± 0 **theory** — Fully Integrated Mathematical Model (Alice Architecture Core)

0. Notation and Assumptions (Continuous Time Dynamics)

Time $t \geq 0$ is continuous. Expectation $\mathbb{E}[\cdot]$, Variance $\mathrm{Var}[\cdot]$.
Exponential decay kernel $k_\beta(t-\tau) = e^{-\beta(t-\tau)}\,\mathbf{1}_{\{\tau \leq t\}}$ $\quad(\beta > 0)$.
Wiener process $W(t)$, Poisson process $N(t)$ (Jumps).
Subjective parameter set $\boldsymbol{\theta}(t)$, Environmental/Stress state $X(t) \in \mathbb{R}^p$.
Uncorrected cumulative quantities $H(t), U(t)$. Corrected states $H'(t), U'(t)$.
$\mathbf{L_M}$: Abstracted Memory Blocks (Mid/Long-Term Tendency).
$\mathbf{L_S}$: Singular Memory Points (Trauma/Peak Events).

1 Emotional Core Cumulative Quantities and Environmental Terms

1.1 Definition of Cumulative Quantities (Convolution Integral & HALM Principle)

The cumulative quantities $\mathbf{H(t)}$ and $\mathbf{U(t)}$ are rigorously defined as the convolution integral of instantaneous inputs (Theoretical Rigor).

$$H(t) = \int_0^t H_{\mathrm{inst}}(\tau)\, e^{-\beta_H(t-\tau)}\, d\tau + H_{\mathrm{env}}(t) \quad (\beta_H > 0)$$
$$U(t) = \int_0^t U_{\mathrm{inst}}(\tau)\, e^{-\beta(t-\tau)}\, d\tau \;+\; U_{\mathrm{env}}(t) \quad (\beta > 0)$$

However, for Computational Implementation, the convolution integral is approximated as the accumulation of a short-term SDE and the influence term μ^{HALM} from the Hierarchical Abstraction Memory (HALM) model.

$$H(t) \approx H(t-\Delta t) + \Delta t \cdot \mu_{\mathbf{H}}^{\mathrm{HALM}}(\cdot) + \sigma_H(\cdot)\, dW_H(t)$$
$$U(t) \approx U(t-\Delta t) + \Delta t \cdot \mu_{\mathbf{U}}^{\mathrm{HALM}}(\cdot) + \sigma_U(\cdot)\, dW_U(t)$$

Definition of Cumulative Quantity Drift Term $\mu^{\mathrm{HALM}}(\cdot)$ based on HALM Calculation

$$\mu_{\mathbf{H}}^{\text{HALM}}(\cdot) = \underbrace{\left[H_{\text{inst}}(t) - \beta_H H(t) + \mu_H^{\text{base}}(\cdot)\right]}_{\text{SDE Short-Term Dynamics}} + \underbrace{\sum_{m \in L_M} \overline{H}_{\text{inst},m} \cdot k_{\beta_M}(t - t_m) + \sum_{s \in L_S} H_{\text{inst},s} \cdot k_{\beta_S}(t - t_s)}_{\text{HALM Memory Influence Term}}$$

$$\mu_{\mathbf{U}}^{\text{HALM}}(\cdot) = \underbrace{\left[U_{\text{inst}}(t) - \beta U(t) + \mu_U^{\text{base}}(\cdot)\right]}_{\text{SDE Short-Term Dynamics}} + \underbrace{\sum_{m \in L_M} \overline{U}_{\text{inst},m} \cdot k_{\beta_M}(t - t_m) + \sum_{s \in L_S} U_{\text{inst},s} \cdot k_{\beta_S}(t - t_s)}_{\text{HALM Memory Influence Term}}$$

1.2 Non-linear Composition of Environmental Terms $H_{\mathbf{env}}(t)$ and $U_{\mathbf{env}}(t)$

$$H_{\text{env}}(t) = \sum_m \rho_m \tanh\big(\tilde{\gamma}_m \eta_m(t)\big) \quad (\eta_m(t) \text{ is a happiness contributing factor})$$

$$U_{\text{env}}(t) = \sum_k \sigma_k \tanh\big(\gamma_k h_k(t)\big) \quad (h_k(t) \text{ is an unhappiness contributing stressor})$$

