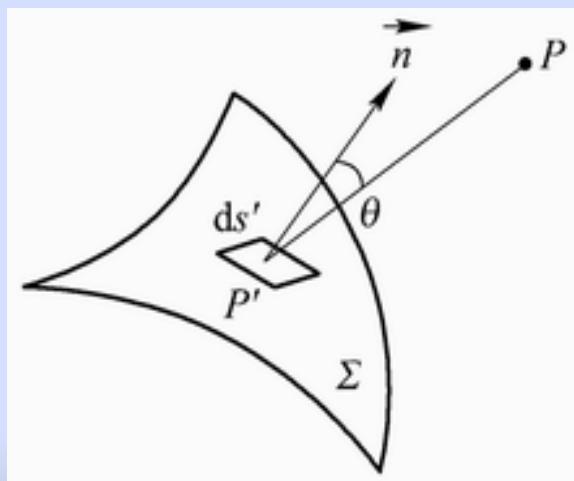


第3章 激光器的输出特性

- 3. 1光学谐振腔的衍射理论
- 3. 2对称共焦腔内外的光场分布
- 3. 3高斯光束的传播特性
- 3. 4稳定球面腔的光束传播特性
- 3. 5激光器的输出功率
- 3. 6激光器的线宽极限
- 3. 7激光光束质量的品质因子

3.1 光学谐振腔的衍射理论

一、菲涅耳-基尔霍夫衍射公式

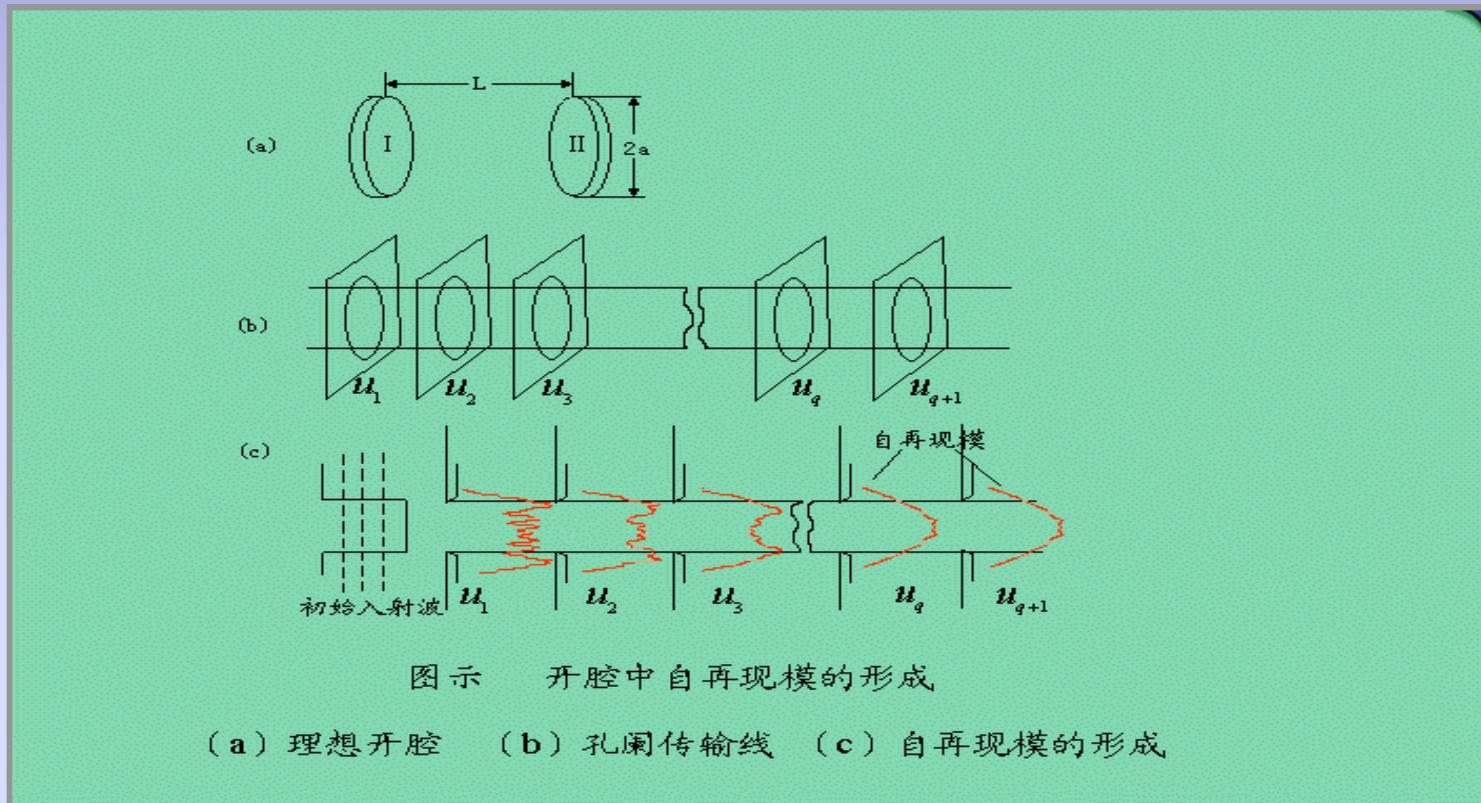


$$u(P) = \frac{ik}{4\pi} \iint_{\Sigma} \frac{u'(P') e^{-ik\rho}}{\rho} \cdot (1 + \cos\theta) ds'$$

图3-1 惠更斯-菲涅耳原理

二. 光学谐振腔的自再现模积分方程

■ 自再现模

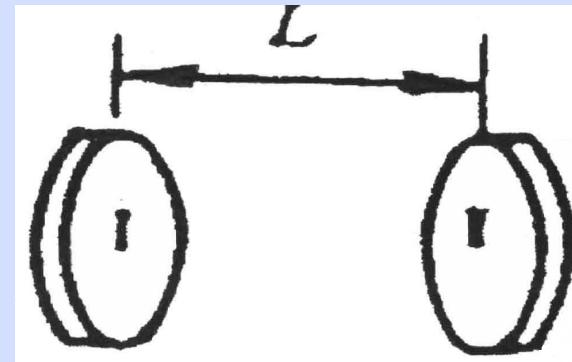


平面波在平行平面镜腔中的传输

腔模：光腔中可能存在的电磁场空间分布状态

横模

纵模



■自再现模积分方程

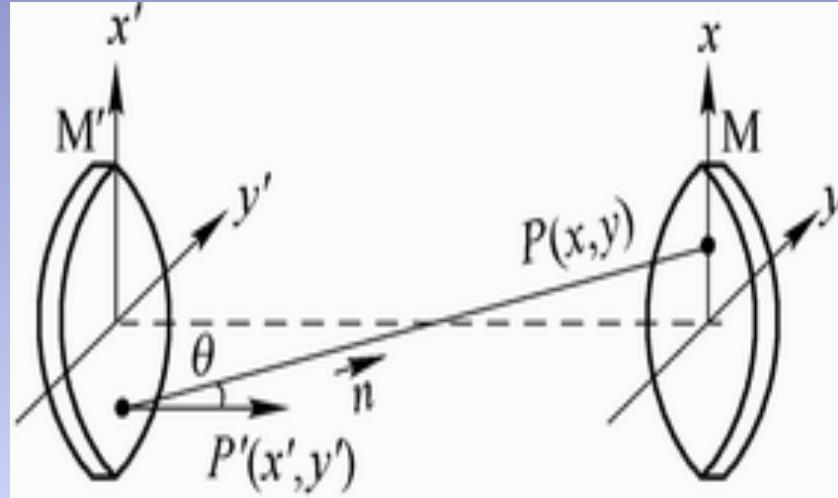


图3-2 镜面上场分布的计算示意图

$$u_{q+1}(x, y) = \frac{ik}{4\pi} \iint_M u_q(x', y') \frac{e^{-ik\rho}}{\rho} \cdot (1 + \cos\theta) ds' \quad (3-2)$$

$$u_{q+1} = \sigma u_q \quad (3-3)$$

$$\sigma u(x, y) = \frac{ik}{4\pi} \iint_{M'} u(x', y') \frac{e^{-ik\rho}}{\rho} (1 + \cos \theta) ds'$$

$$\sigma_{mn} u_{mn}(x, y) = \iint K(x, y, x', y') u_{mn}(x', y') ds' \quad (3-6)$$

$$K(x, y, x', y') = \frac{ik}{2\pi L} e^{-ik\rho(x, y, x', y')} = \frac{i}{\lambda L} e^{-ik\rho(x, y, x', y')}$$

自再现模积分方程

本征函数 u_{mn} : 横模 TEM _{$_{mn}$}

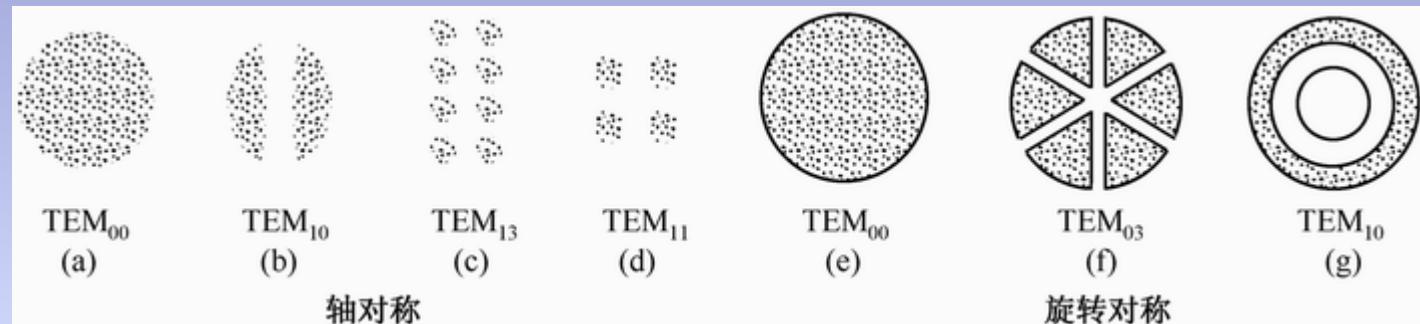


图3-3 横模光斑示意图

单程衍射损耗，用 δ 表示。

$$\left. \begin{aligned} \delta &= \frac{\left| u_q \right|^2 - \left| u_{q+1} \right|^2}{\left| u_q \right|^2} \\ u_{q+1} &= \sigma u_q \end{aligned} \right\} \Rightarrow \delta_{mn} = 1 - |\sigma_{mn}|^2$$

自再现模单程渡越所产生的总相移

$$\delta\Phi = \arg u_{q+1} - \arg u_q = \arg \sigma$$

$$\left. \begin{aligned} \delta\Phi &= -kL + \Delta\phi \\ \delta\Phi &= \arg \sigma \end{aligned} \right\} \Rightarrow \Delta\phi_{mn} = kL + \arg \sigma_{mn}$$

附加相移

三、光学谐振腔谐振频率和激光纵模

Ne原子的0.6328 μm 谱线的频率宽度为

$$\nu = \frac{c}{\lambda} = \frac{3 \times 10^8}{0.6328 \times 10^{-6}} \approx 5 \times 10^{14} \text{ Hz}$$

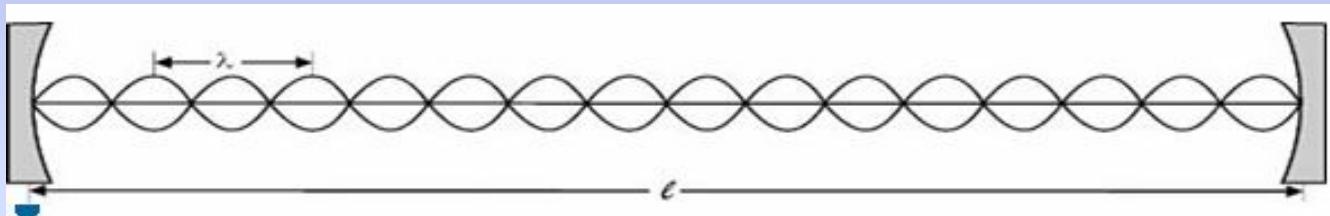
$$\Delta\nu \approx 1.3 \times 10^9 \text{ Hz}$$

$$\frac{\Delta\nu}{\nu} = \frac{1.3 \times 10^{-9}}{5 \times 10^{-14}} \approx 3 \times 10^{-6}$$

实际上He—Ne激光器 $\frac{\Delta\nu}{\nu} < 10^{-15}$

谐振条件：光波在腔内往返一周的总相移应等于 2π 的整数倍

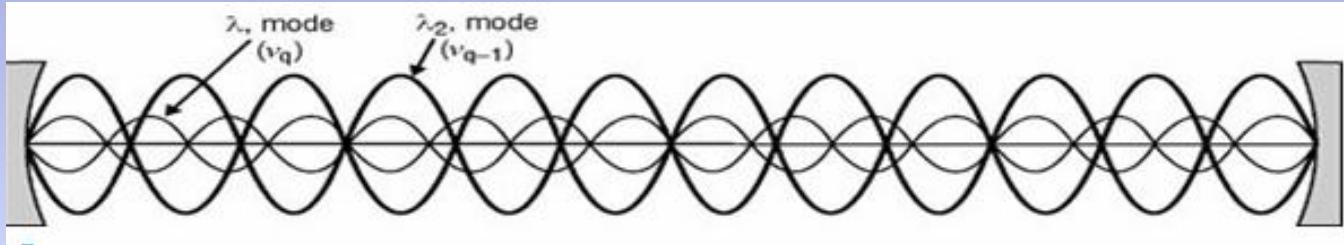
$$2\delta\Phi = 2q\pi \quad q = 1, 2, 3, \dots$$



$$\left. \begin{array}{l} \delta\Phi = -kL + \Delta\phi \\ 2\delta\Phi = 2q\pi \\ k = 2\pi\mu\nu/c \end{array} \right\} \Rightarrow v_{mnq} = \frac{qc}{2\mu L} + \frac{c}{2\pi\mu L} \Delta\phi_{mn} \approx \frac{qc}{2\mu L}$$

$$v_{mnq} = \frac{qc}{2\mu L} \quad \text{——谐振频率}$$

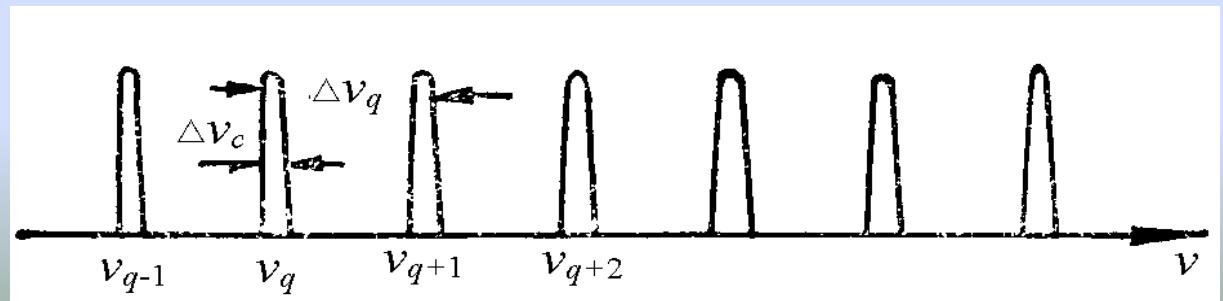
纵模:由纵模序数q所表征的腔内纵向稳定场分布

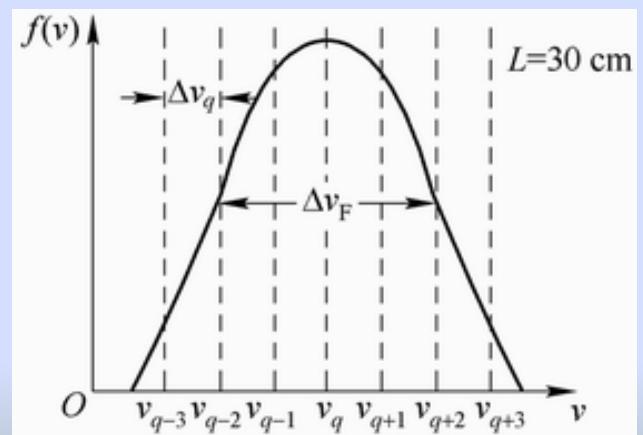


纵模的频率间隔

$$\nu_{mnq} = \frac{qc}{2\mu L} + \frac{c}{2\pi\mu L} \Delta\phi_{mn} \quad \Rightarrow \Delta\nu_q = \nu_{q+1} - \nu_q = \frac{c}{2\mu L}$$

