

# Transformer

## Sequence to Sequence Model

Transformer 모델은 입력 시퀀스 (Context)를 기반으로 출력 시퀀스를 생성하는 모델입니다.

구성 요소

- Word Embedding**
  - 단어를 임베딩 ID로 변환 (512차원)
  - PyTorch: `nn.Embedding`
- Positional Encoding**
  - Transformer는 RNN과 달리 순서를 모르므로 위치 정보를 추가
  - 구현 방법
    - 정현/부정현 코사인 함수
    - 학습된 임베딩

## Encoder

Encoder는 입력 시퀀스를 block 단위로 6번 반복 처리합니다.

각 block은 다음을 포함합니다:

- Multi-Head Self Attention**
  - 단어 간의 관계를 여러 헤드에서 학습
- Feed Forward Network**
  - MLP 구조
  - 비선형 활성화 함수

추가 구성 요소

- Dropout
- Layer Normalization**

## Decoder

Decoder는 다음을 포함합니다:

- Masked Self Attention (자기 자신과의 연결을 차단)
- Auto-regressive** (이전 출력을 입력으로 사용)

Decoder 구조

Decoder는 3개의 Sub-layer + LayerNorm으로 구성됩니다.

- Masked Multi-Head Self Attention
- Encoder-Decoder Attention

### 1. Masked Multi-Head Self-Attention

- Decoder가 자기 자신을 참조할 수 없도록 마스크
- Decoder가 Encoder의 출력을 참조할 수 있도록

### 2. Encoder-Decoder Attention

- Decoder가 Encoder의 출력을 참조

- Query Decoder Key Value Encoder
- Decoder " " Decoder " " / /

### 3. Feed Forward Network

- **全连接层 + 非线性激活函数ReLU/GeLU**

#### 4. $\text{ReLU} + \text{LayerNorm}$

□□□□□□□

- `output = input + Sublayer(input)`
- `LayerNorm`

- $\frac{1}{n} \sum_{i=1}^n x_i$



## Train□□□□□□

## 1. Encoder

- [illegible]

## 2. Decoder

- 00000000000000000000000000000000 label 00000000
- 00000000000000000000000000000000 "0 0 \_\_\_\_"00"0"0

### 3. Teacher Forcing□

- `tokenizer.tokenize(sentence)` → `tokens`
- `tokenizer.decode(tokens)` → `sentence`
- `tokenizer.encode(sentence)` → `tokens`

## Attention

- **Decoder Self-Attention** Mask 00000000000000000000000000000000
- **Encoder-Decoder Attention** Decoder 00000000000000000000000000000000 Encoder 00000000000000000000000000000000

1111

- $\text{softmax} \rightarrow \text{Cross Entropy Loss}$
- one-hot  $\rightarrow \text{Cross Entropy Loss}$

□□□□

□ □ □ □ □ □ □ □ □ □ □ □ □ □ □

□ □

## Teaching Forcing

Decoder	“ ”	“ ”	Teacher Forcing

Encoder Decoder

- 输入序列 <BOS> 输出序列
- 输入序列 输出序列“ ”
- 输入序列“ ”
- ...

在 输入序列 输出序列

Teacher Forcing

## Residual Connection

在 输入序列

输入序列 输出序列“ ” 输入序列 输出序列 $F(x)$  输入序列 输出序列 $x + F(x)$

在 输入序列

- 输入序列  $y = F(x) + x$  输出序列
- 输入序列  $\partial L / \partial x$  输出序列

$$\frac{\partial L}{\partial y} \cdot \frac{\partial y}{\partial x} = \frac{\partial L}{\partial y} \cdot (\frac{\partial F(x)}{\partial x} + 1)$$

- 在  $\frac{\partial F(x)}{\partial x}$  输入序列 1 输出序列

## Layer Normalization


在 输入序列

- 在 **BatchNorm** 输入序列 输出序列
- 在 NLP 输入序列 BatchNorm 输入序列 batch 输入序列
- **Layer Normalization** 输入序列 **feature** 输入序列 **dimension** 输入序列 **Batch Normalization** 输入序列 **feature** 输入序列 **dimension** 输入序列

## Explanation

在 输入序列 **self-attention** 输入序列

在 输入序列

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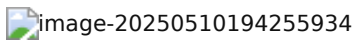
1. 输入序列  $a^1, a^2, a^3, a^4$
2. 输入序列

$$Q = \begin{bmatrix} q^1 & q^2 & q^3 & q^4 \end{bmatrix} \quad W^q \quad K = \begin{bmatrix} k^1 & k^2 & k^3 & k^4 \end{bmatrix} \quad W^k \quad V = \begin{bmatrix} v^1 & v^2 & v^3 & v^4 \end{bmatrix} \quad W^v$$

