

Linear Regression

Gradient Descent

- **Model** $y = w \cdot x + b$
 - Model \rightarrow **training data** \rightarrow **testing data** \rightarrow **Overfitting**
- **Loss** $L(w, b) = \sum_{i=1}^n (y - (w \cdot x + b))^2$
- **Update rule**
 - $w_1 = w_0 - \alpha \frac{\partial L}{\partial w} |_{w=w_0}$
 - $b_1 = b_0 - \alpha \frac{\partial L}{\partial b} |_{b=b_0}$
 - α **Learning rate**

Regularization

- **Loss function**
 - $L(w, b) = \sum_{i=1}^n (y - (w \cdot x + b))^2 + \lambda \sum (w_i)^2$
 - λ **Loss function**
 - λ \rightarrow w \rightarrow 0
 - λ \rightarrow w \rightarrow ∞

Classification

- **1. Bayes' Theorem**
 - $P(y=k|x) = \frac{P(x|y=k) P(y=k)}{P(x)}$
 - $P(y=k|x)$ \rightarrow $P(y=k)$ \rightarrow $P(x|y=k)$
 - $P(x)$ \rightarrow $P(y=k)$ \rightarrow $P(x|y=k)$

2. Bayes' Theorem

- $P(y=k|x)$ \rightarrow $P(x|y=k)$ \rightarrow $P(y=k)$
- $P(x|y=k)$ \rightarrow $P(y=k)$ \rightarrow $P(x|y=k)$
- $P(y=k)$ \rightarrow $P(x|y=k)$ \rightarrow $P(y=k)$

3. Bayes' Theorem

1. $P(y=k)$ \rightarrow $P(x|y=k)$ \rightarrow $P(y=k)$
2. $P(x|y=k)$ \rightarrow $P(y=k)$ \rightarrow $P(x|y=k)$

3. ****** $P(y=k|x)$ ******

4.

- MLE μ Σ
 - $L(\mu, \Sigma) = f_{\mu, \Sigma}(x^1) f_{\mu, \Sigma}(x^2) \dots f_{\mu, \Sigma}(x^N)$
 - $L(\mu, \Sigma) \propto \mu^N \Sigma^{-N/2} \exp\left(-\frac{1}{2\sigma^2} \sum_{i=1}^N (x_i - \mu)^2\right)$
 - $\mu = \frac{1}{N} \sum_{i=1}^N x_i$ $\Sigma = \frac{1}{N} \sum_{i=1}^N (x_i - \mu)(x_i - \mu)^T$
- x $P(y=1|x)$ $P(y=2|x)$

5.

- $P(C_1|x) = \frac{P(x|C_1)P(C_1)}{P(x|C_1)P(C_1) + P(x|C_2)P(C_2)} = \frac{1}{1 + \frac{P(x|C_2)P(C_2)}{P(x|C_1)P(C_1)}} = \frac{1}{1 + e^{-z}} = \sigma(z)$
 - $\sigma(z)$ **sigmoid** $[0, 1]$
- $z = wx + b$
 - $z = \ln \frac{P(x|C_1)P(C_1)}{P(x|C_2)P(C_2)}$
 - $w^T = (\mu^1 - \mu^2)^T \Sigma^{-1}$
 - $b = -\frac{1}{2} (\mu^1)^T \Sigma^{-1} \mu^1 + \frac{1}{2} (\mu^2)^T \Sigma^{-1} \mu^2 + \ln \frac{P(C_1)}{P(C_2)}$
 - $\mu^1 \mu^2 \Sigma^{-1} \Sigma^{-1} N_1 N_2$

Logistic Regression

Loss function

$f_{w,b}(X) = P_{w,b}(C_1|x)$

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| | x^1 | x^2 | x^3 | \cdots |
|--|---------------|---------------|---------------|----------|
| | C_1 | C_1 | C_2 | \cdots |
| | $\hat{y} = 1$ | $\hat{y} = 1$ | $\hat{y} = 0$ | \cdots |

- $f(w,b) = f_{w,b}(x^1) f_{w,b}(x^2) (1 - f_{w,b}(x^3)) \dots f_{w,b}(x^N)$
- $f_{w,b}(x)$ x C_1

$f_{w,b}(x) = P_{w,b}(C_1 | x)$

C_1 C_2 C_2

$P_{w,b}(C_2 | x) = 1 - P_{w,b}(C_1 | x) = 1 - f_{w,b}(x)$

- w, b $w^*, b^* = \arg\max_{w,b} L(w,b) = \arg\min_{w,b} (-\ln L(w,b))$
- $-\ln L(w,b) = -\ln f_{w,b}(x^1) - \ln f_{w,b}(x^2) - \ln(1 - f_{w,b}(x^3)) - \dots = \sum_{n=1}^N [-\hat{y}^n \ln f_{w,b}(x^n) + (1 - \hat{y}^n) \ln(1 - f_{w,b}(x^n))]$

- **Cross entropy** $C(f(x^n), \hat{y}^n) = \sum_n -[\hat{y}^n \ln f_{w,b}(x^n) + (1 - \hat{y}^n) \ln (1 - f_{w,b}(x^n))]$

Gradient Descent

Cross entropy $C(f(x^n), \hat{y}^n) = \sum_n -[\hat{y}^n \ln f_{w,b}(x^n) + (1 - \hat{y}^n) \ln (1 - f_{w,b}(x^n))]$