频率梳

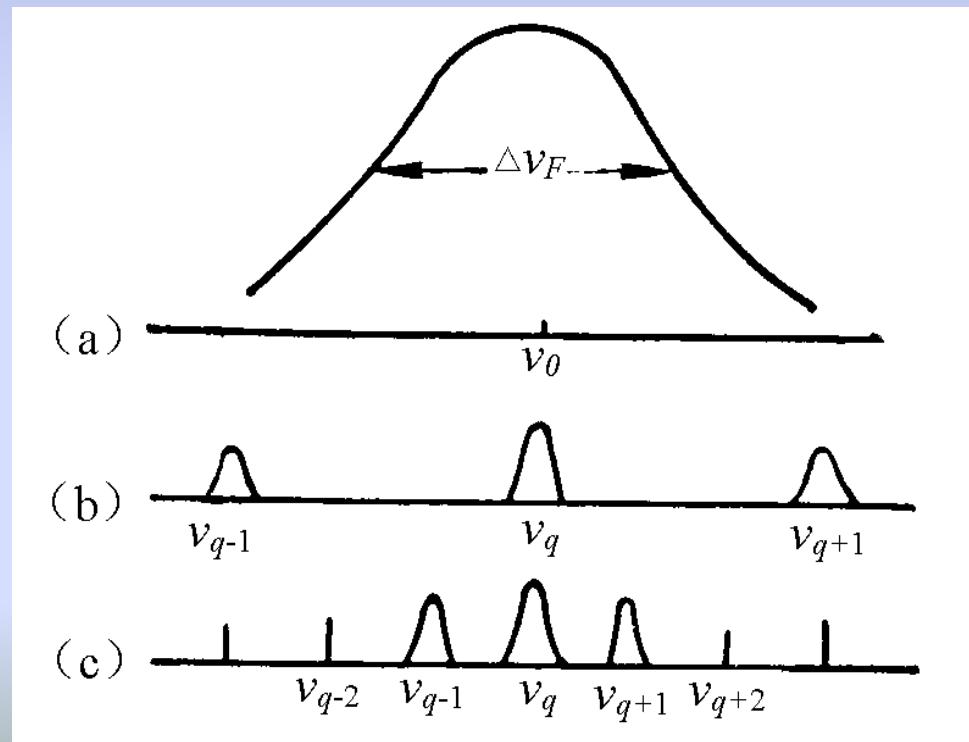




图(3-4) 腔中允许的纵模数

例: $L=10\text{cm}$ 的 H_e-N_e 激光器, 单模

例: $L=30\text{cm}$, $\Delta\nu_q=5\times 10^8\text{Hz}$, 三纵模



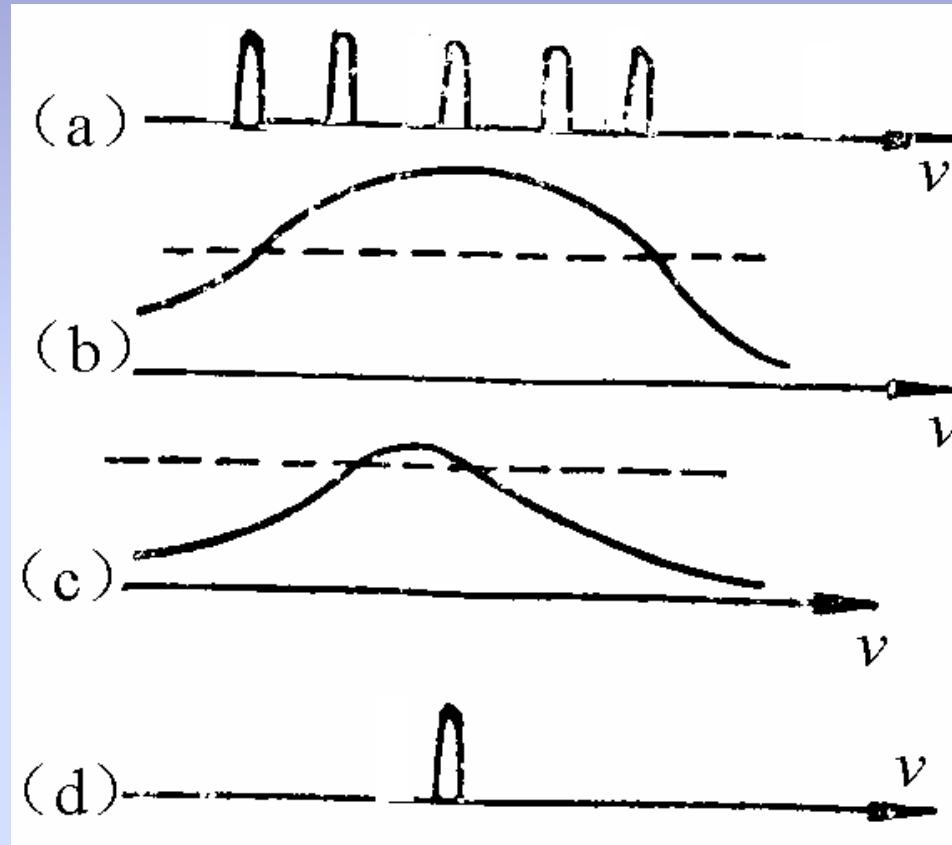
CO_2 激光器: $\lambda=10.6\mu\text{m}$ $\Delta v_F \approx 10^8\text{s}^{-1}$ $L=1\text{m}$

$\Delta v_q = 1.5 \times 10^8\text{s}^{-1}$ 激光器输出单模

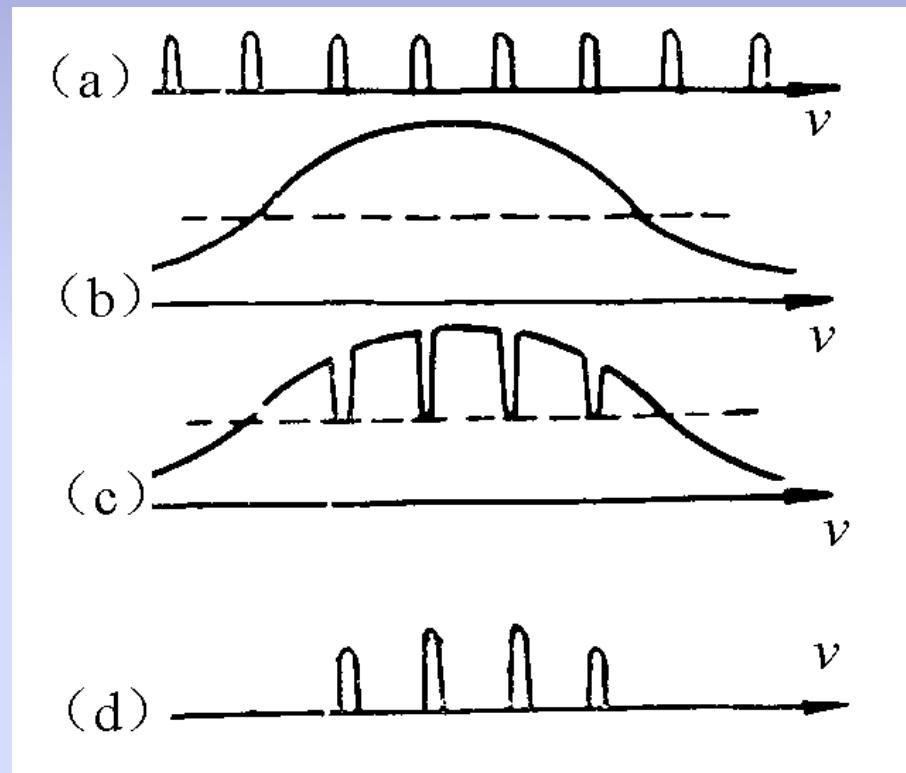
氩离子激光器: $\lambda=0.5145\mu\text{m}$ $\Delta v_F \approx 6 \times 10^8\text{s}^{-1}$ $L=1\text{m}$

$\Delta v_q = 1.5 \times 10^8\text{s}^{-1}$ 激光器多模输出

均匀增宽工作物质



非均匀增宽介质

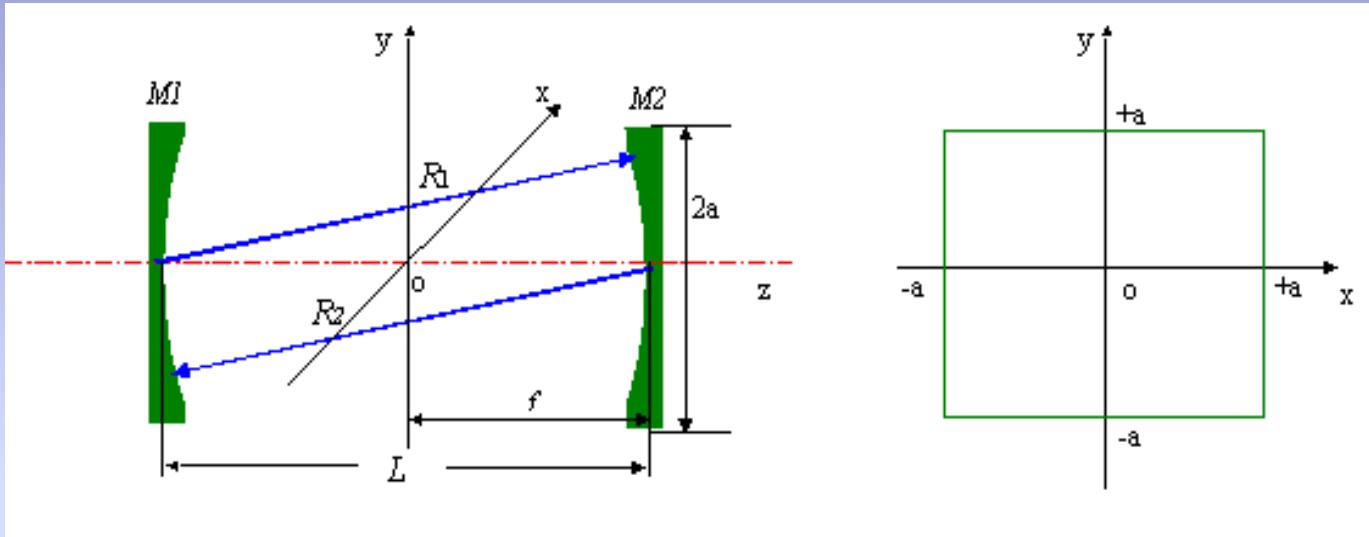


3.2 对称共焦腔内外的光场分布

$$\sigma_{mn} u_{mn}(x, y) = \iint K(x, y, x', y') u_{mm}(x', y') ds' \quad (3-7)$$

$$K(x, y, x', y') = \frac{ik}{2\pi L} e^{-ik\rho(x, y, x', y')} = \frac{i}{\lambda L} e^{-ik\rho(x, y, x', y')}$$

方形镜面对称共焦腔



$$L \gg a \gg \lambda,$$

$$R_1 = R_2 = L = 2f$$

$$\frac{a^2}{L\lambda} \gg \left(\frac{L}{a}\right)^2$$

$$\sigma_{mn} u_{mn}(x, y) = \iint K(x, y, x', y') u_{mm}(x', y') ds' \\ u_{mn} \approx C_{mn} H_m(X) H_n(Y) e^{-\frac{X^2 + Y^2}{2}} \quad (3-18)$$

其中 $X = x\sqrt{\frac{2\pi}{\lambda L}}$, $Y = y\sqrt{\frac{2\pi}{\lambda L}}$

$H_m(X)$ 和 $H_n(Y)$ 均为厄密多项式

$$\sigma_{mn} = \exp[-i(kL - (m+n+1)\frac{\pi}{2})] \quad (3-19)$$

$$u_{mn}(x,y)=C_{mn}H_m(\sqrt{\frac{2\pi}{L\lambda}}x)H_n(\sqrt{\frac{2\pi}{L\lambda}}y)e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

$$H_0(\zeta)=1 \qquad \qquad H_1(\zeta)=2\zeta$$

$$H_2(\zeta)=4\zeta^2-2$$

$$H_3(\zeta)=8\zeta^3-12\zeta$$

$$H_4(\zeta)=16\zeta^4-48\zeta^2+12$$

$$H_m(\xi)=(-1)^m\,e^{\xi^2}\,\frac{{\rm d}^m}{{\rm d}\xi^m}\,e^{-\xi^2}$$

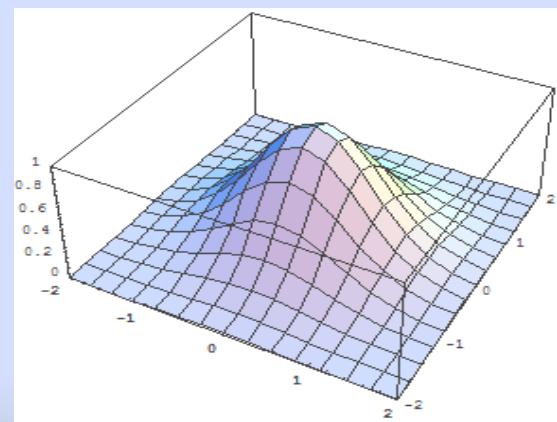
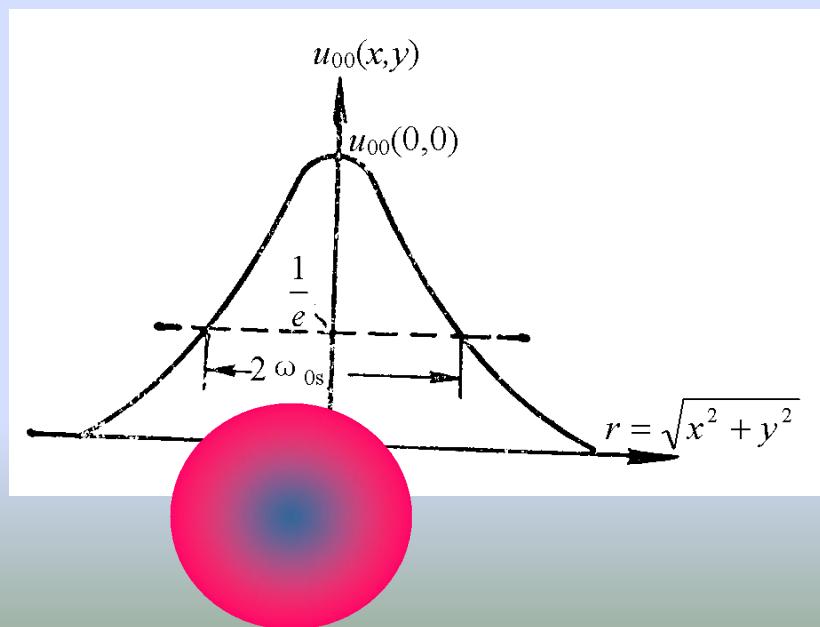
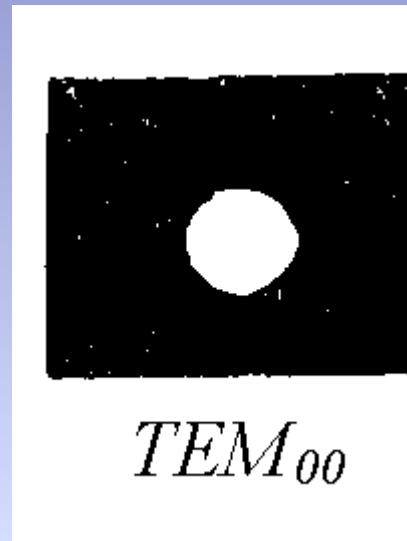
$$u_{mn}(x, y) = C_{mn} H_m \left(\sqrt{\frac{2\pi}{L\lambda}} x \right) H_n \left(\sqrt{\frac{2\pi}{L\lambda}} y \right) e^{-\frac{x^2+y^2}{L\lambda/\pi}} \quad (3-18)$$

