[illegible]

Diffusion probabilistic Model[illegible]

[illegible]

1. [illegible]
2. [illegible]
   - Denoise[illegible]step
   - Denoise[illegible]**Noise Predicter**[illegible]

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[illegible]

> ### [illegible]
>
> [illegible]*VAE*[illegible]
>
> *image-20250511132150467*
>
> $q(z_t|z_{t-1})$[illegible]$p(z_t|z_{t-1})$[illegible]
>
> [illegible] $$ p(x,z_1,z_2,\dots ,z_T) = p(x,z_{1:T}) = p(z_T)p_{\theta}(x|z_1)\prod_{t=2}^{T}p_{\theta}(z_{t-1}|z_t) $$
>
> [illegible]$Z_{1:T}$[illegible] $$ q_{\phi}(z_1,z_2,\dots,z_T | x) = q_{\phi}(z_{1:T}|x) = q_{\phi}(z_1|x)\prod_{t=2}^{T}q_{\phi}(z_t|z_{t-1}) $$ [illegible],[illegible] $p(x)$ $$ \begin{aligned} \ln p(x) &= \ln \int{p(x,z_{1:T})dz_{1:T}} \ &= \ln \int{\frac{p(x,z_{1:T})q_{\phi}(z_{1:T}|x)}{q_{\phi}(z_{1:T}|x)}dz_{1:T}} \ &= \ln \mathbb{E}{q{\phi}(z_{1:T}|x)}(\frac{p(x,z_{1:T})}{q_{\phi}(z_{1:T}|x)}) \ &\geq \mathbb{E}{q{\phi}(z_{1:T}|x)} (\ln{\frac{p(x,z_{1:T})}{q_{\phi}(z_{1:T}|x)}}) \end{aligned} $$
>
> ### [illegible]
>
> [illegible]$q(x_t|x_{t-1})$[illegible]
>
> [illegible]***x***[illegible]***z***[illegible]***x_t***[illegible]
>
> [illegible]T[illegible]x_T[illegible]T[illegible],x_T[illegible] *1* [illegible] [illegible] x_T[illegible]
>
> ### [illegible]-[illegible]
>
> [illegible]
>
> [illegible]x_0[illegible]x_T[illegible] $$ p(x_{0:T}) = q(x_0)\prod_{t=1}^{T}q(x_t|x_{t-1}) $$
>
> [illegible]x_T[illegible]$x_0 $[illegible] $$ p(x_{0:T}) = p(x_T)\prod_{t=T}^1p(x_{t-1}|x_t) $$
>
> #### [illegible]
>
> [illegible]x_0[illegible]$p(x_0)$[illegible]x_0[illegible] [illegible]x_0[illegible] [illegible]$x_{1:T}$ [illegible] [illegible]$q(x_{1:T}|x_0)$[illegible] $$ q(x_{1:T}|x_0) =

\frac{q(x_{0:T})}{q(x_0)} = \frac{q(x_0)\prod_{t=1}^{T} q(x_t|x_{t-1})}{q(x_0)} = \prod_{t=1}^{T} q(x_t|x_{t-1}) $$

[illegible]$q(x_t|x_{t-1})$[illegible] $q(x_t|x_{t-1})$[illegible]$x_{t−1}$ [illegible]β_tI[illegible] [illegible]$0<\beta_1<\beta_2<\dots\beta_T<1$ [illegible]β_t[illegible]

[illegible]$q(x_t|x_{t-1})$[illegible]$\mu {x_t}$[illegible]$x{t-1}$[illegible]α_t[illegible]$\alpha_t = 1 - \beta_t$[illegible]μ_t[illegible]x_{t-1}[illegible]

