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2. **SFT**□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
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3. **RLHF**□□□□□□□□□□
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SFT□Supervised Finetune□

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- **Few-Shot Prompting** □□□□□□□□□□□□□□1-5□□□□□□□□□□□□□□□□□□□□□

- **Seed Prompt** □□□□□□□□□`task_type`□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

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    - answer□□□□□□□□□□□□□□□□
    - task_type □□□□□sft□□□□□□□□□□□□□□□

- □□□□

    - □□prompt
        - □□□task_type□□□□□□□□□seed prompt□□□□□□□seed□□□□□□□□□**pretrain**□□□□□□
          □□□**seed**□□□□□□□□□□**prompt**
    - □□answer
        - □□□□□□□GPT4/Claude3
        - □□□□□Qwen_72B/deepseek_MoE

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RL□Reinforcement Learning□

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- **Agent**□□□□□□□□□□□□□□□
- **action**□□□
- **Environment**□□□□□□□reward□□□□□
- **reward**□□
- **State**□□□□□□

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1. □□M^2□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
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3. □□M^2□□□□□□□□□□□□□□□□□

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$$ \begin{bmatrix} S_1 \to S_1 & S_1 \to S_2 & S_1 \to S_3 \\ S_2 \to S_1 & S_2 \to S_2 & S_2 \to S_3 \\ S_3 \to S_1 & S_3 \to S_2 & S_3 \to S_3 \end{bmatrix} $$

- □□□□□t□□□S_t□□□□□**t**□□□□□□□□□□□□□□□$(S_1,\dots,S_t)$□□□□□□□□□□S_{t+1}□□□□□□ $p(S_{t+1}|S_1,\dots,S_t)$
- □□

$$ p(S_{t+1}|S_t) =p(S_{t+1}|S_1,\dots,S_t) $$

- □□□□□□□□□□□□□□□□□□□□□

  - □□□□□□$<S,P>$□S□□□□□□□P□□□□□□□

  $$ P_{ss'} = p(S_{t+1}=s'|S_t = s) $$

□□□□□□□□□MRP□

□□□□□□$<S,P,R,\gamma>$

- S□□□□□□

- P□□□□□□□□□

- R□□□□□ $R_S = E[R_{t+1}|S_t=s]$

  - □□□□□□□□□□□□
  - □□□□□s□□□s'□□□
  - □□□□□□□□s'□□□□□□s□□□□□□□□□□□□□□□□□□□□□□

- γ□□□□□/□□□□ $\gamma \in [0,1]$
- □□□Return□□G_t□□□□t□□□□□□□□
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  $$ G_t = R_{t+1}+\gamma R_{t+2}+\gamma^2 R_{t+3}+\dots = \sum_{k=0}^{\infty}\gamma^kR_{t+k+1} $$

- □□□□$V(s)$□□□□□□s□□□□□□□
  - □□□□□□□□□□
  - □□s□□

$$ V(s) = E[G_t|S_t=s] $$

MRPs□□□□□□

□□□□□□□□□ $$ V(s) = E[G_t|S_t=s]\ G_t = R_{t+1} + \underbrace{\gamma R_{t+2}+\gamma^2 R_{t+3}+\dots}{\gamma V(S{t+1})} = \sum_{k=0}^{\infty}\gamma^kR_{t+k+1} $$ □□□ $$ V(s) = E[R_{t+1}+\gamma V(S_{t+1})|S_t = s] $$

□□□□□□□□□□**MDP**□

□□□□□□□□□□□□□□□$<S,A,P,R,\gamma>$

- S□□□□□□
- A□□□□
- P□□□□□□□□□ $P_{ss'}^a = p[S_{t+1}=s'|S_t=s,A_t=a]$
- R□□□□□ $R_S^a = E[R_{t+1}|S_t=s,A_t=a]$
  - □□□□□□□□□□□□□
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- γ□□□□□/□□□□ $\gamma \in [0,1]$

□□

π□□□□□□□□□□□□$\pi(a|s)=P[A_t=a|S_t=s]$

- □□□□□□□□□□□□□□s□□□□□□□a□□□□□
- □□□□□□□□□□
- MDP□□□□□□□□□□□□□□□□□□□□□□

□□□□□□□□□□□□□$M=<S,A,P,R,\gamma>$□□□□π□□□□□□□□□□□□□□□□□□□□□□

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$$ P_{s,s'}^{\pi} = \sum_{a\in A}\pi(a|s)P_{s,s'}^a \ R_s^{\pi} = \sum_{a \in A} \pi(a|s)R_s^a $$

