

# Linear Regression

## Gradient Descent

- **Model**  $y = w \cdot x + b$ 
  - Model  $\rightarrow$  **training data**  $\rightarrow$  **testing data**  $\rightarrow$  **Overfitting**
- **Loss**  $L(w, b) = \sum_{i=1}^n (y - (w \cdot x + b))^2$
- **Update rule**
  - $w_1 = w_0 - \alpha \frac{\partial L}{\partial w} |_{w=w_0}$
  - $b_1 = b_0 - \alpha \frac{\partial L}{\partial b} |_{b=b_0}$
  - $\alpha$  **Learning rate**

## Regularization

- **Loss function**
  - $L(w, b) = \sum_{i=1}^n (y - (w \cdot x + b))^2 + \lambda \sum (w_i)^2$
  - $\lambda$  **Loss function**
    - $\lambda$   $\rightarrow$   $w$   $\rightarrow$  0
    - $\lambda$   $\rightarrow$   $w$   $\rightarrow$   $\infty$

# Classification

- **1. Bayes' Theorem**
  - $P(y=k|x) = \frac{P(x|y=k) P(y=k)}{P(x)}$
  - $P(y=k|x)$   $\rightarrow$   $P(y=k)$   $\rightarrow$   $P(x|y=k)$
  - $P(x)$   $\rightarrow$   $P(y=k)$   $\rightarrow$   $P(x|y=k)$

## 2. Bayes' Theorem

- $P(y=k|x)$   $\rightarrow$   $P(x|y=k)$   $\rightarrow$   $P(y=k)$
- $P(x|y=k)$   $\rightarrow$   $P(y=k)$   $\rightarrow$   $P(x|y=k)$
- $P(y=k)$   $\rightarrow$   $P(x|y=k)$   $\rightarrow$   $P(y=k)$

## 3. Bayes' Theorem

1.  $P(y=k)$   $\rightarrow$   $P(x|y=k)$
2.  $P(x|y=k)$   $\rightarrow$   $P(y=k)$ 
  - $P(x|y=k)$   $\rightarrow$   $P(y=k)$

3. **\*\***  $P(y=k|x)$  **\*\***

#### 4.

- MLE  $\mu$   $\Sigma$ 
  - $L(\mu, \Sigma) = f_{\mu, \Sigma}(x^1) f_{\mu, \Sigma}(x^2) \dots f_{\mu, \Sigma}(x^N)$
  - $L(\mu, \Sigma) \propto \mu \Sigma$
  - $\mu_1, \mu_2, \Sigma$
- $x$   $P(y=1|x)$   $P(y=2|x)$

#### 5.

- $P(C_1|x) = \frac{P(x|C_1)P(C_1)}{P(x|C_1)P(C_1) + P(x|C_2)P(C_2)} = \frac{1}{1 + \frac{P(x|C_2)P(C_2)}{P(x|C_1)P(C_1)}} = \frac{1}{1 + e^{-z}} = \sigma(z)$ 
  - $\sigma(z)$  **sigmoid**  $[0, 1]$
- $z = wx + b$ 
  - $z = \ln \frac{P(x|C_1)P(C_1)}{P(x|C_2)P(C_2)}$ 
    - $w^T = (\mu^1 - \mu^2) \Sigma^{-1}$
    - $b = -\frac{1}{2} (\mu^1)^T \Sigma^{-1} \mu^1 + \frac{1}{2} (\mu^2)^T \Sigma^{-1} \mu^2 + \ln \frac{P(N_1)}{P(N_2)}$ 
      - $\mu^1 \Sigma^{-1} \mu^2 \Sigma^{-1} N_1 N_2$

## Logistic Regression

### Loss function

$f_{w,b}(X) = P_{w,b}(C_1|x)$

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	$x^1$	$x^2$	$x^3$	$\cdots$
	$C_1$	$C_1$	$C_2$	$\cdots$
	$\hat{y} = 1$	$\hat{y} = 1$	$\hat{y} = 0$	$\cdots$

- $f(w,b) = f_{w,b}(x^1) f_{w,b}(x^2) (1 - f_{w,b}(x^3)) \dots f_{w,b}(x^N)$

- $f_{w,b}(x)$   $x$   $C_1$

$f_{w,b}(x) = P_{w,b}(C_1 | x)$

$C_1$   $C_2$   $C_2$

$P_{w,b}(C_2 | x) = 1 - P_{w,b}(C_1 | x) = 1 - f_{w,b}(x)$

- $w, b$   $w^*, b^* = \arg \max L(w, b) = \arg \min (-\ln L(w, b))$
- $-\ln L(w, b) = \ln f_{w,b}(x^1) + \ln f_{w,b}(x^2) + \ln(1 - f_{w,b}(x^3)) = \sum_n [-\hat{y}^n \ln f_{w,b}(x^n) + (1 - \hat{y}^n) \ln(1 - f_{w,b}(x^n))]$

- **Cross entropy**  $C(f(x^n), \hat{y}^n) = \sum_n -[\hat{y}^n \ln f_{w,b}(x^n) + (1 - \hat{y}^n) \ln (1 - f_{w,b}(x^n))]$

## Gradient Descent

**Cross entropy**  $C(f(x^n), \hat{y}^n) = \sum_n -[\hat{y}^n \ln f_{w,b}(x^n) + (1 - \hat{y}^n) \ln (1 - f_{w,b}(x^n))]$