- [illegible]x_{t-1}[illegible]$\sqrt{\alpha_t}$[illegible]μ_{x_t}

$$ \mu_{x_t} = \sqrt{\alpha_t}x_{t-1} $$ x_t[illegible]x_{t-1}[illegible] $$ \Sigma_{x_t} = \beta_tI = (1 - \alpha_t)I $$ [illegible]**$q(x_t|x_{t-1})$**[illegible]**$\sqrt{\alpha_t}x_{t-1}$**[illegible]**$(1 - \alpha_t)I$**[illegible]$\sqrt{\alpha_t}x_{t-1}$[illegible]$\mathcal{N}(0,(1-\alpha_t)I)$[illegible]$t$ [illegible]**x_t**[illegible] $$ q(x_t|X_{t-1}) = \mathcal{N}(\sqrt{\alpha_t}x_{t-1},(1-\alpha_t)I) $$

$$ \begin{align} x_t &= \sqrt{\alpha_t}x_{t-1} + \mathcal{N}(0,(1-\alpha_t)I)\ &= \sqrt{\alpha_t}x_{t-1} + \sqrt{1-\alpha_t}\epsilon,\epsilon\sim \mathcal{N}(0,I) \end{align} $$

α_t[illegible]*t*[illegible]

[illegible]

[illegible]x_0[illegible]t[illegible]x_t[illegible]ϵ_t[illegible] $$ \begin{align} x_t &= \sqrt{\alpha_t}x_{t-1} + \sqrt{1-\alpha_t}\epsilon_t \ &= \sqrt{\alpha_t}(\sqrt{\alpha_{t-1}}x_{t-2} + \sqrt{1-\alpha_{t-1}}\epsilon_{t-1}) + \sqrt{1-\alpha_t}\epsilon_t\ &= \sqrt{\alpha_t\alpha_{t-1}}x_{t-2} + \underbrace{\sqrt{\alpha_t-\alpha_t\alpha_{t-1}}\epsilon_{t-1}+ \sqrt{1-\alpha_t}\epsilon_t}{[illegible]0[illegible]}\ &= \sqrt{\alpha_t\alpha{t-1}}x_{t-2} + \underbrace{\sqrt{\sqrt{\alpha_t-\alpha_t\alpha_{t-1}}^2+ \sqrt{1-\alpha_t}^2}\epsilon}{[illegible]} \ &= \sqrt{\alpha_t\alpha{t-1}}x_{t-2} +{\sqrt{1 - \alpha_t\alpha_{t-1}}\epsilon} \ &= \sqrt{\prod_{i=1}^t{\alpha_i}}x_0 + \sqrt{1 - \prod_{i=1}^t{\alpha_i}}\epsilon \ & = \sqrt{\overline{\alpha_t}}x_0 + \sqrt{1 - \overline{\alpha_t}}\epsilon ,\overline{\alpha_t} = \prod_{i=1}^t{\alpha_i},\epsilon\sim \mathcal{N}(0,I) \ &\sim\mathcal{N}(\sqrt{\overline{\alpha_t}}x_0,(1 -\overline{\alpha_t})I) \end{align} $$ [illegible]$\alpha_{0:T}$[illegible] [illegible]

[illegible]

[illegible]$p(x_{0:T})$[illegible] $$ p(x_{0:T}) = p(x_T)\prod_{t=T - 1}^0p(x_{t}|x_{t+1}) $$

[illegible]$p(x_T)$[illegible]$p_{\theta}(x_t|x_{t+1})$[illegible] $$ \begin{align} p_{\theta}(x_t|x_{t+1}) &= \frac{p(x_{t+1}|x_t)p(x_t)}{p(x_{t + 1})}\ &=\frac{p(x_{t+1}|x_t)p(x_t)}{\int{p(x_{t + 1}|x_t)p(x_t)dx_t}} \end{align} $$ [illegible]**x_t**[illegible]$p_{\theta}(x_t|x_{t+1})$[illegible]