- □□□□□□□□□□□□s□□□□□□□□□□□□s'□□□□□□□□□□□□□
- □□□□□□□□□□□□□□□□□□□□□

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□□□□□□s□□□□□□□**π**□□□□□□

image-20250817175244201 $$ V_{\pi}(s) = E_{\pi}[G_t|S_t=s] $$ □□□□□□□ $$ G_t =

\underbrace{R_{t+1}}{\text{□□□□}} + \underbrace{\gamma R{t+2}+\gamma^2 R_{t+3}+\dots}{{\text{□□□□□□□□□□□}}} = \sum_{k=0}^{\infty}\gamma^kR_{t+k+1} $$ □□□□□□□□□□□□□□□□□□□□□□□□ $$ V_{\pi}(s) = E_{\pi}[R_{t+1}+\gamma V_{\pi}(S_{t+1})|S_t = s] $$

□□□□□□

□□□□□□s□□□□□□**a**□□□□□□□**π**□□□□□□

image-20250817175237288 $$ q_{\pi}(s,a) = E_{\pi}[G_t|S_t = s,A_t = a] $$ □□□□□□□□□□□□□□□□□□□□□□□□

□□□□□ $$ q_{\pi}(s,a) = E_{\pi}[R_{t+1} +\gamma q_{\pi}(S_{t+1},A_{t+1})|S_t = s,A_t = a] $$

□□□□□□□□□□□□□□□□□□□□□□

□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)q_{\pi}(s,a) $$

image-20250817181356989

□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ $$ q_{\pi}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s') $$

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□□□□□□□□

□□□□□□□□ $$ V_{*}(s) = \max_{\pi}V_{\pi}(s) $$ □□□□□□□□ $$ q_*(s,a) = \max_{\pi}q_{\pi}(s,a) $$ □□□□

- □□□□□□□□□□□□$\pi_* \ge any \pi$
- □□□□□□□□□□□□□□□□□□□□
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$$ \pi_* \ge any \pi $$

□□□$V_{\pi'}(s)\ge V_{\pi}(s)$□□$\pi' > \pi$

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□□□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)q_{\pi}(s,a) $$ □ $$ q_{\pi}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s') $$ □□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)(R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s')) $$ □□□□□□□□□□□□□□□□□□□□□□□□□□□□

□□□□□□□□□□□□□□□□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)q_{\pi}(s,a) $$

□□□□π_*□□□□□□□□s□□□□□□$q_*(s,a)$

- □□□□π_*□□□□$V_{\pi_*}(s) \ge V_{\pi}(s)$
- □□□$q_{\pi_*}(s,a) \ge q_{\pi}(s,a)$
- □□□$V_{\pi}(s)$□□□□□□□$q_{\pi}(s,a)$

$$ \pi_{}(a \mid s) = \begin{cases} 1 & \text{if } a = \arg \max_{a} q_{}(s, a), \ 0 & \text{otherwise}. \end{cases} $$ □□□□□□□□□□□□□ $$ V_*(s) = \max_aq_*(s,a) $$ □□□□□□□ $$ q_*(s,a)=R_s^a+\gamma\sum_{s'}P_{s,s'}^aV_*(s') $$ □□□□□□□□□□□□□□□□□□□ $$ V_*(s) = \max_a(R_s^a+\gamma\sum_{s'}P_{s,s'}^aV_*(s')) $$

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□□□□□□ $$ q_{\pi}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s') $$

□□□□□□□□□□□□□□□□□□□□□□V_{π}□□ $$ q_{}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{}(s') $$ □□$V_*(s')$□□□□□□□□□□□□□□□□ $$ q_{}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^a\max_{a'}q_(s',a') $$

LLaMA

□GPT□□□LLaMA□□□□□**Transformer**□□□□□□Decoder-only□□□□□□□□□□□□□□□□□□□□□

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RMSNorm

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□□□□ $$ RMSNorm(x) = \frac{x}{\sqrt{\frac{1}{d}\sum_{i=1}^dx_i^2+ \epsilon}} \cdot\gamma $$ □□□

