Rethinking Sparse Optimization Through Deep Learning

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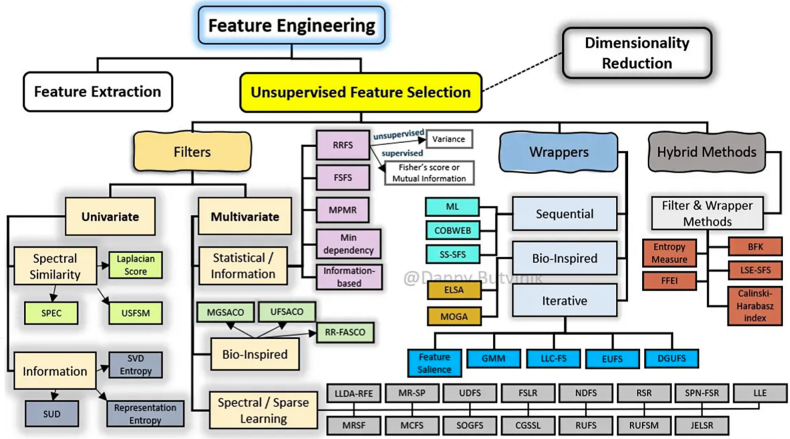
Mathematical Optimization Society, May 16-19, 2025

Joint work with Chenyi Huang (SHU), Long Chen (SHU), and others

Outline

UFS

- Unsupervised feature selection *vs.* Feature extraction
- Select a subset of input features without labels



PCA

- Given $X = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n) \in \mathbb{R}^{d \times n}$, principal component analysis (PCA) is

$$\begin{aligned} \min_{W \in \mathbb{R}^{d \times p}} \quad & \frac{1}{2}\|X - WW^\top X\|_{\mathrm{F}}^2 \\ \text{s.t.} \quad & W^\top W = I_p \end{aligned}$$

$$\Updownarrow$$

$$\begin{aligned} \min_{W \in \mathbb{R}^{d \times p}} \quad & -\operatorname{Tr}(W^\top X X^\top W) \\ \text{s.t.} \quad & W^\top W = I_p \end{aligned}$$

- Unsupervised feature selection by sparse PCA

$$\begin{aligned} \min_{W \in \mathbb{R}^{d \times p}} \quad & -\operatorname{Tr}(W^\top X X^\top W) \\ \text{s.t.} \quad & W^\top W = I_p,\ \|W\|_{2,0} \le s \end{aligned}$$

- The i-th feature can be measured by $\|\mathbf{w}^i\|$ since $\mathbf{z}_i = (\mathbf{w}^{1\top},\ \mathbf{w}^{2\top},\ \ldots\ ,\mathbf{w}^{d\top})\mathbf{x}_i$
- The dimension number is often omitted when it does not cause ambiguity

Motivation

- Li-Nie-Bian et al, Sparse PCA via $\ell_{2,p}$-Norm Regularization for Unsupervised Feature Selection, IEEE TPAMI, 2023

$$\begin{aligned} \min_{W} \quad & -\operatorname{Tr}(W^\top X X^\top W) + \lambda \|W\|_{2,p}^p \ (0 < p < 1) \\ \text{s.t.} \quad & W^\top W = I \end{aligned}$$

- Li-Sun-Zhang, Unsupervised Feature Selection via Nonnegative Orthogonal Constrained Regularized Minimization, arXiv, 2024

$$\begin{aligned} \min_{W,Y} \quad & \operatorname{Tr}(Y^\top L Y) + \alpha \|Y - X^\top W\|_{2,1} + \beta \|W\|_{2,1} + \gamma \|W\|_{\mathrm{F}}^2 \\ \text{s.t.} \quad & Y^\top Y = I, \ Y \geq 0 \end{aligned}$$

- Hu-Wang-Zhang et al, Bi-Level Spectral Feature Selection, IEEE TNNLS, 2025
- Jiao-Xue-Zhang, Sparse Learning-Based Feature Selection in Classification: A Multi-Objective Perspective, IEEE TETCI, 2025
- Li-Yu-Yang et al, Exploring Feature Selection With Limited Labels: A Comprehensive Survey of Semi-Supervised and Unsupervised Approaches, IEEE TKDE, 2024