[illegible]*(ELBO)*

[illegible]***VAE***[illegible]$p(x_0)$[illegible] $$ p(x_0) = \int{p(x_{0:T})d{x_{1:T}}} $$ [illegible]$p(x_0)$[illegible]***ELBO***[illegible] $$ \begin{align} \ln{p(x_0)} &= \ln{\int{p(x_{0:T})d{x_{1:T}}}} \ &= \ln{\int{\frac{p(x_{0:T})q(x_{1:T}|x_0)}{q(x_{1:T}|x_0)}}d{x_{1:T}}}\ &= \ln{\mathbb{E} {q(x{1:T}|x_0)}[\frac{p(x_{0:T})}{q(x_{1:T}|x_0)}]} \ &\ge \mathbb{E}{q(x{1:T}|x_0)}[\ln{\frac{p(x_{0:T})}{q(x_{1:T}|x_0)}}]\ &= \mathbb{E}{q(x{1:T}|x_0)}[\ln{\frac{p(x_T)\prod_{t=1}^Tp_{\theta}(x_{t-1}|x_{t})}{\prod_{t=1}^{T} q(x_t|x_{t-1})}}]\ &= \mathbb{E}{q(x{1:T}|x_0)}[\ln{\frac{p(x_T)p_{\theta}(x_0|x_1)\prod_{t=2}^Tp_{\theta}(x_{t-1}|x_{t})}{q(x_{T}|x_{T-1})\prod_{t=1}^{T-1} q(x_t|x_{t-1})}}]\ &=\mathbb{E}{q(x{1:T}|x_0)}[\ln{\frac{p(x_T)p_{\theta}(x_0|x_1)\prod_{t= 1}^{T - 1}p_{\theta}(x_{t}|x_{t+1})}{q(x_{T}|x_{T-1})\prod_{t=1}^{T-1} q(x_t|x_{t-1})}}]\ &=\mathbb{E}{q(x{1:T}|x_0)}[\ln{\frac{p(x_T)p_{\theta}(x_0|x_1)}{q(x_{T}|x_{T-1})}}] + \mathbb{E}{q(x{1:T}|x_0)}[\ln\prod_{t=1}^{T - 1}{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}]\ &= \mathbb{E}{q(x{1:T}|x_0)}[\ln{p_{\theta}(x_0|x_1)}] + \mathbb{E}{q(x{1:T}|x_0)}[\ln{\frac{p(x_{T})}{q(x_{T}|x_{T-1})}}] + \mathbb{E}{q(x{1:T}|x_0)}[{\sum_{t=1}^{T-1}{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}}}]\ &= \mathbb{E}{q(x{1}|x_0)}[\ln{p_{\theta}(x_0|x_1)}] + \mathbb{E}{q(x{T - 1},x_T|x_0)}[\ln{\frac{p(x_{T})}{q(x_{T}|x_{T-1})}}] + \sum_{t=1}^{T-1}\mathbb{E}{q(x{t-1},x_{t},x_{t+1}|x_0)}[{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}}]\ &= \underbrace{\mathbb{E}{q(x{1}|x_0)}[\ln{p_{\theta}(x_0|x_1)}]}{□□□} - \underbrace{\mathbb{E}{q(x_{T - 1},x_T|x_0)}[\ln{\frac{p(x_{T})}{q(x_{T}|x_{T-1})}}] }{□□□□□} - \underbrace{\sum{t=1}^{T-1}\mathbb{E}{q(x{t-1},x_{t},x_{t+1}|x_0)}[{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}}] }_{□□□} \end{align} $$ □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

□□□□□ $$ \begin{align} q(x_{T-1}|x_0) &= \int{q(x_1|x_0)q(x_2|x_1)\dots q(x_{T-1}|x_{T-2})}dx_{1:T-2} \ &= \int{q(x_1|x_0)q(x_2|x_1,x_0)\dots q(x_{T-1}|x_{T-2},x_{T-3}\dots,x_0)}dx_{1:T-2} \ &= \int{\frac{q(x_{0:T})}{q(x_0)}}dx_{1:T-2} \ &= \int{q(x_{1:T}|x_0)}dx_{1:T-2} \ &= q(x_{T-1}|x_0) \end{align} $$

$$ \begin{align} \mathbb{E}{q(x{1:T}|x_0)}[\ln{p_{\theta}(x_0|x_1)}] &= \int{\ln{p_{\theta}(x_0|x_1)}q(x_{1:T}|x_0)}d{x_{1:T}}\ &=\int{\ln{p_{\theta}(x_0|x_1)}q(x_1|x_0)}d{x_1}\underbrace{\int{\prod_{2}^{T-1}{q(x_t|x_{t-1})}}d{x_{2:T}}}{1(□□□□□)}\ &=\mathbb{E}{q(x_1|x_0)}[{\ln{p_{\theta}(x_0|x_1)}}] \end{align} $$

