Lecture 20: Decision Networks

CMPSCI 383: Artificial Intelligence
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Today's lecture

- The value of information
- Decision trees
- Decision networks

Value of perfect information

- The general case: We assume that exact evidence can be obtained about the value of some random variable E_j.
- The agent's current knowledge is E.
- The value of perfect information is:

$VPI_E(E_j)$ = V(best-action|E,E_j) - V(best-action|E)

Properties of VPI

- In general:
  $VPI_E(E_j,E_k) \neq VPI_E(E_j) + VPI_E(E_k)$
- But the order is not important:
  $VPI_E(E_j,E_k) =$
  $VPI_E(E_j) + VPI_{E,E_j}(E_k) = VPI_E(E_k) + VPI_{E,E_k}(E_j)$
- What about the value of imperfect information?

Example 1: Oil drilling

Suppose an oil company is hoping to buy one of n blocks of ocean drilling rights.

- Exactly one block contains oil worth C dollars.
- The price of each block is C/n dollars.
- If the company is risk-neutral, it will be indifferent between buying a block or not.
- A seismologist offers the company a survey indicating whether block #3 contains oil.
- How much should the company be willing to pay for the information?

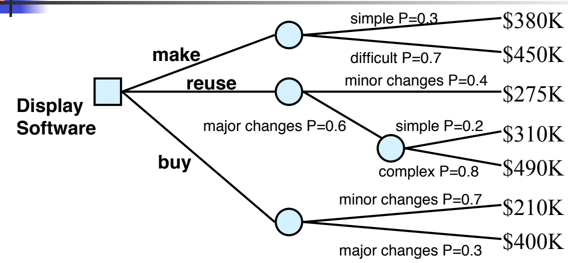
Example 1: Oil drilling cont.

- What can the company do with the information?
- Case 1: block #3 contains oil (p=1/n).
  Company will buy it and make a profit of:
  C - C/n = (n-1) C/n dollars.
- Case 2: block #3 contains no oil (p=(n-1)/n).
  Company will buy different block and make:
  C/(n-1) - C/n = C/(n (n-1)) dollars.
- Now, the overall expected profit is C/n.
- Q. What is the value of information?

Decision trees

- A decision tree is an explicit representation of all the possible scenarios from a given state.
- Each path corresponds to decisions made by the agent, actions taken, possible observations, state changes, and a final outcome node.
- Similar to a game played against "nature"

Example 2: Software development



- EU(**make**) = 0.3 ∗ $380K + 0.7 ∗ $450K = **$429K**
- EU(**reuse**) = 0.4 ∗ $275K + 0.6 ∗ [0.2 ∗ $310K + 0.8 ∗ $490K] = **$382.4K**
- EU(**buy**) = 0.7 ∗ $210K + 0.3 ∗ $400K = **$267K**

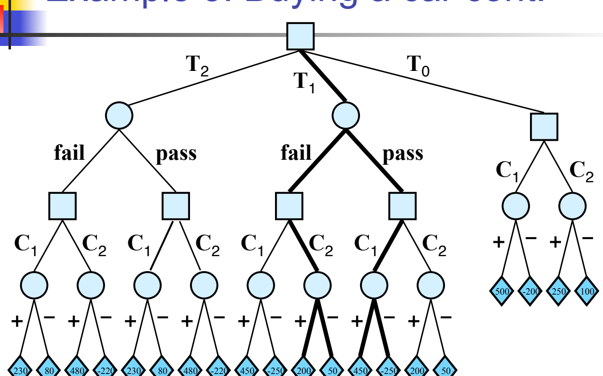
Example 3: Buying a car

- There are two candidate cars C_1 and C_2, each can be of good quality (+) or bad quality (−).
- There are two possible tests, T_1 (costs $50) and T_2 (costs $20).
- C_1 costs $1500 ($500 below market value) but if it is of bad quality repair cost is $700.
- C_2 costs $1150 ($250 below market value) but if it is of bad quality repair cost is $150.
- Buyer must buy one of the cars and can perform at most one test.

