi > 1$ denotes the gross markup of the intermediate goods firms. The parameter $\psi \leq 0$ governs the degree of curvature of the intermediate firm's demand curve.[7] In Figure 1 we show how relative demand is affected by the relative price under alternative assumptions about ψ, given a value for the gross markup of $\phi = 1.1$. When $\psi = 0$, the demand curve exhibits constant elasticity as under the standard Dixit–Stiglitz aggregator, implying a log-linear relationship between relative demand and relative prices. When $\psi < 0$—as in Smets and Wouters, for example—a firm instead faces a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a larger fall in demand compared to the $\psi = 0$ case. Relative to the standard Dixit–Stiglitz aggregator, this introduces more strategic complementarity in price setting, which causes intermediate firms to adjust prices by less to a given change in marginal cost. Finally, we note that $G(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in product and factor markets. Thus final goods producers minimize the cost of producing a given quantity of the output index Y_t, taking as given the price $P_t(f)$ of each

[6] In Section 4 below we also examine the implications of a discount factor shock instead of the consumption preference shock. The impulse responses of both shocks are virtually identical. We use the consumption preference shock in our baseline specification to remain as close as possible to Erceg and Lindé (2014).

[7] The parameter used in Smets and Wouters (2007) to characterize the curvature of the Kimball aggregator can be mapped to our model using the following formula: $\epsilon = -\frac{\phi}{\phi-1}\psi$.

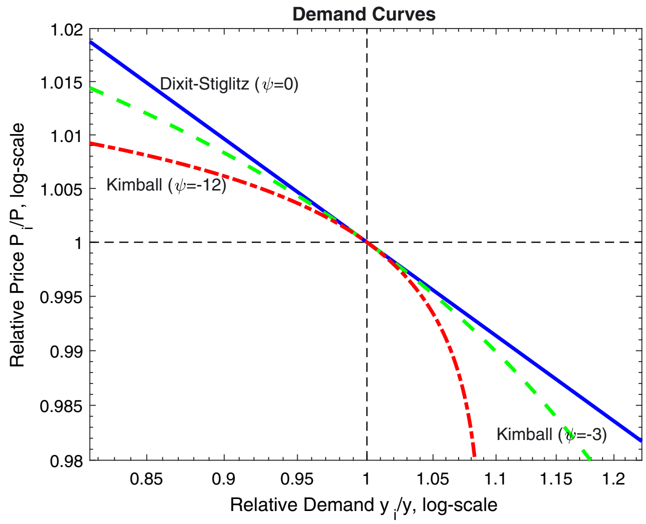

FIGURE 1 Demand curves: implications of Kimball versus Dixit–Stiglitz aggregators [Colour figure can be viewed at wileyonlinelibrary.com]

intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price P_t, and hence solve the following profit maximization problem:

$$\max_{Y_t, Y_t(f)} P_t Y_t - \int_0^1 P_t(f) Y_t(f) \, \mathrm{d}f, \tag{7}$$

subject to the constraint (Equation 5). The first-order conditions can be written as

$$\begin{aligned} \frac{Y_t(f)}{Y_t} &= \frac{1}{1+\psi}\left(\left(\frac{P_t(f)}{P_t}\frac{1}{\Lambda_t}\right)^{\frac{\phi}{1-\phi}(1+\psi)} + \psi\right), \\ P_t \Lambda_t &= \left(\int P_t(f)^{\frac{1+\psi\phi}{1-\phi}} \mathrm{d}f\right)^{\frac{1-\phi}{1+\psi\phi}}, \\ \Lambda_t &= 1 + \psi - \psi \int \frac{P_t(f)}{P_t} \mathrm{d}f, \end{aligned} \tag{8}$$

where Λ_t denotes the Lagrange multiplier on the aggregator constraint (Equation 6). Note that for $\psi = 0$ it follows that $\Lambda_t = 1 \; \forall t$ and the first-order conditions in Equation 8 simplify to the usual Dixit and Stiglitz (1977) expressions:

$$\frac{Y_t(f)}{Y_t} = \left(\frac{P_t(f)}{P_t}\right)^{\frac{\phi}{1-\phi}}, P_t = \left(\int P_t(f)^{\frac{1}{1-\phi}} \mathrm{d}f\right)^{1-\phi}.$$

2.2.2 | Intermediate goods production

A continuum of intermediate goods $Y_t(f)$ for $f \in [0, 1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand schedule from the final goods firms through the solution to the problem in Equation 7 that varies inversely with its output price $P_t(f)$ and directly with aggregate demand Y_t.

Aggregate capital K is assumed to be fixed, so that production of the intermediate good firm is given by

$$Y_t(f) = K(f)^\alpha N_t(f)^{1-\alpha}. \tag{9}$$

Despite the fixed aggregate stock $K \equiv \int K(f) \, \mathrm{d}f$, fractions of the aggregate capital shock can be freely allocated across firms, implying that real marginal cost, $\mathrm{MC}_t(f)/P_t$, is identical across firms and equal to

$$\frac{\mathrm{MC}_t}{P_t} \equiv \frac{W_t/P_t}{\mathrm{MPL}_t} = \frac{W_t/P_t}{(1-\alpha) K^\alpha N_t^{-\alpha}}, \tag{10}$$

where W_t is the nominal wage, MPL_t is the marginal product of labour, and $N_t = \int N_t(f) \, \mathrm{d}f$.

The prices of the intermediate goods are determined by Calvo (1983) style staggered nominal contracts. In each period, each firm f faces a constant probability, $1-\xi$, of being able to reoptimize its price $P_t(f)$. The probability that a firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price set in the previous period as follows:

$$\tilde{P}_t = (1+\pi) P_{t-1}, \tag{11}$$

where π is the steady-state (net) inflation rate and $\tilde{P}_t$ is the updated price. In Section 4 we examine the implications of not allowing for price indexation.

Given Calvo-style pricing frictions, firm f that is allowed to reoptimize its price, $P_t^{\text{opt}}(f)$, solves the following problem:

$$\max_{P_t^{\text{opt}}(f)} E_t \sum_{j=0}^{\infty} (\beta\xi)^j \Theta_{t,t+j} \left((1+\pi)^j P_t^{\text{opt}}(f) - \text{MC}_{t+j} \right) Y_{t+j}(f),$$

where $\Theta_{t,t+j}$ is the stochastic discount factor (the conditional value of future profits in utility units, recalling that the household is the owner of the firms), and demand $Y_{t+j}(f)$ from the final goods firms is given by the equations in Equation 8.

2.3 | Monetary and fiscal policies

The evolution of nominal government debt is determined by the government budget constraint:

$$B_t = (1+i_{t-1}) B_{t-1} + P_t G_t - \tau_N W_t N_t - T_t, \tag{12}$$

where G_t denotes real government expenditures on the final good Y_t. Following Erceg and Lindé (2014) we assume that net lump-sum taxes as a share of nominal steady-state gross domestic product (GDP), $\tau_t \equiv \frac{T_t}{P_t Y}$, stabilize government debt as a share of nominal steady-state GDP, $b_t \equiv \frac{B_t}{P_t Y}$:

$$\tau_t - \tau = \varphi (b_{t-1} - b). \tag{13}$$

Here τ and b denote the steady states of τ_t and b_t. Finally, real government consumption, G_t, is assumed to be exogenous.

Turning to the central bank, we assume that it sets the nominal interest rate by using the following Taylor rule that is subject to the zero lower bound:

$$1+i_t = \max\left(1, (1+i)\left(\frac{1+\pi_t}{1+\pi}\right)^{\gamma_\pi}\left(\frac{Y_t}{Y_t^{\text{pot}}}\right)^{\gamma_x}\right), \tag{14}$$

where Y_t^{pot} denotes the level of output that would prevail if prices were flexible, and i the steady-state net nominal interest rate, which is given by $r+\pi$, where $r \equiv 1/\beta - 1$.

2.4 | Aggregate resources

It is straightforward to show that aggregate output Y_t is given by

$$Y_t = \left(p_t^*\right)^{-1} K^\alpha N_t^{1-\alpha}, \tag{15}$$

where

$$p_t^* \equiv \int_0^1 \frac{1}{1+\psi}\left(\left(\frac{P_t(f)}{P_t}\frac{1}{\Lambda_t}\right)^{\frac{\phi}{1-\phi}(1+\psi)} + \psi\right) \mathrm{d}f.$$

The variable $p_t^* \geq 1$ denotes the Yun (1996) aggregate price distortion term.

Aggregate output can be used for private consumption and government consumption, so that

$$C_t + G_t = \left(p_t^*\right)^{-1} K^\alpha N_t^{1-\alpha}. \tag{16}$$

The price distortion term introduces a wedge between the use of production inputs and the output that is available for private and government consumption. Note, however, that p_t^* vanishes when the model is linearized.

2.5 | Parametrization

Our benchmark calibration—essentially adopted from Erceg and Lindé (2014)—is fairly standard at a quarterly frequency. We set the discount factor $\beta = 0.995$, and the steady-state net inflation rate $\pi = 0.005$, which implies a steady-state nominal interest rate $i = 0.01$ (i.e., 4% at an annualized rate).[8] We set the capital share parameter $\alpha = 0.3$ and the Frisch elasticity of labor supply $\frac{1}{\chi} = 0.4$. We set the steady-state value for the consumption preference shock $\nu = 0.01$.[9] Three parameters determine the direct sensitivity of prices to marginal costs: the gross markup ϕ, the stickiness parameter ξ, and the Kimball parameter ψ. We have direct evidence on two of these: ϕ and ξ. A large body of *microeconomic* evidence (see, e.g., Klenow & Malin, 2010; Nakamura & Steinsson, 2008; and references therein) suggest that firms change their prices rather frequently—on average, somewhat more often than once a year. Based on this micro-evidence, we set $\xi = 0.667$, implying an average price contract duration of 3 quarters. We set the gross markup $\phi = 1.1$ as a compromise between the low estimate of ϕ in Altig, Christiano et al. (2011) and the higher estimated value by Smets and Wouters (2007). To pin down the Kimball parameter ψ, consider the log-linearized New Keynesian Phillips Curve in our model:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \widehat{\mathrm{mc}}_t, \tag{17}$$

where $\widehat{\mathrm{mc}}_t$ denotes the log-deviation of marginal cost from its steady state. $\hat{\pi}_t$ denotes the log-deviation of gross inflation from its steady state. The parameter κ denotes the slope of the Phillips curve and is given by

$$\kappa \equiv \frac{(1-\xi)(1-\beta\xi)}{\xi} \frac{1}{1-\phi\psi}. \tag{18}$$

The *macroeconomic* evidence suggests that the sensitivity of aggregate inflation to variations in marginal cost is very low (see, e.g., Altig et al., 2011). To capture this, we set the Kimball parameter $\psi = -12.2$ so that the slope of the Phillips curve is $\kappa = 0.012$ given the values for β, ξ, and ϕ discussed above.[10] This calibration allows us to match micro- and macro-evidence about firms' price-setting behavior and is aimed to capture the resilience of core inflation, and measures of expected inflation, to a deep downturn such as the Great Recession.

Consistent with the pre-crisis US federal debt level, we assume a government debt-to-annualized-output ratio of 0.6. We assume that government consumption accounts for 20% of GDP. Further, we set net lump-sum taxes $\tau = 0$ in steady state. The above assumptions imply a steady-state labor income tax $\tau_N = 0.33$. The parameter φ in the tax rule (Equation 13) is set equal to 0.0101, which implies that the contribution of lump-sum taxes to the response of government debt is quantitatively negligible in the first couple of years following a shock. For monetary policy, we use the standard Taylor (1993) rule parameters $\gamma_\pi = 1.5$ and $\gamma_x = 0.125$.

In order to facilitate comparison between the nonlinear and linearized model, we specify processes for the exogenous shocks such that there is no loss in precision due to an approximation. In particular, the consumption preference and government spending shocks are assumed to follow AR(1) processes:

$$\begin{aligned} G_t - G &= \rho_G (G_{t-1} - G) + \varepsilon_{G,t} \\ \nu_t - \nu &= \rho_\nu (\nu_{t-1} - \nu) + \varepsilon_{\nu,t} \end{aligned} \tag{19}$$

where $\varepsilon_{G,t}$ and $\varepsilon_{\nu,t}$ are normally distributed i.i.d. shocks. In our baseline parametrization we assume $\rho_\nu = \rho_G = 0.95$. We also investigate the sensitivity of our results when we assume moving average processes instead of autoregressive processes. Those results are reported in the Appendix.

2.6 | Model solution

Our benchmark results are based on the solution of the linearized and nonlinear model using the solution method developed in Fair and Taylor (1983). For robustness, we also compute the solution of the linearized and nonlinear model using the global solution method developed by Judd (1998) and (Coleman, 1990, 1991), which allows shock uncertainty to affect the decision rules of households and firms.[11]

[8]We rule out steady-state multiplicity by restricting our attention to the steady state with a positive inflation rate.

[9]By setting the steady value of the consumption taste shock to a small value, we ensure that the dynamics for the other shocks are roughly invariant to the presence of $-C\nu_t$ in the period consumption utility function.

[10]The median estimates of the Phillips curve slope in recent empirical studies (e.g., Adolfson, Laséen, Lindé, & Villani, 2005; Altig et al., 2011; Galí & Gertler, 1999; Galí, Gertler, & López-Salido, 2001; Lindé, 2005; Smets & Wouters, 2003, 2007) are in the range of 0.009–0.014.

[11]The replication codes are available in the Journal of Applied Econometrics code archive.

