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2. **SFT**□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
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3. **RLHF**□□□□□□□□□□□
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SFT□Supervised Finetune□

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- **Few-Shot Prompting** □□□□□□□□□□□□□□1-5□□□□□□□□□□□□□□□□□□□□□
- **Seed Prompt** □□□□□□□□□`task_type`□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□
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    - answer□□□□□□□□□□□□□□□□
    - task_type □□□□□□sft□□□□□□□□□□□□□□□
- □□□□
    - □□prompt
        - □□□task_type□□□□□□□□□□□□seed prompt□□□□□□□seed□□□□□□□□□**pretrain**□□□□□□ □□□**seed**□□□□□□□□□□**prompt**
    - □□answer
        - □□□□□□□GPT4/Claude3
        - □□□□□Qwen_72B/deepseek_MoE

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RL□Reinforcement Learning□

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- **Agent**□□□□□□□□□□□□
- **action**□□□
- **Environment**□□□□□□reward□□□□
- **reward**□□□
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2. □□□□□□□□□□□□□□□□□□□□□□□□□□□□□
3. □□M^2□□□□□□□□□□□□□

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- □□□□□□□t□□□□□□□□□□□□S_t□□ □□□□□□□□□□□□

$$ \begin{bmatrix} S_1 \to S_1 & S_1 \to S_2 & S_1 \to S_3 \ S_2 \to S_1 & S_2 \to S_2 & S_2 \to S_3 \ S_3 \to S_1 & S_3 \to S_2 & S_3 \to S_3 \end{bmatrix} $$

- □□□□t□□□□S_t□□□□**t**□□□□□□□□□□□□$(S_1,\dots,S_t)$□□□□□□□□S_{t+1}□□□□□□ $p(S_{t+1}|S_1,\dots,S_t)$
- □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□

$$ p(S_{t+1}|S_t) =p(S_{t+1}|S_1,\dots,S_t) $$

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  - □□□□□$<S,P>$□S□□□□□□P□□□□□

  $$ P_{ss'} = p(S_{t+1}=s'|S_t = s) $$

□□□□□□□MRP□

□□□□□$<S,P,R,\gamma>$

- S□□□□□
- P□□□□□□□
- R□□□□ $R_S = E[R_{t+1}|S_t=s]$
  - □□□□□□□□□
  - □□□□s□□s'□□□
  - □□□□□□s'□□□□□s□□□□□□□□□□□□□□□□□

- γ□□□□□/□□□□ $\gamma \in [0,1]$

- □□□Return□□G_t□□□□t□□□□□□□□

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    $$ G_t = R_{t+1}+\gamma R_{t+2}+\gamma^2 R_{t+3}+\dots = \sum_{k=0}^{\infty}\gamma^kR_{t+k+1} $$

- □□□□$V(s)$□□□□□□s□□□□□

    - □□□□□□□
    - □□s□□□

$$ V(s) = E[G_t|S_t=s] $$

MRPs□□□□□□

□□□□□□□□□ $$ V(s) = E[G_t|S_t=s]\ G_t = R_{t+1} + \underbrace{\gamma R_{t+2}+\gamma^2 R_{t+3}+\dots}{\gamma V(S{t+1})} = \sum_{k=0}^{\infty}\gamma^kR_{t+k+1} $$ □□□ $$ V(s) = E[R_{t+1}+\gamma V(S_{t+1})|S_t = s] $$

□□□□□□□□□MDP□

□□□□□□□□□□□□□□$<S,A,P,R,\gamma>$

- S□□□□□□

- A□□□□

- P□□□□□□□□□ $P_{ss'}^a = p[S_{t+1}=s'|S_t=s,A_t=a]$

- R□□□□□ $R_S^a = E[R_{t+1}|S_t=s,A_t=a]$

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- γ□□□□□/□□□□ $\gamma \in [0,1]$