Example 3: Buying a car cont.

- The chances that the cars are of good quality are 0.70 for C_1 and 0.80 for C_2.
- Test T_1 will confirm good quality with probability 0.80 and will confirm bad quality with probability 0.65.
- Test T_2 will confirm good quality with probability 0.75 and will confirm bad quality with probability 0.70.

Example 3: Buying a car cont.



Evaluating decision trees

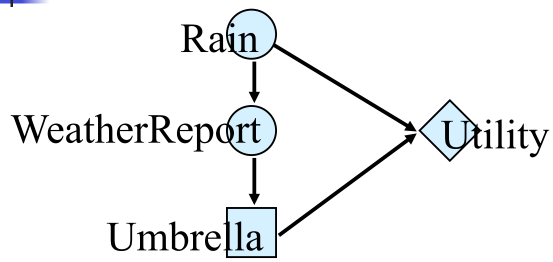
Expecti-max

1. Traverse the tree in a depth-first manner:
   (a) Assign a value to each leaf node based on outcome
   (b) Calculate the average utility at each chance node
   (c) Calculate the maximum utility at each decision node, while marking the maximum branch
2. Trace back the marked branches, from the root node down to find the desired optimal plan.

Decision networks

- Decision networks or influence diagrams are an extension of belief networks that allow for reasoning about actions and utility.
- The network represents information about the agent's current state, its possible actions, the possible outcomes of these actions, and their utility.

Example 4: Taking an umbrella



Parameters: P(Rain), P(WeatherReport|Rain), P(WeatherReport|¬Rain), Utility(Rain,Umbrella)

Nodes in a decision network

- **Chance nodes** (ovals) have CPTs (conditional probability tables) that depend on the states of the parent nodes (chance or decision).
- **Decision nodes** (squares) represent options available to the decision maker.
- **Utility nodes** (Diamonds) or value nodes represent the overall utility based on the states of the parent nodes.

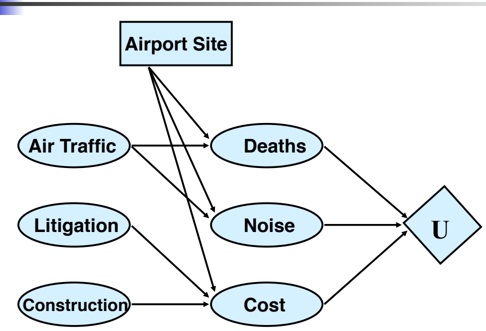
Topology of decision networks

1. The directed graph has no cycles.
2. The utility nodes have no children.
3. There is a directed path that contains all of the decision nodes.
4. A CPT is attached to each chance node specifying P(A|parents(A)).
5. A real valued function over parents(U) is attached to each utility node.

Semantics

- Links into decision nodes are called "information links," and they indicate that the state of the parent is known prior to the decision.
- The directed path that goes through all the decision nodes defines a temporal sequence of decisions.
- It also partitions the chance variables into sets: I_0 is the vars observed before any decision is made, I_1 is the vars observed after the first and before the second decision, etc. I_n is the set of unobserved vars.
- The "no-forgetting" assumption is that the decision maker remembers all past observations and decisions.

Example 5: Airport siting problem



Evaluating decision networks

1. Set the evidence variables for the current state.
2. For each possible value of the decision node:
   (a) Set the decision node to that value.
   (b) Calculate the posterior probabilities for the parent nodes of the utility node.
   (c) Calculate the expected utility for the action.
3. Return the action with the highest utility.

Example 6: Mildew

Two months before the harvest of a wheat field, the farmer observes the state Q of the crop, and he observes whether it has been attacked by mildew, M. If there is an attack, he will decide on a treatment with fungicides.

There are five variables:

- Q: fair (f), not too bad (n), average (a), good (g)
- M: no (no), little (l), moderate (m), severe (s)\
- H: state of Q plus rotten (r),bad (b), poor (p)
- OQ: observation of Q
- OM: observation of M

Mildew decision model