基模 TEM_{00} $u_{00}(x, y) = C_{00} e^{-\frac{x^2+y^2}{L\lambda/\pi}}$

基模振幅最
大值的 $1/e$ 处

基模光斑半径 $\omega_{0s} = \sqrt{\frac{L\lambda}{\pi}}$

$$u_{00}(x, y) = C_{00} e^{-\frac{x^2 + y^2}{L\lambda/\pi}}$$



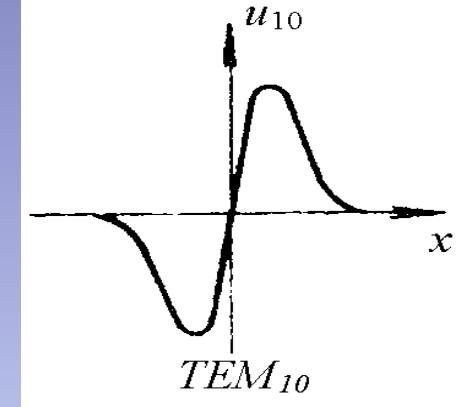
数值例： $L=1m$, $\lambda=10.6\mu m$,

共焦腔的 CO_2 激光器 $\omega_{0s} \approx 1.84mm$

$L=30cm$, $\lambda=0.6328\mu m$, 共焦腔的 H_e-N_e 激光器

$\omega_{0s} \approx 0.25mm$ 可见，共焦腔的光斑半径非常小。

$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

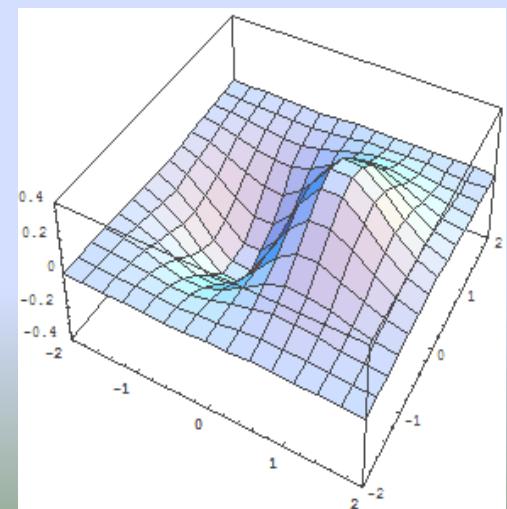


$$u_{10} = C_{10} \frac{2\sqrt{2}}{\omega_{0s}} x \cdot \exp\left(-\frac{x^2 + y^2}{L\lambda/\pi}\right)$$

$$= C'_{10} x \cdot \exp\left(-\frac{x^2 + y^2}{\omega_{0s}}\right)$$



TEM_{10}



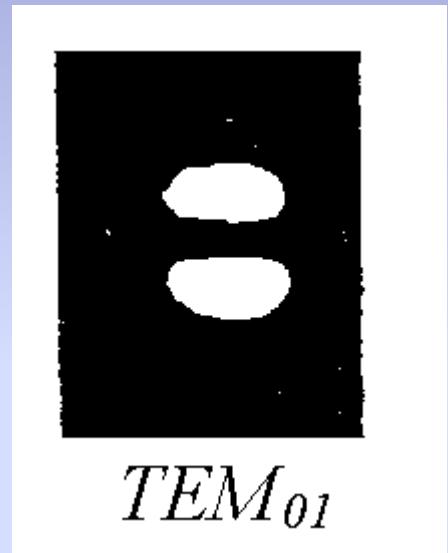
$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

因为 $H_1(\xi) = 2\xi$ $H_0(\xi) = 1$

$$u_{01} = C_{01} \frac{2\sqrt{2}}{\omega_{0s}} y \cdot \exp\left(-\frac{x^2 + y^2}{L\lambda/\pi}\right)$$

$$= C_{01}' y \cdot \exp\left(-\frac{x^2 + y^2}{\omega_{0s}}\right)$$

当 $y = 0$ 时 $u_{01} = 0$, 出现一条暗线



$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

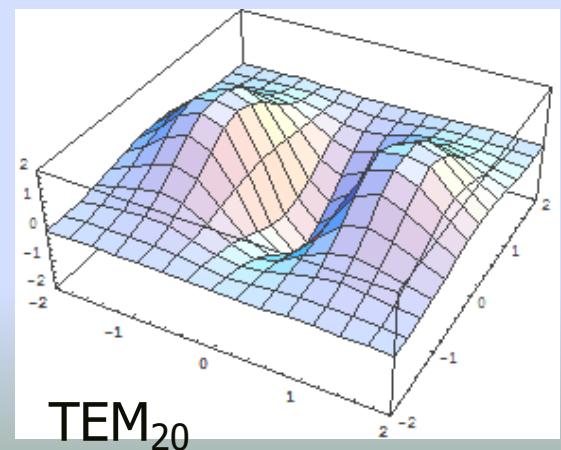
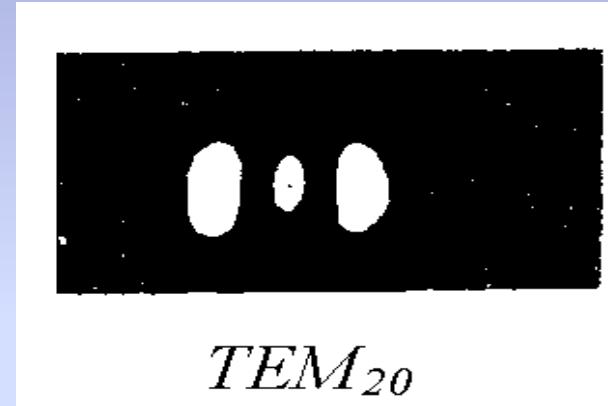
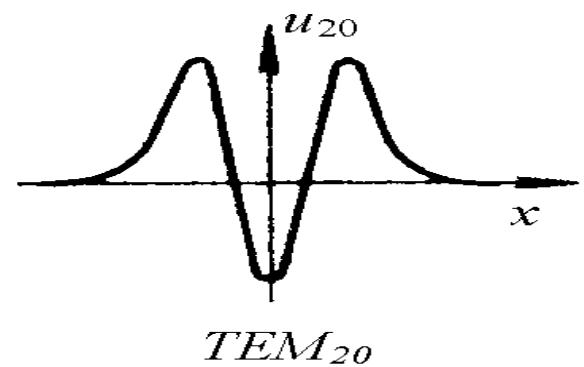
因为 $H_2(\xi) = 4\xi^2 - 2$ $H_0(\xi) = 1$ 故

$$u_{20} = C_{20} \left[4 \frac{2x^2}{\omega_{0s}^2} - 2 \right] \cdot \exp\left(-\frac{x^2 + y^2}{L\lambda}\right)$$

$$= C_{20}' (4x^2 - \omega_{0s}^2) \cdot \exp\left(-\frac{x^2 + y^2}{\omega_{0s}^2}\right)$$

$$x = \pm \frac{\omega_{0s}}{2} \quad \text{时} \quad u_{20} = 0$$

x 方向出现两条暗线



$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

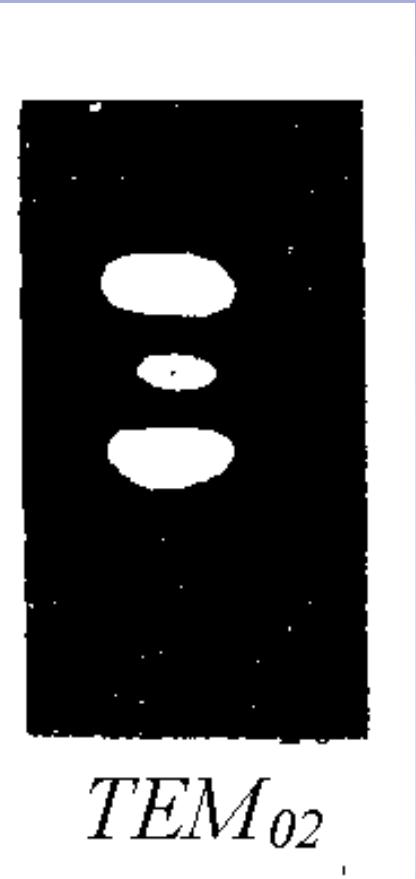
因为 $H_2(\xi) = 4\xi^2 - 2$ $H_0(\xi) = 1$ 故

$$u_{02} = C_{02} \left[4 \frac{2y^2}{\omega_{0s}^2} - 2 \right] \cdot \exp\left(-\frac{x^2 + y^2}{L\lambda}\right)$$

$$= C_{02}' (4y^2 - \omega_{0s}^2) \cdot \exp\left(-\frac{x^2 + y^2}{\omega_{0s}^2}\right)$$

$$y = \pm \frac{\omega_{0s}}{2} \quad \text{时} \quad u_{02} = 0$$

y 方向出现两条暗线



$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

因为 $H_1(\xi) = 2\xi$ 故

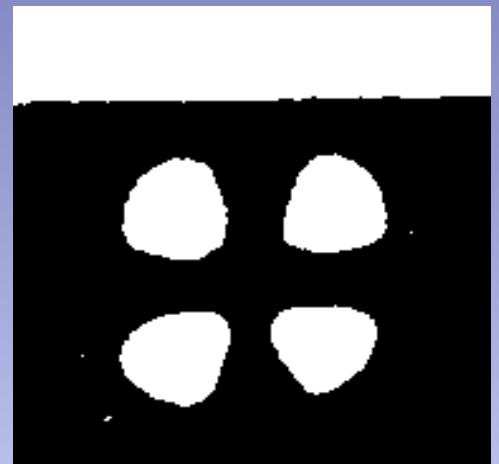
$$u_{11} = C_{11} \cdot 4 \cdot \frac{2}{\omega_{0s}^2} xy \cdot \exp\left(-\frac{x^2 + y^2}{\omega_{0s}^2}\right)$$

$$= C_{11} xy \cdot \exp\left(-\frac{x^2 + y^2}{\omega_{0s}^2}\right)$$

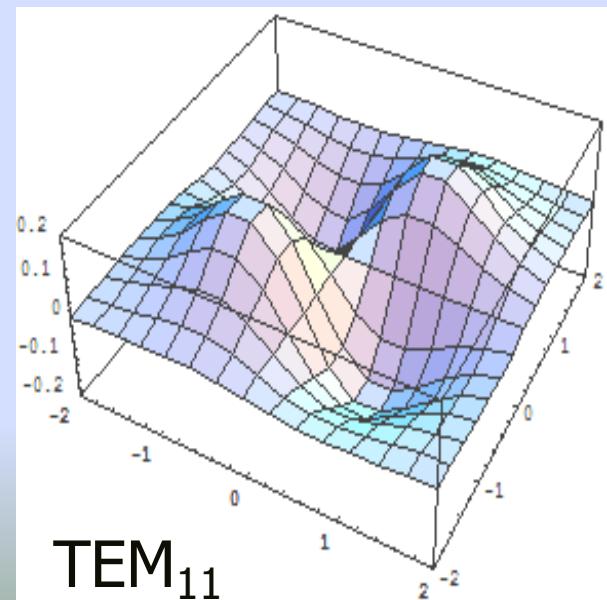
当 $x = 0$ 时 $u_{11} = 0$

$y = 0$ 时 $u_{11} = 0$

在 x 、 y 方向各出现一条暗线



TEM_{11}



TEM_{11}

$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

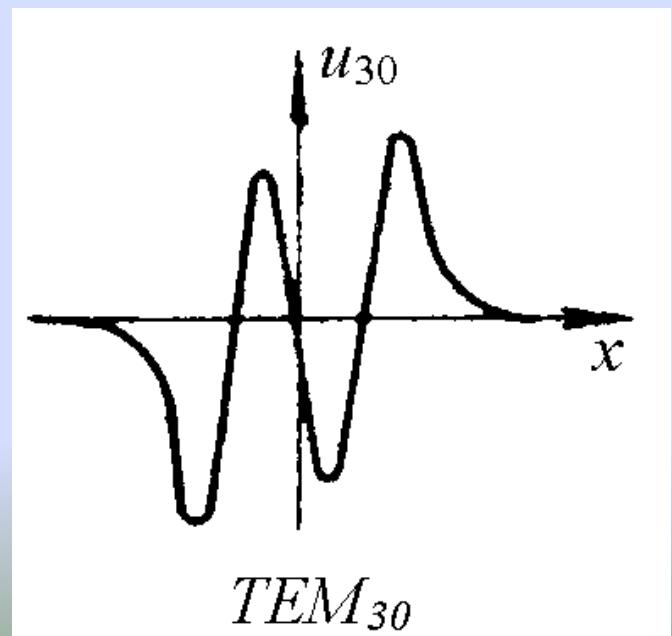
因为 $H_3(\xi) = 8\xi^3 - 12\xi$ $H_0(\xi) = 1$ 故

$$u_{30} = C_{30} \left(\frac{16\sqrt{2}}{\omega_{0s}^3} x^3 - \frac{12\sqrt{2}}{\omega_{0s}} x \right) \exp\left(-\frac{x^2 + y^2}{\omega_{0s}^2}\right)$$

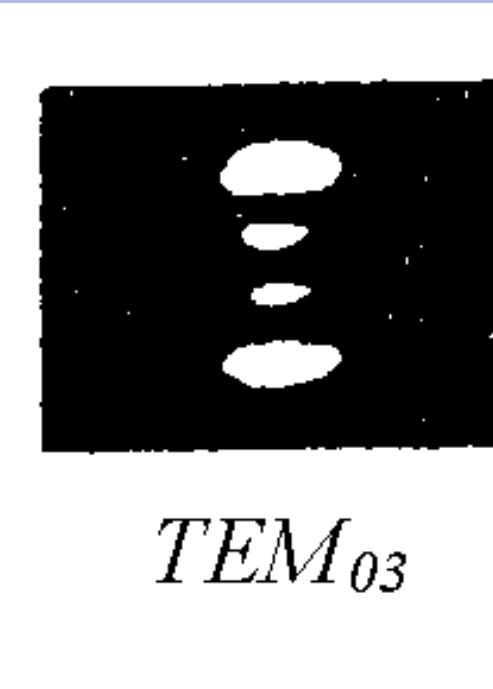
当 $x = 0$ 和 $x = \pm \frac{\sqrt{3}}{2} \omega_{0s}$ 时

$$u_{30} = 0$$

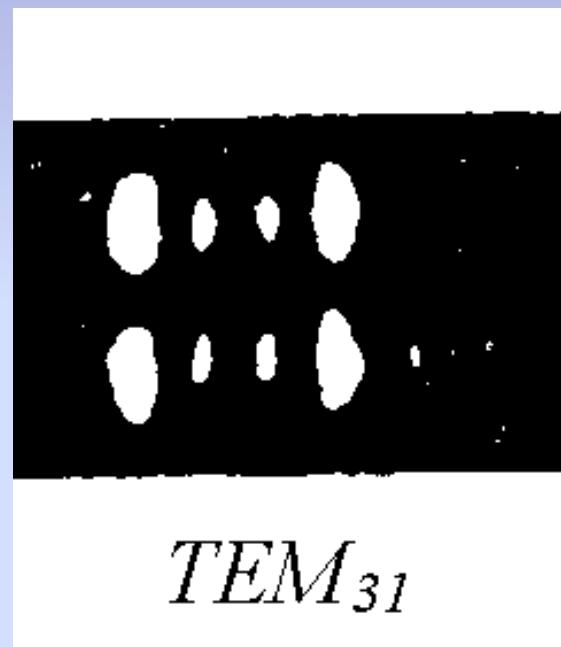
在 x 方向各出现三条暗线



$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$



TEM_{03}

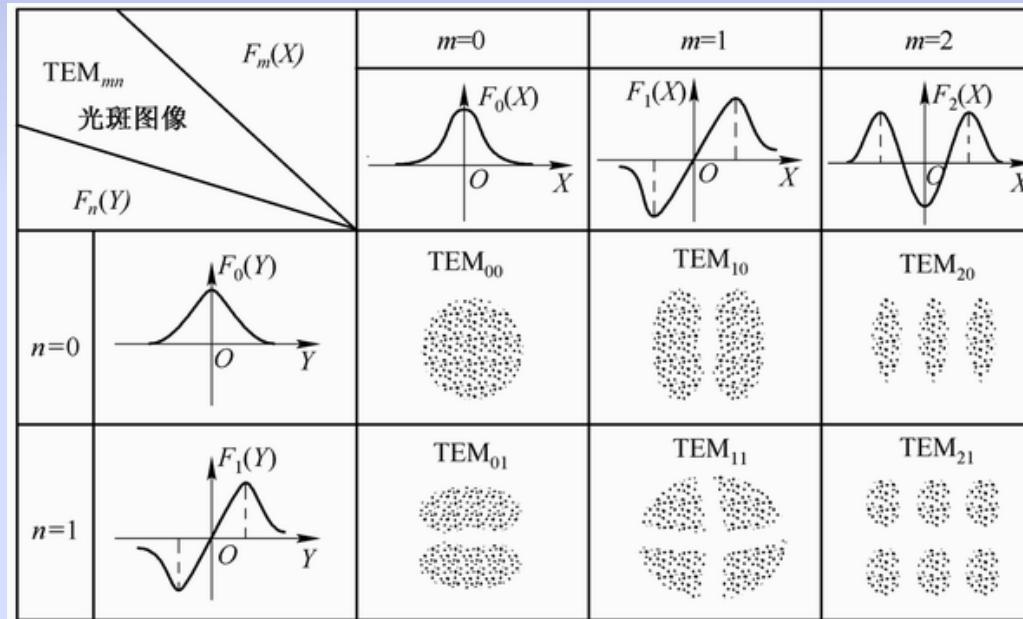


TEM_{31}

高阶模的光斑尺寸与基模的关系

$$\left. \begin{array}{l} \omega_{ms} = \sqrt{2m+1}\omega_{0s} \\ \omega_{ns} = \sqrt{2n+1}\omega_{0s} \end{array} \right\}$$

$$F_m = H_m(X) e^{-\frac{X^2}{2}}$$



图(3-5) $F_m(X)-X$ 及 $F_n(Y)-Y$ 的变化曲线及相应的光强分布

镜面上场位相分布:共焦腔反射镜面本身构成光场的一个等相位面。

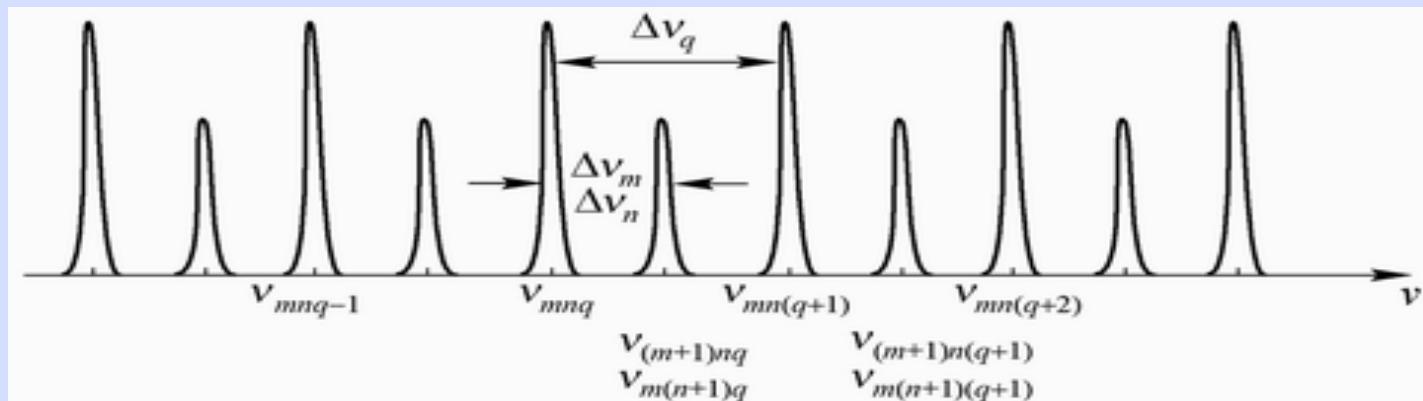
$$u_{mn}(x, y) = C_{mn} H_m\left(\frac{\sqrt{2}}{\omega_{0s}} x\right) H_n\left(\frac{\sqrt{2}}{\omega_{0s}} y\right) e^{-\frac{x^2+y^2}{L\lambda/\pi}}$$