**Gradient Descent**

- $\frac{\partial \ln(1 - f_{w,b}(x))}{\partial w_i} = \frac{\partial \ln(1 - f_{w,b}(x))}{\partial z} \frac{\partial z}{\partial w_i} = -\sigma(z) x_i$ 
  - $\frac{\partial \ln(1 - \sigma(z))}{\partial z} = -\frac{1}{1 - \sigma(z)} \sigma(z)$
  - $\frac{\partial z}{\partial w_i} = x_i$
- $\frac{\partial \ln f_{w,b}(x)}{\partial w_i} = \frac{\partial \ln f_{w,b}(x)}{\partial z} \frac{\partial z}{\partial w_i} = (1 - \sigma(z)) x_i$
- $\frac{\partial (-\ln(w, b))}{\partial w_i} = \sum_n -(\hat{y}^n - f_{w,b}(x^n)) x_i^n$ 
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## Square Error

- **Loss function**  $L(f) = \frac{1}{2} \sum_n (f_{w,b}(x^n) - \hat{y}^n)^2$
- $\frac{\partial (f_{w,b}(x) - \hat{y})^2}{\partial w_i} = 2(f_{w,b}(x) - \hat{y}) f_{w,b}(x) (1 - f_{w,b}(x)) x_i$ 
  - $\hat{y} = 0 \implies f_{w,b}(x) = 1$
  - $\hat{y} = 1 \implies f_{w,b}(x) = 0$

## Multi-class Classification

**weight bias**

- $C_1: w^1, b_1 \implies z_1 = w^1 \cdot x + b_1$
  - $C_2: w^2, b_2 \implies z_2 = w^2 \cdot x + b_2$
  - $C_3: w^3, b_3 \implies z_3 = w^3 \cdot x + b_3$
1. **softmax**
    - $f(z) = \frac{e^z}{\sum_{i=1}^n e^i}$
    - $f(z) \in [0, 1]$
  2. **Cross entropy**
    - $-\sum_{i=1}^n \hat{y}_i \ln y_i$

## Limitation of Logistic Regression

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- $\sigma(z) = 0.5 \iff w \cdot x + b = 0$

**Feature transformation**

## Discriminative and Generative

**Discriminative**

Discrete-time stochastic processes

- Discrete-time stochastic processes  $\mathbf{x}$
- Discrete-time  $P(y|x)$
- Discrete-time stochastic processes

## Generative models

Discrete-time stochastic processes

- Discrete-time  $P(x|y)$
- Discrete-time stochastic processes  $w, b$
- Discrete-time stochastic processes

## Deep Learning

### Three steps for Deep Learning

#### 1. Define a set of funtion

- Discrete-time stochastic processes
- Discrete-time
  - Discrete-time stochastic processes
  - Discrete-time stochastic processes ReLU, Sigmoid, Tanh
  - Discrete-time stochastic processes
  - Discrete-time stochastic processes Adam

Discrete-time

- Discrete-time stochastic processes Sigmoid, ReLU, ReLU

#### Sigmoid

- $(0, 1)$ 
  - Discrete-time stochastic processes 0
- Discrete-time stochastic processes

#### ReLU

- $[0, \infty)$
- Discrete-time stochastic processes sigmoid
- Discrete-time stochastic processes

#### ReLU

- ReLU Discrete-time stochastic processes 0

#### 2. Goodness of function

- Discrete-time stochastic processes
  - Discrete-time stochastic processes
  - Discrete-time stochastic processes  $k$
  - Discrete-time stochastic processes
  - Discrete-time stochastic processes

### 3. Pick the best function

- 二次関数
- 二次関数
- 二次関数
- 二次関数
- 二次関数

## Gradient Descent

- Learning Rate
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Taylor Series

## Adagrad

## Stochastic Gradient Descent

$$L = \sum_n (\hat{y}^n - (b + \sum(w_{ix} \cdot x_i^n)))^2$$

- **Gradient Descent**
- **Stochastic Gradient Descent** example
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## Feature Scaling

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- $\frac{x_i - m_i}{\sigma_i}$
- $m_i$   $\sigma_i$
- 0 1

## Backpropagation

- Forward pass Backword pass
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## Forward pass

## Backward pass

## BERT

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## Unsupervised Learning

## Self-supervised Learning

## Word Embedding

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1. 词嵌入 (word embeddings)
  - 将单词映射为低维向量 (one-hot encoding) 的改进 (降低维度)
  - 使用 GloVe 或 Word2Vec 模型 (通常 50-300 维)
2. 句子表示 (sentence representations)
  - 使用平均池化 (averaging) 或加权平均 (weighted averaging)
  - 使用 Recurrent Neural Networks (RNNs) 或 Long Short-Term Memory (LSTMs)

## Predication-based

**Prediction-based**

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- one-hot encoding
  - "0" "0" "0" "0" "0" "0" "0" "0"
- "0" "0" "0" "0" × "0" "0" "0" "0" → "0" "0" "0" "0"
  - "0" "0" "0" "0" × "0" "0" "0" "0" → "0" "0" "0" "0"
  - "0" "0" "0" "0" × "0" "0" "0" "0" → "0" "0" "0" "0"
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- 時間系列データの予測

## 5. 時間系列データの予測

- 線形モデル
- 非線形モデル

時間系列データの予測

1. 線形モデル
  - 線形回帰
  - 線形ニューラルネットワーク
2. 非線形モデル
  - 線形ニューラルネットワーク
  - 非線形ニューラルネットワーク

## Seq2Seq

**Sequence to Sequence Model** 時間系列データの予測

時間系列

Seq2Seq 時間系列データの予測

1. エンコーダー 時間系列データの予測
2. デコーダー 時間系列データの予測

**AT (Auto-regressive) VS NAT (Non-Autoregressive Transformer)** 時間系列

時間系列 **AR/AT**

- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測 **Transformer** 時間系列

時間系列 **NAT**

- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測