第五章 对策论

5.1 引言

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引言

- 对策现象和对策论
  - 对策论: 竞赛论或博弈论，研究对策现象中各方是否存在最合理的行动方案，以及如何找到合理的行动方案的数学理论和方法。

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NON-COOPERATIVE GAMES

JOHN NASH

(Received October 11, 1950)

Introduction

Von Neumann and Morgenstern have developed a very fruitful theory of two-person zero-sum games in their book *Theory of Games and Economic Behavior*. This book also contains a theory of *n*-person games of a type which we would call cooperative. This theory is based on an analysis of the interrelationships of the various coalitions which can be formed by the players of the game.

Our theory, in contradistinction, is based on the *absence* of coalitions in that it is assumed that each participant acts independently, without collaboration or communication with any of the others.

The notion of an *equilibrium point* is the basic ingredient in our theory. This notion yields a generalization of the concept of the solution of a two-person zero-sum game. It turns out that the set of equilibrium points of a two-person zero-sum game is simply the set of all pairs of opposing "good strategies."

In the immediately following sections we shall define equilibrium points and prove that a finite non-cooperative game always has at least one equilibrium point. We shall also introduce the notions of solvability and strong solvability of a non-cooperative game and prove a theorem on the geometrical structure of the set of equilibrium points of a solvable game.

As an example of the application of our theory we include a solution of a simplified three person poker game.

Formal Definitions and Terminology

In this section we define the basic concepts of this paper and set up standard terminology and notation. Important definitions will be preceded by a subtitle indicating the concept defined. The non-cooperative idea will be implicit, rather than explicit, below.

Finite Game:

For us an *n-person game* will be a set of *n players*, or *positions*, each with an associated finite set of *pure strategies*; and corresponding to each player, *i*, a *payoff function*, p_i, which maps the set of all *n*-tuples of pure strategies into the real numbers. When we use the term *n-tuple* we shall always mean a set of *n* items, with each item associated with a different player.

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A *mixed strategy* of player *i* will be a collection of non-negative numbers which have unit sum and are in one to one correspondence with his pure strategies.

We write $s_i = \sum_\alpha c_{i\alpha}\pi_{i\alpha}$ with $c_{i\alpha} \geq 0$ and $\sum_\alpha c_{i\alpha} = 1$ to represent such a mixed strategy, where the $\pi_{i\alpha}$'s are the pure strategies of player *i*. We regard the s_i's as points in a simplex whose vertices are the $\pi_{i\alpha}$'s. This simplex may be re-

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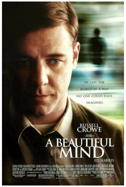

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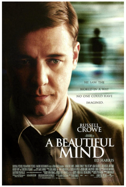

- 对策现象: 具有竞争或对抗性质的现象，如竞技体育 (下棋、打牌以及足球等)、国际政治谈判、商业谈判等。

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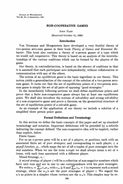

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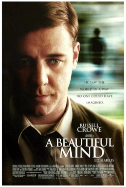

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引言

- 局中人 (players)
  - 有权决定自己行动方案的对策参加者称为局中人，通常用 I 表示局中人的集合。如果有 n 个局中人，则

$$I = \{1, 2, \ldots, n\}$$

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  - 局中人可以为个人或集体;
  - 对策论中对局中人的假设是每个局中人都是理智的，不存在侥幸心理，不存在利用局中人决策的失误来扩大自身利益的行为。

引言

- 策略 (strategies)
  - 可供局中人选择的一个实际可行的完整的行动方案称为一个策略。参加对策的每一局中人 i 的策略记为 s_i，一般每一局中人的策略集中至少应包括两个策略。

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  - 在“齐王赛马”中，若用 (上, 中, 下) 表示以上马、中马、下马依次参赛，就是一个完整的行动方案，即为一个策略。

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- 齐王和田忌各自都有 6 个策略
  - (上, 中, 下)
  - (上, 下, 中)
  - (中, 上, 下)
  - (中, 下, 上)
  - (下, 中, 上)
  - (下, 上, 中)

引言

- 赢得函数 (支付函数)(payoff function)
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  - 当一个局势 s 出现后，应该为每一局中人 i 规定一个赢得值 (或所失值)$H_i(s)$，称为局中人 i 的赢得函数。