□□

π□□□□□□□□□□□$\pi(a|s)=P[A_t=a|S_t=s]$

- □□□□□□□□□□□□□s□□□□□□□a□□□□
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- MDP□□□□□□□□□□□□□□□□□□□□□

□□□□□□□□□□□□$M=<S,A,P,R,\gamma>$□□□π□□□□□□□□□□□□□□□□□□□□□

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$$ P_{s,s'}^{\pi} = \sum_{a\in A}\pi(a|s)P_{s,s'}^a \ R_s^{\pi} = \sum_{a \in A} \pi(a|s)R_s^a $$

- □□□□□□□□□□□s□□□□□□□□□□□s'□□□□□□□□□□□□
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□□□□□□s□□□□□□□**π**□□□□□□

image-20250817175244201 $$ V_{\pi}(s) = E_{\pi}[G_t|S_t=s] $$ □□□□□□□ $$ G_t =

\underbrace{R_{t+1}}{\text{□□□□}} + \underbrace{\gamma R{t+2}+\gamma^2 R_{t+3}+\dots}{{\text{□□□□□□□□□□□}}} = \sum_{k=0}^{\infty}\gamma^kR_{t+k+1} $$ □□□□□□□□□□□□□□□□□□□□□□□□ $$ V_{\pi}(s) = E_{\pi}[R_{t+1}+\gamma V_{\pi}(S_{t+1})|S_t = s] $$

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□□□□□□s□□□□□□**a**□□□□□□□**π**□□□□□□

image-20250817175237288 $$ q_{\pi}(s,a) = E_{\pi}[G_t|S_t = s,A_t = a] $$ □□□□□□□□□□□□□□□□□□□□□□□□

□□□□□ $$ q_{\pi}(s,a) = E_{\pi}[R_{t+1} +\gamma q_{\pi}(S_{t+1},A_{t+1})|S_t = s,A_t = a] $$

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□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)q_{\pi}(s,a) $$

image-20250817181356989

□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ $$ q_{\pi}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s') $$

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□□□□□□□□ $$ V_{*}(s) = \max_{\pi}V_{\pi}(s) $$ □□□□□□□□ $$ q_*(s,a) = \max_{\pi}q_{\pi}(s,a) $$ □□□□

- □□□□□□□□□□□□$\pi_* \ge any \pi$
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$$ \pi_* \ge any \pi $$

□□□$V_{\pi'}(s)\ge V_{\pi}(s)$□□$\pi' > \pi$

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□□□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)q_{\pi}(s,a) $$ □ $$ q_{\pi}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s') $$ □□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)(R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s')) $$ □□□□□□□□□□□□□□□□□□□□□□□□□□□□

□□□□□□□□□□□□□□□□□ $$ V_{\pi}(s) = \sum_{a \in A}\pi(a|s)q_{\pi}(s,a) $$

□□□□π_*□□□□□□□□s□□□□□□$q_*(s,a)$

- □□□□π_*□□□□$V_{\pi_*}(s) \ge V_{\pi}(s)$
- □□□$q_{\pi_*}(s,a) \ge q_{\pi}(s,a)$
- □□□$V_{\pi}(s)$□□□□□□□$q_{\pi}(s,a)$

$$ \pi_{}(a \mid s) = \begin{cases} 1 & \text{if } a = \arg \max_{a} q_{}(s, a), \ 0 & \text{otherwise}. \end{cases} $$ [illegible] $$ V_*(s) = \max_aq_*(s,a) $$ [illegible] $$ q_*(s,a)=R_s^a+\gamma\sum_{s'}P_{s,s'}^aV_*(s') $$ [illegible] $$ V_*(s) = \max_a(R_s^a+\gamma\sum_{s'}P_{s,s'}^aV_*(s')) $$

[illegible]

[illegible] $$ q_{\pi}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{\pi}(s') $$

[illegible]V_{π}[illegible] $$ q_{}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^aV_{}(s') $$ [illegible]$V_*(s')$[illegible] $$ q_{}(s,a) = R_s^a+\gamma \sum_{s' \in S}P_{ss'}^a\max_{a'}q_(s',a') $$