2.6.1 | Benchmark solution method: Fair and Taylor (1983)

The Fair and Taylor (1983) method solves the linearized and nonlinear equilibrium equations—including kinks such as the ZLB—by solving a two-point boundary value problem. The Fair and Taylor method is often referred to as "extended path," "deterministic simulation," or "perfect foresight solution." To solve a model, the method assumes that after a shock the model economy converges back to its steady state in a finite number of periods. In addition, the solution method assumes certainty equivalence. That is, the variance of shocks does not affect the decision rules of households and firms. By imposing certainty equivalence on both the linearized and nonlinear model, the Fair and Taylor solution method allows us to trace out implications of using nonlinear equilibrium equations instead of linearized equilibrium equations for the resulting multiplier.

We check the robustness of our results by also using a global solution method that allows shock uncertainty to explicitly affect the model solution. However, our benchmark results are based on Fair and Taylor (1983) for the following three reasons.

First, because much of the existing literature has often used a perfect foresight approach that imposes certainty equivalence to solve a model, retaining this approach allows us to parse out the effects of going from a linearized to a nonlinear model framework. Second, the real rigidity that we introduce to fit the micro- and macroeconomic evidence implies that expected inflation adjusts slowly, which in turn means that the impact of future shock uncertainty is modest. As shown below, allowing for shock uncertainty does not affect the solution of the nonlinear model noticeably. By contrast, allowing for shock uncertainty in the linearized model greatly affects the model solution, implying even bigger differences between the linearized and nonlinear model for the resulting multiplier. Third, the Fair and Taylor (1983) method allows us to solve the nonlinear model in fractions of a second, whereas the nonlinear model solution with shock uncertainty takes several hours to calculate. Moreover, the Fair and Taylor method also allows us to calculate the solution of larger-scale models with many state variables very quickly. So far, the solution algorithms used to solve models with shock uncertainty have typically not been applied to models with more than four or five state variables.[12]

We use Dynare to solve the nonlinear and linearized model equations that are provided in the Appendix. Dynare is a preprocessor and a collection of MATLAB routines that can solve linear and nonlinear models with occasionally binding constraints. The details about the implementation of the algorithm used can be found in Juillard (1996). The perfect foresight simulation algorithm implemented in Dynare is Fair and Taylor (1983). To solve a model using it, one just has to use the "simul" command. The algorithm can easily handle the ZLB constraint: one just writes the Taylor rule including the max operator in the model equations, and the solution algorithm reliably calculates the model solution in fractions of a second. Thus, except for perhaps obtaining intuition about the economics embedded into models, there is no longer any need to linearize models to solve and simulate them.

For the linearized model, we used the algorithm outlined in Hebden, Lindé, and Svensson (2011) to check for uniqueness of the local equilibrium associated with a positive steady-state inflation rate and to impose the ZLB.[13] We did not find evidence of multiplicity of the local equilibrium in the linearized model as well as in the nonlinear model.

As noted earlier, we rule out the well-known problems associated with steady-state multiplicity emphasized by Benhabib, Schmitt-Grohe, and Uribe (2001) by restricting our attention to the steady state with a positive inflation rate. Our choice of focusing on the positive inflation steady state is in part motivated by recent work by Christiano et al. (2016), who find that alternative solutions in the New Keynesian model may not be economically relevant; that is, these solutions are not stable under learning.

2.6.2 | Alternative solution method: Global solution

For robustness, we also solve the linearized and nonlinear model using the global solution method developed by Coleman (1990, 1991). This solution method is based on a time iteration method on the decision rules of households and firms. With this method, the variance of shocks affects the decision rules of households and firms. The time iteration method has been used recently, for example, by Nakata (2016) and Richter and Throckmorton (2016).

A growing body of work such as Adam and Billi (2006, 2007) within a linearized model framework and Fernández-Villaverde et al. (2015), Gust, Herbst, López-Salido, and Smith (2017), Nakata (2016), and Richter and Throckmorton (2016) within a nonlinear model framework have studied the effects of allowing for shock uncertainty for the decision rules of households and firms in New Keynesian models. These authors have shown that allowing for shock

[12] A recent paper by Judd, Maliar, and Maliar (2011) provides a promising avenue to compute the stochastic solution of larger-scale models efficiently.
[13] When the local equilibrium is unique, this algorithm is equivalent to the OccBin algorithm developed by Iacoviello and Guerrieri (2015) for use with Dynare.

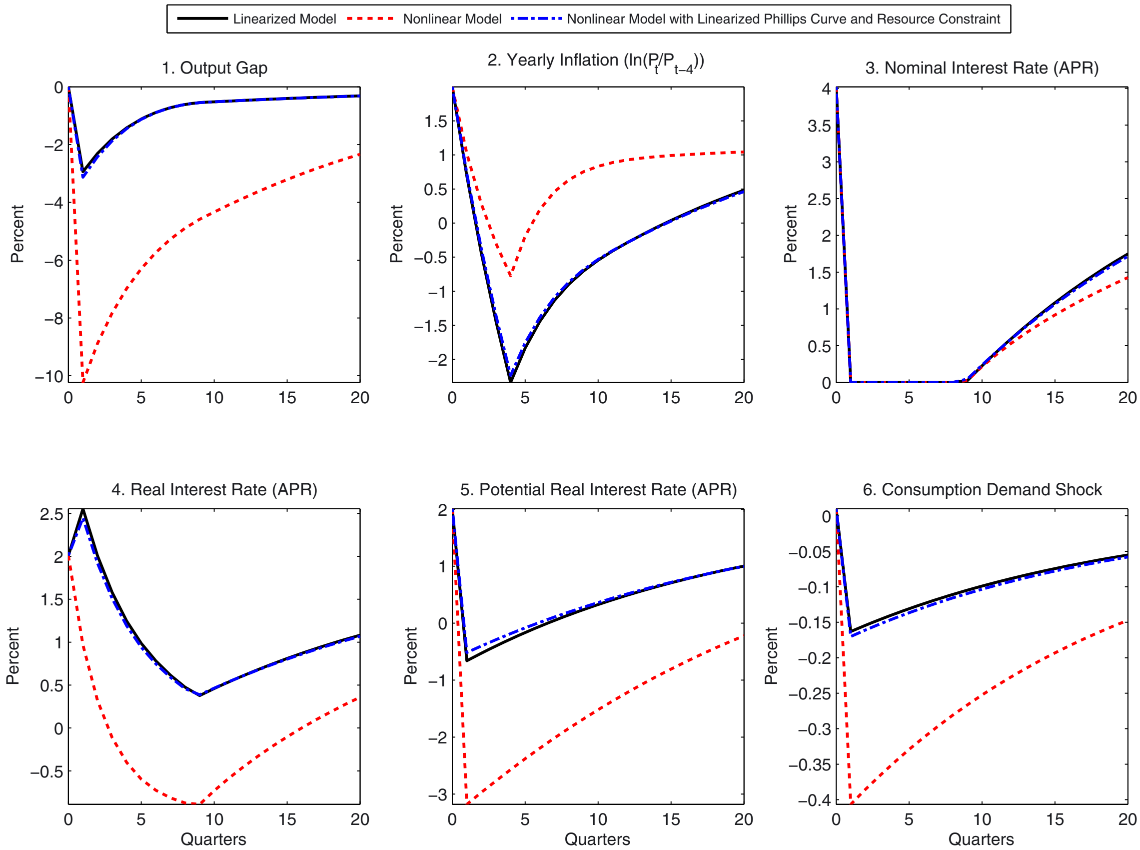

FIGURE 2 Baselines in linearized and nonlinear model for an 8-quarter liquidity trap [Colour figure can be viewed at wileyonlinelibrary.com]

uncertainty can potentially have important implications for equilibrium dynamics, especially when inflation expectations are not well anchored due to nonoptimal monetary policy or when aggregate prices adjust slowly. As we will show below, however, the solution of our nonlinear model is almost unaffected by the presence of substantial shock uncertainty due to the Kimball (1995) aggregator and an empirically realistic low slope of the Phillips curve.

3 | RESULTS

In this section, we report our benchmark results. Our aim is to compare spending multipliers in linearized and nonlinear versions of the model economy. Specifically, we seek to characterize how the difference between the multiplier in the linearized and nonlinear solutions varies with the expected duration of the liquidity trap. We start by reporting how we construct a baseline in which the model economy is driven to the zero lower bound and then report the fiscal multipliers.

3.1 | Construction of baseline

To construct a baseline where the nominal interest rate is bounded at zero for a number of periods—say $\text{ZLB}_{\text{DUR}} = 1, 2, 3, \ldots, M$—we follow Lindé and Trabandt (2018) and the fiscal multiplier literature (e.g., Christiano et al., 2011) and assume that the economy is hit by a large adverse shock that triggers a deep recession and drives the nominal interest rate to zero. The longer the expected liquidity trap duration—that is, the larger value of ZLB_{DUR}—we want to consider, the larger the adverse shock has to be. The particular shock we consider is a negative realization of the consumption preference shock ν_t discussed above.[14]

As an example, Figure 2 shows the baseline generated by the adverse consumption preference shock in the linearized and nonlinear model when the ZLB is binding for eight quarters—that is, $\text{ZLB}_{\text{DUR}} = 8$. The solid black lines depict the

[14]In Section 4 below we show that the type of shock that we use to generate the baseline is immaterial for our results. For example, we show that if the baseline is generated by a discount factor shock instead of a consumption demand shock, the resulting fiscal multipliers are nearly identical.

baseline in the linearized model, and the dotted red lines depict the baseline in the nonlinear model. The economy is in steady state in period 0, and then the shock hits the economy in period 1. As shown in Figure 2, we need to subject the nonlinear model to a more negative consumption preference shock—as shown by the dotted red line in panel 6—to generate $\text{ZLB}_{\text{DUR}} = 8$ for the nominal interest rate shown in panel 3.[15]

Figure 2 provides an important insight into the differences between the linearized and nonlinear solutions. To generate an eight-quarter liquidity trap in the nonlinear model, the potential real rate (panel 5) has to drop much more in the nonlinear model than in the linearized model. Accordingly, the output gap (panel 1) is much more negative in the nonlinar model. Even so, and perhaps most important, we see that the drop in inflation (panel 2) is substantially smaller in the nonlinear model. This suggests that the difference between the linearized and nonlinear model is driven to a large extent by the nonlinearities embedded in the price setting equations.

Interestingly, as in Lindé and Trabandt (2018), the linearized model predicts a protracted period of deflation in response to the Great Recession type shock. In the data, however, a long period of deep deflation after the onset of the Great Recession was not observed. This observation is commonly referred to as the "missing deflation" puzzle; that is, actual inflation did not fall nearly as much as predicted by the linearized New Keynesian model. By contrast, as emphasized in Lindé and Trabandt (2018), the nonlinear model based on the Kimball specification does not appear to produce the "missing deflation" puzzle. Inflation in the nonlinear model falls by much less and turns negative for a very brief period only before recovering relative to the linearized model. Based on these results we argue that the "missing deflation" puzzle is not a puzzle: it arises due to an approximation error when one extrapolates the predictions of a linearized model to very large shocks. The underlying true nonlinear model predicts that macroeconomists should not have expected a long period of deep deflation to occur in the aftermath of the Great Recession.

Given the above discussion, we seek to compare fiscal multipliers in liquidity traps of same expected duration in both the linearized and nonlinear model. Accordingly, we allow for differently sized shocks so that each model variant generates a liquidity trap with the same expected duration $\text{ZLB}_{\text{DUR}} = 1, 2, 3, \dots, M$.

Let

$$\left\{ B^{\text{linear}} \left(\varepsilon_{v,i}^{\text{linear}} \right) \right\}_{i=1}^{M} \text{ and } \left\{ B^{\text{nonlinear}} \left(\varepsilon_{v,i}^{\text{nonlinear}} \right) \right\}_{i=1}^{M}$$

denote matrices of time series with simulated variables of the linearized and nonlinear models, where i indexes the set of time series for a given ZLB duration. The notation reflects that the innovations, $\varepsilon_{v,i}$, to the consumption preference shock v_t, in Equation 19 are set so that

$$\varepsilon_{v,i}^{\text{linearized}} \Rightarrow \text{ZLB}_{\text{DUR}} = i,$$

and

$$\varepsilon_{v,i}^{\text{nonlinear}} \Rightarrow \text{ZLB}_{\text{DUR}} = i,$$

where we consider $i = 1, 2, \dots, M$. In the specific case of $i = 8$, panel 6 in Figure 2 shows that $\varepsilon_{v,8}^{\text{linear}} = -0.17$ and $\varepsilon_{v,8}^{\text{nonlinear}} = -0.41$.

3.2 | Marginal fiscal multipliers

Conditional on the set of baseline scenarios that we have constructed above, we add an increase in government spending g_t in the period when the ZLB starts to bind.

Let

$$\left\{ S^{\text{linear}} \left(\varepsilon_{v,i}^{\text{linear}}, \varepsilon_G \right) \right\}_{i=1}^{M} \text{ and } \left\{ S^{\text{nonlinear}} \left(\varepsilon_{v,i}^{\text{nonlinear}}, \varepsilon_G \right) \right\}_{i=1}^{M}$$

denote matrices of time series with simulated variables of the linearized and nonlinear models, where i indexes the set of time series for a given ZLB duration and ε_G denotes the positive government spending shock that hits the economy when the ZLB starts to bind.

We then compute the marginal impact of the fiscal spending shock as

$$I^{\text{linear}} \left(\text{ZLB}_{\text{DUR}} \right) = S^{linear} \left(\varepsilon_{v,i}^{\text{linear}}, \varepsilon_G \right) - B^{\text{linear}} \left(\varepsilon_{v,i}^{\text{linear}} \right)$$

and

$$I^{\text{nonlinear}} \left(\text{ZLB}_{\text{DUR}} \right) = S^{\text{nonlinear}} \left(\varepsilon_{v,i}^{\text{nonlinear}}, \varepsilon_G \right) - B^{\text{nonlinear}} \left(\varepsilon_{v,i}^{\text{nonlinear}} \right),$$

[15]Figure 2 also depicts a third line "Nonlinear model with linearized NKPC and Resource Constraint," which we will discuss further in Section 3.2.