1.3 SDE for Environmental Factors

$$dh_k(t) = \Big[-\alpha_k\big(h_k(t) - m_k(t)\big)\Big]dt + \nu_k\, dW_k(t) + dJ_k(t)$$

$$d\eta_m(t) = \Big[-\tilde{\alpha}_m\big(\eta_m(t) - \tilde{m}_m(t)\big)\Big]dt + \tilde{\nu}_m\, d\tilde{W}_m(t) + d\tilde{J}_m(t)$$

($m_k(t) = A_k \sin(\frac{2\pi}{T_k}t) + B_k$ etc., are periodic functions)

2 Composition of Instantaneous Contributions $\mathbf{H}_{\mathbf{inst}}(\mathbf{t})$ and $\mathbf{U}_{\mathbf{inst}}(\mathbf{t})$

2.1 Instantaneous Happiness $H_{\mathbf{inst}}(t)$

$$H_{\text{inst}}(t) = \sum_i \mathbb{E}[w_i(t)] \cdot \mathbb{E}[f_i(t)] \cdot \mathbb{E}[s_i(t)]$$

(Using $\mathbb{E}[w_i(t)] = \mu_i(t)$)

2.2 Multiplicative Structure of Happiness Degree $\mu_i(t)$

$$\mu_i(t) = w_{i0} \cdot q_i(t) \cdot r_i(t) \cdot c_i(t) \cdot v_i(t) \cdot d_i(t) \quad (w_{i0} = 1)$$

2.3 Instantaneous Unhappiness $U_{\mathbf{inst}}(t)$ (Including Interaction λ_{jk})

$$U_{\text{inst}}(t) = \sum_j \mathbb{E}[u_j(t)]\, g_j(t) \;+\; \sum_{j,k} \lambda_{jk}\, \mathbb{E}[u_j(t)]\, \mathbb{E}[u_k(t)]$$

(Using $\mathbb{E}[u_j(t)] = \nu_j(t)$)

2.4 Dynamic Learning Rule for Unhappiness Interaction Term λ_{jk}

$$\frac{d\lambda_{jk}}{dt} = \alpha_\lambda \cdot \nu_j(t) \cdot \nu_k(t) - \rho_\lambda \cdot (\lambda_{jk} - \lambda_{jk}^{\text{base}})$$

3 SDE Definition of Dynamic Factors

All dynamic factors $X_i(t) \in \{q_i, r_i, c_i, v_i, d_i\}$ and $Y_j(t) \in \{s_j, l_j, a_j, c_j, r_j, v_j, i_j\}$ follow an SDE of the form:

$$dX(t) = \mu_X(\cdot)\,dt + \sigma_X(\cdot)\,dW_X(t)$$

3.1 SDE for Happiness Side $\mu_i(t)$ Components

$$\begin{aligned}
dq_i(t) &= \Big[\alpha_i H'(t) - \beta_i U'(t) - \gamma_i(q_i(t) - q_{i0})\Big]dt + \sigma_{q,i}\,dW_{q,i}(t) \\
dr_i(t) &= \Big[\alpha_r \cdot \text{match_event}(t) - \beta_r \cdot (r_i(t) - r_{i0})\Big]dt + \sigma_{r,i}\,dW_{r,i}(t) \\
dc_i(t) &= \Big[\phi_i(t) \cdot v_{\text{val}}(t) - \psi_i(t) \cdot (c_i(t) - c_{i0})\Big]dt + \sigma_{c,i}\,dW_{c,i}(t) \\
dv_i(t) &= \Big[\alpha_v \cdot H_{\text{env}}(t) - \beta_v \cdot U_{\text{env}}(t) - \gamma_v \cdot (v_i(t) - v_{i0})\Big]dt + \sigma_{v,i}\,dW_{v,i}(t) \\
dd_i(t) &= \Big[-\theta_i \cdot f_{\text{past}}(t) - \lambda_i \cdot (d_i(t) - d_{i0})\Big]dt + \sigma_{d,i}\,dW_{d,i}(t)
\end{aligned}$$