LLaMA

[illegible]GPT[illegible]LLaMA[illegible]**Transformer**[illegible]Decoder-only[illegible]

- [illegible]
- [illegible]
- [illegible]

RMSNorm

[illegible]

[illegible] $$ RMSNorm(x) = \frac{x}{\sqrt{\frac{1}{d}\sum_{i=1}^dx_i^2+ \epsilon}} \cdot\gamma $$ [illegible]

- x[illegible]
- d[illegible]
- ϵ[illegible]
- γ[illegible]

SwiGLU

[illegible]**Swish**[illegible]

Swish[illegible] $$ Swish(x) = x \cdot \sigma(x) $$

- σ[illegible]Sigmoid[illegible]

GLU[illegible] $$ GLU(x) = \sigma(W_1x+b_1) \odot(W_2x+b_2) $$

- $\odot$[illegible]
- $W_1[illegible]W_2$[illegible]$b_1[illegible]b_2$[illegible]

SwiGLU[illegible] $$ SwiGLU(x) = Swish(Linear_1(x))\odot Linear_2(x) $$ [illegible]

[illegible]

- [illegible]Swish[illegible]
- [illegible]GLU[illegible]

RoPE

[illegible]

GQA

□LLaMA2□3□□□

□□□□□□□

GPT

Deepseek

MLA□□□□□□□□

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- □□□GPU□□□□□□

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- **DP□Data Parallel□**
- **DDP□Distributed Data Parallel□**
- **FSDP□Fully Sharded Data Parallel□**

All-Reduce□□□□□□

- □□□□□□□□GPU□□□□□□□□□□□□□□□□□□□□□□□□
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  - □□□□□□□□GPU□□□□□□□□□□□`SUM`□`MAX`□
  - □□□□□GPU□□□□□□□□□□□

All-Gather□□□□□□

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DP□Data Parallel□

□□□□□□□**Python GIL□□□□□□□□□□□□□□CPU□**

image-20250814165126760

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1. □CPU□□□□□□□□□□□□□GPU□□
2. □□GPU□□□□□□□□□□□□□□□□□□□□□□□□□GPU0□
3. GPU0□□□□□□□□□□□□□□□□□□□□□□
4. □□□□□□□□□□□□□□□**GPU**□

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- □□□□GPU□□□□□□□ψ□□□□□□ψ

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- □□□□□□□□□Python **GIL**□□□□□□**CPU**□
- GPU0□□□□□□□□□□□□□□□□□□□□□□□□□□□□

DDP□Distributed Data Parallel□

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- □□□□□□□□**py**□□□□□**rank**□□□□□□□□□□□□□□□□□□□□□□□
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- GPU0□□□□□□□□□□□□□□□□□GPU
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- □□□**GPU**□□□□□□□□□□□□□□□□□□□□□□□□**Ring-AllReduce**□□□□□
- □□□□□□□□□□□□□□□□□GPU□□□□□□□□□□□□□□□□□□□□

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□□□□GPU□□

- Scatter-Reduce□□□□□/□□□$(N-1)\frac{\psi}{N} \approx \psi$
- All-Gather□□□□□/□□□$(N-1)\frac{\psi}{N} \approx \psi$

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□□□□□□□□**Ring-AllReduce**

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Scatter-Reduce

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FSDP□Fully Sharded Data Parallel□