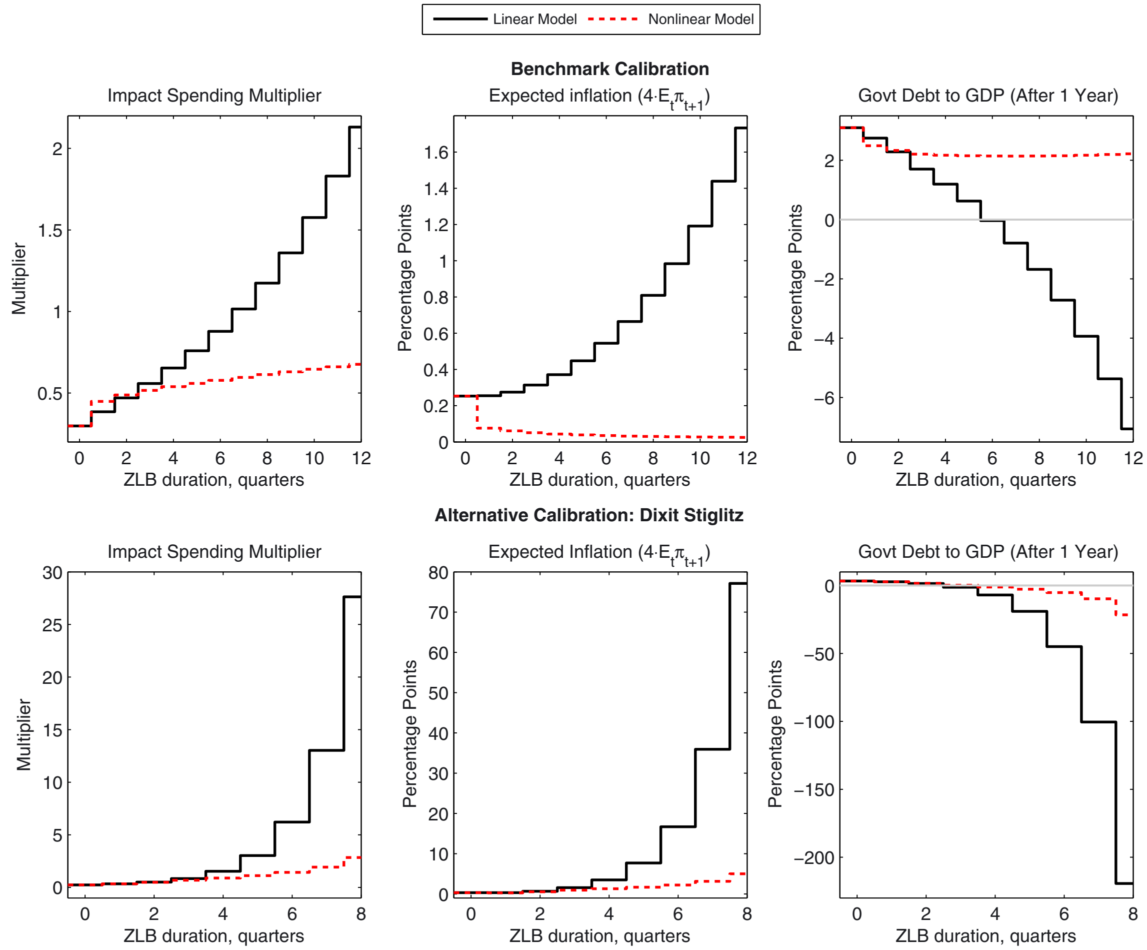

FIGURE 3 Marginal multipliers for government spending [Colour figure can be viewed at wileyonlinelibrary.com]

where we write $I^{\text{linear}}(\text{ZLB}_{\text{DUR}})$ and $I_i^{\text{nonlinear}}(\text{ZLB}_{\text{DUR}})$ to highlight their dependence on the liquidity trap duration. Note that the fiscal spending shock is the same for all i and is scaled such that ZLB_{DUR} is the same as in the baseline. By setting the fiscal impulse so that the liquidity trap duration remains unaffected we calculate "marginal" spending multipliers in the sense that they show the impact of a small change in the fiscal instrument.[16]

Figure 3 contains the main results of the paper. The upper panels report results for the benchmark calibration with the Kimball aggregator. The lower panels report results under the Dixit–Stiglitz aggregator, in which case $\psi = 0$. This parametrization implies a substantially higher slope of the linearized Phillips curve (see Equation 18) and thus a much stronger sensitivity of expected inflation to current and expected future marginal costs (and output gaps). We will first discuss the results under the Kimball parametrization, and then turn to the Dixit–Stiglitz results.

The left-hand panels of Figure 3 report the impact GDP multiplier of government spending; that is,

$$m_i = \frac{\Delta Y_{t,i}}{\Delta G_{t,i}},$$

where the Δ-operator represents the difference between the scenario with the government spending change and the baseline without the spending change. We compute m_i for $\text{ZLB}_{\text{DUR}} = 1, \ldots, 12$. We also compute results for the case when the economy is at the steady state, so that $\text{ZLB}_{\text{DUR}} = 0$.

The top left panel in Figure 3 reports that if the economy is close to or at the steady state (e.g., the ZLB is not binding, $\text{ZLB}_{\text{DUR}} = 0$), the linearized and nonlinear multipliers coincide. In other words, the linear approximation is accurate if the economy is close to or at the steady state. By contrast, if the economy if far away from its steady state—that is, the economy is in a deep recession and experiences a long-lived liquidity trap—the differences between the linearized and nonlinear multipliers become large. For example, in a 3-year liquidity trap, the multiplier in the nonlinear model is

[16]See Erceg and Lindé (2014) for a discussion of the differences between the marginal and average fiscal multiplier.

about 0.65, whereas the multiplier is about 2.1 in the linearized model. So, in a 3-year liquidity trap, the multiplier of the linearized solution is more than three times larger (2.1/0.65). The difference in terms of the response of government debt after the fiscal stimulus, depicted in the upper right panel, largely follows the pattern for m_i: The difference between the linearized and nonlinear model increases with the duration of the ZLB.[17]

The substantial differences in the GDP multiplier and government debt responses between the linearized and nonlinear solutions raises the question of which factors account for them. The middle top panel, which shows the response of the one-period-ahead expected annualized inflation rate (i.e., $4{\times}\mathrm{E}_t\pi_{t+1}$), sheds light on the driving forces behind the differences in the GDP multiplier and government debt. The increasing sensitivity of expected inflation as a function of the liquidity trap duration in the linearized solution is consistent with the existing literature on fiscal multipliers at the ZLB (e.g., Erceg & Lindé, 2014). In the nonlinear solution, expected inflation rises much less in a long-lived trap. This happens as the adverse baseline shock generating the liquidity trap implies that many firms perceive that their demand elasticity is much higher than in normal times (close to the steady state). As a consequence, they are reluctant to change prices much (if at all) in response to changes in marginal costs when they get a chance to re-optimize their prices. In terms of Figure 1, the relevant demand and pricing segment is the upper left quadrant where it is optimal for firms to change prices little.[18] The sharp increase in expected inflation in the linearized model triggers a larger reduction in the actual real interest rate (not shown), and thereby induces a more favorable response of private consumption which helps to boost output relative to the nonlinear model.[19]

Let us turn to the Dixit–Stiglitz case—that is, setting $\psi = 0$ and keeping all other parameters unchanged. The bottom panels of Figure 3 show that the differences between the linearized and nonlinear model are even more pronounced in this case.[20] The larger differences in the Dixit–Stiglitz case are driven by a substantially higher slope of the New Keynesian Phillips curve (Equation 17) when setting $\psi = 0$ and keeping all other parameters unchanged. In other words, expected inflation reacts even more in response to fiscal stimulus, which implies an even larger fiscal multiplier in long-lived liquidity traps. Section 4.2.3 provides further sensitivity analysis for the comparison of Kimball vs. Dixit-Stiglitz aggregation. All told, the results in Figure 3 and the remainder of the paper suggest that the findings reported in the previous literature—which mostly relied on using linearized models—are likely to be biased upward from the perspective of the underlying nonlinear model.

3.2.1 | Accounting for the differences

Given the results described above, the following key questions arise: Why are the GDP multipliers so different and why does expected inflation respond so much more in the linearized solution, and particularly so in the Dixit–Stiglitz case? To shed light on these questions we simulate and report two additional variants of the nonlinear model. In the first, we linearize the pricing equations of the model; that is, we replace all nonlinear pricing equations with the standard linearized Phillips curve. In the second, we linearize all nonlinear pricing equations and the aggregate resource constraint such that the price distortion term disappears from the model (Equation 16). Following the approach outlined in Section 3.1, we construct baseline scenarios for the two additional variants of the nonlinear model for $\mathrm{ZLB}_{\mathrm{DUR}} = 1, \ldots, 12$.

The blue dash-dotted line in Figure 2 depicts the 8-quarter liquidity trap baseline in the variant with linearized pricing equations and resource constraint—that is, the second additional variant described above. Clearly, the simulated paths of the variables in this variant of the model are very similar to those in the completely linearized solution. Therefore, the nonlinearities of the price-setting block appear to account for virtually all differences between the nonlinear and linearized model. Intuitively, in response to the adverse shock, firms perceive their demand elasticity to be high in the nonlinear model with the Kimball aggregator. Therefore, firms are reluctant to change prices much in response to changes in relative demand. In terms of Figure 1, many firms are located in the upper left quadrant after the adverse shock hits the model economy.

Figure 4 examines the implications of partially linearizing the nonlinear model with respect to the fiscal multipliers. The blue dashed-dotted lines—referred to as "Linearized Resource Constraint and New Keynesian Phillips Curve (NKPC)"—are very similar to those obtained with the completely linearized model, both under Kimball and Dixit–Stiglitz

[17]For ease of interpretability, we have normalized the response of debt and inflation so that they correspond to an initial change in government spending as share of steady state output by 1%.

[18]Note that this means that inflation and output behave asymmetrically in expansions and recessions under the Kimball aggregator. Recessions are associated with relatively modest declines in inflation but booms can be associated with notably larger upward swings in inflation as discussed in detail in Lindé and Trabandt (2018).

[19]The small rise in inflation expectations in response to fiscal stimulus with the nonlinear model specification is consistent with the evidence provided by Dupor and Li (2015). These authors argue that expected inflation reacted little to spending shocks in the USA during the Great Recession.

[20]We only show results up to 8 quarters with the Dixit–Stiglitz aggregator to be able to show the differences more clearly in the graph.

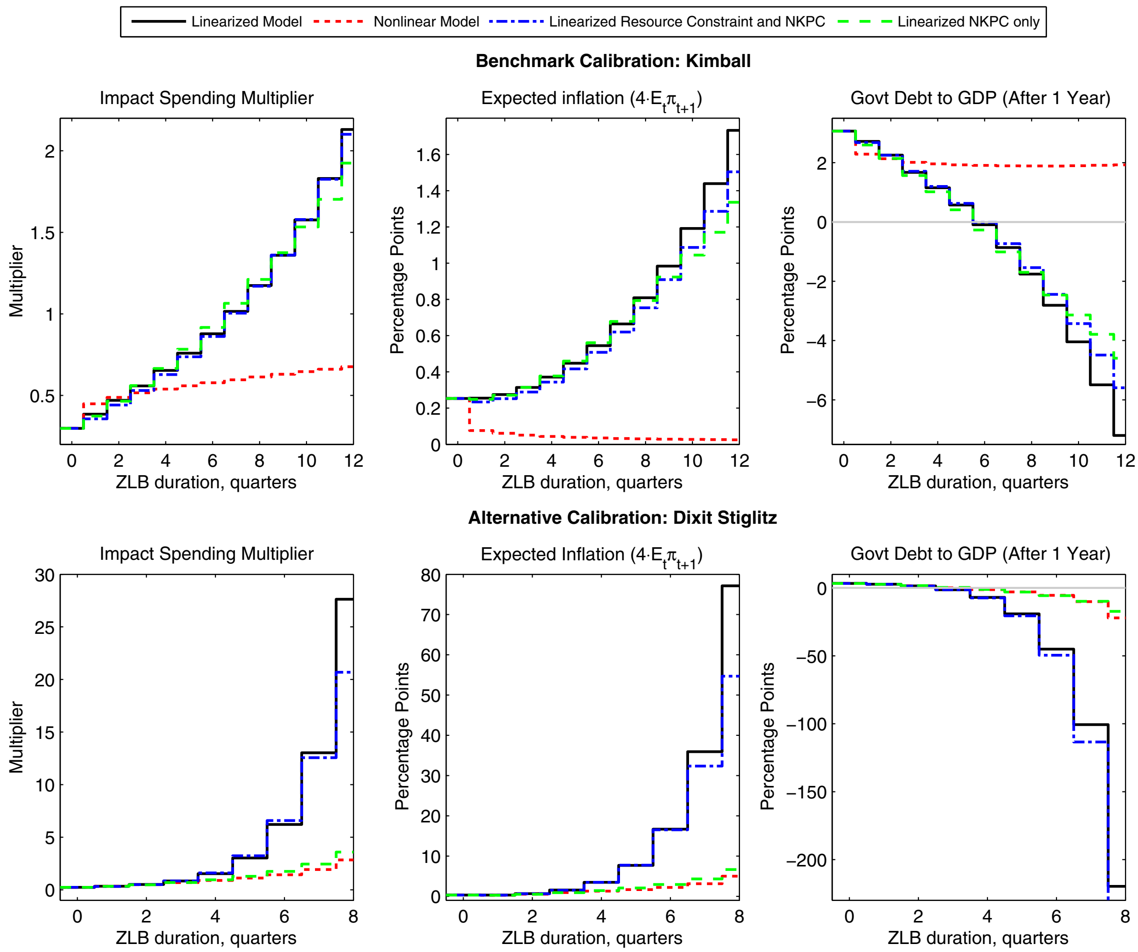

FIGURE 4 Decomposition of marginal multipliers for government spending [Colour figure can be viewed at wileyonlinelibrary.com]

aggregators. Hence, and in line with the results in Figure 2, we draw the conclusion that it is the linearization of the resource constraint and the Phillips curve (Equation 17) which account for the bulk of the differences in fiscal multipliers in the linear and nonlinear models in a long-lived liquidity trap.