单程相移与谐振频率：

$$\left. \begin{aligned} \Delta\phi_{mn} &= kL + \arg \sigma_{mn} \\ \sigma_{mn} &= e^{-i[kL - (m+n+1)\frac{\pi}{2}]} \end{aligned} \right\} \Rightarrow \Delta\phi_{mn} = (m+n+1)\pi/2$$

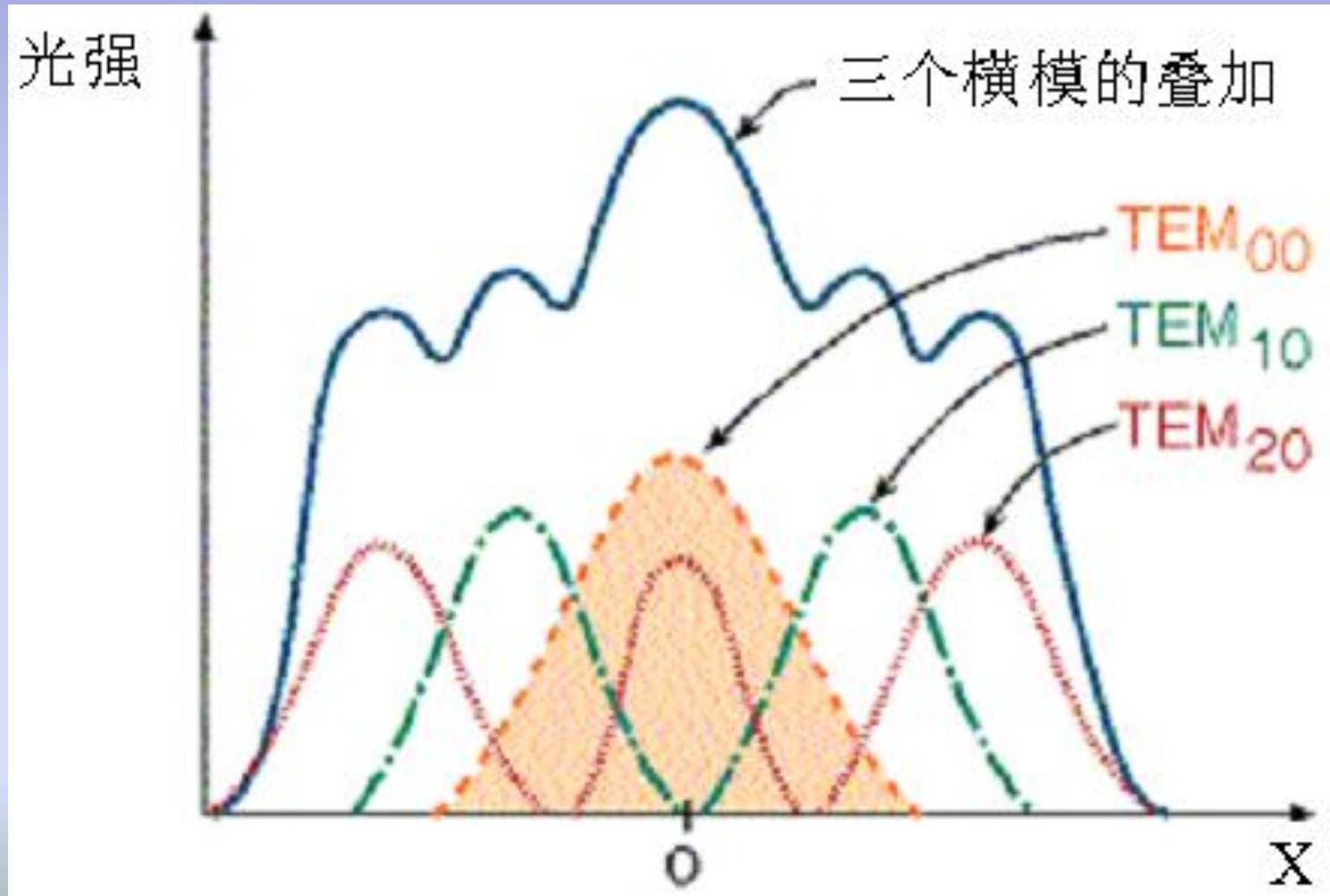
$$\Rightarrow v_{mnq} = \frac{qc}{2\mu L} + \frac{c}{2\pi\mu L} \Delta\phi_{mn}$$

$$\Rightarrow v_{mnq} = \frac{c}{2\mu L} \left[q + \frac{1}{2}(m+n+1) \right] \Rightarrow$$



图(3-6) 方形镜共焦腔的振荡频谱

激光多横模振荡示意图



2、共焦腔中的行波场与腔内外的光场分布

$$u_{mn}(x, y, z) = C_{mn} H_m \left(\sqrt{\frac{2}{1 + \zeta^2}} \cdot \frac{\sqrt{2}}{w_s} x \right) H_n \left(\sqrt{\frac{2}{1 + \zeta^2}} \cdot \frac{\sqrt{2}}{w_s} y \right) \exp \left(-\frac{2}{1 + \zeta^2} \cdot \frac{x^2 + y^2}{w_s^2} \right) \exp(-i\phi(x, y, z))$$

$$\zeta = 2z/L$$

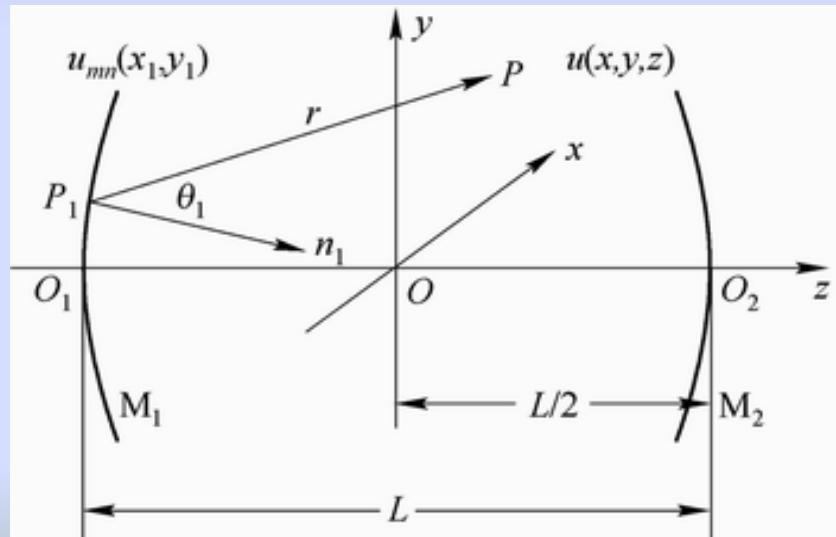


图3-7 计算腔内外光场分布的示意图

$$u_{mn}(x,y,z) = C_{mn} H_m\left(\sqrt{\frac{2}{1+\zeta^2}} \cdot \frac{\sqrt{2}}{w_s} x\right) H_n\left(\sqrt{\frac{2}{1+\zeta^2}} \cdot \frac{\sqrt{2}}{w_s} y\right).$$

$$\underline{\exp\left(-\frac{2}{1+\zeta^2}\cdot\frac{x^2+y^2}{w_s^2}\right)\exp\left(-i\phi(x,y,z)\right)}$$

$$\phi(x,y,z)=k\left[\frac{L}{2}(1+\zeta)+\frac{\zeta}{1+\zeta^2}\frac{x^2+y^2}{L}\right]-(m+n+1)(\frac{\pi}{2}-\varphi)$$

$$\varphi=arctg\frac{1-\zeta}{1+\zeta}=arctg\frac{L-2z}{L+2z}$$

3.3 高斯光束的传播特性

一、高斯光束的振幅

$$u_{mn}(x, y, z) = C_{mn} H_m \left(\sqrt{\frac{2}{1 + \zeta^2}} \frac{\sqrt{2}}{w_s} x \right) H_n \left(\sqrt{\frac{2}{1 + \zeta^2}} \frac{\sqrt{2}}{w_s} y \right) \\ \times \exp \left(-\frac{2}{1 + \zeta^2} \frac{x^2 + y^2}{w_s^2} \right) \exp[-i\phi(x, y, z)]$$

基横模 TEM_{00} 振幅

$$U_{00} = C_{00} \exp \left(-\frac{2}{1 + \zeta^2} \frac{x^2 + y^2}{w_s^2} \right)$$

截面半径：振幅下降为最大值 $1/e$

$$w(z) = \frac{w_s}{\sqrt{2}} \sqrt{1 + \zeta^2} = \frac{w_s}{\sqrt{2}} \sqrt{1 + \frac{4z^2}{L^2}}$$

基模高斯光束在镜面上的截面半径

$$w_s = \sqrt{\frac{\lambda L}{\pi}}$$

$$w(z) = \sqrt{\frac{\lambda L}{2\pi} \left[1 + \left(\frac{2z}{L} \right)^2 \right]}$$

基模高斯光束的束腰半径 $w_0 = \frac{1}{\sqrt{2}} w_s = \sqrt{\frac{\lambda L}{2\pi}}$

$$w(z) = w_0 \sqrt{1 + \left(\frac{2z}{L} \right)^2}$$

$$\frac{w^2}{w_0^2} - \frac{z^2}{(\pi w_0^2 / \lambda)^2} = 1$$

双曲线方程

$$\frac{w^2}{w_0^2} - \frac{z^2}{(\pi w_0^2/\lambda)^2} = 1$$

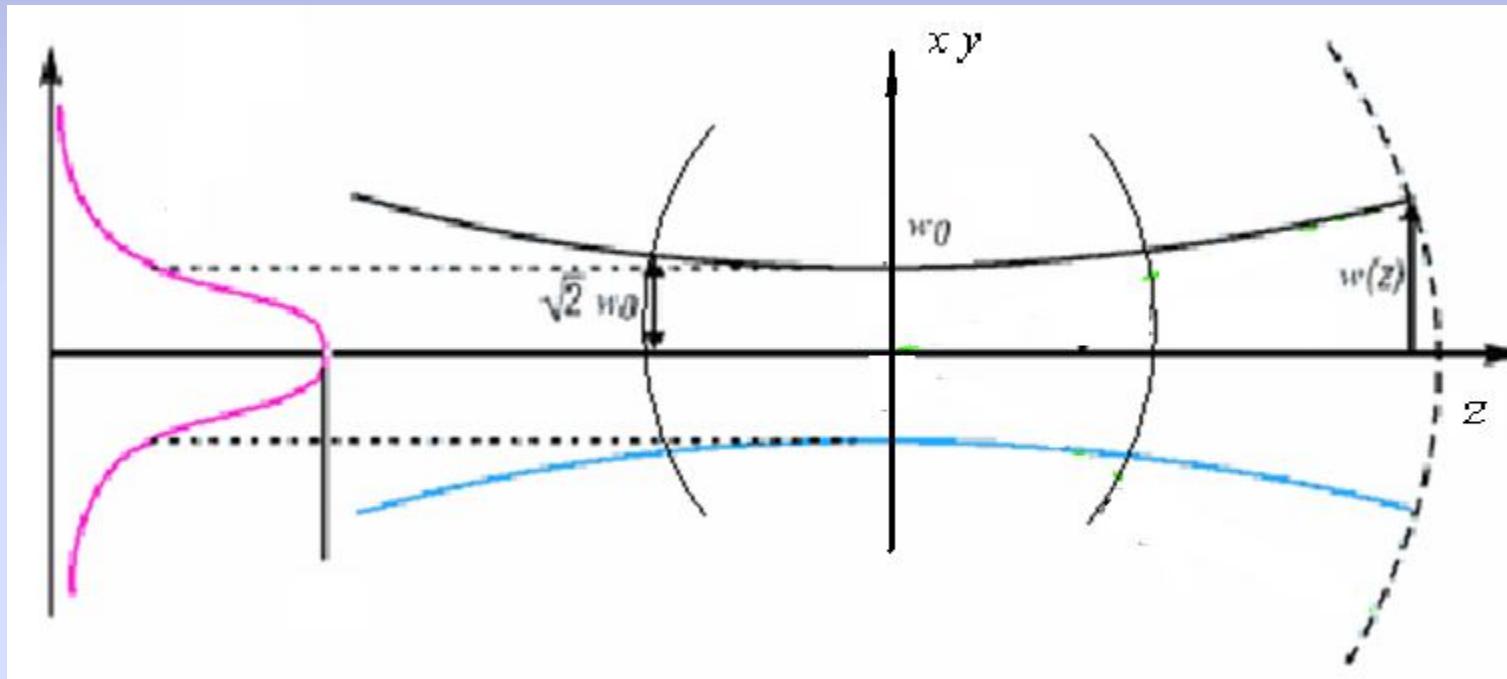


图3-8 基模高斯光束光斑半径随z按照双曲线规律变化

厄密高斯光束

$$w_m(z) = \sqrt{2m+1} w_{m=0}(z)$$

$$w_n(z) = \sqrt{2n+1} w_{m=0}(z)$$

二、高斯光束的相位分布

$$u_{mn}(x, y, z) = C_{mn} H_m \left(\sqrt{\frac{2}{1+\zeta^2}} \frac{\sqrt{2}}{w_s} x \right) H_n \left(\sqrt{\frac{2}{1+\zeta^2}} \frac{\sqrt{2}}{w_s} y \right) \\ \times \exp \left(-\frac{2}{1+\zeta^2} \frac{x^2 + y^2}{w_s^2} \right) \exp[-i\phi(x, y, z)]$$

共焦场的相位分布

$$\phi(x, y, z) = k \left[\frac{L}{2} \left(1 + \frac{2z}{L} \right) + \frac{2z/L}{1 + (2z/L)^2} \frac{x^2 + y^2}{L} \right] - (m+n+1) \left(\frac{\pi}{2} - \varphi \right)$$

$$\varphi = \arctan \frac{L - 2z}{L + 2z}$$

z_0 处等相位面的方程

$$\phi(x, y, z) = \phi(0, 0, z_0)$$

$$k\left[\frac{L}{2}\left(1+\frac{2z}{L}\right)+\frac{2z/L}{1+(2z/L)^2}\frac{x^2+y^2}{L}\right]-(m+n+1)\left[\frac{\pi}{2}-\varphi(z)\right]$$

$$= k\left[\frac{L}{2}\left(1+\frac{2z_0}{L}\right)\right]-(m+n+1)\left[\frac{\pi}{2}-\varphi(z_0)\right]$$

$$k\left[z+\frac{2z/L}{1+(2z/L)^2}\frac{x^2+y^2}{L}\right]+\varphi(z)=kz_0+\varphi(z_0)$$

$$z - z_0 = -\frac{x^2 + y^2}{2z_0 \left[\left(\frac{L}{2z_0} \right)^2 + 1 \right]}$$

$$R_0 = z_0 \left[\left(\frac{L}{2z_0} \right)^2 + 1 \right]$$

$$z - z_0 = -\frac{x^2 + y^2}{2R_0} \approx R_0 \left(1 - \frac{x^2 + y^2}{R_0^2} \right)^{1/2} - R_0$$