- x□□□□
- d□□□□□□□□
- ϵ□□□□□□□□□□
- γ□□□□□□□□□□□

SwiGLU

□□□□□□□□□□□□□□□□□□**Swish**□□□□□□□□□□□□

Swish□□□□ $$ Swish(x) = x \cdot \sigma(x) $$

- σ□Sigmoid□□

GLU□□□□ $$ GLU(x) = \sigma(W_1x+b_1) \odot(W_2x+b_2) $$

- $\odot$□□□□□
- $W_1□W_2$□□□□□□$b_1□b_2$□□□□

SwiGLU□□□□ $$ SwiGLU(x) = Swish(Linear_1(x))\odot Linear_2(x) $$ □□□□□□

□□□

- □□□□□□Swish□□□□□□□□□□□□□
- □□□□□GLU□□□□□□□□□□□□□□□

RoPE

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GQA

□LLaMA2□3□□□

□□□□□□□

GPT

Deepseek

MLA□□□□□□□

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- □□□□□□□□GPU□□□□
- □□□GPU□□□□□□

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- **DP□Data Parallel□**
- **DDP□Distributed Data Parallel□**
- **FSDP□Fully Sharded Data Parallel□**

All-Reduce□□□□□□

- □□□□□□□□GPU□□□□□□□□□□□□□□□□□□□□□□□□
- □□□□
  - □□□□□GPU□□□□□□□
  - □□□□□□□□GPU□□□□□□□□□□□`SUM`□`MAX`□
  - □□□□□GPU□□□□□□□□□□□

All-Gather□□□□□□

- □□□□□□□□GPU□□□□□□□□□□□□□□□□□GPU□□□□□□□□□□□□□□
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  - □□□□□GPU□□□□□□□□□□□□

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DP□Data Parallel□

□□□□□□□**Python GIL**□□□□□□□□□□□□□□**CPU**□

image-20250814165126760

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1. □CPU□□□□□□□□□□□□□GPU□□
2. □□GPU□□□□□□□□□□□□□□□□□□□□□□□□□GPU0□
3. GPU0□□□□□□□□□□□□□□□□□□□□□□
4. □□□□□□□□□□□□□□□**GPU**□

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- □□GPU0□□□□□□□$(N-1)\psi$□□□□□□$(N-1)\psi$
- □□□□GPU□□□□□□□ψ□□□□□□ψ

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- □□□□□□□□□Python **GIL**□□□□□□**CPU**□
- GPU0□□□□□□□□□□□□□□□□□□□□□□□□□□□□

DDP□Distributed Data Parallel□

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- □□□□□□□□**py**□□□□□**rank**□□□□□□□□□□□□□□□□□□□□□□□
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- GPU0□□□□□□□□□□□□□□□□□GPU
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FSDP□Fully Sharded Data Parallel□

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DeepSpeed ZeRO-1

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DeepSpeed ZeRO-2

ZeRO-2□□□**FP16**□□□□□□□□□□□□GPU□□□□□□□□□□□□□

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DeepSpeed ZeRO-3

ZeRO-3□**FP16**□□□□□□□□

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□□□ $$ Y = XW $$ □□□

- $X \in R^{2 \times 2}□W \in R^{2 \times 2}$

□□□□ $$ \begin{bmatrix} y_1&y_2 \y_3&y_4 \end{bmatrix} = \begin{bmatrix} x_1&x_2 \x_3&x_4 \end{bmatrix} \begin{bmatrix} w_1&w_2 \w_3&w_4 \end{bmatrix} $$ □□□□□□□□□□□□□□□GPU□□□□□ $$ W = \begin{bmatrix} W_0&W_1
\end{bmatrix} $$ □□ $$ Y_0 = XW_0 = \begin{bmatrix} x_1&x_2 \x_3&x_4
\end{bmatrix}\begin{bmatrix} w_1 \ w_3
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\end{bmatrix} $$ image-20250815105857273

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- $X \in R^{2 \times 2}□W \in R^{2 \times 2}$

□□□□ $$ \begin{bmatrix} y_1&y_2 \y_3&y_4 \end{bmatrix} = \begin{bmatrix} x_1&x_2 \x_3&x_4 \end{bmatrix} \begin{bmatrix} w_1&w_2 \w_3&w_4 \end{bmatrix} $$ □□□□□□□□□□□□□□GPU□□□□□ $$ W = \begin{bmatrix} W_0\W_1
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PipeDream --- DeepSpeed

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- **Weight stashing**□□□□□□□□□□□□□□□□□□□□□□ Micro-batch □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
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□□□□□ `Vertical Sync` □□