3.2 SDE for Unhappiness Side $\nu_j(t)$ Components

$$\begin{aligned}
ds_j(t) &= \Big[\alpha_s U'(t) - \beta_s H'(t) - \gamma_s(s_j - s_{j0})\Big]dt + \sigma_{s,j}\,dW_{s,j}(t) \\
dl_j(t) &= \Big[\alpha_l U'(t) - \gamma_l(l_j - l_{j0})\Big]dt + \sigma_{l,j}\,dW_{l,j}(t) \\
da_j(t) &= \Big[\alpha_a \cdot \text{Recur}_j(t) - \gamma_a(a_j - a_{j0})\Big]dt + \sigma_{a,j}\,dW_{a,j}(t) \\
dc_j(t) &= \Big[\alpha_c \cdot \text{Impact}_j(t) - \gamma_c(c_j - c_{j0})\Big]dt + \sigma_{c,j}\,dW_{c,j}(t) \\
dr_j(t) &= \Big[\alpha_r U'(t) - \beta_r H'(t) - \gamma_r(r_j - r_{j0})\Big]dt + \sigma_{r,j}\,dW_{r,j}(t) \\
dv_j(t) &= \Big[\alpha_v U_{\text{env}}(t) - \gamma_v(v_j - v_{j0})\Big]dt + \sigma_{v,j}\,dW_{v,j}(t) \\
di_j(t) &= \Big[\alpha_i \cdot \text{Isolation}(t) - \gamma_i(i_j - i_{j0})\Big]dt + \sigma_{i,j}\,dW_{i,j}(t)
\end{aligned}$$

4 Strict Definition of Correction and Recovery Terms $\mathbf{P(t)}$ and $\mathbf{R(t)}$

4.1 Positive Correction Term $\mathbf{P(t)}$

$$
\begin{aligned}
P(t) &= A_P(t) \cdot C_P(t) \cdot S_P(t) \cdot M_P(t) \\
A_P(t) &= \frac{1}{1 + e^{-\gamma_P \left(\overline{H}_{\text{env}}(t) - \delta_P\right)}} \\
C_P(t) &= 1 + \epsilon_P \cdot \cos\!\left(\frac{2\pi}{T_{\text{day}}} t + \phi_P\right) \\
S_P(t) &= \alpha_P e^{-t/\tau_{P1}} + (1 - \alpha_P) e^{-t/\tau_{P2}} \\
M_P(t) &= \tanh\!\big(\beta_P \cdot I_P(t)\big)
\end{aligned}
$$

4.2 Recovery Term $\mathbf{R(t)}$ (HALM $\mathbf{S_R(t)}$ Integrated Version)

$$
\begin{aligned}
R(t) &= A_R(t) \cdot C_R(t) \cdot T_R(t) \cdot H_R(t) \cdot \mathbf{S_R(t)} \cdot M_R(t) \\
A_R(t) &= \frac{1}{1 + e^{-\gamma_R \left(\overline{H}_{\text{env}}(t) - \delta_R\right)}} \\
C_R(t) &= 1 + \epsilon_R \cdot \cos\!\left(\frac{2\pi}{T_{\text{day}}} t + \phi_R\right) \\
T_R(t) &= \frac{1}{1 + e^{-\kappa_R \left(U(t) - \theta_R\right)}} \\
H_R(t) &= e^{-\lambda_R \int_0^t U(\tau)\, d\tau} \\
\mathbf{S_R(t)} &= \exp\left(-\lambda_S \sum_{s \in L_S} U_{\text{inst},s} \cdot k_{\beta_S}(t - t_s)\right) \quad \text{(Singular Memory Dependence)} \\
M_R(t) &= \tanh\!\big(\beta_R \cdot I_R(t)\big)
\end{aligned}
$$

5 Final Dynamics and Objectives of the ±0 theory

5.1 Corrected States $H'(t), U'(t)$ after Normalization and Psychological Correction

$$
\begin{cases}
H'(t) = \kappa_H H(t) + P\big(t; \boldsymbol{\theta}(t), X(t)\big) \\
U'(t) = \kappa_U U(t) - R\big(t; \boldsymbol{\theta}(t), X(t)\big)
\end{cases}
$$