Gradient Descent

- $\frac{\partial \ln(1 - f_{w,b}(x))}{\partial w_i} = \frac{\partial \ln(1 - f_{w,b}(x))}{\partial z} \frac{\partial z}{\partial w_i} = -\sigma(z) x_i$
 - $\frac{\partial \ln(1 - \sigma(z))}{\partial z} = -\frac{1}{1 - \sigma(z)} \sigma(z)$
 - $\frac{\partial z}{\partial w_i} = x_i$
- $\frac{\partial \ln f_{w,b}(x)}{\partial w_i} = \frac{\partial \ln f_{w,b}(x)}{\partial z} \frac{\partial z}{\partial w_i} = (1 - \sigma(z)) x_i$
- $\frac{\partial (-\ln(w, b))}{\partial w_i} = \sum_n -(\hat{y}^n - f_{w,b}(x^n)) x_i^n$
 -

Square Error

- **Loss function** $L(f) = \frac{1}{2} \sum_n (f_{w,b}(x^n) - \hat{y}^n)^2$
- $\frac{\partial (f_{w,b}(x) - \hat{y})^2}{\partial w_i} = 2(f_{w,b}(x) - \hat{y}) f_{w,b}(x) (1 - f_{w,b}(x)) x_i$
 - $\hat{y} = 0 \implies f_{w,b}(x) = 1$
 - $\hat{y} = 1 \implies f_{w,b}(x) = 0$

Multi-class Classification

weight bias

- $C_1: w^1, b_1 \quad z_1 = w^1 \cdot x + b_1$
 - $C_2: w^2, b_2 \quad z_2 = w^2 \cdot x + b_2$
 - $C_3: w^3, b_3 \quad z_3 = w^3 \cdot x + b_3$
1. **softmax**
 - $f(z) = \frac{e^z}{\sum_{i=1}^n e^i}$
 - $f(z) \in [0, 1]$
 2. **Cross entropy**
 - $-\sum_{i=1}^n \hat{y}_i \ln y_i$

Limitation of Logistic Regression

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- $\sigma(z) = 0.5 \iff w \cdot x + b = 0$

Feature transformation

Discriminative and Generative

Discriminative

Discrete-time stochastic processes

- Discrete-time stochastic processes \mathbf{x}
- Discrete-time $P(y|x)$
- Discrete-time stochastic processes

Generative models

Discrete-time stochastic processes

- Discrete-time $P(x|y)$
- Discrete-time stochastic processes w, b
- Discrete-time stochastic processes

Deep Learning

Three steps for Deep Learning

1. Define a set of function

- Discrete-time stochastic processes
- Discrete-time
 - Discrete-time stochastic processes
 - Discrete-time stochastic processes ReLU, Sigmoid, Tanh
 - Discrete-time stochastic processes
 - Discrete-time stochastic processes Adam

Discrete-time

- Discrete-time stochastic processes Sigmoid, ReLU, ReLU

Sigmoid function

- $\sigma(x) = \frac{1}{1 + e^{-x}}$
 - Discrete-time stochastic processes 0
- Discrete-time stochastic processes

ReLU function

- $\max(0, x)$
- Discrete-time stochastic processes sigmoid
- Discrete-time stochastic processes

ReLU function

- ReLU function

2. Goodness of function

- Discrete-time stochastic processes
 - Discrete-time stochastic processes
 - Discrete-time stochastic processes k
 - Discrete-time stochastic processes
 - Discrete-time stochastic processes

3. Pick the best function

- 二次函数
- 二次函数
- 二次函数
- 二次函数
- 二次函数

Gradient Descent

- Learning Rate
-
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Taylor Series

Adagrad

Stochastic Gradient Descent

$$L = \sum_n (\hat{y}^n - (b + \sum(w_{ix} i^n)))^2$$

- Gradient Descent
- Stochastic Gradient Descent example
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Feature Scaling

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- $\frac{x_i - m_i}{\sigma_i}$
- m_i σ_i
- 0 1

Backpropagation

- Forward pass Backword pass
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-
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Forward pass

Backward pass

BERT

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Unsupervised Learning

Self-supervised Learning

Word Embedding

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1. 词嵌入 (Word Embedding)
 - 将单词映射为低维向量 (通常使用 GloVe 或 Word2Vec)
 - 词嵌入的维度通常设置为 50-300
2. 句子表示 (Sentence Representation)
 - 使用平均池化 (Average Pooling) 或最大池化 (Max Pooling) 将词嵌入聚合为句子表示
 - 也可以使用更复杂的模型 (如 LSTM 或 Transformer) 来生成句子表示

Predication-based

Prediction-based

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1111

1. one-hot encoding
 - "0" → [1, 0, 0, 0]
2.
 - "0" × "0" → "0"
 - "0" × "1" → "1"
 - "1" × "0" → "0"
 - "1" × "1" → "1"
3.
 - "0" × "0" → "0"
 - "0" × "1" → "1"
 - "1" × "0" → "0"
 - "1" × "1" → "1"
- 4.

- 時間系列データの予測

5. 時間系列データの予測

- 線形モデル
- 非線形モデル

時間系列データの予測

1. 線形モデル
 - 線形回帰
 - 線形ニューラルネットワーク
2. 非線形モデル
 - 線形ニューラルネットワーク
 - 非線形ニューラルネットワーク

Seq2Seq

Sequence to Sequence Model 時間系列データの予測

時間系列

Seq2Seq 時間系列データの予測

1. エンコーダー 時間系列データの予測
2. デコーダー 時間系列データの予測

AT **Auto-regressive** 時間系列 **VS** **NAT** **Non-Autoregressive Transformer** 時間系列

時間系列 **AR/AT**

- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測 **Transformer** 時間系列

時間系列 **NAT**

- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測
- 時間系列データの予測