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DeepSpeed ZeRO-1

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DeepSpeed ZeRO-2

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DeepSpeed ZeRO-3

ZeRO-3□**FP16**□□□□□□□□

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□□□ $$ Y = XW $$ □□□

- $X \in R^{2 \times 2}□W \in R^{2 \times 2}$

□□□□ $$ \begin{bmatrix} y_1&y_2 \y_3&y_4 \end{bmatrix} = \begin{bmatrix} x_1&x_2 \x_3&x_4 \end{bmatrix} \begin{bmatrix} w_1&w_2 \w_3&w_4 \end{bmatrix} $$ □□□□□□□□□□□□□□□GPU□□□□□ $$ W = \begin{bmatrix} W_0&W_1
\end{bmatrix} $$ □□ $$ Y_0 = XW_0 = \begin{bmatrix} x_1&x_2 \x_3&x_4
\end{bmatrix}\begin{bmatrix} w_1 \ w_3
\end{bmatrix}= \begin{bmatrix} y_1 \y_3 \end{bmatrix}\ Y_1 = XW_1 = \begin{bmatrix} x_1&x_2 \x_3&x_4 \end{bmatrix}\begin{bmatrix} w_2 \ w_4
\end{bmatrix}= \begin{bmatrix} y_2 \y_4 \end{bmatrix} $$ □□□□□□ $$ Y = \begin{bmatrix} Y_0 &Y_1

\end{bmatrix} $$ image-20250815105857273

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□□□ $$ Y = XW $$ □□□

- $X \in R^{2 \times 2}□W \in R^{2 \times 2}$

□□□□ $$ \begin{bmatrix} y_1&y_2 \y_3&y_4 \end{bmatrix} = \begin{bmatrix} x_1&x_2 \x_3&x_4 \end{bmatrix} \begin{bmatrix} w_1&w_2 \w_3&w_4 \end{bmatrix} $$ □□□□□□□□□□□□□□GPU□□□□□ $$ W = \begin{bmatrix} W_0\W_1 \end{bmatrix} $$ □□X□□□□□□ $$ X = \begin{bmatrix} X_1 & X_2\end{bmatrix} $$ □□□□□□ $$ Y_1 = \begin{bmatrix} x_1 \ x_3\end{bmatrix} \begin{bmatrix} w_1 & w_2\end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \y_{31} &y_{32}\end{bmatrix}\ Y_2 = \begin{bmatrix} x_2 \ x_4\end{bmatrix} \begin{bmatrix} w_3 & w_4\end{bmatrix} = \begin{bmatrix} y_{23} & y_{24} \y_{43} &y_{44}\end{bmatrix} $$ □□□Y_1□Y_2□□□□□□□□□ $$ Y = Y_1 +Y_2 $$

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PipeDream --- DeepSpeed

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- □□□□□□5□Micro-batch□Machine1□□□□□□□□□□□□□□□□□□□□Micro-batch 1□□□□□□□□□□□□□□□□□FW2-4□□□□□□□
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- **Weight stashing**□□□□□□□□□□□□□□□□□□□ Micro-batch □□□□□□□□□□□□□□□□□□□□□□□□□□
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    1. **Micro-batch □□□□□□□Machine 1 □□□□□□□□□□□□□□ W_1□□□ □□□□□□□□□□□□ Stage**

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3. □□□□□□□□□□□□□□□□□□□□□□□ □□□□□□□□□□ **W_1** □□□□□□□□□□□□□□□□□□□□

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□□□□□ `Weight Stashing` □□

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- Device 1 □ □□□5□□□ □□□□□□□□□□1□□□□□□□□□□□□□1□□□□□□□□
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□□□□□ `Vertical Sync` □□

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- **Device 1**□□□□5□□□□□□□□□□□□□□□□1□□□□□□□□□□□□1□□□□□□□□
- **Device 2**□□□□5□□□□□□□□□□□□□□□□1□□□□□□□□□□□□□1□□□□□□□□

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1. □**Master-Weight**□□□**fp16**
2. □inputs□□□□□□□□□□□□□□□□□□□□□□□□□□FP16
3. □□□□□□□□□□□□□□□fp32□□□□□□□

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Loss Scaling

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3. □□□□□□□□fp16□□□□□□□□□□□□□□□□□□□□□□fp32□□□
4. □□□□□□□□□□**fp32**□□□□□□□□□□□□