$$ \begin{align} \mathbb{E}{q(x{1:T}|x_0)}[\ln{\frac{p(x_{T})}{q(x_{T}|x_{T-1})}}] &= \int{\ln{\frac{p(x_{T})}{q(x_{T}|x_{T-1})}}q(x_{1:T}|x_0)}d{x_{1:T}} \ &= \int{\ln{\frac{p(x_T)}{q(x_T|x_{T-1})}}}q(x_T|x_{T-1})dx_{T-1,T} \underbrace{\int{\prod_{t=1}^{T-1}q(x_t|x_{t-1})}dx_{1:T-2}}{□□□□□:q(x{T-1}|x_0)} \ &= \int{\ln{\frac{p(x_T)}{q(x_T|x_{T-1})}}}q(x_T|x_{T-1})q(x_{T-1}|x_0)dx_{T-1,T}\ &= \int{\ln{\frac{p(x_T)}{q(x_T|x_{T-1})}}}q(x_{T-1},x_{T}|x_0)dx_{T-1,T}\ &= \mathbb{E}{q(x{T - 1},x_T|x_0)}[\ln{\frac{p(x_{T})}{q(x_{T}|x_{T-1})}}] \end{align} $$

$$ \begin{align} \mathbb{E}{q(x{1:T}|x_0)}[{\sum_{t=1}^{T-1}{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}}}] &= \sum_{t=1}^{T-1}\int{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}q(x_{1:T}|x_0)}d_{x_{1:T}} \ &= \sum_{t=1}^{T-1}\int{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}}\prod_{k=1}^Tq(x_{k-1}|x_k)dx_{1:T}\ &= \sum_{t=1}^{T-1}\int{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}}q(x_{t}|x_{t-1})q(x_{t+1}|x_{t})d{x_{t-1},x_{t},x_{t+1}} \int{\prod_1^{t}} \end{align} $$

□□□□□□ELBO□□□□□□

- $\mathbb{E}{q(x{1}|x_0)}[\ln{p_{\theta}(x_0|x_1)}]$□□□□□□□□□□VAE□□□□□□□□□□□□□□□□ x_1□□□□□□□□□x_0
- $\mathbb{E}{q(x{T - 1},x_T|x_0)}[\ln{\frac{p(x_{T})}{q(x_{T}|x_{T-1})}}]$□□□□□□□□□□□□ □□□□□□□□$T$□□□□□□□□□□□□□□□□0

> - $\sum_{t=1}^{T-1}\mathbb{E}{q(x{t-1},x_{t},x_{t+1}|x_0)}[{\ln{\frac{p_{\theta}(x_{t}|x_{t+1})}{ q(x_t|x_{t-1})}}}]$[illegible]KL[illegible]$t$[illegible]

[illegible]

- [illegible]
- [illegible]
- [illegible]DDPI[illegible]
- [illegible]VAE[illegible]T[illegible]T[illegible]VAE[illegible]
- [illegible]MCMC[illegible]

Denoising Diffusion Probabilistic Models([illegible] DDPM)

[illegible]

[illegible]

1. [illegible]$q(x_0)$[illegible]x_0
2. [illegible]$Uniform({1, ..., T})$[illegible]t
3. [illegible]$\mathcal{N}(0,I)$[illegible]ϵ
4. [illegible]$\nabla_\theta |\epsilon - \epsilon_\theta(\sqrt{\alpha_t}x_0 + \sqrt{1-\alpha_t}\epsilon, t)|^2$[illegible]
   - α_t[illegible]x_0[illegible]
   - [illegible]

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[illegible]

[illegible]

1. [illegible]X_T
2. [illegible]z
3. [illegible]$x_{t-1} = \frac{1}{\sqrt{\alpha_t}}(x_t - \frac{1-\alpha_t}{\sqrt{1-\overline{\alpha}_t}}\epsilon_{\theta}(x_t,t)) + \sigma_tz$
   - x_{t-1}[illegible]$t-1$[illegible]
   - $\alpha_t,\overline{\alpha}_t$[illegible]
   - $\epsilon_{\theta}(x_t,t)$[illegible]

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