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- **Device 1**□□□□5□□□□□□□□□□□□1□□□□□□□□□1□□□□□□□
- **Device 2**□□□□5□□□□□□□□□□□□1□□□□□□□□□1□□□□□□□

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□□□□□□□□□□□□**float16**□□□□□□□□□□□□□□**float32**□□□□□**Master-Weight**□

□□□□

1. □**Master-Weight**□□□**fp16**
2. □inputs□□□□□□□□□□□□□□□□□□□□□□FP16
3. □□□□□□□□□□□□fp32□□□□□□

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Loss Scaling

1. [illegible]fp16[illegible]fp32
2. [illegible]fp32[illegible]
3. [illegible]fp16[illegible]fp32[illegible]
4. [illegible]**fp32**[illegible]

[illegible]**fp16**[illegible]FP16[illegible]

image-20250813121114207

[illegible]RNN[illegible]fp16[illegible]fp32

[illegible]

[illegible]

image-20250813115950009

[illegible]

image-20250813115500851

[illegible]

1. [illegible]
2. [illegible]

image-20250813115927153

[illegible]

MHA[illegible]MQA[illegible]GQA

image-20250816120438735

MHA

[illegible]

MQA[illegible]Multi-Query Attention[illegible]

[illegible]

[illegible]

- [illegible]Transformer[illegible]key[illegible]value[illegible]K[illegible]V[illegible]Q[illegible]
- [illegible]**K[illegible]V**[illegible]
- [illegible]KV[illegible]

[illegible]

- □□□□□□□□□□□□□□□□□□
- □□□□□□□□□□□□□□□□□□□□□□□□

GQA□Grouped-Query Attention□

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□□□

- **□Query□□□□□□□□□□□□□Key□Value**
- □□□□□□N□N=1□□**MQA**□N□□Query□□□□□□**MHA**

□□□

- □□□□□□□□□□□□□□□□□□□□□**MHA**□□□□□□□□□□

MLA□Multi-Head Latent Attention□

□□□□□□□□□**Deepseek-V3**□□□

□□□□□

- KV Cache□□□□□□□□□□□□□□□□□□□□□□
- □□□□□**MQA**□**GQA**□□□□□**KV Cache**□□□□□□□□□□□□□□

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- □□□□□□□□KV Cache□□□□
- □□MQA□GQA□□□□□□

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- **□Key□Value**□□□□□□□□□□□□□□□□□□□□□□□□□□□KV□□□□□□□KV□□□□□□□□
- □□□□□□□□□□□□□KV□□□□MLA□□KV□□□□□□□

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□Query□Key□□□□□□□□$\begin{bmatrix}q_t^R& q_t^C\end{bmatrix}$□$\begin{bmatrix}k_t^R& k_t^C\end{bmatrix}$□□□□□□□□□□□$(q_t^C,k_t^C)$□□□□□□RoPE□□$(q_t^R,k_t^R)$

1. □□□□□□□h_t
2. □□□□□□□□c_t^{KV}□c_t^Q
   - c_t^{KV}□KV□□□□□
   - c_t^Q□Q□□□□
3. □Key□Value□□□□□
   - □□□□□□□□□□□□□□□□ Key □ Value□□$k_{t,i}^C,v_{t,i}^C$
   - □□□□□□□□□□□□□□□□□□□□□□□$k_{t,i}^R$□□□□□□□□□□□□
4. □□□Query□□□□□□□□□□□□□□□$q_{t,i}^R$□□□□□□□□□□$q_{t,i}^C$□□□□□□□□□□□□□□□□
5. □Q□K□V□□□□□□□□□□□□□□□□□□□□

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□KV□□□□□□

image-20250816133715163

□□□□$c_t^{KV}□k_t^C□v_t^{C}$□□□□□□□□□□□□ $$ c_t^{KV} = W^{DKV}h_t \ k_t^C = W^{UK}c_t^{KV} \ v_t^C = W^{UV}c_t^{KV} $$ □□□□KV□□□□□□□□□□□□

□□□□□□□□□□□□□□□□□□c_t^{KV}□□□□□□□□□□□□□**K□V**□□

□**Q**□□□□□□□□□

image-20250816135113159

□□□□ $$ c_t^Q = W^{DQ}h_t\ q_t^C = W^{UQ}c_t^Q $$ □□□□□□**KV Cache**□□□□□□□□□□□□□□□□□□