□□□□□□□□□□□□□□**fp16**□□□□□□□□□□□□□□□□□□□□□□FP16□□□□□□

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1. □□□□□□
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MHA□MQA□GQA

image-20250816120438735

MHA

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MQA□Multi-Query Attention□

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- □□□Transformer□□□□□□□□□□key□□□value□□□□□□□□□□□□□□□□K□V□□□□□□□□□□Q□□□□□□□□
- □□□□□□□□□□□□□□□□□□□□□□□**K□V**□□
- □□KV□□□□□□□□□□□

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GQA□Grouped-Query Attention□

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- **□Query□□□□□□□□□□□□Key□Value**
- □□□□□□N□N=1□□**MQA**□N□□Query□□□□□□□**MHA**

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- □□□□□□□□□□□□□□□□□□□□**MHA**□□□□□□□□□□

MLA□Multi-Head Latent Attention□

□□□□□□□□□**Deepseek-V3**□□□

□□□□□

- KV Cache□□□□□□□□□□□□□□□□□□□□□
- □□□□□**MQA**□**GQA**□□□□□**KV Cache**□□□□□□□□□□□□□□

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- □□□□□□□□KV Cache□□□□
- □□MQA□GQA□□□□□□

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- **□Key□Value**□□□□□□□□□□□□□□□□□□□□□□□□□□KV□□□□□□KV□□□□□□□□
- □□□□□□□□□□□□KV□□□□MLA□□KV□□□□□□

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□Query□Key□□□□□□□□$\begin{bmatrix}q_t^R& q_t^C\end{bmatrix}$□$\begin{bmatrix}k_t^R& k_t^C\end{bmatrix}$□□□□□□□□□□$(q_t^C,k_t^C)$□□□□□□RoPE□□$(q_t^R,k_t^R)$

1. □□□□□□□h_t
2. □□□□□□□□c_t^{KV}□c_t^Q
   - c_t^{KV}□KV□□□□
   - c_t^Q□Q□□□□
3. □Key□Value□□□□□
   - □□□□□□□□□□□□□□□ Key □ Value□□$k_{t,i}^C,v_{t,i}^C$
   - □□□□□□□□□□□□□□□□□□□□□□$k_{t,i}^R$□□□□□□□□□□□□□□
4. □□□Query□□□□□□□□□□□□□$q_{t,i}^R$□□□□□□□□□□$q_{t,i}^C$□□□□□□□□□□□□□□□
5. □Q□K□V□□□□□□□□□□□□□□□□□□□□□□

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□KV□□□□□□

image-20250816133715163

□□□□$c_t^{KV}□k_t^C□v_t^{C}$□□□□□□□□□□□□□ $$ c_t^{KV} = W^{DKV}h_t \ k_t^C = W^{UK}c_t^{KV} \ v_t^C = W^{UV}c_t^{KV} $$ □□□□KV□□□□□□□□□□□□□□

□□□□□□□□□□□□□□□□□□c_t^{KV}□□□□□□□□□□□□□**K□V**□□

□Q□□□□□□□□

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□□□□ $$ c_t^Q = W^{DQ}h_t\ q_t^C = W^{UQ}c_t^Q $$ □□□□□□**KV Cache**□□□□□□□□□□□□□□□□□

MLA□Q□K□RoPE□□

MLA□□□k_t^R□q_t^R□□RoPE□□□□□**RoPE**□□□□□□□□□□

- RoPE□Q□K□□□□□□□□□□□□□□□□□□**RoPE**□□□□□□
- □□□□□□□c_t^{KV}□□RoPE□□□□□□□□□□R_n□□□□□W_{QK}□□□□□□□□□□K_{rot}□□□RoPE□□□□□□□□**KV Cache**□□□□□

$$ K_{rot} = R_n(W_{QK}\cdot c_t^{KV}) $$

- □□□□□□□□□□□□□□□W_{QK}□□□□□□□□

□□□□□□□ $$ S = (W^{UQ})^T(c_t^Q)^TR_m^TR_nc_t^{KV}W^{UK} $$ □□□□ $$ S = (c_t^Q)^TR_m^T(W^{UQ})^TW^{UK}R_nc_t^{KV}\ =(c_t^Q)^TR_m^TW_{merge}R_nc_t^{KV} $$ □□□□□□□□□□□□□□□□W_{QK}□□□□□□□□