Interestingly, as shown by the green dashed line in the top panels of Figure 4, it is almost sufficient just to linearize the NKPC to account for most of the differences in terms of the fiscal multipliers between the linearized and nonlinear solution with the Kimball aggregator. Therefore, the nonlinearities implied by the price dispersion term do not matter much quantitatively for the Kimball aggregator specification of the nonlinear model.

On the other hand, the bottom panels in Figure 4 show that linearization of the New Keynesian Phillips curve alone is not sufficient to explain the large discrepancies between the linearized and nonlinear model when the Dixit–Stiglitz aggregator is used. Put differently, with the Dixit–Stiglitz aggregator the tables are turned: The nonlinearities in the price dispersion term account for most of the differences between the linearized and nonlinear models, while the nonlinearities of the price-setting block are of second-order importance. The driving force behind the differences between the Kimball and the Dixit–Stiglitz aggregators is that the price distortion variable moves much more for the latter specification. Reflecting the insights from Figure 1, reoptimizing firms will adjust their prices much more under Dixit–Stiglitz compared to Kimball for a given value of ξ. So, in a model with Dixit–Stiglitz aggregation firms adjust prices a lot when they reoptimize, so that the bulk of the difference between the linearized and nonlinear model is driven by movements in the price distortion term. By contrast, firms adjust prices only a little in response to shocks with the Kimball aggregator specification. so that the price distortion term is much less important.

3.2.2 | Relation to existing work

Our results are very helpful in understanding the differences between the results reported in Boneva et al. (2016) and Eggertsson and Singh (2016). The former authors argue that it is key to account for the price distortion term as the main difference between the linearized and nonlinear solutions. Our results are in line with their finding given that Boneva et al. consider a model framework that incorporates the Dixit–Stiglitz aggregator. In terms of the magnitude of the multiplier it is important to note that we report lower multipliers in our nonlinear solution (the red dotted line in Figure 4) than Boneva et al. for the same degree of price adjustment. The reason for our lower multipliers is our government spending process, which is assumed to be a fairly persistent AR(1) process. As an alternative to our benchmark specification we follow Boneva et al. and assume that government spending follows a moving average (MA) process and is increased only when the nominal policy rate is constrained by the ZLB. With this specification we obtain a marginal multiplier of unity in both the linearized and nonlinear model already in a one-quarter liquidity trap.[21] More details concerning the results based on the MA process are provided in Section 4.2.5. There we show that the important differences between the linearized and nonlinear model hold up for longer-lived liquidity traps.

Our results can also be used to understand the results reported by Eggertsson and Singh (2016). These authors consider a model with a Dixit–Stiglitz aggregator and assume firm-specific labor, which implies that the price distortion term does not affect equilibrium allocations. Their model specification implies that they are effectively working with a nonlinear variant of our model without the price distortion—that is, the blue dashed-dotted line in the bottom part of Figure 4. The results reported in the bottom part of Figure 4 indeed indicate that the linearized solution is very similar to the nonlinear solution when the price dispersion term is kept constant. Thus our results confirm the conclusions by Eggertsson and Singh for this variation of the New Keynesian model.[22] However, our analysis also makes clear that their findings do not necessarily extend to alternative model environments—for example, the variation of the New Keynesian model considered by Boneva et al. (2016).

4 | ROBUSTNESS

In this section, we examine the robustness of the results. We focus on the sensitivity of our results when solving the model with global methods to allow future shock uncertainty to affect the decision rules of households and firms. We also summarize the results of further robustness checks, including the effects of other shocks, the sensitivity of our results with respect to the baseline shock, the aggregator specification (Kimball vs. Dixit–Stiglitz), price indexation and the exogenous process for government spending.

4.1 | Global solution allowing for shock uncertainty

Jung et al. (2005), Adam and Billi (2006, 2007), Fernández-Villaverde et al. (2015), Gust et al. (2017), Nakata (2016), Richter and Throckmorton (2016), and others have studied the solutions of the linearized and nonlinear New Keynesian model, focusing on the effects of shock uncertainty on the decision rules of households and firms. In this subsection we show that our key findings hold up—and are even strengthened—when we allow for substantial future shock uncertainty.

We solve the stochastic linearized and nonlinear models using the global solution method developed by Judd (1998) and (Coleman, 1990, 1991). This solution method is based on a time iteration method on the decision rules of households and firms. The variance of shocks can affect the policy functions. The time iteration method has been used recently, by Nakata (2016) and Richter and Throckmorton (2016), for example.[23] We solve the nonlinear and linearized model subject to stochastic shocks to consumption preferences and government consumption. At the respective model solutions, the stochastic nonlinear and stochastic linearized model economies observe a probability of being at the ZLB of 10%. Details about the computation of the stochastic global solution are provided in Appendix A3.

Figure 5 provides the results when solving the model with shock uncertainty compared to the deterministic solution. Figure 5(a) shows the impulse responses for GDP and annualized inflation in the linearized model for the equivalent of a 1% of GDP hike in government spending in an 8-quarter liquidity trap. In Figure 5(b), we show the corresponding responses in the nonlinear model. In analogy with how we compute the impulse responses in the deterministic solution, we compute the impulse responses under shock uncertainty by first generating a baseline where we subject the model

[21]See Woodford (2011) for a proof of this result.

[22]Strictly speaking, the Eggertsson and Singh (2016) model only omits the price distortion but retains the nonlinear pricing equations. Our blue dashed-dotted line in the bottom part of Figure 4 linearizes the price-setting block in addition to removing the price distortion. However, the dashed green line shows that nonlinearities in the price-setting block matter very little when a Dixit–Stiglitz aggregator is used.

[23]We are grateful to Richter and Throckmorton (2016) for making their codes publicly available. Their codes provided us with a useful starting point for solving our model.

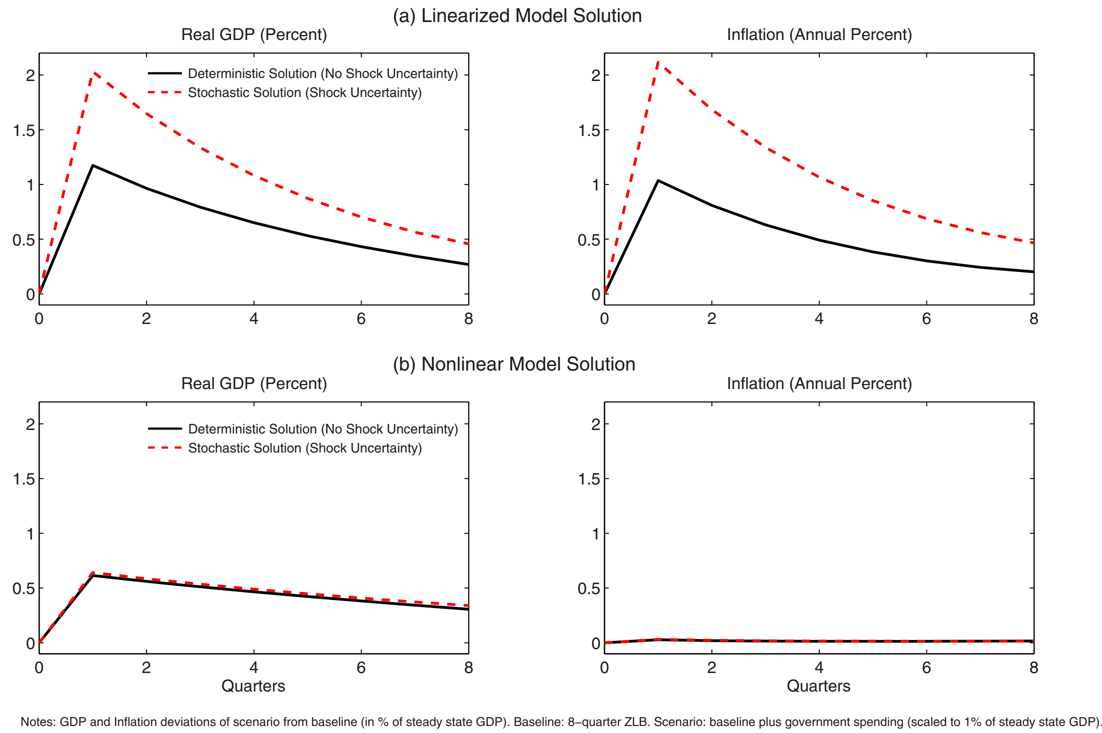

FIGURE 5 Effects of an increase in government spending in an 8-quarter liquidity trap: (A) linearized model solution; (B) nonlinear model solution. *Notes*: GDP and inflation deviation of scenario from baseline (as percentage of steady-state GDP). Baseline: 8-quarter ZLB. Scenario: baseline plus government spending (scaled to 1% of steady-state GDP) [Colour figure can be viewed at wileyonlinelibrary.com]

to a negative consumption preference shock which generates an expected 8-quarter liquidity trap under the assumption that no further consumption preference shocks are realized during the transition back to the steady state. In the quarter in which the liquidity trap is expected to last for 8 quarters, we add a small positive government spending shock and compute the impulse responses in Figure 5 as the difference between the simulation with government spending and the simulation with consumption preference shocks only.[24]

The solid black lines in Figure 5(a) correspond to the linearized model solved with the Fair and Taylor (1983) method—that is, the deterministic solution of the linearized model. The impact of government consumption on GDP in period 1 is the same as the one depicted in the top panel of Figure 3; that is, the impact multiplier is 1.2 in an 8-quarter ZLB episode. The red dashed lines in Figure 5(a) correspond to the case when the linearized model is solved subject to future shock uncertainty. In this case, the impact multiplier increases to about 2.1. Evidently, shock uncertainty elevates the multiplier substantially in the linearized model.

Figure 5(b) shows the comparison of the impulse responses in the deterministic versus stochastic solution in the nonlinear model. The solid black lines correspond to the deterministic solution of the nonlinear model. As in Figure 3, the multiplier is about 0.6 for an 8-quarter liquidity trap. Interestingly, the nonlinear model is not much affected by shock uncertainty. The multiplier increases from 0.61 in the deterministic solution to 0.64 in the fully stochastic nonlinear model solution. The solution of the nonlinear model is not much affected due to the nonlinearities embedded in the Kimball aggregator together with the low slope of the Phillips curve. Both features reduce the incentive of firms to change their prices in response to expectations of adverse shocks in the future even when the economy is stuck in a long–lived liquidity trap. By contrast, the linearized model incorrectly extrapolates the behavior of households and firms such that inflation reacts much more strongly to shocks, leading to an even higher multiplier than in the deterministic solution.

[24]Bodenstein, Hebden, and Nunes (2012) use the same approach when computing impulse responses. Although the assumption that no further shocks are realized on the transition path back to steady state is improbable, this way of computing the impulses makes them directly comparable with how they are computed in the deterministic solution. Importantly, the impulse responses still reflect the impact of future shock uncertainty via the effect that shock uncertainty has on the decision rules of households and firms.

To sum up, our results indicate that the implications of uncertainty in the nonlinear model are quantitatively negligible. By contrast, the multiplier in the linearized model is greatly elevated when shock uncertainty is allowed for in the solution of the model. Consequently, our conclusion of an important difference between the linearized and nonlinear solution in long-lived liquidity traps holds up under future shock uncertainty.

4.2 | Additional robustness analysis

We perform a variety of additional robustness checks. Given space constraints, we summarize the key takeaways from the additional robustness analysis here. Appendices A4–A8 contain further details.

4.2.1 | Effects of other shocks

We examine the implications of the following four additional shocks for the linearized and nonlinear model: discount factor shock, technology shock, markup shock and monetary policy shock. Two key takeaways emerge from this analysis. First, for all shocks considered, there are substantial differences between the linearized and nonlinear model. Second, in the linearized model, the responses of variables to the government consumption shock, the consumption demand shock, the discount factor shock, and the technology shock are observationally equivalent. In the nonlinear model a similar observation arises-the responses of model variables are (nearly) observationally equivalent. Appendix A4 contains the details.

4.2.2 | Choice of baseline shock

We study how our results are affected when a discount factor shock or a technology shock is used to generate the baseline in which the ZLB is binding for a desired number of quarters. For the linearized model, the multiplier results are invariant with respect to the choice of baseline shock (see Erceg & Lindé, 2014, for analytical proofs). That is, the multiplier is identical when the baseline is generated either by a consumption preference shock or by a discount factor shock or by a technology shock. For the nonlinear model we show that the multiplier schedules are nearly invariant with respect to alternative baseline shocks. Appendix A5 contains the details.