MLA□Q□K□RoPE□□

MLA□□□k_t^R□q_t^R□□RoPE□□□□□**RoPE**□□□□□□□□□□□

- RoPE□Q□K□□□□□□□□□□□□□□□□□□□**RoPE**□□□□□□
- □□□□□□□c_t^{KV}□□RoPE□□□□□□□□□□R_n□□□□□W_{QK}□□□□□□□□□□K_{rot}□□□RoPE□□□□□□□□□**KV Cache**□□□□□

$$ K_{rot} = R_n(W_{QK}\cdot c_t^{KV}) $$

- □□□□□□□□□□□□□□□□W_{QK}□□□□□□□□□

□□□□□□□ $$ S = (W^{UQ})^T(c_t^Q)^TR_m^TR_nc_t^{KV}W^{UK} $$ □□□□ $$ S = (c_t^Q)^TR_m^T(W^{UQ})^TW^{UK}R_nc_t^{KV}\ =(c_t^Q)^TR_m^TW_{merge}R_nc_t^{KV} $$ □□□□□□□□□□□□□□□□□□W_{QK}□□□□□□□□□

MLA□□□RoPE□c_t^{KV}□□□□□□□□□□□□□□□□

1. □□□□□□□**k_t^C**□□
   - □□□□□□ K□c_t^{KV}□□□□ **RoPE**□□□□□
   - □□□□□□□□□ c_t^{KV}□□□□□□□
2. □□ **RoPE** □□□**k_t^R**□□
   - □□□ h_t□□□□□□□□□□ **RoPE**
   - □□□□□□□□□□□□□□□□□□
3. □□□□
   - □□$K=[k_t^C;k_t^R]$□□□□□□□□□□□□□□□KV Cache

MFA□Multi-matrix Factorization Attention□

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Paged Attention

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- GPU□CPU□□□□□□□□□□□□□□□
- □□□□/□□GPU□□□□□□□□□□□□□□

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- □□□□□□□□□□□□□
    - □□□□□□□□□□□□□□□□`[□□500MB] [□□1GB] [□□300MB] [□□2GB]`□□□□□□□□500MB□□□□□□□□□□□□□□□□□600MB□□□□□□□□□□□□
    - □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

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- □□□□□□□□□□□□□□□□□□□□□□□Paged Attention□□□□□□□□□□□□□□□□□□□□□□□□**KV Cache**

image-20250819121841203

- □□□□□□□□□□□□**CPU**□□□□□□□□□□□□□□□□□□
- **KV Cache**□□□Prompt□□□□□Paged Attention□□□□KV Cache□□□□□
    - □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ KV □□
    - □□□□**Copy-on-Write**□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□**3**□□□**5**□□□□□□□□□□□□□□□□□□

Flash Attention

□□□□□□□□IO□□□□□□□□□□SRAM□□□□IO□□

□□GPU□□□□□□□□□□□□□□□**SRAM**□□□□□□□□**HBM**□□□□□□□**Flash attention**□□□□□□**Attention**□□□□□**HBM**□□□

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- □□□□□□HBM□□□□□□□□□□□□□□
- □□□□□□□□□□□GPU□□□□□□□□□□□
- □□□□□□□□□□□□□□□□□SRAM□□□□□□□□□Pytorch□□□□□□□□□□□□□□□□□□□□□

□□□□pytorch□attention□□□□

- □HBM□□Q□K□SRAM
- □□□$S = QK^T$
- □S□□HBM□□□□□□□□□□□□□□□□□□□□
- □HBM□□S□HRAM
- □□$P=softmax(S)$
- □P□□HBM□□□□□□□□□□□□□□□□□□□□
- □HBM□□P□SRAM
- □□$O=PV$
- □O□□HBM
- □□O

□□□□□□□□□

- **Compute-Bound**□□□□□□□
  - □□□□□□□□□□□□□□□□
  - □□□□□□□**GPU**□□□□□□□□□□□□□□□□□□□□□□□
- **Memory-Bound**□□□□□□□
  - □□□ReLU□Softmax□Sum□Dropout□
  - □□□□□□□□□□□□□□GPU□□□□□□□□

□□□□**Safe-Softmax**

□□softmax $$ softmax(x_i) = \frac{e^{x_i}}{\sum_{j=1}^n e^{x_j}} $$ □□□□□□□□□□□□□□□□□**x_i**□□□□□□□□□□□□