MLA□□□RoPE□c_t^{KV}□□□□□□□□□□□□□□□

1. □□□□□□□**k_t^C□□**
   - □□□□□□ K□c_t^{KV}□□□□ **RoPE**□□□□□
   - □□□□□□□□□ c_t^{KV}□□□□□□□
2. □□ **RoPE** □□□**k_t^R□□**
   - □□□ h_t□□□□□□□□□□ **RoPE**
   - □□□□□□□□□□□□□□□□□□
3. □□□□
   - □□$K=[k_t^C;k_t^R]$□□□□□□□□□□□□□□□KV Cache

MFA□Multi-matrix Factorization Attention□

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Paged Attention

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- GPU□CPU□□□□□□□□□□□□□□□
- □□□□/□□GPU□□□□□□□□□□□□□□

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- □□□□□□□□□□□□□
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- □□□□□□□□□□□□□□□□□□□□□□□Paged Attention□□□□□□□□□□□□□□□□□□□□□□□□**KV Cache**

image-20250819121841203

- □□□□□□□□□□□□**CPU**□□□□□□□□□□□□□□□□□□
- **KV Cache**□□□□Prompt□□□□□Paged Attention□□□□KV Cache□□□□
  - □□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□ KV □□
  - □□□□**Copy-on-Write**□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□□**3**□□□□**5**□□□□□□□□
    □□□□□□□□□□

Flash Attention

□□□□□□□□**IO**□□□□□□□□□□**SRAM**□□□□**IO**□□

□□□GPU□□□□□□□□□□□□**SRAM**□□□□□□□□**HBM**□□□□□□□**Flash attention**□□□□□□**Attention**□□□□
□**HBM**□□□

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- □□□□□□HBM□□□□□□□□□□□□□□
- □□□□□□□□□□□□GPU□□□□□□□□□□
- □□□□□□□□□□□□□□□□□SRAM□□□□□□□□Pytorch□□□□□□□□□□□□□□□□□□□□□□

□□□□pytorch□attention□□□□

- □HBM□□Q□K□SRAM
- □□□$S = QK^T$
- □S□□HBM□□□□□□□□□□□□□□□□□□□□□
- □HBM□□S□HRAM
- □□$P=softmax(S)$
- □P□□HBM□□□□□□□□□□□□□□□□□□□□□
- □HBM□□P□SRAM
- □□$O=PV$
- □O□□HBM
- □□O

□□□□□□□□□□

- **Compute-Bound**□□□□□□□□
  - □□□□□□□□□□□□□□□□□□
  - □□□□□□□□**GPU**□□□□□□□□□□□□□□□□□□□□□□□□□□
- **Memory-Bound**□□□□□□□□
  - □□□ReLU□Softmax□Sum□Dropout□
  - □□□□□□□□□□□□□□□□GPU□□□□□□□□□

□□□□**Safe-Softmax**

□□softmax $$ softmax(x_i) = \frac{e^{x_i}}{\sum_{j=1}^n e^{x_j}} $$ □□□□□□□□□□□□□□□□□**x_i**□□
□□□□□□□□□□

Safe Softmax $$ softmax(x_i) = \frac{e^{x_i-\max(x)}}{\sum_{j=1}^n e^{x_j- \max(x)}} $$

