作业 5

2、a) $\hat{y}^{(n)} = \sigma \left(\sum_{h=1}^{2} W_{ho} V_h^{(n)} + \sum_{i=1}^{3} W_{io} x_i^{(n)} \right)$

$= \sigma \left(\sum_{h=1}^{2} \left(W_{ho} \sigma \left(\sum_{i=1}^{3} W_{ih} x_i^{(n)} \right) \right) + \sum_{i=1}^{3} W_{io} x_i^{(n)} \right)$

b) $\dfrac{\partial Loss}{\partial W_{io}} = \dfrac{1}{N} \sum_{n=1}^{N} \left(\dfrac{\partial loss}{\partial \hat{y}^{(n)}} \dfrac{\partial \hat{y}^{(n)}}{\partial W_{io}} \right)$

$= \dfrac{1}{N} \sum_{n=1}^{N} \left(2 (\hat{y}^{(n)} - y^{(n)}) \, \hat{y}^{(n)} (1 - \hat{y}^{(n)}) \, x_i^{(n)} \right)$.

$i \in \{1, 2, 3\}$

c) 求梯度 $\dfrac{\partial Loss}{\partial x_i^{(n)}} = \dfrac{\partial Loss}{\partial \hat{y}^{(n)}} \cdot \dfrac{\partial \hat{y}^{(n)}}{\partial x_i^{(n)}}$

$= 2 (\hat{y}^{(n)} - y^{(n)}) \, \hat{y}^{(n)} (1 - \hat{y}^{(n)}) \cdot \left(W_{io} + \sum_{h=1}^{2} W_{ho} V_h^{(n)} (1 - V_h^{(n)}) W_{ih} \right)$

更新公式：

$$x_i^{(n)} \leftarrow x_i^{(n)} - \eta \dfrac{\partial loss}{\partial x_i^{(n)}}$$

3. 网络输出值为
$$\begin{matrix} 1 & 1.5 \\ 1 & 1.5 \end{matrix}$$

$$\frac{\partial L}{\partial H_j} = \sum_{i=1}^{4} \frac{\partial L}{\partial y_i} \cdot \frac{\partial y_i}{\partial H_j} = \sum_{i=1}^{4} (y_i - d_i) \frac{\partial y_i}{\partial H_j}$$

可计算得 $\frac{\partial L}{\partial H_1} = 0$ $\frac{\partial L}{\partial H_2} = 1$

$\frac{\partial L}{\partial H_3} = 0$ $\frac{\partial L}{\partial H_4} = 1$

∴ 更新权值 $H_i \leftarrow H_i - \alpha \frac{\partial L}{\partial H_i}$

得更新后 H_m 为 $\begin{bmatrix} 0.2 & -0.3 \\ 0.3 & -0.2 \end{bmatrix}$