4.2.3 | Kimball versus Dixit–Stiglitz

In the linearized model, we show that the Kimball and Dixit–Stiglitz aggregators yield identical multiplier schedules when the degree of price stickiness and the Kimball elasticity parameter are parametrized such that the slope of the linearized New Keynesian Phillips curve (κ in Equation 17) is kept constant. So going from Kimball to Dixit–Stiglitz by making prices more sticky yields identical multipliers in the linearized model. In the nonlinear model, the same conclusion is not true. There, a reparametrization of the Dixit–Stiglitz version of the model with higher price stickiness does not produce the same multipliers as under Kimball. This demonstrates that the modeling of price frictions matters importantly within a nonlinear framework. Appendix A6 contains the details.

4.2.4 | Price indexation

We examine the consequences of not allowing prices of non-optimizing firms to be indexed to the steady-state rate of inflation. We show that our benchmark results are little affected by the indexation assumption. Appendix A7 contains the details.

4.2.5 | Government spending process

Finally, we examine the implications of adopting a moving average (MA) process for government spending at the ZLB instead of a general AR(1) process. We show that our benchmark results hold up well for an MA process for government spending. If anything, an MA process magnifies the differences between the linearized and nonlinear model in terms of the multiplier. Appendix A8 contains the details.

5 | EMPIRICAL IMPLICATIONS

A key feature of the Great Recession in the USA and other advanced economies was a large, sharp and persistent fall in GDP of nearly 10% relative to the precrisis trend. As oil prices fell sharply in response to the recession, headline inflation slowed down substantially. However, measures of core inflation—the relevant benchmark for standard macroeconomic models without commodities—slowed down only by a modest amount of about 1 percentage point (see, e.g., Figure 8 in Christiano, Eichenbaum, & Trabandt, 2015).

Estimated standard New Keynesian models which target to explain all variation in the data using full-information Bayesian likelihood methods have difficulties in accounting for the low elasticity between output and inflation observed during the Great Recession.

One way to account for the moderate drop in inflation in the face of the large contraction in GDP is to resort to large offsetting effects on inflation stemming from positive price markup shocks (see, e.g., Lindé, Smets, & Wouters, 2016). Some researchers have emphasized that financial frictions may be responsible for the small elasticity between output and inflation witnessed during the crisis. Christiano et al. (2015) use a model to show that the observed fall in total factor productivity and the rise in firms' cost to borrow funds for working capital played critical roles in accounting for the small drop in inflation that occurred during the Great Recession. Gilchrist, Schoenle, Sim, and Zakrajšek (2017) develop a model in which firms face financial frictions when setting prices in an environment with customer markets. Financial distortions create an incentive for financially constrained firms to raise prices in response to adverse financial or demand shocks in order to preserve internal liquidity and avoid accessing external finance. While financially unconstrained firms cut prices in response to these adverse shocks, the share of financially constrained firms is sufficiently large in their model to attenuate the fall in inflation in response to fluctuations in GDP. Gilchrist et al. (2017) examine a micro-data set which supports the implications of their model.

The mechanism based on the nonlinear Kimball (1995) aggregator that we have used in our paper following Lindé and Trabandt (2018) offers an alternative explanation for understanding the small elasticity of inflation and output observed during the Great Recession. To examine the empirical potency of the mechanism in a rigorous way, one would have to follow the work of Gust et al. (2017), Richter and Throckmorton (2016), and Kulish and Pagan (2017), and estimate the nonlinear model with likelihood methods. Given the strong nonlinearities associated with the Kimball (1995) aggregator and the fact that embedding the nonlinear Kimball (1995) aggregator into a standard New Keynesian model requires the introduction of several endogenous state variables, this will be a tough but potentially very rewarding challenge, as suggested by the recent work of Aruoba, Boccola, and Schorfheide (2017). To begin with, one could use the perfect foresight approach to likelihood evaluation developed by Iacoviello and Guerrieri (2016). An interesting extension in this context would also be to examine the possibility of the existence of a deflationary regime, as in Aruoba, Cuba-Borda, and Schorfheide (2018), as this could have important implications for the size of the fiscal multiplier.

Ideally, one should also complement the empirical approach based on macroeconomic data with firm-level data on prices and quantities. Using micro-data would allow us to examine empirically the properties of the nonlinear Kimball (1995) aggregator shown in Figure 1. In addition, one could possibly also draw and extend empirical findings in the industrial organization literature to shed further light on the properties of the Kimball (1995) aggregator. While we are excited about these empirical applications, we leave them to future research.

6 | CONCLUSIONS

All told, our paper provides an example of a potential first-order policy mistake that is based on using a linear approximation to solve a model to calculate a fiscal spending multiplier. The mistake involves a nearly three times as large multiplier (2 instead of 0.7) as well as the implication of a self-financing fiscal stimulus in a long-lived liquidity trap. Our analysis of the true underlying nonlinear model arrives at very different conclusions: a small multiplier and no self-financing. Therefore, our results caution against the common practice of using linearized models to calculate fiscal multipliers in long-lived liquidity traps.

Using our benchmark model with real rigidities following Kimball (1995), we have documented that it is the linearization of the Phillips curve which accounts for the bulk of the difference between the linearized and nonlinear model.

The results in our model imply large differences between the linearized and nonlinear model, supporting the findings in Boneva et al. (2016). In contrast to Boneva et al., however, it is important to point out that we consider a model which matches macroeconomic evidence of a low Phillips curve slope *and* microeconomic evidence of frequent price changes by firms.

Even so, the way nonlinearities are introduced into a model can matter. Specifically, our analysis has shown that it is possible to construct New Keynesian models in which the difference between the linearized and nonlinear model is relatively small even in long-lived liquidity traps. More precisely, confirming the results in Eggertsson and Singh (2016), our analysis documents that this is the case in the Eggertsson and Woodford (2003) "Yeoman farmer" New Keynesian sticky price model with firm-specific labor. In that model the price dispersion term is irrelevant for equilibrium dynamics. As this model fits the macro- and microevidence on price setting equally well as our benchmark model using the Kimball (1995) aggregator, an important issue that remains to be studied is which of the competing frameworks best fits the data.

It would also be interesting to study the robustness of our results in an empirically realistic framework such as Christiano et al. (2005), where one would allow for a nonlinear Kimball (1995) aggregator in both price setting and wage setting and nonlinearities originating from financial frictions following, for example, the Bernanke, Gertler, and Gilchrist (1999) financial accelerator mechanism. Such a framework would allow us to study the robustness of our findings in a framework which has a spending multiplier in the mid-range of the VAR evidence when monetary policy is unconstrained. Doing so is important for the substantive issue whether the spending multiplier can be sufficiently elevated in a long-lived liquidity trap so that a transient hike in spending is associated with a fiscal free lunch (and, conversely, whether a spending cut could be self-defeating in a long-lived trap). We leave these extensions to future research.

ACKNOWLEDGMENTS

A previous version of this paper was titled "Fiscal multipliers in a nonlinar world." We are grateful for helpful comments by our discussants Anton Braun, Robert Kollmann, Jinill Kim, and Andresa Lagerborg, and participants of the Federal Reserve Macro System Committee meeting at the Federal Reserve Bank of Boston in November 2013, the ECB/EACBN/Atlanta Fed conference "Nonlinearities in macroeconomics and finance in the light of crises" hosted by the European Central Bank in December 2014, the Korean Economic Association and Social Science conference "Recent Issues in Monetary Economics" in Seoul October 2016, and the European University Institute workshop "Economic Policy Challenges" in Florence November 2016, respectively. In addition, we have benefited from comments from participants in seminars at Norges Bank, Oslo University, Sveriges Riksbank, University of Vienna, North Carolina State University, Central Bank of Finland, University of Hamburg, Goethe University Frankfurt am Main, European Central Bank, European Commission, University of Halle-Wittenberg, Halle Institute for Economic Research, University of Erlangen-Nürnberg, Central Bank of Korea, Humboldt Universität Berlin, Bank of England, Queen Mary University and the Federal Reserve Bank of Atlanta. Special thanks to Svetlana Chekmasova, Sher Singh, and Mazi Kazemi for outstanding research assistance. Part of this paper was written when the authors were affiliated with the International Finance Division of the Board of Governors of the Federal Reserve System (Lindé and Trabandt), the International Monetary Fund (Lindé) and the 2016/2017 DG-ECFIN Fellowship Initiative of the European Commission (Trabandt). The authors are grateful to these institutions for their support of this project. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank.

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SUPPORTING INFORMATION

Additional supporting information may be found online in the Supporting Information section at the end of the article.

How to cite this article: Lindé J, Trabandt M. Should we use linearized models to calculate fiscal multipliers?. *J Appl Econ*. 2018;1–29. https://doi.org/10.1002/jae.2641

APPENDIX A

Below we state the nonlinear and linearized equilibrium conditions of the model.[25] We also provide a detailed description of the additional robustness analyses which we summarize in the main text.

[25]All derivations and the closed form steady state are provided in the Technical Appendix, which is available at https://sites.google.com/site/mathiastrabandt/home/downloads/LindeTrabandt_Multiplier_TechApp.pdf. (See also Supporting Information).

A1: Nonlinear equilibrium equations

$$\text{Marginal utility (n1)}: \quad (c_t - cv_t)^{-1} = \lambda_t$$

$$\text{Leisure/labor (n2)}: \quad n_t^{\chi} = (1-\tau_N)\,\lambda_t w_t$$

$$\text{Euler equation (n3)}: \quad \lambda_t = \beta E_t \frac{1+i_t}{\Pi_{t+1}} \lambda_{t+1}$$

$$\text{Resource constraint/GDP (n4)}: \quad c_t + g_t = y_t$$

$$\text{Production (n5)}: \quad y_t = \left(p_t^*\right)^{-1} k^{\alpha} n_t^{1-\alpha}$$

$$\text{Nonlin. pricing 1 (n6)}: \quad s_t = \frac{(1+\psi)(1+\theta)}{1+\psi+\theta\psi} \lambda_t y_t \vartheta_t^{\frac{1+\theta}{\theta}(1+\psi)} mc_t + \beta\xi E_t(\Pi/\Pi_{t+1})^{-\frac{1+\theta}{\theta}(1+\psi)} s_{t+1}$$

$$\text{Nonlin. pricing 2 (n7)}: \quad f_t = \lambda_t y_t \vartheta_t^{\frac{1+\theta}{\theta}(1+\psi)} + \beta\xi E_t(\Pi/\Pi_{t+1})^{-\frac{(1+\psi+\psi\theta)}{\theta}} f_{t+1}$$

$$\text{Nonlin. pricing 3 (n8)}: \quad a_t = \frac{\psi\theta}{1+\psi+\theta\psi} y_t \lambda_t + \beta\xi E_t\,(\Pi/\Pi_{t+1})\, a_{t+1}$$

$$\text{Nonlin. pricing 4 (n9)}: \quad s_t = f_t \tilde{p}_t - a_t \tilde{p}_t^{1+\frac{1+\theta}{\theta}(1+\psi)}$$

$$\text{Zero profit condition (n10)}: \quad \vartheta_t = 1 + \psi - \psi \Delta_{t,2}$$

$$\text{Aggregate price index (n11)}: \quad \vartheta_t = \Delta_{t,3}$$

$$\text{Overall price dispersion (n12)}: \quad p_t^* = \frac{\vartheta_t^{\frac{1+\theta}{\theta}(1+\psi)}}{1+\psi} \Delta_{t,1}^{-\frac{1+\theta}{\theta}(1+\psi)} + \frac{\psi}{1+\psi}$$

$$\text{Price dispersion 1 (n13)}: \quad \Delta_{t,1}^{-\frac{(1+\theta)(1+\psi)}{\theta}} = (1-\xi)\,\tilde{p}_t^{-\frac{(1+\theta)(1+\psi)}{\theta}} + \xi\left[(\Pi/\Pi_t)\,\Delta_{t-1,1}\right]^{-\frac{(1+\theta)(1+\psi)}{\theta}}$$

$$\text{Price dispersion 2 (n14)}: \quad \Delta_{t,2} = (1-\xi)\,\tilde{p}_t + \xi\,(\Pi/\Pi_t)\,\Delta_{t-1,2}$$

$$\text{Price dispersion 3 (n15)}: \quad \Delta_{t,3}^{-\frac{1+\psi+\psi\theta}{\theta}} = (1-\xi)\tilde{p}_t^{-\frac{1+\psi+\psi\theta}{\theta}} + \xi\left((\Pi/\Pi_t)\,\Delta_{t-1,3}\right)^{-\frac{1+\psi+\psi\theta}{\theta}}$$

$$\text{Marginal cost (n16)}: \quad (1-\alpha)\,\mathrm{mc}_t = w_t k^{-\alpha} n_t^{\alpha}$$

$$\text{Taylor rule (n17)}: \quad 1 + i_t = \max\left(1, (1+i)\left[\Pi_t/\Pi\right]^{\gamma_\pi}\left[\frac{y_t}{y}\Big/\frac{y_t^{\mathrm{pot}}}{y^{\mathrm{pot}}}\right]^{\gamma_x}\right)$$

$$\text{Government budget (n18)}: \quad b_t = \frac{1+i_{t-1}}{\Pi_t} b_{t-1} + \frac{g_t}{y} - \frac{\tau_N w_t n_t}{y} - \tau_t$$

$$\text{Fiscal rule (n19)}: \quad \tau_t = \tau + \varphi\,(b_{t-1} - b)$$

Flex-price (potential) economy: version of the model when prices are flexible; that is, $\xi = 0$.