2. **attention score** 
$$A = \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} & \alpha_{1,3} & \alpha_{1,4} & \alpha_{2,1} & \alpha_{2,2} & \alpha_{2,3} & \alpha_{2,4} & \alpha_{3,1} & \alpha_{3,2} & \alpha_{3,3} & \alpha_{3,4} & \alpha_{4,1} & \alpha_{4,2} & \alpha_{4,3} & \alpha_{4,4} \end{bmatrix} \quad Q \cdot K^T$$

3.  $\sqrt{d_k}$  **softmax**  $d_k$   $key/query$

4.

$$\text{Attention}(Q,K,V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$
[illegible]

- $\text{aaaaaa} \$a^i, a^j, a^m, a^n \$ \text{aaaa} \$W^q W^k W^v \$ \text{aaaaaaaaaaaaaaaaaaaa} \$Q^K V \$$
- $\text{aaa} \$W^q W^k W^v \$ \text{aaaaaa}$

[illegible]

- ☐ ☐ ☐ ☐ ☐ ☐

$$K_1 = \begin{bmatrix} k^{1,1} & k^{1,j} & k^{1,m} & k^{1,n} \\ k^{i,2} & k^{j,2} & k^{m,2} & k^{n,2} \end{bmatrix} \quad K_2 = \begin{bmatrix} k^{1,1} & k^{1,j} & k^{1,m} & k^{1,n} \\ k^{i,2} & k^{j,2} & k^{m,2} & k^{n,2} \end{bmatrix}$$

$$V_1 = \begin{bmatrix} v^{1,1} & v^{2,1} & v^{m,1} & v^{n,1} \end{bmatrix} \quad V_2 = \begin{bmatrix} v^{1,2} & v^{2,2} & v^{m,2} & v^{n,2} \end{bmatrix}$$

- 计算点积

$$\text{head}_1 = \text{softmax}\left(\frac{Q_1 K_1^T}{\sqrt{d_k}}\right) V_1 \quad \text{head}_2 = \text{softmax}\left(\frac{Q_2 K_2^T}{\sqrt{d_k}}\right) V_2$$

- 拼接

$$\text{multihead} = \begin{bmatrix} \text{head}_1 & \text{head}_2 \end{bmatrix}$$

- 线性变换

$$\text{output} = \text{multihead} \cdot W^O$$

## Self-attention

计算每个 token 与所有 token 的点积

通过 **FC** 层将输入映射到 Q, K, V

## Sequence Labeling

输入序列

- 通过 **FC** 层将输入映射到 Q, K, V

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输出序列

通过 **FC** 层将输入映射到 Q, K, V

计算点积

- $Q = X \times W^Q$
- $K = X \times W^K$
- $V = X \times W^V$

计算点积

- $Q \cdot K^T$

- 归一化

- $Q = X \times W^Q$
- $K = X \times W^K$
- $V = X \times W^V$

$$Q \cdot K^T$$

- 计算  $(Q) \cdot (K)$  的 **attention score**

- dot product
  - $(Q) \cdot (K)$
  - $(K) \cdot (Q)$

- $QK^T$  の値を正規化する

- 正規化された値を **softmax** 関数で処理する
- softmax の結果を RELU 関数で処理する
- 処理された結果を (V) と乗算して **attention score** を計算する
  - $V$  の値を乗算する
  - $QK^T$  の値を乗算する
  - $V$  の値を乗算する

## Multi-head Self-attention

Multi-head Self-attention の仕組み

- $a^i$  の値を計算する
  - $b^{i,1}$ 
    - $q^{i,1} k^{i,1} k^{j,1}$  の値を  $b^{i,1}$  と計算する
  - $b^{i,2}$ 
    - $q^{i,2} k^{i,2} k^{j,2}$  の値を  $b^{i,2}$  と計算する
  - $b^i$ 
    - $b^{i,1} b^{i,2}$  の値を  $b^i$  と計算する **attention score**
- 最終的な結果を計算する



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## Masked Multi-Head Self-Attention

Masked Multi-Head Self-attention の仕組み

Masked Multi-Head Self-attention の仕組み "A B C D" token

Masked Multi-Head Self-attention の仕組み  $\begin{bmatrix} 1 & \text{masked} & \text{masked} & \text{masked} \\ 1 & 1 & \text{masked} & \end{bmatrix}$