- □□□□□□□□□□$\max(x)$□□□□□□□
- □□□□□□□□□□□$e^{\max(x)}$□□□□□

□□□□□□□□□□

- **Tiling Algorithm**□□□□□□
  - □□□□□□□□SRAM□□□□□□□□□□□□□□□□□□**SRAM**□□□□
  - □□□□□□□□□□□□□**SRAM**□□Flash Attention□□□□□□□SRAM□□□□□□□□□□
- **Recomputation**□□□□□
  - GPU□□□□□□HBM□□□□□□□□□□□□□□□□□□□□□□□□□□□
  - □□$S=QK^T$□Flash Attention□□□HBM□□□□□□□□□SRAM□□□
  - □□$P = softmax(S)$□□□□□HBM□□□□□□□□□□
- **Kernal Fusion**□□□□□□
  - □□□□□□□□□□□□□□□□□**HBM**□□□
  - □□□□□□□□□Kernal□□□□□□□□□□□□

□□□□□□□□□□SRAM□□□□□□□□□□

- □□□Q□K□V□□□□□□SRAM

- □□$S=QK^T$

- □□$P=softmax(S)$

  - □□□□□□□□softmax□□□□□□softmax

- □□□□□□□□Safe-Softmax
- □□□Safe-Softmax□□□□□□□□□□□□□□□□□□□□□□Safe-Softmax□□
  - □□□□□2□$x_1 = [1,2]$$x_2 = [3,4]$
  - □□□□□□□$m(x_1)=2=m(x)$
  - □□□□□□□□$f(x_1)=[e^{1-m(x_1)},e^{2-m(x_1)}] = [e^{-1},e^0]$
  - □□□□□□□$l(x_1)=e^{-1}+e^0$
  - □□□□□□□□$m(x)=\max(m(x_1),m(x_2)) = 4$
  - □□$f(x_2) = [e^{3-m(x_2)},e^{4-m(x_2)}] =[e^{-1},e^{0}]$
  - □□□□□□□□$l(x_2)=e^{-1}+e^0$
  - □□□$f(x)$□$l(x)$

  $$ f(x) = [e^{m(x_1)-m(x)}f(x_1),e^{m(x_2)-m(x)}f(x_2)]\ f(x) = [e^{-2}(e^{-1},e^0),e^0(e^-1,e^0)]\ l(x)=e^{m(x_1)-m(x)}*l(x_1) + e^{m(x_2)-m(x)}*l(x_2) \ l(x) = e^{-3} + e^{-2} + e^{-1} + e^0 $$

- □□$O=PV$
- □SRAM□□□O□□HBM

GPU□□□

□□ `SIMT` □□□□□□□□□□□□□□□□□□□□□□□

GPU□□□□□□□Warp□32□□□□□□□

SM□□□□□□GPU□□□□

- **CUDA Cores□CUDA □□□□** □□□□□□□
- **Tensor Cores□□□□□** □□□□□□□□□□□□
- **Warp Scheduler□Warp** □□□□□Warp□□□□□□□
- **Shared Memory**□□□□□□□□□□□□□
- **L1 Cache**□□□□□□□□□□□□□□□□
- **Load/Store Units**□□□□□□□HBM□□□□□Shared Memory□□□
- **Register File**□□□□□□□□□□□
- **SFU**□□□□□□□□□

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GPU□□□□□□□□□

- **SRAM**□□□□L1 Cache□□□ **SM**□□□L2 Cache□□□□□**SM**□□□Shared Memory□□□□□□□□ □□□□□□□□□
- **HBM**□□□□□□□□□□□□□□□□□□□□□

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- `NVLink` □**GPU**□□□□□□□□
- `PCIe` □**CPU-GPU**□□□

□□

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1. □□□□□□□PCIe□□□□CPU□□□□□GPU HBM
2. □□□□□□□□**HBM**□□□**L2 Cache**□□□**Shared Memory**□□**L1 Cache**□□□□□□□□□□□□□□□
   - □□HBM□□□□□□□□□SM□□□□□□□□□□**SM**□□□□□□□□□□□□□□□□□□□□□□□□□□
3. □□□□□ **CUDA Core**□□**Tensor Core**□□□□
4. □□□□□□□□□□□□HBM