$$\text{Euler equation, flex-price (n20)}: \quad \left(c_t^{\mathrm{pot}} - c^{\mathrm{pot}} v_t\right)^{-1} = \beta E_t rr_t^{\mathrm{pot}} \left(c_{t+1}^{\mathrm{pot}} - c^{\mathrm{pot}} v_{t+1}\right)^{-1}$$

$$\text{Leisure/labor, flex-price (n21)}: \quad \left(n_t^{\mathrm{pot}}\right)^{\chi} = \left(c_t^{\mathrm{pot}} - c^{\mathrm{pot}} v_t\right)^{-1} \left(1 - \tau_N^{\mathrm{pot}}\right) w_t^{\mathrm{pot}}$$

$$\text{Wage, flex-price (n22)}: \quad \frac{1-\alpha}{1+\theta}\left(k^{\mathrm{pot}}\right)^{\alpha} = w_t^{\mathrm{pot}} \left(n_t^{\mathrm{pot}}\right)^{\alpha}$$

$$\text{Res. constraint, flex-price (n23)}: \quad c_t^{\mathrm{pot}} + g_t^{\mathrm{pot}} = y_t^{\mathrm{pot}}$$

$$\text{Production, flex-price (n24)}: \quad y_t^{\mathrm{pot}} = \left(k^{\mathrm{pot}}\right)^{\alpha} \left(n_t^{\mathrm{pot}}\right)^{1-\alpha}$$

$$\text{Gov. budget, flex-price (n25)}: \quad b_t^{\mathrm{pot}} = rr_{t-1}^{\mathrm{pot}} b_{t-1}^{\mathrm{pot}} + \frac{g_t^{\mathrm{pot}}}{y} - \frac{\tau_N^{\mathrm{pot}} w_t^{\mathrm{pot}} n_t^{\mathrm{pot}}}{y} - \tau_t^{\mathrm{pot}}$$

$$\text{Fiscal rule, flex-price (n26)}: \quad \tau_t^{\mathrm{pot}} = \tau + \varphi\left(b_{t-1}^{\mathrm{pot}} - b\right)$$

In the above equations θ denotes the net markup and is defined as $\theta = \phi - 1$. We have 26 equations in the following 26 unknowns:

$$c_t \lambda_t n_t w_t i_t \Pi_t y_t p_t^* s_t \vartheta_t \mathrm{mc}_t f_t a_t \tilde{p}_t \Delta_{t,1} \Delta_{t,2} \Delta_{t,3} b_t \tau_t$$
$$c_t^{\mathrm{pot}} rr_t^{\mathrm{pot}} n_t^{\mathrm{pot}} w_t^{\mathrm{pot}} y_t^{\mathrm{pot}} b_t^{\mathrm{pot}} \tau_t^{\mathrm{pot}}$$

The processes of the exogenous variables g_t and v_t are provided in Equations 19 in the main text.

A2: Linearized equilibrium equations

$$\text{Leisure/labor (l1)}: \quad \chi \hat{n}_t + \frac{\hat{c}_t - \breve{v}_t}{(1-\nu)} = \hat{w}_t$$

$$\text{Euler equation (l2)}: \quad 0 = E_t \left(\breve{\imath}_t - \hat{\Pi}_{t+1} - \frac{\hat{c}_{t+1} - \breve{v}_{t+1}}{1-\nu} + \frac{\hat{c}_t - \breve{v}_t}{1-\nu} \right)$$

$$\text{Resource constraint (l3)}: \quad c\hat{c}_t + g\hat{g}_t = y\hat{y}_t$$

$$\text{Production (l4)}: \quad \hat{y}_t = (1-\alpha)\,\hat{n}_t$$

$$\text{Phillips curve (l5)}: \quad \hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi} \frac{1}{1-\phi\psi} \widehat{\mathrm{mc}}_t$$

$$\text{Marginal cost (l6)}: \quad \widehat{\mathrm{mc}}_t = \hat{w}_t + \alpha \hat{n}_t$$

$$\text{Taylor rule (l7)}: \quad \breve{\imath}_t = \max\left(-i, \gamma_\pi \hat{\Pi}_t + \gamma_x \left(\hat{y}_t - \hat{y}_t^{\mathrm{pot}} \right) \right)$$

$$\text{Government budget (l8)}: \quad \breve{b}_t = \frac{1+i}{\Pi} b \left(\breve{\imath}_{t-1} - \hat{\Pi}_t \right) + \frac{1+i}{\Pi} \breve{b}_{t-1} + g\hat{g}_t - \tau_N wn \left(\hat{w}_t + \hat{n}_t \right) - \breve{\tau}_t$$

$$\text{Fiscal rule 1 (l9)}: \quad \breve{\tau}_t = \varphi \breve{b}_{t-1}$$

$$\text{Marginal utility (l10)}: \quad \hat{c}_t = -(1-\nu)\,\hat{\lambda}_t + \breve{v}_t$$

$$\text{Leisure/labor (l1)}: \quad \chi \hat{n}_t + \frac{\hat{c}_t - \breve{v}_t}{(1-\nu)} = \hat{w}_t$$

$$\text{Euler equation (l2)}: \quad 0 = E_t \left(\breve{\imath}_t - \hat{\Pi}_{t+1} - \frac{\hat{c}_{t+1} - \breve{v}_{t+1}}{1-\nu} + \frac{\hat{c}_t - \breve{v}_t}{1-\nu} \right)$$

$$\text{Resource constraint (l3)}: \quad c\hat{c}_t + g\hat{g}_t = y\hat{y}_t$$

$$\text{Production (l4)}: \quad \hat{y}_t = (1-\alpha)\,\hat{n}_t$$

$$\text{Phillips curve (l5)}: \quad \hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \frac{(1-\xi)(1-\beta\xi)}{\xi} \frac{1}{1-\phi\psi} \widehat{\mathrm{mc}}_t$$

$$\text{Marginal cost (l6)}: \quad \widehat{\mathrm{mc}}_t = \hat{w}_t + \alpha \hat{n}_t$$

$$\text{Taylor rule (l7)}: \quad \breve{\imath}_t = \max\left(-i, \gamma_\pi \hat{\Pi}_t + \gamma_x \left(\hat{y}_t - \hat{y}_t^{\mathrm{pot}} \right) \right)$$

$$\text{Government budget (l8)}: \quad \breve{b}_t = \frac{1+i}{\Pi} b \left(\breve{\imath}_{t-1} - \hat{\Pi}_t \right) + \frac{1+i}{\Pi} \breve{b}_{t-1} + g\hat{g}_t - \tau_N wn \left(\hat{w}_t + \hat{n}_t \right) - \breve{\tau}_t$$

$$\text{Fiscal rule 1 (l9)}: \quad \breve{\tau}_t = \varphi \breve{b}_{t-1}$$

$$\text{Marginal utility (l10)}: \quad \hat{c}_t = -(1-\nu)\,\hat{\lambda}_t + \breve{v}_t$$

$$\text{Euler equation, flex-price (l11)}: \quad 0 = E_t \left(\widehat{rr}_t^{\mathrm{pot}} - \frac{\hat{c}_{t+1}^{\mathrm{pot}} - \breve{v}_{t+1}}{1-\nu} + \frac{\hat{c}_t^{\mathrm{pot}} - \breve{v}_t}{1-\nu} \right)$$

$$\text{Leisure/labor, flex-price (l12)}: \quad \chi \hat{n}_t^{\mathrm{pot}} + \frac{\hat{c}_t^{\mathrm{pot}} - \breve{v}_t}{(1-\nu)} = \hat{w}_t^{\mathrm{pot}}$$

$$\text{Wage, flex-price (l13)}: \quad \hat{w}_t^{\mathrm{pot}} = -\alpha \hat{n}_t^{\mathrm{pot}}$$

$$\text{Res. constraint, flex-price (l14)}: \quad c^{\mathrm{pot}} \hat{c}_t^{\mathrm{pot}} + g^{\mathrm{pot}} \hat{g}_t = y^{\mathrm{pot}} \hat{y}_t^{\mathrm{pot}}$$

$$\text{Production, flex-price (l15)}: \quad \hat{y}_t^{\mathrm{pot}} = (1-\alpha)\,\hat{n}_t^{\mathrm{pot}}$$

$$\text{Gov. Budget, flex-price (l16)}: \quad \breve{b}_t^{\mathrm{pot}} = rr^{\mathrm{pot}} \times b^{\mathrm{pot}} \widehat{rr}_{t-1}^{\mathrm{pot}} + rr^{\mathrm{pot}} \breve{b}_{t-1}^{\mathrm{pot}} + g^{\mathrm{pot}} \hat{g}_t - \tau_N w^{\mathrm{pot}} n^{\mathrm{pot}} \left(\hat{w}_t^{\mathrm{pot}} + \hat{n}_t^{\mathrm{pot}} \right) - \breve{\tau}_t^{\mathrm{pot}}$$

$$\text{Fiscal rule, flex-price (l17)}: \quad \breve{\tau}_t^{\mathrm{pot}} = \varphi_T \breve{b}_{t-1}^{\mathrm{pot}}$$

where hat variables denote percentage deviations from steady state and breve variables denote absolute deviations from steady state. We have 17 equations in the following 17 unknowns:

$$\hat{c}_t \hat{n}_t \hat{w}_t \breve{\imath}_t \hat{\Pi}_t \hat{y}_t \widehat{\mathrm{mc}}_t \breve{b}_t \breve{\tau}_t \hat{\lambda}_t$$
$$\hat{c}_t^{\mathrm{pot}} \widehat{rr}_t^{\mathrm{pot}} \hat{n}_t^{\mathrm{pot}} \hat{w}_t^{\mathrm{pot}} \hat{y}_t^{\mathrm{pot}} \breve{b}_t^{\mathrm{pot}} \breve{\tau}_t^{\mathrm{pot}}$$

The variables $\hat{g}_t$ and $\breve{v}$ are exogenous and are defined as $\hat{g}_t = \frac{g_t - g}{g}$ and $\breve{v}_t = v_t - \breve{v}$.

The above linearized equilibrium equations can be summarized as follows:

$$x_t = E_t x_{t+1} - \eta(\check{\imath}_t - E_t\hat{\Pi}_{t+1} - \hat{r}_t^{\text{pot}}), \tag{A1}$$

$$\hat{\Pi}_t = \beta E_t\hat{\Pi}_{t+1} + \kappa x_t, \tag{A2}$$

$$\hat{y}_t^{\text{pot}} = \frac{1}{\phi_{mc}\eta}\left(g_y\hat{g}_t + (1-g_y)\check{\nu}_t\right), \tag{A3}$$

$$\hat{r}_t^{\text{pot}} = \frac{1}{\eta}\left(1 - \frac{1}{\phi_{\text{mc}}\eta}\right)\left[g_y(\hat{g}_t - E_t\hat{g}_{t+1}) + (1-g_y)(\check{\nu}_t - E_t\check{\nu}_{t+1})\right], \tag{A4}$$

$$b_t = (1+r)b_{t-1} + (1+r)b(\check{\imath}_{t-1} - \pi_t) + g_y\hat{g}_t - \tau_N s_N(\hat{y}_t + \phi_{\text{mc}}x_t) - \tau_t, \tag{A5}$$

$$\hat{y}_t = x_t + \hat{y}_t^{\text{pot}}, \tag{A6}$$

where η, κ, ϕ_{mc} and s_N are composite parameters defined as

$$\eta = (1-g_y)(1-\nu), \tag{A7}$$

$$\kappa = \frac{(1-\xi)(1-\beta\xi)}{\xi}\frac{1}{1-\phi\psi}\phi_{\text{mc}}, \tag{A8}$$

$$\phi_{\text{mc}} = \frac{\chi}{1-\alpha} + \frac{1}{\eta} + \frac{\alpha}{1-\alpha}, \tag{A9}$$

$$s_N = \frac{1-\alpha}{\phi}. \tag{A10}$$

Equation A1 expresses the "New Keynesian" IS curve in terms of the output and real interest rate gaps. Thus the output gap x_t depends inversely on the deviation of the real interest rate $(\check{\imath}_t - E_t\hat{\Pi}_{t+1})$ from its potential rate r_t^{pot}, as well as on the expected output gap in the following period. The parameter η determines the sensitivity of the output gap to the real interest rate; as indicated by Equation A7, it depends on the steady-state government spending share of output g_y, and a (small) adjustment factor ν which scales the consumption preference shock ν_t. The price-setting equation A2 specifies current inflation $\hat{\Pi}_t$ to depend on expected inflation and the output gap, where the sensitivity to the latter is determined by the composite parameter κ. Given the Calvo (1983) contract structure, Equation A8 implies that κ varies directly with the sensitivity of marginal cost to the output gap ϕ_{mc}, and inversely with the mean contract duration $(\frac{1}{1-\xi})$. The marginal cost sensitivity equals the sum of the absolute value of the slopes of the labor supply and labor demand schedules that would prevail under flexible prices: accordingly, as seen in Equation A9, ϕ_{mc} varies inversely with the Frisch elasticity of labor supply $\frac{1}{\chi}$, the interest sensitivity of aggregate demand δ, and the labor share in production $(1-\alpha)$. Equations A3 and A4 determine potential output and the potential (or natural) real rate. The evolution of government debt is determined by Equation A5 and depends on variations in the service cost of debt, government spending, as well as labor income and lump-sum tax revenues. Equation A6 is a simple definitional equation for actual output y_t (in logs). Finally, the policy rate $\check{\imath}_t$ follows a Taylor rule subject to the zero lower bound (linearized version of the policy rule, Equation 14 in the main text) and the exogenous shocks follow the processes in Equations 19.