Masked Multi-Head Self-attention の仕組み  $\begin{bmatrix} \text{masked} & 1 & 1 & \text{masked} \\ 1 & 1 & 1 & 1 \end{bmatrix}$  image-20250702191639250

## Word Embedding

Word Embedding の仕組み **One-Hot Encoding**

- 単語をベクトルに変換する
- 単語をベクトルに変換する

## Positional Encoding

- **Self-attention** の仕組み "A B C D" token
- 単語をベクトルに変換する

Positional Encoding の仕組み

1. **positional vector** を計算する
2. 単語をベクトルに変換する

$$\text{Final\_embedding} = \text{Token\_embedding} + \text{Positional\_encoding}$$

□□□□□□

1111111111111111

10000000000

111

$$PE_{(pos, 2i)} = \sin(pos/10000^{2i/d_{model}}) \setminus PE_{(pos, 2i+1)} = \cos(pos/10000^{2i/d_{model}})$$

1. \$pos\$
  2. \$2i\$  
\$2i \leq d\_{\text{model}}\$
  3. \$d\_{\text{model}}\$
- I am a kid
    - - I pos
      - am pos 1
    - am pos = 1
      - $i = 0$ 
        - $\text{pos}_{(1,0)} = \sin(1/10000^{0/d_{\text{model}}})$
        - $\text{pos}_{(1,1)} = \cos(1/10000^{0/d_{\text{model}}})$
      - $i = 1$ 
        - $\text{pos}_{(1,2)} = \sin(1/10000^{2/d_{\text{model}}})$
        - $\text{pos}_{(1,3)} = \cos(1/10000^{2/d_{\text{model}}})$

[illegible]

- [illegible]

□□□□□□□□

$$\underbrace{\begin{bmatrix} \text{PE}_{(\text{pos} + \Delta, 2i)} \mid \text{PE}_{(\text{pos} + \Delta, 2i+1)} \end{bmatrix}}_{\text{ } (\text{pos} + \Delta) \text{ } }$$

$$\underbrace{\begin{bmatrix} \cos(\Delta \theta_i) & \sin(\Delta \theta_i) \\ -\sin(\Delta \theta_i) & \cos(\Delta \theta_i) \end{bmatrix}}_{\text{}} \underbrace{\begin{bmatrix} PE_{(pos, 2i)} \\ PE_{(pos, 2i+1)} \end{bmatrix}}_{\text{}} \begin{bmatrix} pos \\ pos \end{bmatrix}$$

$$\Delta \theta_i = \frac{1}{10000^{\frac{2i}{d_{\text{model}}}}}$$

$$\underbrace{\begin{bmatrix} \cos(\Delta \theta_i) & \sin(\Delta \theta_i) \\ -\sin(\Delta \theta_i) & \cos(\Delta \theta_i) \end{bmatrix}}_{\text{rot}(\Delta \theta_i)}$$

$$\Delta \theta_i \text{pos}$$



$$\begin{aligned} & \begin{bmatrix} \text{PE}_{(\text{pos} + \Delta, 2i)} \\ \text{PE}_{(\text{pos} + \Delta, 2i+1)} \end{bmatrix} \\ &= \begin{bmatrix} \sin((\text{pos} + \Delta) \cdot \theta_i) \cos((\text{pos} + \Delta) \cdot \theta_i) \\ \sin((\text{pos} + \Delta) \cdot \theta_i) \sin((\text{pos} + \Delta) \cdot \theta_i) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\Delta \theta_i) \sin(\text{pos} \cdot \theta_i) + \sin(\Delta \theta_i) \cos(\text{pos} \cdot \theta_i) \\ \cos(\Delta \theta_i) \cos(\text{pos} \cdot \theta_i) - \sin(\Delta \theta_i) \sin(\text{pos} \cdot \theta_i) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\Delta \theta_i) & \sin(\Delta \theta_i) \\ -\sin(\Delta \theta_i) & \cos(\Delta \theta_i) \end{bmatrix} \begin{bmatrix} \sin(\text{pos} \cdot \theta_i) \cos(\text{pos} \cdot \theta_i) \\ \sin(\text{pos} \cdot \theta_i) \sin(\text{pos} \cdot \theta_i) \end{bmatrix} \\ &= \begin{bmatrix} \cos(\Delta \theta_i) & \sin(\Delta \theta_i) \\ -\sin(\Delta \theta_i) & \cos(\Delta \theta_i) \end{bmatrix} \begin{bmatrix} \text{PE}_{(\text{pos}, 2i)} \\ \text{PE}_{(\text{pos}, 2i+1)} \end{bmatrix} \end{aligned}$$

