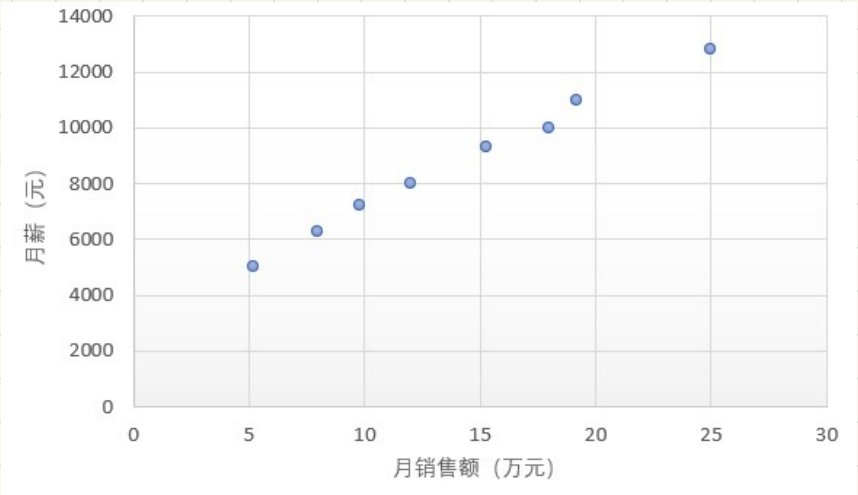
人智原 HW4

1、(1) 如图所示.



(2) 由最小二乘法知

$$\hat{w} = \frac{\sum_{i=1}^{8}(x_i-\bar{x})(y_i-\bar{y})}{\sum_{i=1}^{8}(x_i-\bar{x})^2} = \frac{S_{xy}}{S_{xx}}$$

又 $\bar{x} = 14.0625$, $\bar{y} = 8700$,

$S_{xy} = 32791.75 + 6393.75 + 742.5 + 11816.25 + 44843.75 + 14550 + 10443.75 + 5118.75 = 117700$

$S_{xx} = 78.5439 + 18.1689 + 1.5314 + 26.3939 + 119.6289 + 36.7539 + 4.2539 + 15.5039 = 300.7788$

$\therefore \hat{w} = 391.3$, $\hat{b} = \bar{y} - \bar{x}\hat{w} = 3197$

∴ 线性回归方程

$$y = 391.3x + 3197$$

回归系数 $r^2 = \frac{S_{xy}^2}{S_{xx} S_{yy}} = 0.9939$

(3) $MAE = \frac{1}{8}\sum_{i=1}^{8} |y_i - \hat{y}_i| = 170.1$

$MSE = \frac{1}{8}\sum_{i=1}^{8} (y_i - \hat{y}_i)^2 = 35240.85$

由 MAE 和 $\sqrt{MSE}$ 都远小于 $\bar{y}$，
相对误差非常小，故(2)中线性函数拟合的很好.

4. 推导如下：

由 $\log P(Y=k) = \beta_k x - \log Z$

得 $\log ZP(Y=k) = \beta_k x$

即 $P(Y=k) = \frac{1}{Z} e^{\beta_k x}$ (1)

又 $\sum_{j=1}^{k} P(Y=j) = \sum_{j=1}^{k} \frac{1}{Z} e^{\beta_j x} = 1$

∴ $Z = \sum_{j=1}^{k} e^{\beta_j x}$

代入(1)，$P(Y=k) = \frac{e^{\beta_k x}}{\sum_{j=1}^{k} e^{\beta_j x}}$