A3: Details of the global solution method

In order to solve the fully stochastic nonlinear and linearized model, we discretize the state space. Solving the stochastic nonlinear model is computationally challenging due to the nonlinearities embedded in the Kimball aggregator, the size of the state space, and the non-availability of closed-form solutions for some of the model variables needed to evaluate expectations. In the nonlinear model we use the Rouwenhorst (1995) method with 9 gridpoints to approximate the government consumption process. We use 19 gridpoints to discretize the consumption preference process. For the endogenous state variables that result from the Kimball (1995) formulation we use 9 gridpoints for each state variable. In total we have about 14,000 gridpoints in the policy functions. For points that are not exactly on the nodes of the policy functions, we use linear interpolation/extrapolation. To approximate expectations we use numerical integration based on the trapezoid rule, for which we evaluate future realizations of the consumption preference process on 9 gridpoints together with a truncated normal distribution with ±6 standard deviations. It takes about 24 hours on a workstation with a dual Intel Xeon processor E5-2637 v3 (four cores, hyperthreading technology, 15 MB cache, 3.5 GHz turbo) to solve the nonlinear model.

The specification for solving the stochastic linearized model is similar to that of the stochastic nonlinear model. However, given the absence of the endogenous state variables and the availability to solve for endogenous variables in closed form to evaluate expectations in the linearized model, we solve the linearized model using 200 gridpoints for each exogenous process—that is, a total of 40,000 gridpoints. It takes about 1 hour on the above workstation to solve the linearized model.

We calibrate the two exogenous processes as follows:

$$\frac{G_t - G}{G} = 0.95\left(\frac{G_{t-1} - G}{G}\right) + \frac{0.01}{G}\varepsilon_{G,t}, \varepsilon_G \overset{\text{i.i.d.}}{\sim} N(0,1)$$

$$\nu_t - \nu = 0.80\,(\nu_{t-1} - \nu) + \sigma_\nu \varepsilon_{\nu,t}, \varepsilon_\nu \overset{\text{i.i.d.}}{\sim} N(0,1).$$

The autocorrelation of 0.95 and the standard deviation of 0.01 for the government consumption process are estimated using the cyclical component of HP-filtered US data from 1955:Q1 to 2017:Q2. The autocorrelation of the consumption demand shock is set to 0.80 following Nakata (2016). Finally, we tune the standard deviation of the consumption demand shock in the linearized and nonlinear model such that the model economies observe a probability of being at the ZLB of 10%. The corresponding values are $\sigma_\nu^{\text{linear}} = 0.023$ and $\sigma_\nu^{\text{nonlinear}} = 0.042$. At the solution, the mean duration of ZLB episodes is about 2 quarters in the linear and nonlinear model. The maximum observed duration of ZLB episodes are 17 quarters in the linearized model and 11 quarters in the nonlinar model.

A4: Robustness: Effects of other shocks

In this paper we have focused on the implications of a consumption demand shock and a government consumption shock in a linearized versus nonlinear version of a New Keynesian model. In this subsection we provide insights into how other shocks affect the dynamics of the linearized and nonlinear model. Figures A1 and A2 provide impulse responses of the linearized and nonlinear model to the following six shocks: government consumption shock, consumption demand shock, discount factor shock, technology shock, markup shock, and monetary policy shock.

To introduce the discount factor shock δ_t we specify the household utility function as follows:

$$\max_{\{C_t,N_t,B_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \varsigma_t \left(\log\left(C_t - C\nu_t\right) - \frac{N_t^{1+\chi}}{1+\chi} \right), \tag{A11}$$

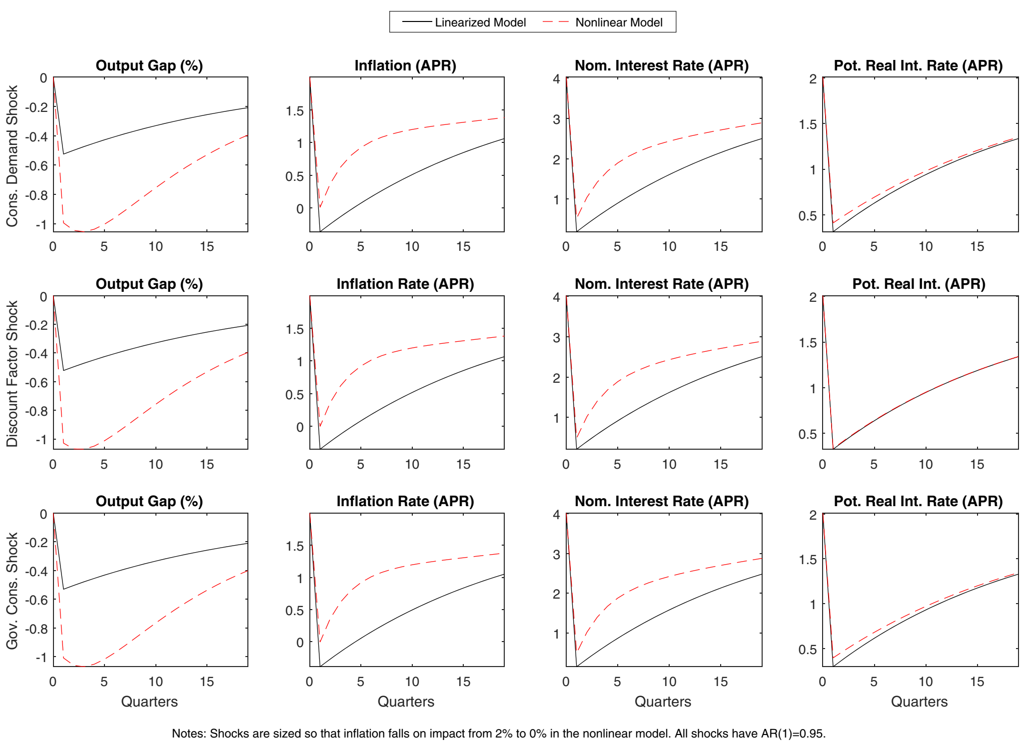

FIGURE A1 Impulse responses to shocks I: linearized versus nonlinear model. *Notes*: Shocks are sized so that inflation falls on impact from 2% to 0% in the nonlinear model. All shocks have AR(1) = 0.95 [Colour figure can be viewed at wileyonlinelibrary.com]

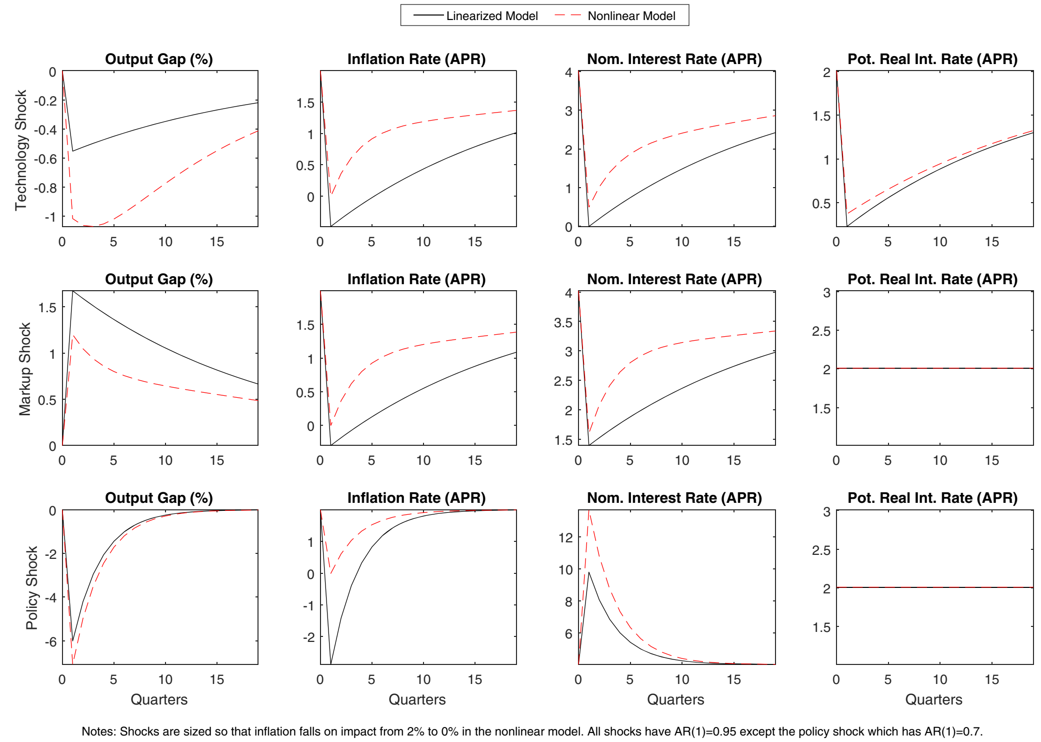

FIGURE A2 Impulse responses to shocks II: linearized versus Nonlinear model. *Notes*: Shocks are sized so that inflation falls on impact from 2% to 0% in the nonlinear model. All shocks have AR(1) = 0.95 except the policy shock, which has AR(1) = 0.7 [Colour figure can be viewed at wileyonlinelibrary.com]

where

$$\delta_t \equiv \frac{E_t \varsigma_{t+1}}{\varsigma_t}. \tag{A12}$$

The technology shock z_t is introduced into the production function:

$$Y_t = z_t \left(p_t^*\right)^{-1} k^{\alpha} N_t^{1-\alpha}.$$

The markup shock ϱ_t is introduced into the equation for marginal cost:

$$\mathrm{mc}_t = \varrho_t \frac{1}{1-\alpha} \frac{w_t}{z_t} k^{-\alpha} N_t^{\alpha}$$

The monetary policy shock ε_t is introduced into the Taylor rule:

$$1 + i_t = \max\left(1, (1+i)\left[\Pi_t/\Pi\right]^{\gamma_\pi} \left[\frac{Y_t}{Y} \Big/ \frac{Y_t^{\mathrm{pot}}}{Y^{\mathrm{pot}}}\right]^{\gamma_x}\right) e^{\varepsilon_t}$$

All shocks are assumed to follow AR(1) processes with autocorrelation 0.95, except the monetary policy shock, which we assume to have an autocorrelation of 0.7. We subject the linearized and the nonlinear models to the same shock. We size the shock such that the inflation rate in the nonlinear model falls from its steady state of 2% to 0% in response to each shock. Figures A1 and A2 show the results.

There are two takeaways from Figures A1 and A2. First, for all six shocks considered, there are substantial differences between the linearized and nonlinear solutions. Second, in the linearized model, the responses of inflation, GDP, and the nominal interest rate to the government consumption shock, the consumption demand shock, the discount factor shock, and the technology shock are (nearly) observationally equivalent. In the nonlinear model the same observation arises; that is, the responses of inflation, GDP, and the nominal interest rate to these shocks are (nearly) observationally equivalent in the nonlinear model.

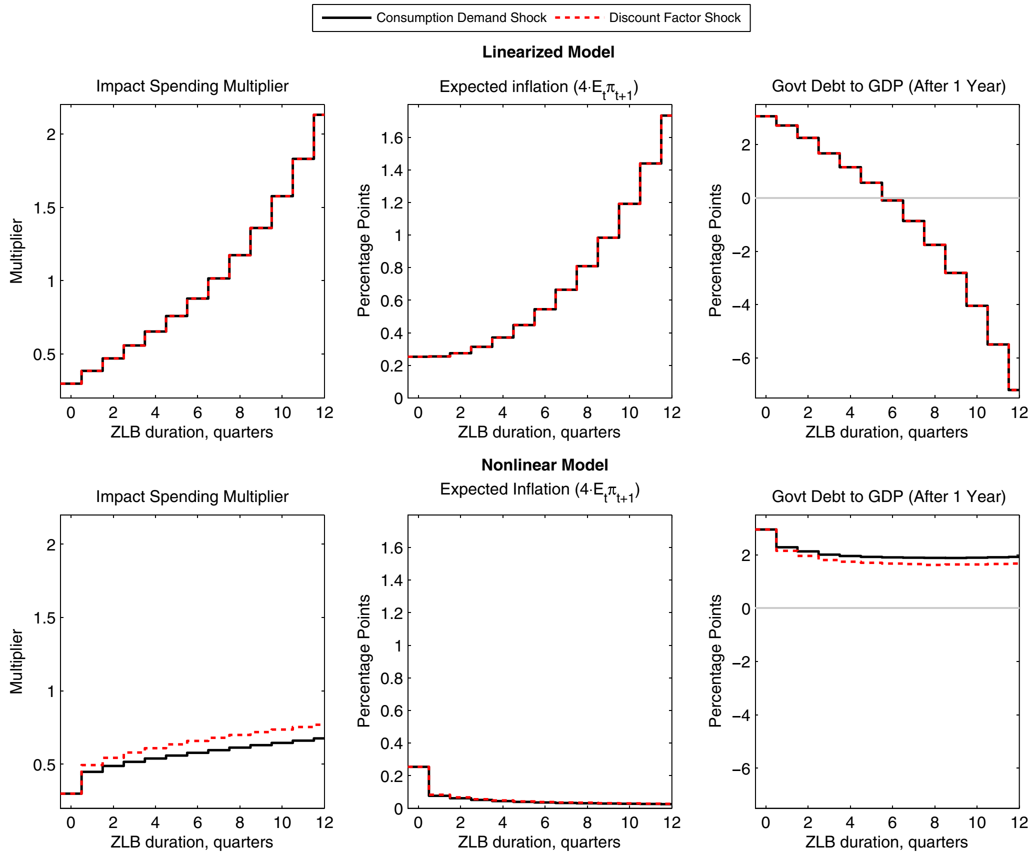

FIGURE A3 Sensitivity of marginal multipliers with respect to baseline shock [Colour figure can be viewed at wileyonlinelibrary.com]

A5: Robustness: Choice of baseline shock

In line with Erceg and Lindé (2014) we use the consumption preference shock ν_t to generate our baselines. A negative shock to ν_t implies that both potential output and the real interest rate fall (see Figure 2). In contrast, most papers in the literature on fiscal multipliers have assumed that an increased desire to save, represented by a higher discount factor, causes the natural real rate to fall below zero and thereby triggers the economy to enter into a liquidity trap. A higher discount factor leaves potential output unchanged, and consequently has the flavor of a negative demand shock when output (and the output gap) contracts because monetary policy cannot cut the policy rate below zero to mimic the fall in the natural real rate.