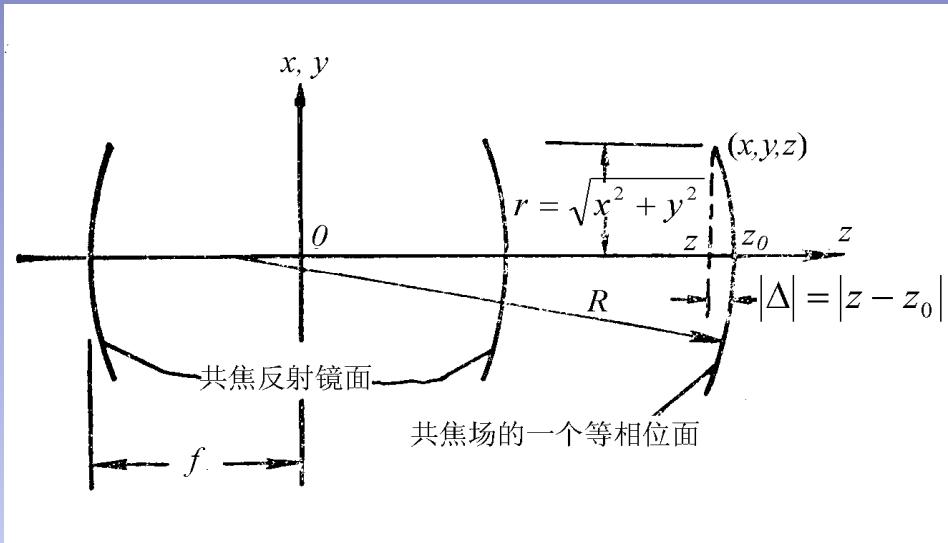
球面方程

$$(z - z_0 + R_0)^2 + x^2 + y^2 = R_0^2$$

等相位面在近轴区域可看成半径为 R_0 的球面

高斯光束等相面的曲率中心并不是一个固定点，它要随着光束的传播而移动。

$$R_0 = z_0 \left[1 + \left(\frac{L}{2z_0} \right)^2 \right]$$



$$R_0^2 = x^2 + y^2 + (z - z_0 + R_0)^2$$

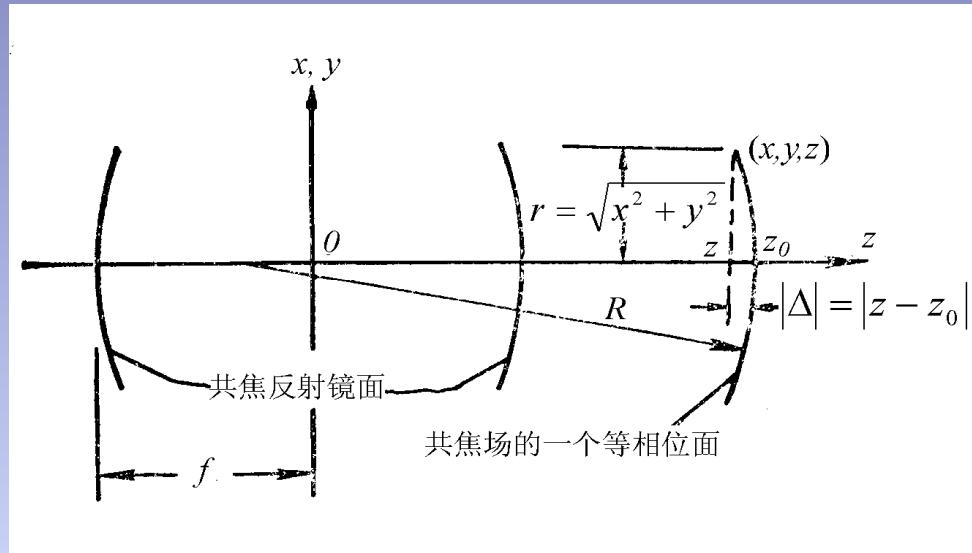
$$R_0 = z_0 \left[1 + \left(\frac{L}{2z_0} \right)^2 \right]$$

共焦腔的等相面是凹面向着腔的中心的球面

在 $z < 0$ 处, 光束是沿着 z 的方向传播的会聚球面波;

在 $z = 0$ 处变成一个平面波;

在 $z > 0$ 处又变成立发散球面波。



$z_0 = 0 \quad R(z_0) \rightarrow \infty$ 束腰处的等相位面为平面

$z_0 \rightarrow \infty \quad R(z_0) \rightarrow \infty$ 无穷远处等相位面为平面

$z_0 = \pm L/2 \quad R(z_0) = L$ 曲率半径达到最小值

$|z_0| \gg L/2 \quad R(z_0) \rightarrow z_0$

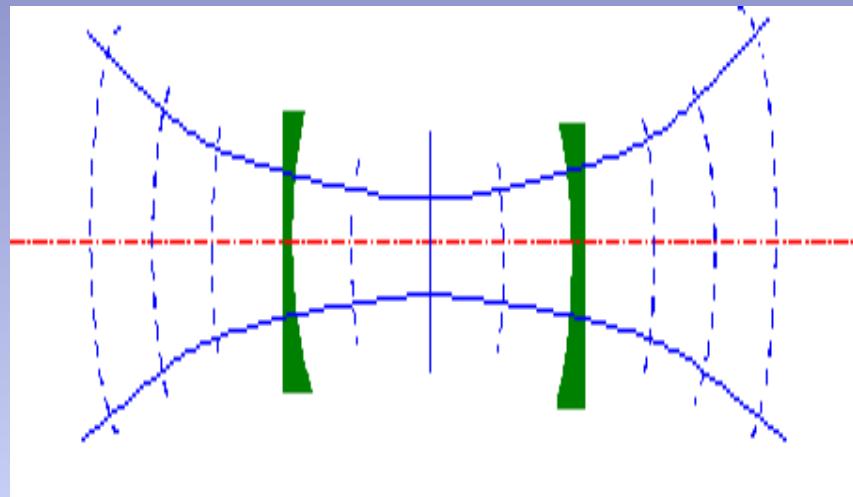
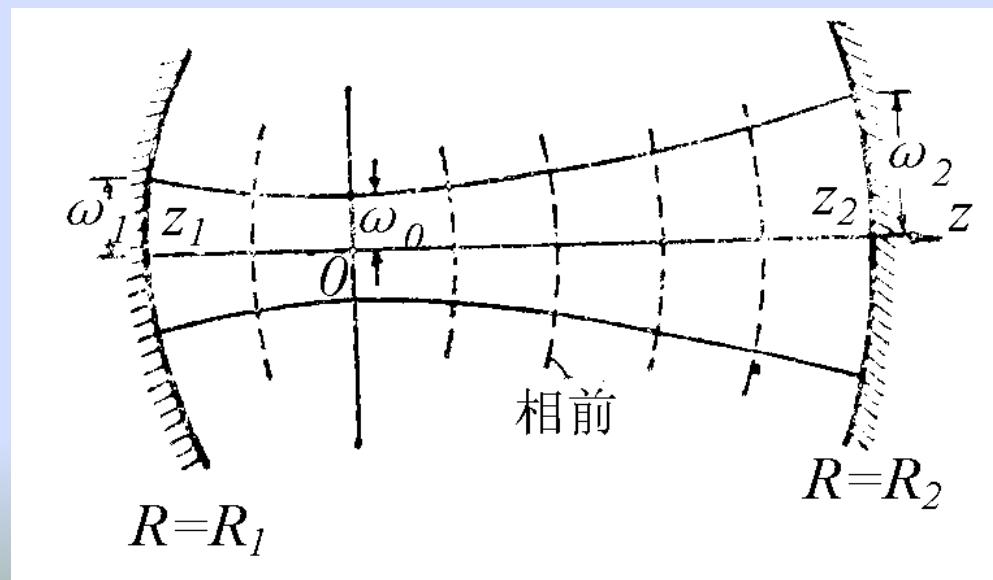
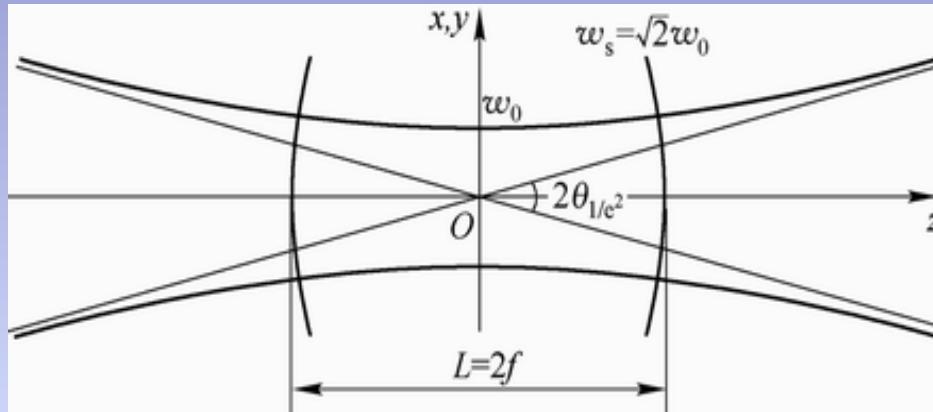


图3-9 共焦场中等相位面的分布



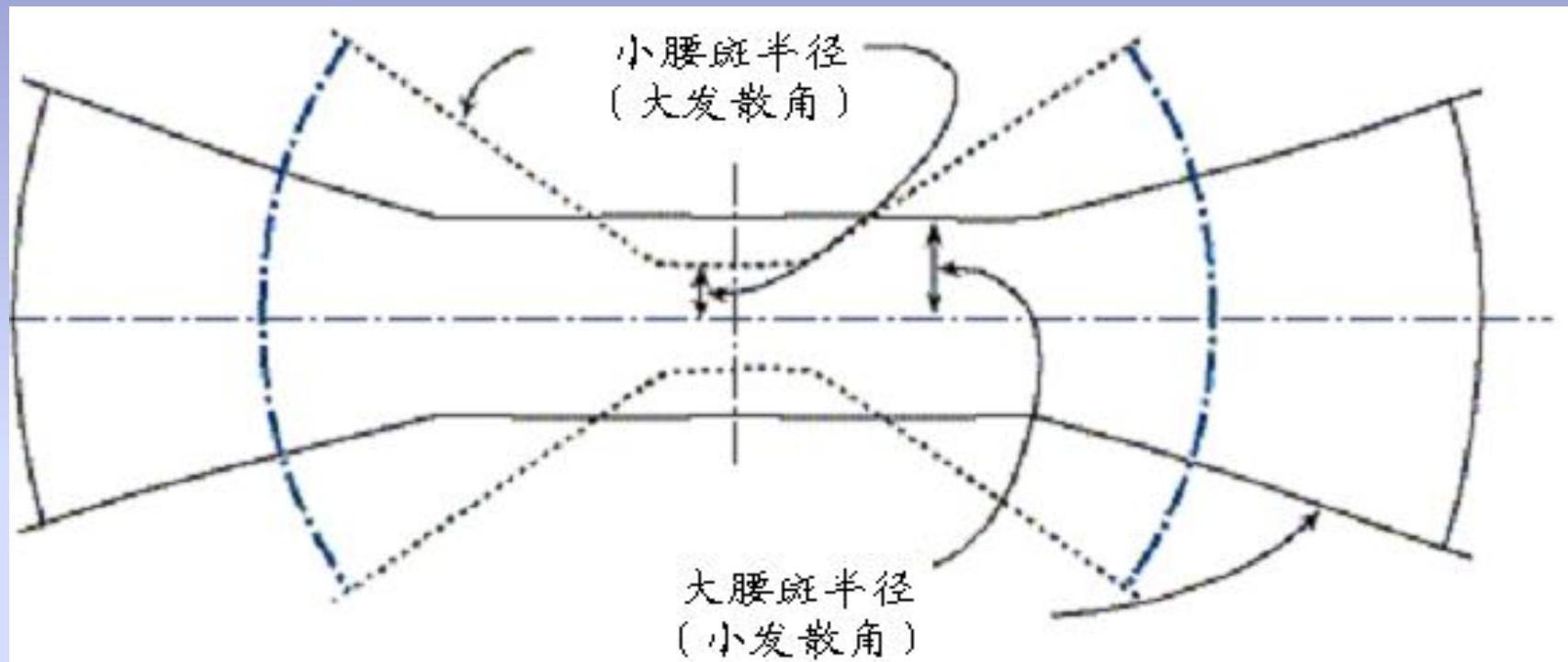
三、高斯光束的远场发散角



图(3-8) 基模光斑半径随z按双曲线规律的变化

基模远场发散角 2θ

$$\left. \begin{aligned} 2\theta &= \lim_{z \rightarrow \infty} \frac{2w(z)}{z} \\ w(z) &= w_0 \sqrt{1 + \left(\frac{\lambda z}{\pi w_0^2}\right)^2} \end{aligned} \right\} \Rightarrow 2\theta = \frac{2\lambda}{\pi w_0}$$



不同束腰半径的激光光束远场发散角对比图

例：某共焦腔氦氖激光器， $L=30\text{cm}$, $\lambda = 0.638\mu\text{m}$

$$2\theta = 2\sqrt{\frac{2\lambda}{L\pi}} \approx 2.3 \times 10^{-3} \text{ rad}$$

某共焦腔二氧化碳激光器， $L=1\text{m}$, $\lambda = 10.6\mu\text{m}$

$$2\theta \approx 5.2 \times 10^{-3} \text{ rad}$$

四、高斯光束的高亮度

亮度 B :单位面积的发光面在其法线方向上单位立体角范围内输出去的辐射功率。

$$B = \frac{\Delta I}{\Delta S \Delta \Omega}$$

$$\Delta \Omega = \pi \theta^2$$

例：测量实验室中的一台 Ar^+ 激光器，获得如下数据：

(1) 在输出镜上光斑半径 $r \approx 0.5 mm$, (2) 光束发散角 $\theta \approx 0.5 \times 10^{-3} rad$, (3) 连续输出功率 $\Delta I = 1 W$, 求激光器辐射亮度。

解：输出镜面上光束截面积 $\Delta S = \pi r^2 = 7.5 \times 10^{-3} cm^2$

光束所占立体角 $\Delta\Omega = \pi\theta^2 \approx 7.5 \times 10^{-7} sr$

$$B = \frac{\Delta I}{\Delta S \Delta\Omega} \approx 2 \times 10^8 W / (cm^2 \cdot sr)$$

而 $B_{\text{太阳}} = 1.2 \times 10^3 W / (cm^2 \cdot sr)$

$$\frac{B_{Ar^+}}{B_{\text{太阳}}} = 10^5 \quad (\text{十万倍})$$

3.4 稳定球面腔的光束传播特性

任何一个共焦腔与无穷多个稳定球面腔等价，而任何一个球面腔唯一地等价于一个共焦腔。

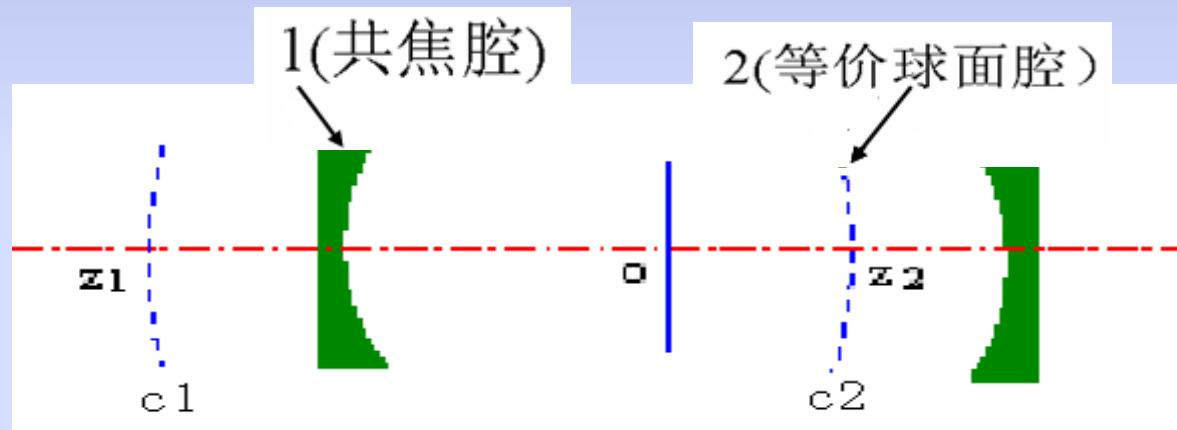


图3-10 共焦腔与稳定球面腔的等价性

3.4.1 稳定球面腔的等价共焦腔

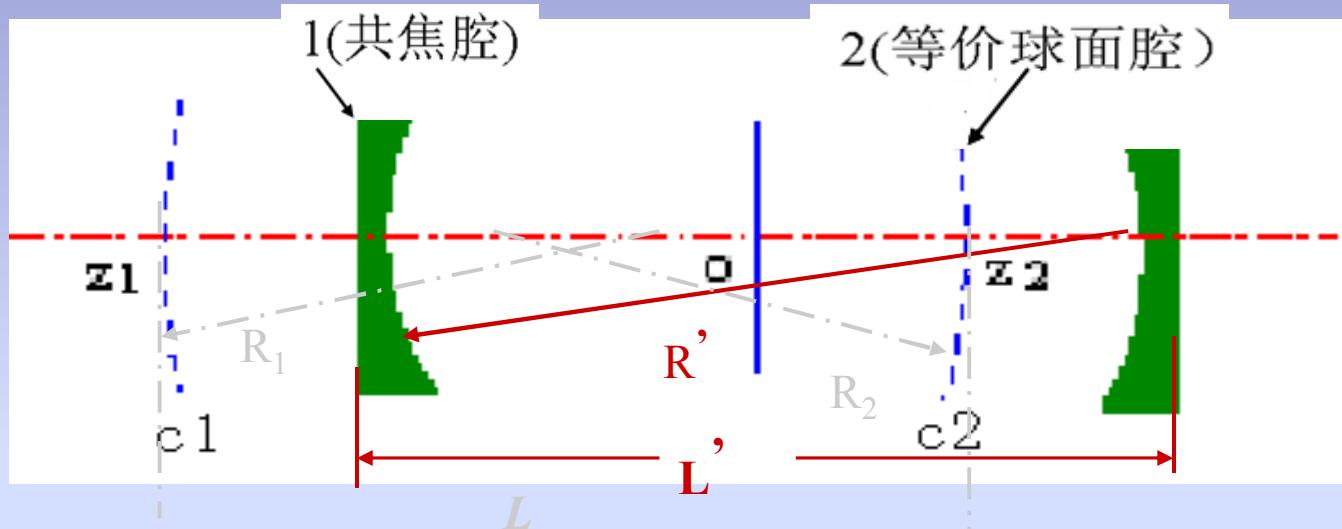


图3-10 共焦腔与稳定球面腔的等价性

焦距 f 、中心在 $z=0$ 的对称共焦腔(R' , R' , L')