Outline

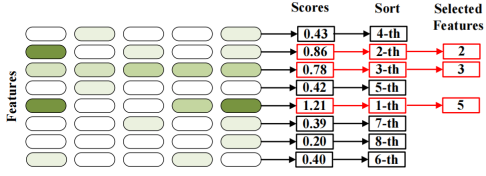
New Formulation

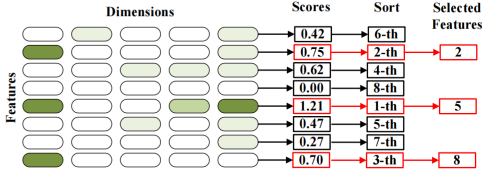
- Xiu-Huang-Shang et al, Bi-Sparse Unsupervised Feature Selection, IEEE TIP, 2025

$$\min_{W} \; -\mathrm{Tr}(W^\top SW) + \lambda \|W\|_{2,p}^{p} \; (0 < p < 1)$$
$$\text{s.t.} \;\; W^\top W = I$$

$$\Downarrow$$

$$\min_{W} \; -\mathrm{Tr}(W^\top SW) + \lambda_1 \|W\|_{2,p}^{p} + \lambda_2 \|W\|_{q}^{q} \; (0 \le p, q < 1)$$
$$\text{s.t.} \;\; W^\top W = I$$





Optimization Algorithm

- First-order algorithm: PAM (Proximal Alternating Method)
- Model reformulation

$$\begin{aligned}
&\min_{W} \; -\mathrm{Tr}(W^\top SW) + \lambda_1 \|W\|_{2,p}^p + \lambda_2 \|W\|_q^q \\
&\text{s.t.} \;\; W^\top W = I \\
&\qquad\qquad \Downarrow \\
&\min_{W,U,V} \; -\mathrm{Tr}(W^\top SW) + \lambda_1 \|V\|_{2,p}^p + \lambda_2 \|U\|_q^q \\
&\text{s.t.} \;\; W^\top W = I, V = W, U = W \\
&\qquad\qquad \Downarrow \\
&\min_{W,U,V} \; -\mathrm{Tr}(W^\top SW) + \lambda_1 \|V\|_{2,p}^p + \lambda_2 \|U\|_q^q \\
&\qquad + \frac{\beta_1}{2}\|W - U\|_{\mathrm{F}}^2 + \frac{\beta_2}{2}\|W - V\|_{\mathrm{F}}^2 + \Phi(W)
\end{aligned}$$

Optimization Algorithm

- Input: X, λ_1, λ_2, β_1, β_2, p,q, τ_1, τ_2, τ_3
- Initialize: W^0, U^0, V^0
- While not converged do
  - Update W^{k+1} by

$$\begin{aligned} \min_{W} \ & -\mathrm{Tr}(W^\top S W) + \frac{\beta_1}{2}\|W - U^k\|_{\mathrm{F}}^2 + \frac{\beta_2}{2}\|W - V^k\|_{\mathrm{F}}^2 + \frac{\tau_1}{2}\|W - W^k\|_{\mathrm{F}}^2 \\ \text{s.t.} \ & W^\top W = I \end{aligned}$$

  - Update U^{k+1} by

$$\min_{U} \ \lambda_2\|U\|_q^q + \frac{\beta_1}{2}\|W^{k+1} - U\|_{\mathrm{F}}^2 + \frac{\tau_2}{2}\|U - U^k\|_{\mathrm{F}}^2$$

  - Update V^{k+1} by

$$\min_{V} \ \lambda_1\|V\|_{2,p}^p + \frac{\beta_2}{2}\|W^{k+1} - V\|_{\mathrm{F}}^2 + \frac{\tau_3}{2}\|V - V^k\|_{\mathrm{F}}^2$$