To ensure that our results hold up when we follow the bulk of the literature, we present results when the recession is assumed to be triggered by a discount factor shock, as used in the seminal papers by Eggertsson and Woodford (2003) and Christiano et al. (2011). For the linearized model, we establish that the results are invariant with respect to the the choice of baseline shock (see Erceg & Lindé, 2014, for analytical proofs). For the nonlinear model, Figure A3 shows that the multiplier schedules are nearly invariant with respect to the baseline shock.[26]

Figure A3 reports results when the discount factor shock δ_t defined in Equation A12 is driving the baseline in Figure 2. For ease of comparison, the benchmark results with the consumption preference shock ν_t driving the baseline are also included. The upper panels of Figure A3 confirm the results by Erceg and Lindé (2014) by showing that the fiscal spending multiplier is independent of the shock driving the baseline when the model is linearized, as long as the different baseline shocks generate an equally long-lived ZLB episode. Thus our choice to work with the consumption preference shock ν_t instead of the discount factor shock δ_t has no consequences for our results in the linearized model. As for the nonlinear model, the lower panels in Figure A3 show that the results are very similar even in the nonlinear solution, so our choice of the baseline shock appears to be unproblematic.

[26]We have also checked the robustness of the nonlinear multiplier schedule w.r.t. technology shocks generating the baseline, and found that the results are robust in this case as well.

A6: Robustness: Kimball versus Dixit–Stiglitz aggregator

To further tease out the difference between the Kimball versus Dixit–Stiglitz aggregator, Figure A4(a) compares outcomes when the sticky price parameter ξ is adjusted in the Dixit–Stiglitz version so that the slope of the linearized Phillips curve (Equation 17) is the same as in our benchmark Kimball calibration. Both the Kimball and Dixit–Stiglitz versions hence now feature a linearized Phillips curve with an identical slope coefficient ($\kappa = 0.012$; see Equation 18), but the Dixit–Stiglitz version of the model achieves this with a substantially higher value of $\xi = 0.90$. However, since only the value of κ matters in the linearized solution, the multiplier schedules are invariant w.r.t. the mix of ξ and ψ that achieves a given κ in the linearized model. Consequently, the linearized solution for the Dixit–Stiglitz aggregator is thus identical to the Kimball solution depicted by the solid black line in the upper panel in Figure 3.

Even so, the nonlinear solutions shown in Figure A4(a) differ. In particular, we see that the Dixit–Stiglitz aggregator implies that expected inflation and the output multiplier respond more when the duration of the liquidity trap increases. Thus, when the Kimball parameter ψ goes toward zero, the more will expected inflation and the output multiplier respond when ZLB_{DUR} increases; conversely, making ψ more negative and lowering ξ flattens the output multiplier schedule even more. The explanation behind this finding is that a more negative value of ψ induces the elasticity of demand to vary more with the relative price differential among the intermediate good firms as shown in Figure 1, and this price differential increases when the economy is far from the steady state. Thus intermediate goods firms which only infrequently are able to re-optimize their price will optimally choose to respond less to a given fiscal impulse far from the steady state when price differentials are larger, as they perceive that they may have a much larger impact on their demand for a given change in their relative price. As a result, aggregate inflation and expected inflation are less affected far from the steady state in the Kimball case relative to the Dixit–Stiglitz case for which the elasticity of demand is independent of the relative price differential. This demonstrates that the modeling of price-setting frictions matters importantly within a nonlinear framework.

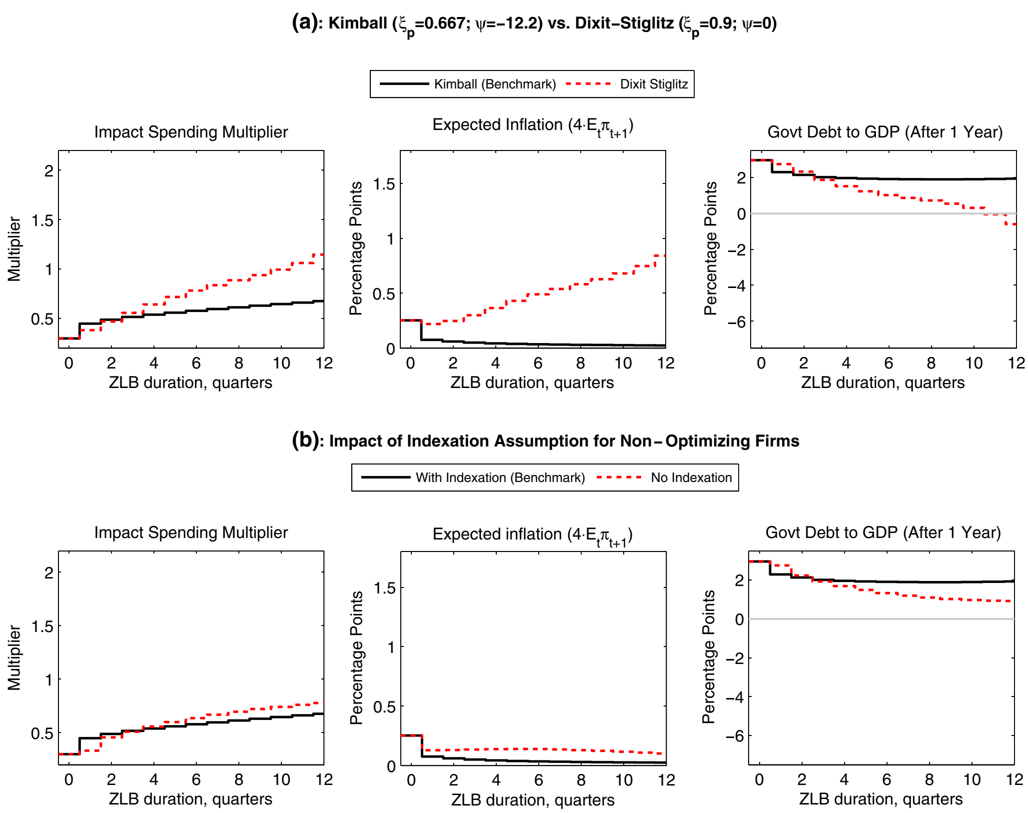

FIGURE A4 Sensitivity analysis of marginal multipliers in nonlinear model: (a) Kimball ($\xi_p = 0.667$; $\phi = -12.2$) versus Dixit–Stiglitz ($\xi_p = 0.9$; $\phi = 0$); (b) impact of indexation assumption for nonoptimizing firms [Colour figure can be viewed at wileyonlinelibrary.com]

A7: Robustness: Price indexation

So far, we have followed the convention in the literature and assumed that non-optimizing firms index their prices to the steady-state rate of inflation (see Equation 11). This is a convenient benchmark modeling assumption as it simplifies the analysis by removing steady-state price distortions. However, the indexation assumption has been criticized for being inconsistent with the microeconomic evidence on price-setting behavior of firms. According to micro evidence on price setting, prices set by firms remain unchanged for several quarters. By contrast, the indexation scheme in our model (as well as in most of the literature) implies that prices changes in each quarter: either because firms can choose an optimal price or because of mechanical indexation of the price set in the previous period.

To examine the importance of the indexation assumption for the resulting fiscal multiplier we reformulate the model. In particular, following Ascari and Ropele (2007) and Christiano, Eichenbaum, and Trabandt (2015, 2016), for example, we do not allow non-optimizing firms to index their prices. These firms must keep their price unchanged, that is:

$$\tilde{P}_t = P_{t-1}. \tag{A13}$$

Figure A5(b) reports the results for the benchmark nonlinear model (black solid lines) with the version of the nonlinear model when indexation is not allowed (red dotted lines). From the panels, we see that abandoning the conventional assumption of full indexation results in a somewhat steeper fiscal multiplier schedule. The steeper fiscal multiplier schedule is due to the higher sensitivity of expected inflation in the "no-indexation" model since firms take into account in their price-setting decisions that their prices will not automatically adjust in response to shocks. We verified that the fiscal marginal multipliers in the linearized model are also roughly unchanged (not shown in the figure). All told, our benchmark results are robust with respect to the price indexation assumption.

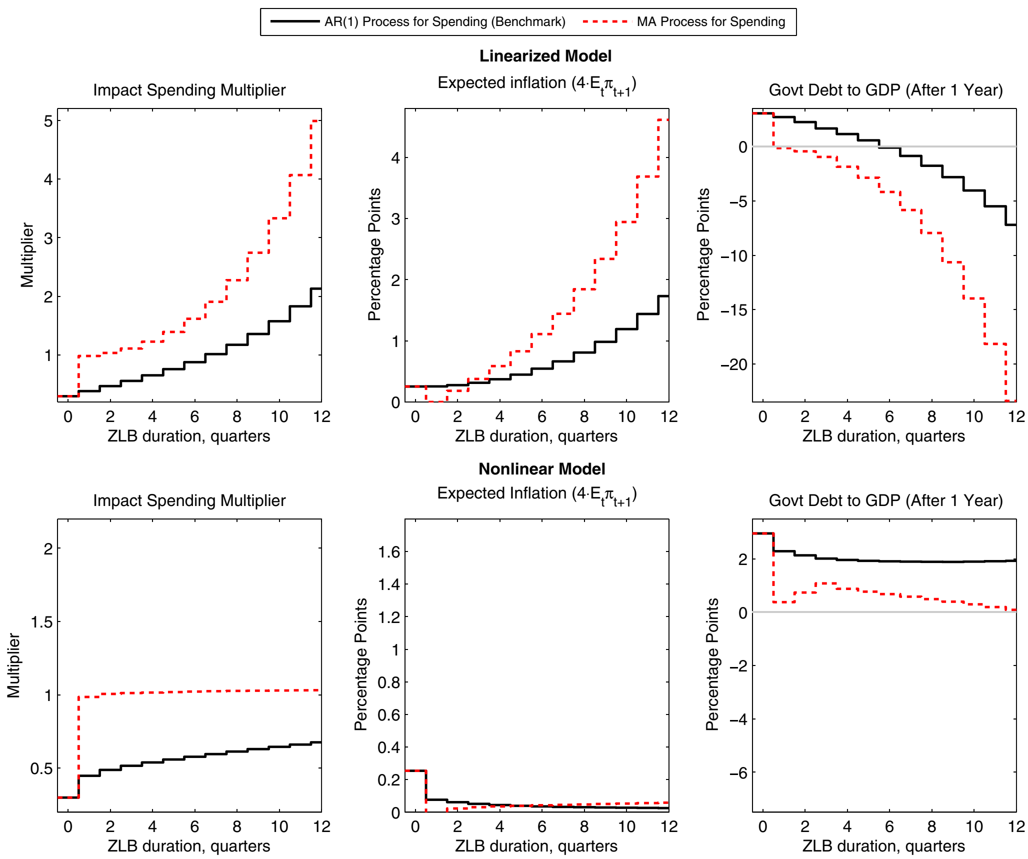

FIGURE A5 Sensitivity of marginal multipliers with respect to specification of spending process [Colour figure can be viewed at wileyonlinelibrary.com]

A8: Robustness: Government spending process

Another aspect we want to understand is how our results differ from Boneva et al. (2016) due to our AR(1) assumption for government spending instead of the moving average (MA) process they work with. Figure A5 assesses this issue by comparing the results of our benchmark AR(1) process for G_t against an MA process in which G_t is elevated to a higher level as long as the policy rate is bounded at zero and set to its steady-state value otherwise. Apart from the fact that our benchmark solution procedure does not account for shock uncertainty, this approach of modeling government spending is identical to Boneva et al. (2016), who in turn follow Eggertsson (2010).

As can be seen from the upper panels of Figure A5, the MA process increases the marginal spending multiplier at the ZLB substantially relative to the AR(1) process. The multiplier is higher because increases in government spending have very benign effects on the potential real interest rate when the duration of the spending hike equals the expected duration of the liquidity trap (see, e.g., Erceg & Lindé, 2014). For a one-quarter liquidity trap the multiplier equals unity, as shown analytically by Woodford (2011). Our fairly persistent AR(1) process tends to dampen the multiplier schedule, since a relatively large fraction of spending occurs when the ZLB is no longer binding. This feature explains why the AR(1) multiplier is substantially lower in a short-lived liquidity trap. However, the AR(1) process is also associated with a substantially lower multiplier even in a fairly long-lived trap compared to the MA process because its has less benign effects on the potential real rate.

All this is well known in the body of work focusing on linearized models. However, the results for the nonlinear model, shown in the lower panels of Figure A5, are much less explored. We have already discussed the AR(1) case at length in the text. What we see is that the results for the MA process are quite different for longer ZLB durations, because the MA schedule for the nonlinear model stays essentially flat at unity, in line with the findings of Boneva et al. (2016); for a 12-quarter trap the multiplier only increases to 1.03 from a multiplier of unity in a one-period liquidity trap. This is in sharp contrast to the multiplier schedule for the linearized model, where the multiplier is as high as 5 in a liquidity trap lasting 3 years. All told, the results show that our benchmark results hold up well for an MA process for government spending. If anything, an MA process magnifies the differences between the linearized and nonlinear solution in terms of the multiplier. Moreover, the linear and nonlinear model results in Figure A5 are in line with the existing literature.