(z_1, z_2)的等价稳定球面腔参数(R_1, R_2, L)为

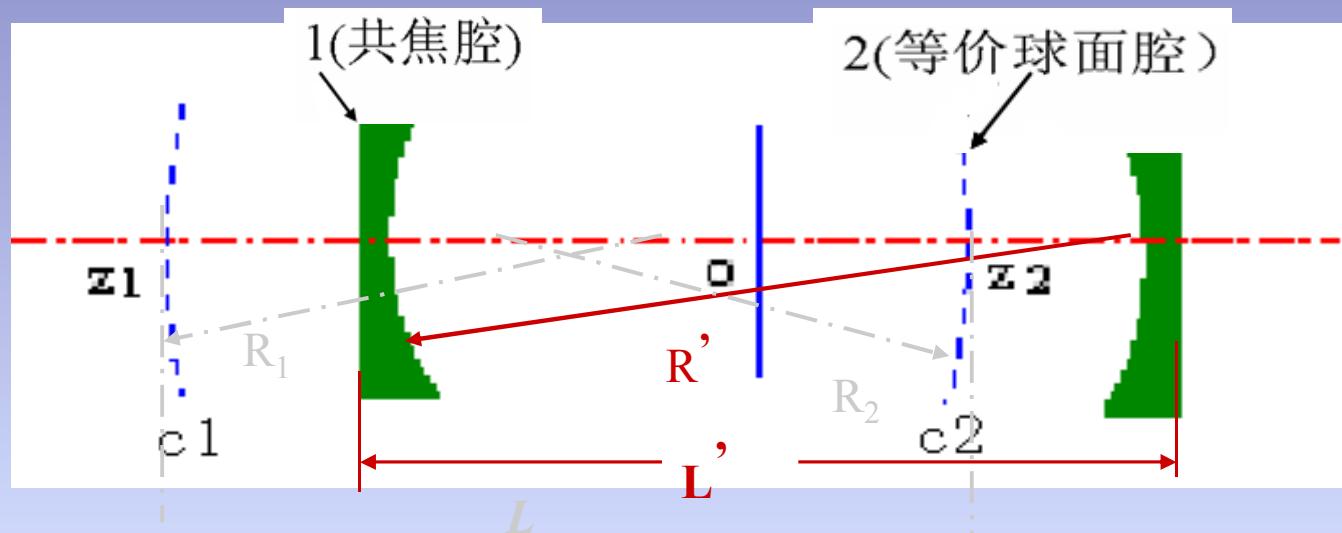


图3-10 共焦腔与稳定球面腔的等价性

$$\left\{ \begin{array}{l} R_1 = R(z_1) = -(z_1 + \frac{f^2}{z_1}) \\ R_2 = R(z_2) = +(z_2 + \frac{f^2}{z_2}) \\ L = z_2 - z_1 \end{array} \right. \quad f: \text{对称共焦腔焦距}$$

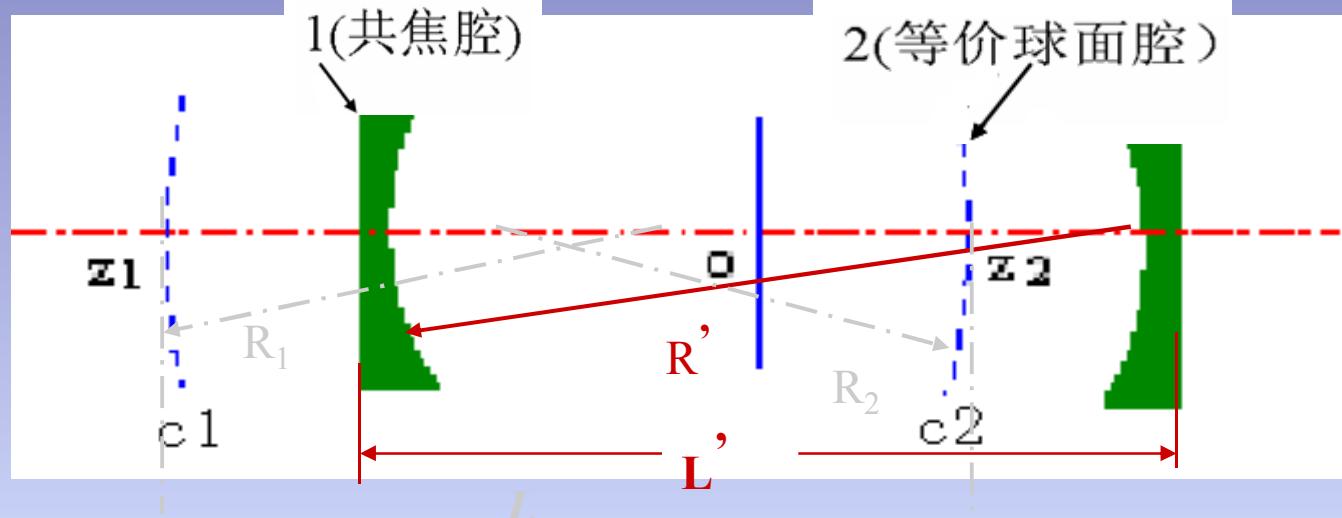


图3-10 共焦腔与稳定球面腔的等价性

$$\left\{ \begin{array}{l} R_1 = R(z_1) = -(z_1 + \frac{f^2}{z_1}) \\ R_2 = R(z_2) = +(z_2 + \frac{f^2}{z_2}) \\ L = z_2 - z_1 \end{array} \right. \quad \xrightarrow{\hspace{1cm}} \quad \left\{ \begin{array}{l} g_1 = 1 - \frac{L}{R_1} = \frac{f^2 + z_1 z_2}{z_1^2 + f^2} \\ g_2 = 1 - \frac{L}{R_2} = \frac{f^2 + z_1 z_2}{z_2^2 + f^2} \end{array} \right. \quad \Rightarrow \quad 0 < g_1 g_2 < 1$$

即放置在 c_1 、 c_2 处的反射镜构成稳定腔

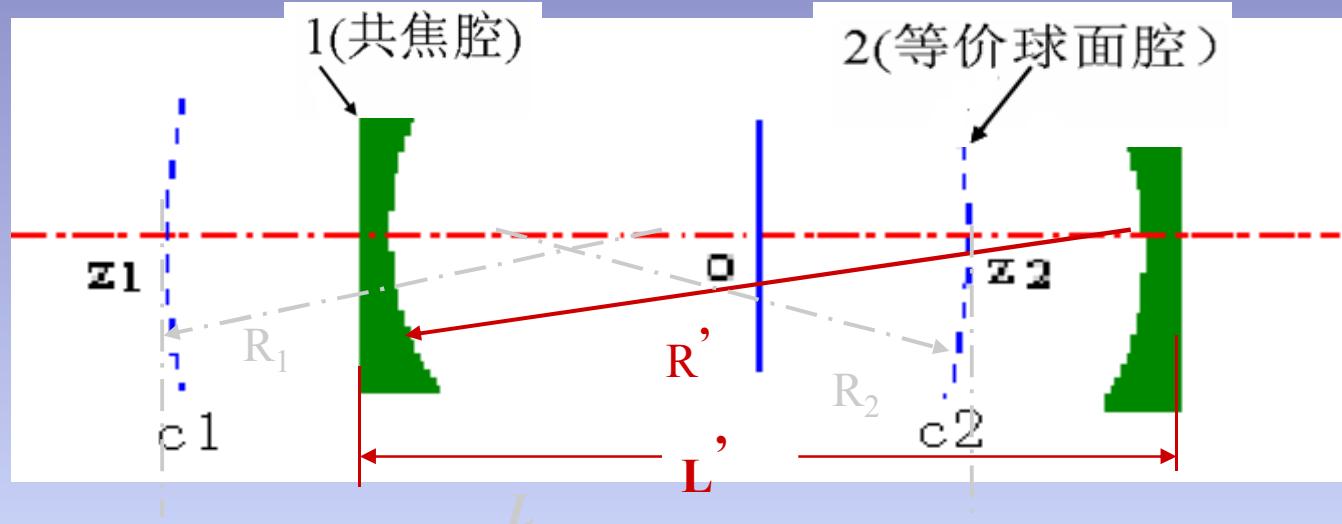


图3-10 共焦腔与稳定球面腔的等价性

$$\left\{ \begin{array}{l} R_1 = |z_1| \left[1 + \left(\frac{f}{z_1} \right)^2 \right] \\ R_2 = |z_2| \left[1 + \left(\frac{f}{z_2} \right)^2 \right] \\ |z_1| + |z_2| = L \end{array} \right.$$



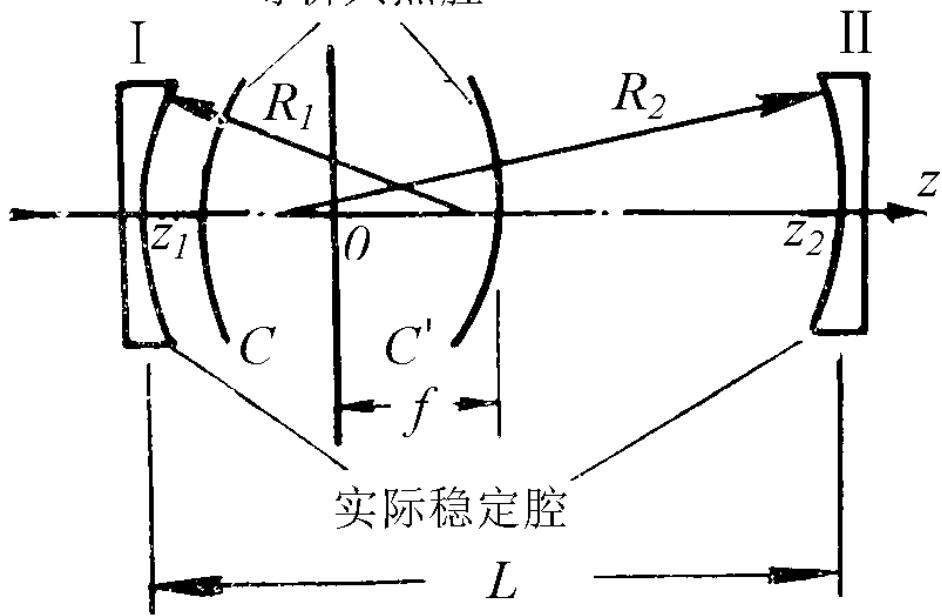
$$\begin{aligned} |z_1| &= \frac{L(R_2 - L)}{R_1 + R_2 - 2L} \\ |z_2| &= \frac{L(R_1 - L)}{R_1 + R_2 - 2L} \\ f &= \frac{\sqrt{L(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}}{R_1 + R_2 - 2L} \end{aligned}$$

可以证明，当 R_1, R_2, L 满足稳定腔条件 $0 < g_1 g_2 < 1$ 时，

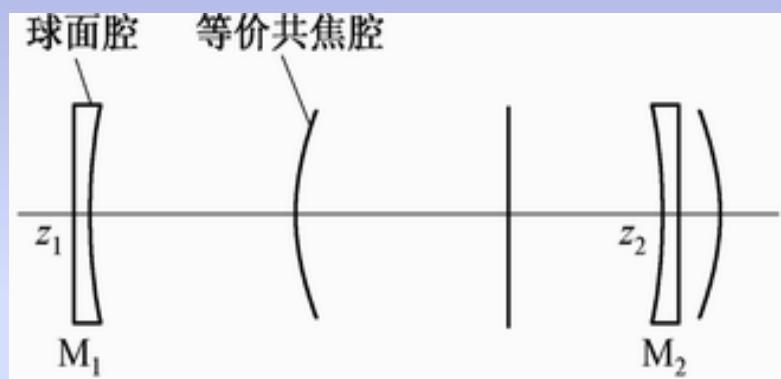
则有 $f > 0, z_1 < 0, z_2 > 0$

如果一个球面满足稳定条件，则可以找到一个，而且也只能找到一个共焦腔，其行波场的某两个等相位面与给定球面腔的两个镜面相重合，从而，这两个腔的模式完全相同

等价共焦腔



实际稳定腔



3.4.2 稳定球面腔的光束传播特性

$$\left\{ \begin{array}{l} R_1 = |z_1| \left[1 + \left(\frac{f}{z_1} \right)^2 \right] \\ R_2 = |z_2| \left[1 + \left(\frac{f}{z_2} \right)^2 \right] \\ |z_1| + |z_2| = L \end{array} \right. \quad \begin{aligned} |z_1| &= \frac{L(R_2 - L)}{R_1 + R_2 - 2L} \\ |z_2| &= \frac{L(R_1 - L)}{R_1 + R_2 - 2L} \\ f &= \frac{\sqrt{L(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}}{R_1 + R_2 - 2L} \end{aligned}$$

等效共焦腔的束腰半径

$$w_0 = \sqrt{\frac{\lambda L'}{2\pi}} \quad L' = 2f$$

稳定球面腔镜面处的光斑半径

$$w(z) = w_0 \sqrt{1 + \left(\frac{z}{f}\right)^2} \quad w_{s_1}, w_{s_2}$$

方形镜一般稳定球面腔的两个反射镜面顶点处的位相因子分别为：

$$\phi(x, y, z) = k \left[\frac{L}{2} (1 + \zeta) + \frac{\zeta}{1 + \zeta^2} \frac{x^2 + y^2}{L} \right] - (m + n + 1) \left(\frac{\pi}{2} - \varphi \right)$$

$$\phi(0, 0, z_1) = kf \left(1 + \frac{z_1}{f} \right) - (m + n + 1) \left(\frac{\pi}{2} - \arctan \frac{f - z_1}{f + z_1} \right)$$

$$\phi(0, 0, z_2) = kf \left(1 + \frac{z_2}{f} \right) - (m + n + 1) \left(\frac{\pi}{2} - \arctan \frac{f - z_2}{f + z_2} \right)$$

谐振条件 $\phi(0, 0, z_2) - \phi(0, 0, z_1) = q\pi$

$$k(z_1 - z_2) + (m + n + 1) \left(\arctan \frac{f - z_1}{f + z_1} - \arctan \frac{f - z_2}{f + z_2} \right) = q\pi \Rightarrow$$

$$\frac{2\pi}{\lambda}L + (m+n+1)\arctan \frac{fL}{f^2 - z_1 z_2} = q\pi \Rightarrow$$

$$\frac{2\pi}{\lambda}L - (m+n+1)\arccos \sqrt{g_1 g_2} = q\pi \Rightarrow$$

谐振频率

$$\nu_{mnq} = \frac{c}{\mu\lambda} = \frac{c}{2\mu L} \left[q + \frac{1}{\pi} (m+n+1) \cos^{-1} \sqrt{g_1 g_2} \right]$$

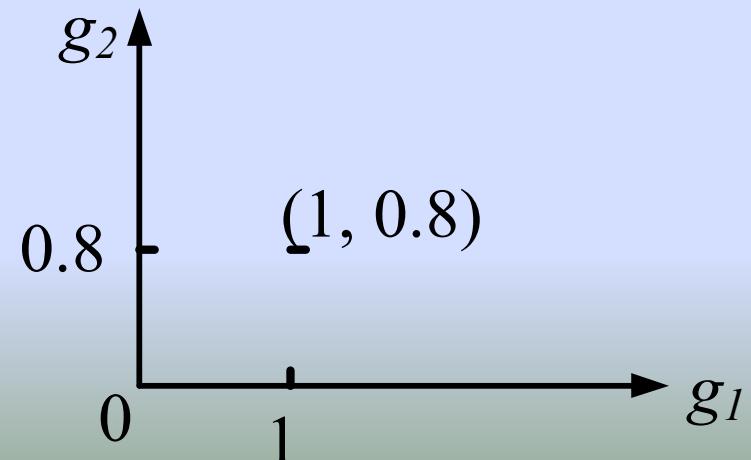
例: 平凹腔 $R_1 \rightarrow \infty$, $R_2 = 5m$, $L=1m$.