# Transformer

$$\Delta \theta_i$$

$$w_i = 10000^{\frac{2i}{d_{\text{model}}}} \theta_i = \text{pos} / 10000^{\frac{2i}{d_{\text{model}}}} = \text{pos} / w_i$$

$$\Delta \theta_i$$

- $w_i \theta_i$
- $w_i \theta_i$
- $w_i \sin(\text{pos} / 10000) \text{pos} \approx 20000 \pi$

$$\Delta \theta_i$$

- $QK^T$
- $QK^T$

$$q_i = W_q(x_i + p_i) \quad k_j = W_k(x_j + p_j)$$

$$q_i^T k_j = (x_i + p_i)^T W_q^T W_k (x_j + p_j) = \underbrace{x_i^T W_q^T W_k x_j}_{\text{}} + \underbrace{x_i^T W_q^T W_k p_j + p_i^T W_q^T W_k x_j}_{\text{}} + \underbrace{p_i^T W_q^T W_k p_j}_{\text{}}$$

$$q_i^T k_j = x_i^T W_q^T W_k x_j + \underbrace{p_i^T W_q^T W_k p_j}_{\text{}} \quad \beta_{i-j} = x_i^T W_q^T W_k x_j + \underbrace{\beta_{i-j}}_{\text{}} \quad \beta_{i-j} \text{head}_h \beta_{i-j}^h$$

# T5

$$\Delta \theta_i$$

$$\Delta \theta_i$$

# ALibi



QK\$



$m_h=2^{-\frac{8\times h}{n_{\text{head}}}}$   
 $m=\begin{bmatrix}2^{-\frac{8\times 1}{n_{\text{head}}}}2^{-\frac{8\times 2}{n_{\text{head}}}}\cdots2^{-\frac{8\times n_{\text{head}}}{n_{\text{head}}}}\end{bmatrix}$   
 $D_{ij}=-(i-j)$   
 $\text{softmax}(q_iK^T+m\cdot[-(i-1),\dots,-2,-1,0])$

RoPE

QK

Attention\_score =  $q_m\cdot k_n^T$   
 $q^{\text{rot}} = q_m\cdot R_m\cdot k^{\text{rot}} = k_n\cdot R_n$   
 $q^{\text{rot}}\cdot k^{\text{rot}} \&= q_mR_mR_n^Tk_n^T \&= q_mR_{mR_{-n}}k_n^T \&= q_mR_{m-n}k_n^T$   
RoPE

1.

2.

$m,n$   $m-n$

- $d$ RoPE $d/2$  $d$

$R_{\theta,m} = \begin{bmatrix} \cos m\theta_1 & -\sin m\theta_1 & 0 & \cdots & 0 \\ \sin m\theta_1 & \cos m\theta_1 & 0 & \cdots & 0 \\ 0 & 0 & \cos m\theta_2 & -\sin m\theta_2 & 0 \\ 0 & 0 & \sin m\theta_2 & \cos m\theta_2 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix}$

- 

$\begin{bmatrix} x_i \\ x_{i+1} \end{bmatrix}$

$\begin{bmatrix} \cos m\theta_i & -\sin m\theta_i \\ \sin m\theta_i & \cos m\theta_i \end{bmatrix}$   
 $\begin{bmatrix} x_i \\ x_{i+1} \end{bmatrix}$

$(m-n)$

$\theta_i = \frac{1}{10000^{\frac{2i}{d_{\text{model}}}}}$   
 $R_{m-n} = \begin{bmatrix} \cos((m-n)\theta_i) & \sin((m-n)\theta_i) \\ -\sin((m-n)\theta_i) & \cos((m-n)\theta_i) \end{bmatrix}$   
 $R_n^TR_m = R_{m-n}$   
 $R_{-m} = R^T_m$

$$R_{nq}^{TR_{mk}} = q^{TR_{m-n}k}$$

$$q_n^{\text{rot}} = R_{nq} \setminus k_m^{\text{rot}} = R_{mk}$$

$$QK\$$$

**KV Cache**

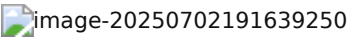
- 
- 
- $QK\$V\$$
- 

token $KV\$$

- 
- 
- 
- 

KV Cache“” $KV\$$

**KV Cache**



**KV Cache**



**Pre-NormPost-Norm**

**Pre-NormPost-Norm**TransformerLNLayer Normalization

- 
- Transformer\*\* (warm-up)\*\*
-