Safe Softmax $$ softmax(x_i) = \frac{e^{x_i-\max(x)}}{\sum_{j=1}^n e^{x_j- \max(x)}} $$

- □□□□□□□□□□$\max(x)$□□□□□□□
- □□□□□□□□□□□$e^{\max(x)}$□□□□□□

□□□□□□□□□□

- **Tiling Algorithm**□□□□□□
  - □□□□□□□□SRAM□□□□□□□□□□□□□□□□□□□**SRAM**□□□□
  - □□□□□□□□□□□□□**SRAM**□□Flash Attention□□□□□□□SRAM□□□□□□□□□□□
- **Recomputation**□□□□□
  - GPU□□□□□HBM□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
  - □□$S=QK^T$□Flash Attention□□□HBM□□□□□□□□□SRAM□□□
  - □□$P = softmax(S)$□□□□□HBM□□□□□□□□□□
- **Kernal Fusion**□□□□□□
  - □□□□□□□□□□□□□□□□□**HBM**□□□
  - □□□□□□□□□Kernal□□□□□□□□□□□□

□□□□□□□□□□SRAM□□□□□□□□□□□

- □□□Q□K□V□□□□□□SRAM

- □□$S=QK^T$

- □□$P=softmax(S)$

  - □□□□□□□□softmax□□□□□□□□softmax

- □□□□□□□□□□□□Safe-Softmax
- □□□□□Safe-Softmax□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□Safe-Softmax□□□
    - □□□□□2□□□□$x_1 = [1,2]$$x_2 = [3,4]$
    - □□□□□□□□□□$m(x_1)=2=m(x)$
    - □□□□□□□□□□□□$f(x_1)=[e^{1-m(x_1)},e^{2-m(x_1)}] = [e^{-1},e^0]$
    - □□□□□□□□□□□$l(x_1)=e^{-1}+e^0$
    - □□□□□□□□□□□□$m(x)=\max(m(x_1),m(x_2)) = 4$
    - □□□$f(x_2) = [e^{3-m(x_2)},e^{4-m(x_2)}] =[e^{-1},e^{0}]$
    - □□□□□□□□□□□□$l(x_2)=e^{-1}+e^0$
    - □□□□$f(x)$□$l(x)$

  $$ f(x) = [e^{m(x_1)-m(x)}f(x_1),e^{m(x_2)-m(x)}f(x_2)]\ f(x) = [e^{-2}(e^{-1},e^0),e^0(e^-1,e^0)]\ l(x)=e^{m(x_1)-m(x)}*l(x_1) + e^{m(x_2)-m(x)}*l(x_2) \ l(x) = e^{-3} + e^{-2} + e^{-1} + e^0 $$

- □□$O=PV$
- □SRAM□□□O□□HBM

GPU□□□□

□□ `SIMT` □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

GPU□□□□□□□□□□Warp□32□□□□□□

SM□□□□□□□□□GPU□□□□□□

- **CUDA Cores□CUDA** □□□□ □□□□□□□□□□□
- **Tensor Cores**□□□□□□□ □□□□□□□□□□□□□□□□□
- **Warp Scheduler□Warp** □□□□□□□Warp□□□□□□□□□□
- **Shared Memory**□□□□□□□□□□□□□□□□□□□
- **L1 Cache**□□□□□□□□□□□□□□□□□□□□□□□
- **Load/Store Units**□□□□□□□□□HBM□□□□□□□□Shared Memory□□□□□
- **Register File**□□□□□□□□□□□□□□□□□
- **SFU**□□□□□□□□□□□□□

image-20250819141437101

GPU□□□□□□□□□□□□□

- **SRAM**□□□□□□L1 Cache□□□□□ **SM**□□□□L2 Cache□□□□□□□□**SM**□□□□Shared Memory□□□□□□□□□□□□□□□□□□□□□□□□□□
- **HBM**□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

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image-20250819132629103

- `NVLink` □**GPU**□□□□□□□□□□□□
- `PCIe` □**CPU-GPU**□□□□

[illegible]

[illegible]

1. [illegible]PCIe[illegible]CPU[illegible]GPU HBM
2. [illegible]**HBM**[illegible]**L2 Cache**[illegible]**Shared Memory**[illegible]**L1 Cache**[illegible]
    - [illegible]HBM[illegible]SM[illegible]**SM**[illegible]
3. [illegible] **CUDA Core**[illegible]**Tensor Core**[illegible]
4. [illegible]HBM