- (1) 证明此腔稳定, 指出它在稳定图中位置;
- (2) 求等价对称共焦腔参数(R_1', R_2', L');
- (3) 作平凹腔与等价对称共焦腔相对位置图.

解:(1) $\because g_1 = 1 - \frac{L}{R_1} = 1, \quad g_2 = 1 - \frac{L}{R_2} = 0.8$

$$\therefore 0 < g_1 g_2 = 0.8 < 1, \quad \text{稳定}$$

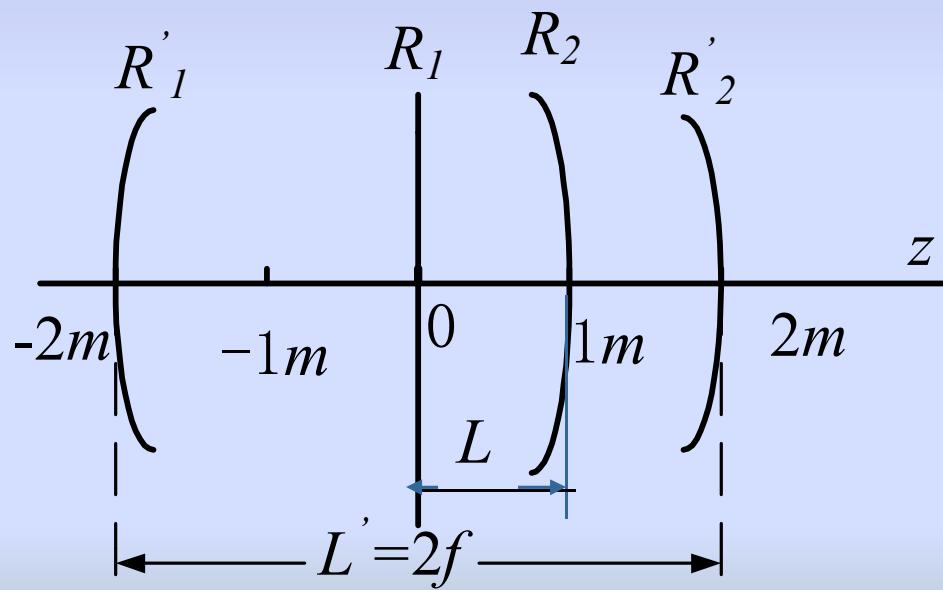
所求位置为(1,0.8)



(2) $z_1 = 0, z_2 = 1m, f = 2m$

故 $R_1' = R_2' = L' = 2f = 4m$

(3) $z_1 = 0$ 由可知, 平凹腔平面镜 R_1 位于等价共焦腔中心.



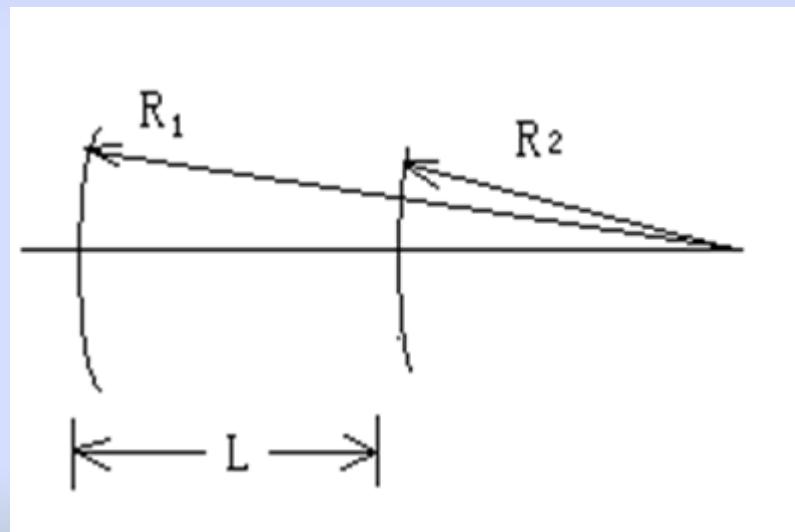
例：有一凹凸腔，腔长 $L=30\text{cm}$, 两个反射镜的曲率半径大小分别为 $R_1=50\text{cm}$, $R_2=-30\text{cm}$, 见下图。使用He-Ne做激光工作物质。

- 求：
- (1) 利用稳定性条件证明此腔为稳定腔；
 - (2) 此腔的等价共焦腔焦距；
 - (3) 此腔产生的高斯光束的腰斑半径及腰位置
 - (4) 此腔产生的高斯光束的远场发散角。

解：(1) 共轴球面谐振腔的稳定性条件为：

$$0 < g_1 g_2 < 1$$

对于凹凸腔，由已知条件有： $R_1 = 50\text{cm}$, $R_2 = -30\text{cm}$



$$g_1 = 1 - \frac{L}{R_1} = 1 - \frac{30}{50} = \frac{2}{5}$$

$$g_2 = 1 - \frac{L}{R_2} = 1 - \frac{30}{(-30)} = 2$$

故 $0 < g_1 g_2 = \frac{4}{5} < 1$ 可见此腔为稳定腔

(2)此腔的等价共焦腔焦距:

$$f = \frac{\sqrt{L(R_1 - L)(R_2 - L)(R_1 + R_2 - L)}}{R_1 + R_2 - 2L}$$

代入各数值得 $f = 15\text{cm}$

(3)此腔产生的高斯光束的腰斑半径及腰位置

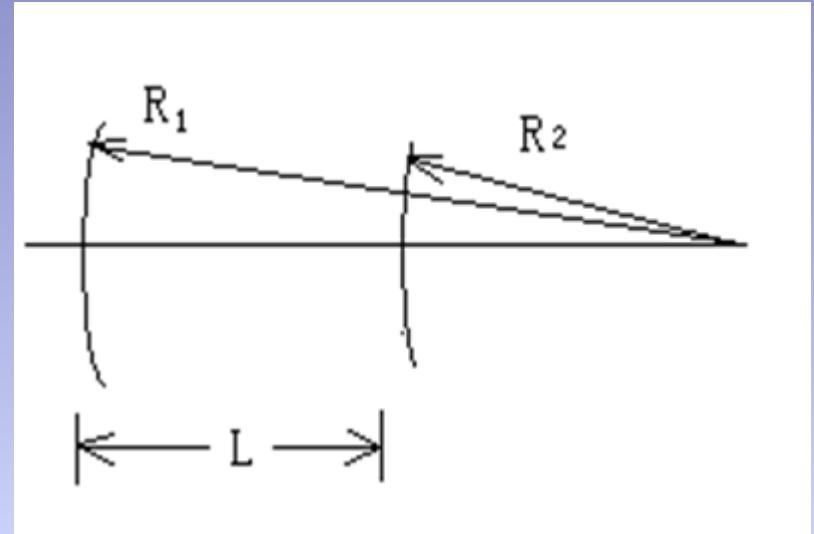
He-Ne激光器的波长 $\lambda = 0.6328 \times 10^{-4} cm$

腰斑半径

$$w_0 = \sqrt{\frac{\lambda f}{\pi}} = \sqrt{\frac{0.6328 \times 10^{-4} \times 15}{3.14}} = 1.74 \times 10^{-2} cm$$

以其等价共焦腔的中点为原点，腰位置即z=0处，设凹镜z坐标为z₁，凸镜z坐标为z₂，有：

$$\begin{cases} z_1 = \frac{L(R_2 - L)}{(L - R_1) + (L - R_2)} \\ z_2 = \frac{-L(R_1 - L)}{(L - R_1) + (L - R_2)} \end{cases}$$



代入各数值有 $z_1 = -45\text{cm}$, $z_2 = -15\text{cm}$

故腰斑在凹凸腔的右边，距凹腔45 cm，距凸腔15cm
处。

(4)此腔产生的高斯光束的远场发散角

远场发散角

$$2\theta = 2 \sqrt{\frac{\lambda}{\pi f}} = 2 \times \sqrt{\frac{0.6328 \times 10^{-4}}{3.14 \times 15}} = 2.32 \times 10^{-3} \text{ rad}$$

3.5. 激光器的输出功率

3.5.1 均匀增宽型介质激光器的输出功率

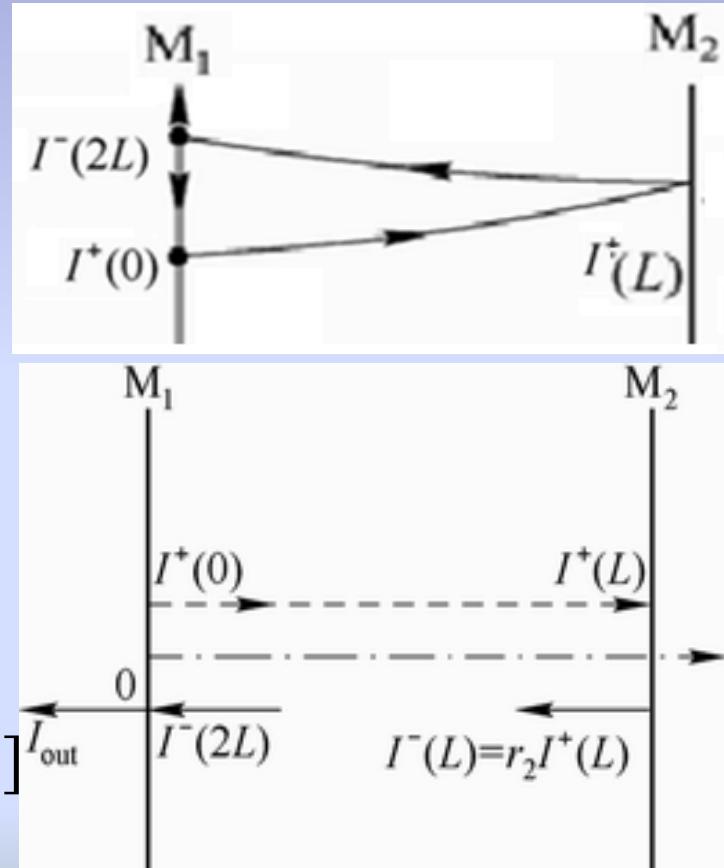
腔内最小的光强 $I^+(0)$

腔内最大光强

$$I^-(2L) = r_2 I^+(0) \exp[2L(G - a_{\text{内}})]$$

输出光强：

$$I_{out} = t_1 I^-(2L) = t_1 r_2 I^+(0) \exp[2L(G - a_{\text{内}})]$$



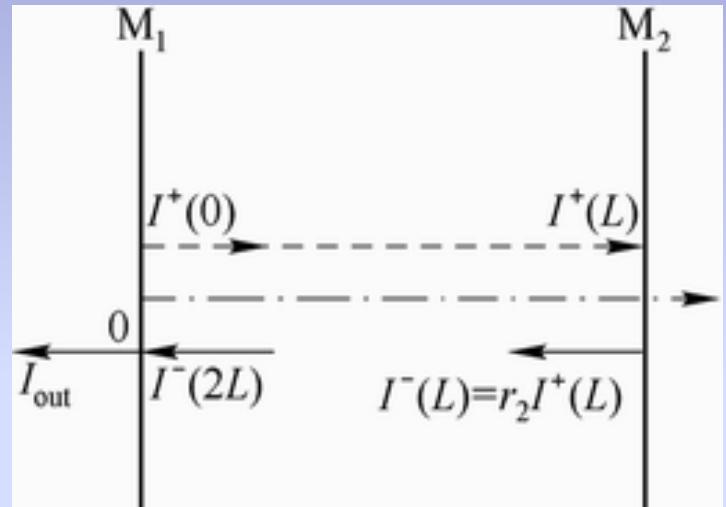
图(3-11) 谐振腔内光强

$$\text{镜面损耗: } I_h = a_1 I^-(2L) = a_1 r_2 I^+(0) \exp[2L(G - a_{\text{内}})]$$

剩余部分:

$$I^+(0) = r_1 I^-(2L)$$

$$= r_1 r_2 I^+(0) \exp[2L(G - a_{\text{内}})]$$



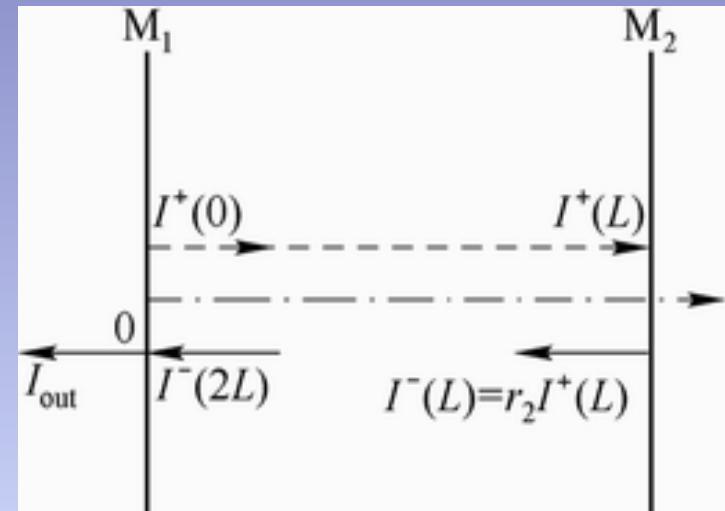
图(3-11) 谐振腔内光强

由能量守恒定律可得:

$$I^-(2L) - I^+(0) = I_{out} + I_h = (a_1 + t_1) I^-(2L)$$

腔内 \mathbf{z} 处： $I^+(\mathbf{z})$ 和 $I^-(2L-\mathbf{z})$,

用平均光强 $2I$ 近似代替腔内光
强 $I^+(\mathbf{z})+I^-(2L-\mathbf{z})$,



图(3-11) 谐振腔内光强

$$G = \frac{G^0}{1 + 2I/I_s} = G_{\text{阈}}$$

腔内平均行波光强

$$I = \frac{I_s}{2} \left(\frac{G^0}{G_{\text{阈}}} - 1 \right) = \frac{I_s}{2} \left(\frac{G^0}{a_{\text{总}}} - 1 \right)$$

$$a_{\text{内}} = 0$$

$$\text{M}_2: r_2 \approx 1, t_2 \approx 0, a_2 \approx 0$$

$$\text{M}_1: r_1 = 1 - (a_1 + t_1)$$

$$\text{激光器的总损耗: } a_{\text{总}} = a_{\text{内}} - \frac{1}{2L} \ln r_1 r_2 = -\frac{1}{2L} \ln[1 - (a_1 + t_1)]$$

$$\left\{ \ln(1-x) \approx -(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots) \right\}$$

$$a_{\text{总}} = \frac{a_1 + t_1}{2L}$$

激光器内行波的平均光强 I

$$I = \frac{I_s}{2} \left(\frac{2LG^0}{a_1 + t_1} - 1 \right)$$

激光器输出光强

$$I_{out} = t_1 I = \frac{I_s}{2} t_1 \left(\frac{2LG^0}{a_1 + t_1} - 1 \right)$$

激光器的平均截面为 A , 则其输出功率为:

$$P = AI_{out} = \frac{1}{2} t_1 I_s A \left(\frac{2LG^0}{a_1 + t_1} - 1 \right)$$

$$P = AI_{out} = \frac{1}{2}t_1I_sA\left(\frac{2LG^0}{a_1+t_1}-1\right)$$

P 与 t_1 的关系：

$$G_{域} = a_{总} = a_{内} - \frac{1}{2L} \ln r_1 r_2 = a_{内} - \frac{1}{2L} \ln [1 - (a_1 + t_1)]$$