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- 在“齐王赛马”中
  - 局中人集合 $I = \{1, 2\}$;
  - 齐王和田忌的策略集可分别用 $S_1 = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6\}$ 和 $S_2 = \{\beta_1, \beta_2, \beta_3, \beta_4, \beta_5, \beta_6\}$ 表示;
  - 齐王的任一策略 α_i 和田忌的任一策略 β_j 就构成了一个局势 s_{ij};
  - 如果 $\alpha_1 = (上, 中, 下)$，$\beta_1 = (上, 中, 下)$，则在局势 s_{11} 下，齐王的赢得值 $H_1(s_{11}) = 3$，田忌的赢得值为 $H_2(s_{11}) = -3$。

引言

- 例 1 (市场购买力争夺问题)
  - 据预测，某乡镇下一年的饮食品购买力将有 4000 万元。乡镇企业和中心城市企业饮食品的生产情况是: 乡镇企业有特色饮食品和低档饮食品两类，中心城市企业有高档饮食品和低档饮食品两类产品。它们争夺这一部分购买力的结局见下表。

乡镇企业策略	出售高档饮食品	出售低档饮食品
出售特色饮食品	2000	3000
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- 问乡镇企业和中心城市企业应如何选择对自己最有利的产品策略。

例 2 (销售竞争问题)

- 假定企业 I，II 均能向市场出售某一产品，不妨假定他们可于时间区间 $[0,1]$ 内任一时间出售。设企业 I 在时刻 x 出售，企业 II 在时刻 y 出售，则企业 I 的收益 (赢得) 函数为

$$H(x,y)=\begin{cases} c(y-x) \text{ 若} x<y \\ \frac{1}{2}c(1-x) \text{ 若} x=y \\ c(1-x) \text{ 若} x>y \end{cases}$$

- 问这两个企业各选择什么时机出售对自己最有利？

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- 问这两个企业各选择什么时机出售对自己最有利？
- 在这个例子中，企业 I, II 可选择的策略均有无穷多个。

引言

- 例 3 (拍卖问题)
  - 最常见的一种拍卖形式是先由拍卖商把拍卖品描述一番，然后提出第一个报价。接下来由买者报价，每一次报价都要比前一次高，最后谁出的价最高，拍卖品即归谁所有。

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  - 假设有 n 个买主给出的报价分别为 $p_1, \ldots, p_n$，且不妨设 $p_n > p_{n-1} > \ldots > p_1$，现买主 n 只要报价略高于 p_{n-1}，就能买到拍卖品，即拍卖品实际上是在次高价格上卖出的。

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  - 现在的问题是，各买主之间可能知道他人的估价，也可能不知道他人的估计，每人应如何报价对自己能以较低的价格得到拍卖品最为有利？最后的结果又会怎样？

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- 例 4 (囚犯问题)
    - 设有两个嫌疑犯因涉嫌某一大案被警官拘留，警官分别对两人进行审讯。根据法律，如果两个人都承认此案是他们干的，则每人各判刑 7 年; 如果两人都不承认，则由于证据不足，两人各判刑 1 年; 如果只有一人承认，则承认者予以宽大释放，则不承认者将判刑 9 年。

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- 对两个囚犯来说，面临着一个在“承认”和“不承认”这两个策略间进行选择的难题？

引言

- 对策的分类
  - 根据局中人的个数，分为二人对策和多人对策
  - 根据各局中人的赢得函数的代数和是否为零，分为零和对策和非零和对策
  - 根据各局中人之间是否允许合作，分为合作对策和非合作对策
  - 根据局中人的策略集中的策略个数，分为有限对策和无限对策

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  - 根据局中人的策略集中的策略个数，分为有限对策和无限对策
- 研究对象
  - 二人有限零和对策，又称为矩阵对策，是目前为止在理论研究和求解方法都比较完善的一个对策分支。
  - 齐王赛马

引言

- 小结
  - 对策论
  - 三要素
    - 局中人 (players)
    - 策略 (strategies)
    - 赢得函数 (payoff function)
  - 对策的分类
    - 二人对策和多人对策
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    - 合作对策和非合作对策
    - 有限对策和无限对策
  - 矩阵对策

Q&A

Thank you!

感谢您的聆听和反馈