($\kappa_H, \kappa_U > 0$ are calibration gains)

5.2 Final SDE for Corrected States (HALM $\mu^{\mathbf{HALM}}$ Integrated Version)

$$
\begin{cases}
dH'(t) = \left[\kappa_H \cdot \mu_{\mathbf{H}}^{\text{HALM}}(\cdot) + \mu_P(\cdot)\right] dt + \sigma_{H'}(\cdot)\, dW_{H'}(t) + J_{H'}(\cdot)\, dN_{H'}(t) \\
dU'(t) = \left[\kappa_U \cdot \mu_{\mathbf{U}}^{\text{HALM}}(\cdot) - \mu_R(\cdot)\right] dt + \sigma_{U'}(\cdot)\, dW_{U'}(t) + J_{U'}(\cdot)\, dN_{U'}(t)
\end{cases}
$$

Here, the SDE drift terms $\mu_{\mathbf{H}}^{\text{HALM}}(\cdot), \mu_{\mathbf{U}}^{\text{HALM}}(\cdot)$ are defined in Section 1.1 based on the integration of short-term SDE dynamics and HALM memory terms:

$$\mu_{\mathbf{H}}^{\text{HALM}}(\cdot) = \Big[H_{\text{inst}}(t) - \beta_H H(t) + \mu_H^{\text{base}}(\cdot)\Big] + \sum_{m \in L_M} \overline{H}_{\text{inst},m} \cdot k_{\beta_M}(t - t_m) + \sum_{s \in L_S} H_{\text{inst},s} \cdot k_{\beta_S}(t - t_s)$$

$$\mu_{\mathbf{U}}^{\text{HALM}}(\cdot) = \Big[U_{\text{inst}}(t) - \beta U(t) + \mu_U^{\text{base}}(\cdot)\Big] + \sum_{m \in L_M} \overline{U}_{\text{inst},m} \cdot k_{\beta_M}(t - t_m) + \sum_{s \in L_S} U_{\text{inst},s} \cdot k_{\beta_S}(t - t_s)$$

($\mu_H^{\text{base}}(\cdot), \mu_U^{\text{base}}(\cdot)$ are stabilization terms including the interaction terms $\lambda_{HU}, \lambda_{UH}$ between $\mathbf{H}$ and $\mathbf{U}$)

5.3 Equilibrium Condition and Limit of Death

$$\begin{aligned}
\text{Equilibrium Condition} \quad & \mathbb{E}\big[H'(t)\big] \;\approx\; \mathbb{E}\big[U'(t)\big] \\
\text{Control Objective Function} \quad & \min_{\text{Intervention}} \quad J = \limsup_{T \to \infty} \frac{1}{T} \int_0^T \Big(\mathbb{E}[H'(t)] - \mathbb{E}[U'(t)]\Big)^2 dt \\
\text{Limit of Death} \quad & \lim_{t \to T} \mathbb{E}[H(t)] = 0, \qquad \lim_{t \to T} \mathbb{E}[U(t)] = 0
\end{aligned}$$

5.4 Integrated Loss Function for Calibration

$$L_{\text{Total}} = L_{\text{Zero}} + \lambda_{\text{Pred}} L_{\text{Predict}} + \lambda_{\text{Scale}} L_{\text{Scale}}$$

$$\begin{aligned}
L_{\text{Zero}} &= \frac{1}{T} \int_0^T \Big(\kappa_H \widetilde{H}(t) - \kappa_U \widetilde{U}(t)\Big)^2 dt \\
L_{\text{Predict}} &= \frac{1}{T} \int_0^T \big(U^{\text{pred}}(t) - U^{\text{obs}}(t)\big)^2 dt \\
L_{\text{Scale}} &= \big(\kappa_H \mathbb{E}[H_{\text{inst}}^A] - \mu_A\big)^2 + \big(\kappa_U \mathbb{E}[U_{\text{inst}}^T] - \mu_A\big)^2
\end{aligned}$$