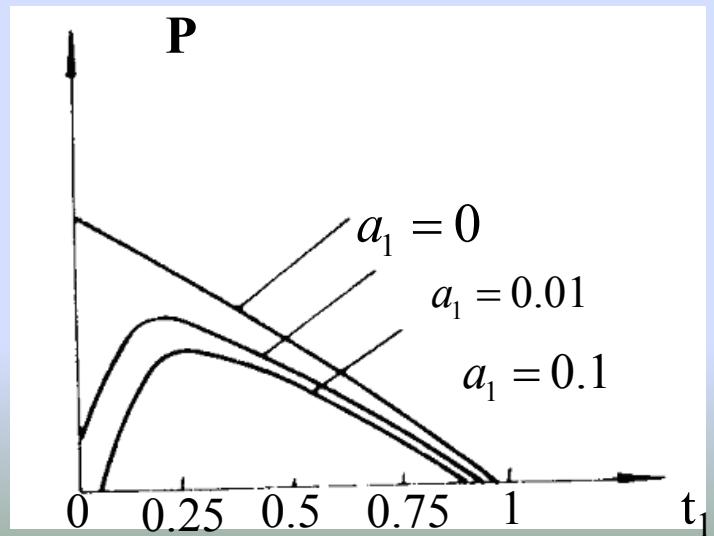
$$I_h = a_1 I^-(2L) = a_1 r_2 I^+(0) \exp[2L(G - a_{内})]$$

$$P = AI_{out} = \frac{1}{2} t_1 I_s A \left(\frac{2LG^0}{a_1 + t_1} - 1 \right)$$

$$\frac{dP}{dt_1} = 0 \Rightarrow \frac{1}{2} A \cdot I_s \left(\frac{2LG^0}{a_1 + t_1} - 1 \right) + \frac{1}{2} t_1 A I_s \left(-\frac{2LG^0}{(a_1 + t_1)^2} \right) = 0$$

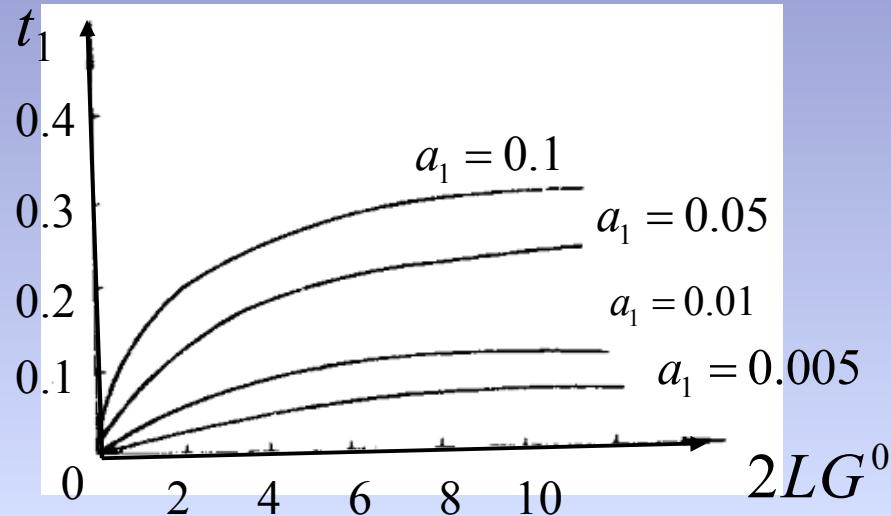
$$t_1 = (2LG^0 a_1)^{1/2} - a_1 = \sqrt{a_1} (\sqrt{2LG^0} - \sqrt{a_1})$$

$$2LG^0 = 3$$



$$t_1 = (2LG^0 a_1)^{1/2} - a_1 = \sqrt{a_1} (\sqrt{2LG^0} - \sqrt{a_1})$$

$$P = AI_{out} = \frac{1}{2} t_1 I_s A \left(\frac{2LG^0}{a_1 + t_1} - 1 \right)$$



输出镜具有最佳透射率时激光器的输出功率：

$$P = \frac{1}{2} I_s A \sqrt{a_1} (\sqrt{2LG^0} - \sqrt{a_1}) \left(\frac{2LG^0}{\sqrt{2LG^0 a_1}} - 1 \right)$$

$$= \frac{1}{2} I_s A (\sqrt{2LG^0} - \sqrt{a_1})^2$$

3.5.2 非均匀增宽型介质激光器的输出功率

腔内最大光强 $I^-(2L, v) = r_2 I^+(0, v) \exp 2L(G - a_{\text{内}})$

输出光强 $I_{out}(v) = t_1 I^-(2L, v) = t_1 r_2 I^+(0, v) \exp 2L(G - a_{\text{内}})$

镜面损耗 $I_h(v) = a_1 I^-(2L, v) = a_1 r_2 I^+(0, v) \exp 2L(G - a_{\text{内}})$

最小光强: $I^+(0, v) = r_1 I^-(2L, v) = r_1 r_2 I^+(0, v) \exp 2L(G - a_{\text{内}})$

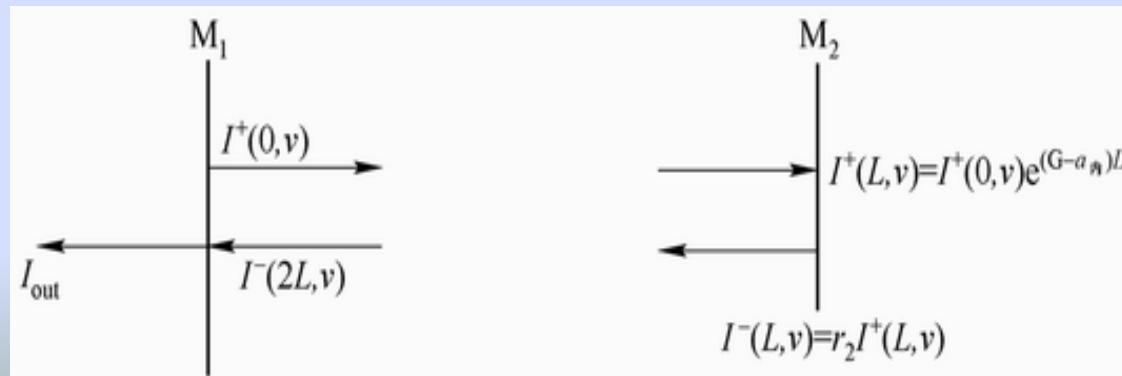


图3-12 非均匀增宽激光器腔内的光强

$$\nu_q \neq \nu_0 \quad G(v) = \frac{G_D^0(\nu)}{\sqrt{1 + I/I_s}}$$

$$\nu_q = \nu_0 \quad G_D(\nu_0) = \frac{G_D^0(\nu_0)}{\sqrt{1 + 2I/I_s}}$$

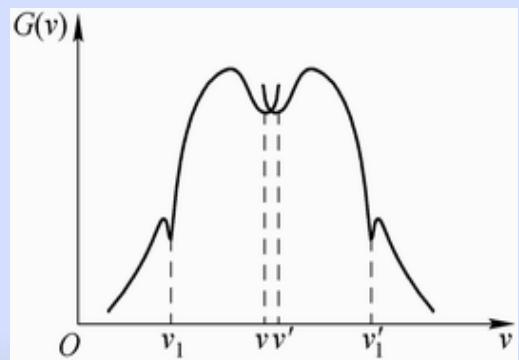


图3-13 非均匀增宽激光器的“烧孔效应”

$$G_D(v_0) = \frac{G_D^0(v_0)}{\sqrt{3}} \text{ 和 } G_D(v) = \frac{G_D^0(v)}{\sqrt{2}}$$

若增益系数的阈值都相等，则 v_0 和 v 光波的平均光强分别：

$$I(v_0) = \frac{1}{2} I_s \left[\left(\frac{G_D^0(v_0)}{G_{\text{阈}}} \right)^2 - 1 \right] \quad \text{和} \quad I(v) = I_s \left[\left(\frac{G_D^0(v)}{G_{\text{阈}}} \right)^2 - 1 \right]$$

$\nu \neq \nu_0$ 单频激光器的输出功率

$$a_{\text{总}} = a_{\text{内}} - \frac{1}{2L} \ln r_1 r_2 = -\frac{1}{2L} \ln [1 - (a_1 + t_1)]$$

$$I(\nu) = I_s \left[\left(\frac{2LG_D^0(\nu)}{a_1 + t_1} \right)^2 - 1 \right]$$

$$I_{\text{out}}(\nu) = t_1 I(\nu) = t_1 I_s \left[\left(\frac{2LG_D^0(\nu)}{a_1 + t_1} \right)^2 - 1 \right]$$

激光器的输出功率 $P(\nu) = A I_{\text{out}}(\nu) = A t_1 I_s \left[\left(\frac{2LG_D^0(\nu)}{a_1 + t_1} \right)^2 - 1 \right]$

腔内单纵模频率 ν_0

$$I(\nu_0) = \frac{1}{2} I_s \left[\left(\frac{2LG_D^0(\nu_0)}{a_1 + t_1} \right)^2 - 1 \right]$$

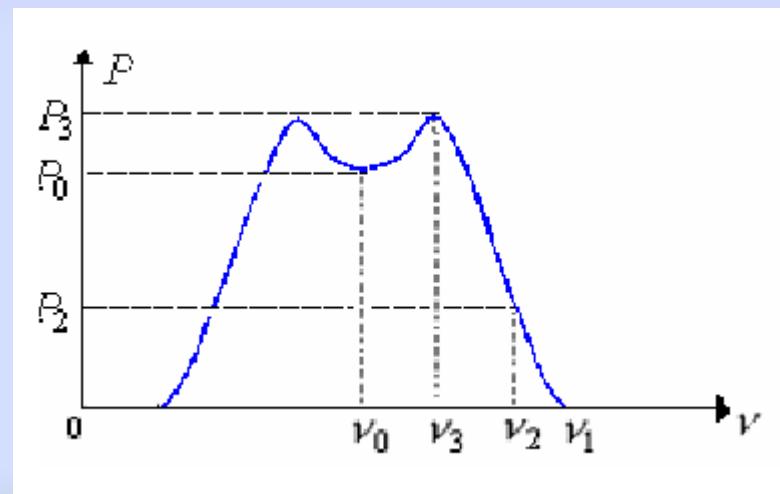
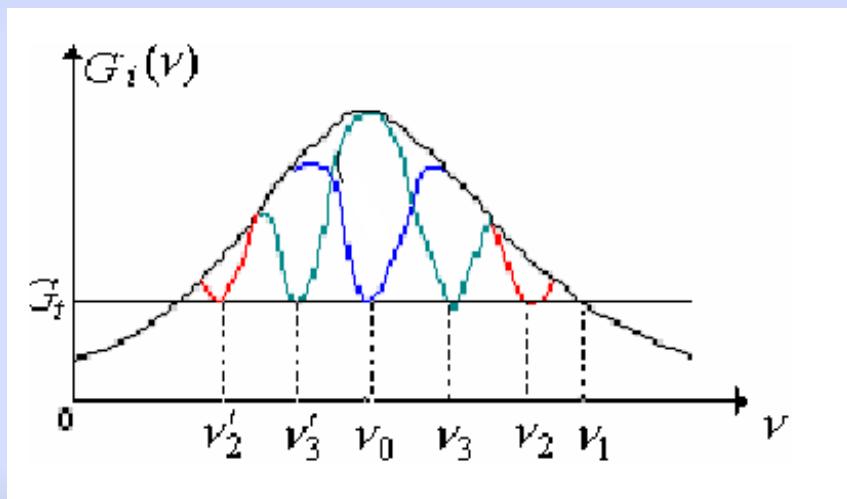
激光器输出光强：

$$I_{\text{out}}(\nu_0) = t_1 I(\nu_0) = \frac{1}{2} t_1 I_s \left[\left(\frac{2LG_D^0(\nu_0)}{a_1 + t_1} \right)^2 - 1 \right]$$

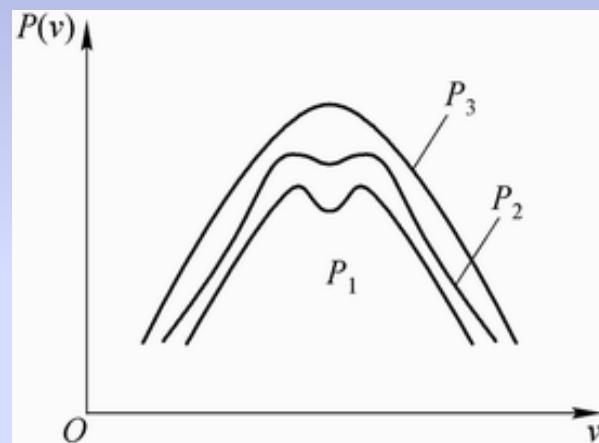
$$\text{激光器的输出功率 } P(\nu_0) = A I_{\text{out}}(\nu_0) = \frac{1}{2} A t_1 I_s \left[\left(\frac{2LG_D^0(\nu_0)}{a_1 + t_1} \right)^2 - 1 \right]$$

单纵模输出的激光器谐振频率由小到大变化，逐渐接近 ν_0 时，输出功率也逐渐变大，但当频率 ν 变到

$$\nu_0 - \sqrt{1 + \frac{I}{I_s}} \frac{\Delta\nu}{2} < \nu < \nu_0 + \sqrt{1 + \frac{I}{I_s}} \frac{\Delta\nu}{2}$$



兰姆凹陷的宽度: $\delta\nu = \sqrt{1 + \frac{I}{I_s}} \Delta\nu$



图(3-15)“兰姆凹陷”与管中气压的关系

若腔内允许多个谐振频率，且相邻两个纵模的频率间隔大于烧孔的宽度以及各频率的烧孔都是彼此独立的，则平均光强为：

$$I(\nu) = \begin{cases} I_s \left[\left(\frac{2LG_D^0(\nu)}{a_1 + t_1} \right)^2 - 1 \right] & \nu \neq \nu_0 \\ \frac{1}{2} I_s \left[\left(\frac{2LG_D^0(\nu_0)}{a_1 + t_1} \right)^2 - 1 \right] & \nu = \nu_0 \end{cases}$$

输出功率为： $P(\nu) = \begin{cases} At_1 I_s \left[\left(\frac{2LG_D^0(\nu)}{a_1 + t_1} \right)^2 - 1 \right] & \nu \neq \nu_0 \\ \frac{1}{2} At_1 I_s \left[\left(\frac{2LG_D^0(\nu_0)}{a_1 + t_1} \right)^2 - 1 \right] & \nu = \nu_0 \end{cases}$

多频激光器的输出功率为： $P = \sum_{i=1}^N P(\nu_i)$

若腔内多纵模的频率 ν 对称的分布在 ν_0 的两侧，也即有一个纵模频率 $\nu = \nu_0 + b$ ，必有另一个纵模频率 $\nu' = \nu_0 - b$ ，则在理想情况下纵模的增益系数为：

$$G(\nu) = \frac{G_D^0(\nu)}{\sqrt{1 + (I(\nu) + I(\nu'))/I_s}} = \frac{G_D^0(\nu)}{\sqrt{1 + 2I/I_s}} = G_{\text{固}}$$

纵模在腔内的平均光强为：

$$\overline{I(\nu)} = \frac{1}{2} I_s \left[\left(\frac{G_D^0(\nu) \cdot 2L}{a_1 + t_1} \right)^2 - 1 \right]$$

纵模 ν 的输出功率为：

$$P(\nu) = \frac{1}{2} A t_1 I_s \left[\left(\frac{G_D^0(\nu) \cdot 2L}{a_1 + t_1} \right)^2 - 1 \right]$$

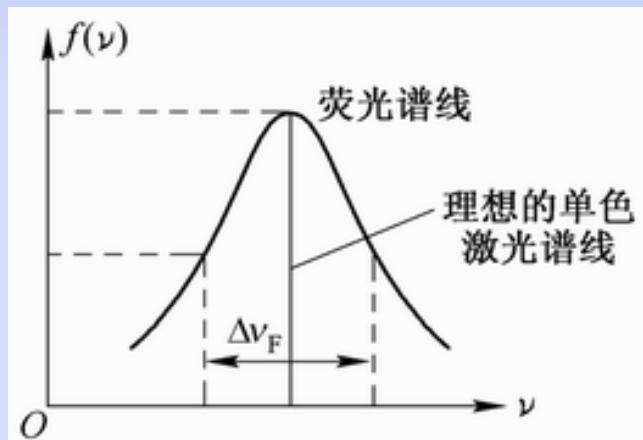
该多模激光器的输出功率为：

$$P = \sum_{i=1}^N P(\nu_i)$$

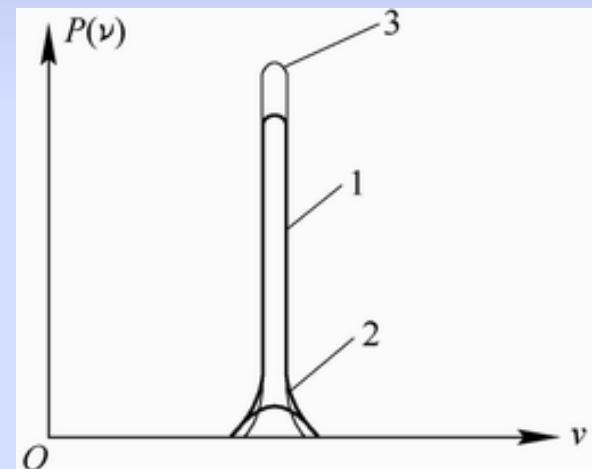
3.6 激光器的线宽极限

发光物质的荧光谱线

荧光线宽



图(3-16) 荧光谱线与理想的单色激光谱线



图(3-17) 激光的极限线宽

3.7 激光光束质量的品质因子 M^2

$$M^2 = \frac{w\theta}{w_0\theta_0}$$

$$w_0\theta_0 = \frac{\lambda}{\pi}$$