- Output: $W^{k+1}, U^{k+1}, V^{k+1}$

Update W

- Riemannian gradient

$$\begin{aligned} \min_{W} \ & -\operatorname{Tr}(W^\top S W) + \frac{\beta_1}{2}\|W - U^k\|_{\mathrm{F}}^2 + \frac{\beta_2}{2}\|W - V^k\|_{\mathrm{F}}^2 + \frac{\tau_1}{2}\|W - W^k\|_{\mathrm{F}}^2 \\ \text{s.t.} \ & W^\top W = I \end{aligned}$$

$$\Downarrow$$

$$\nabla g(W) = -2SW + \beta_1(W - U^k) + \beta_2(W - V^k) + \tau_1(W - W^k)$$

$$\Downarrow$$

$$\begin{aligned} \operatorname{grad} g(W) &= \mathcal{P}_W(\nabla g(W)) \\ &= \nabla g(W) - W \operatorname{sym}(W^\top \nabla g(W)) \end{aligned}$$

Update W

- Riemannian Hessian

$$\nabla^2 g(W) = -2I \otimes S + (\beta_1 + \beta_2 + \tau_1) I$$

$$\Downarrow$$

$$\begin{aligned} \operatorname{Hess} g(W) &= \mathcal{P}_W(\nabla^2 g(W)) \\ &= \nabla^2 g(W) - W \operatorname{sym}(W^\top \nabla^2 g(W)) \end{aligned}$$

$$\Downarrow$$

$$\operatorname{Hess} g(W) \approx \frac{\operatorname{grad} g(W + \varepsilon I) - \operatorname{grad} g(W)}{\varepsilon}$$

Update W

- Input: S, U^k, V^k, β_1, β_2, τ_1, ε, $\Delta' > 0$, $\rho' \in [0, \frac{1}{4})$
- While not converged do
  - Obtain η_i by solving trust domain subproblem

$$\min_{\eta \in \mathrm{T_W\,St}(d,m)} \; m_W(\eta) = g(W) + \mathrm{Tr}(\eta^\top \operatorname{grad} g(W)) + \frac{1}{2}\operatorname{vec}(\eta)^\top \operatorname{Hess} g(W) \operatorname{vec}(\eta)$$
$$\text{s.t.} \qquad \mathrm{Tr}(\eta^\top W \eta^\top) \leq \Delta^2$$

  - Compute the trust ratio ρ_i
  - if $\rho_i < \frac{1}{4}$ then
    $\Delta_{i+1} = \frac{1}{4}\Delta_i$
  - else if $\rho_i > \frac{3}{4}$ and $\|\eta_i\| = \Delta_i$ then
    $\Delta_{i+1} = \min(2\Delta_i, \Delta')$
  - else
    $\Delta_{i+1} = \Delta_i$
  - if $\rho_i > \rho'$ then
    $W_{i+1}^k = R_W(\eta_i)$
  - else
    $W_{i+1}^k = W_i^k$
- Output: W

Update U

$$\min_{U} \ \lambda_2 \|U\|_q^q + \frac{\beta_1}{2}\|W^{k+1} - U\|_{\mathrm{F}}^2 + \frac{\tau_2}{2}\|U - U^k\|_{\mathrm{F}}^2$$

$$\Downarrow$$

$$\min_{U} \ \lambda_2 \|U\|_q^q + \frac{\beta_1 + \tau_2}{2}\|U - \frac{\beta_1}{\beta_1 + \tau_2} W^{k+1} + \frac{\tau_2}{\beta_1 + \tau_2} U^k\|_{\mathrm{F}}^2$$

$$\Downarrow$$

$$\min_{u_{ij}} \ \lambda_2 |u_{ij}|^q + \frac{\beta_1 + \tau_2}{2}(u_{ij} - y_{ij})^2$$

$$\Downarrow$$

$$u_{ij} = \mathrm{Prox}(y_{ij}, \lambda_2/(\beta_1 + \tau_2))$$

Lemma

- Revisiting ℓ_q $(0 \leq q<1)$ Norm Regularized Optimization, arXiv:2306.14394

$$\begin{aligned}\text{Prox}(a,\lambda) &= \text{argmin}_x \frac{1}{2}(x-a)^2 + \lambda|x|^q \ (0 \leq q < 1) \\ &= \begin{cases} \{0\}, & |a| < \kappa(\lambda, q) \\ \{0, \text{sgn}(a)c(\lambda,q)\}, & |a| = \kappa(\lambda,q) \\ \{\text{sgn}(a)\varpi_q(|a|)\}, & |a| > \kappa(\lambda,q) \end{cases}\end{aligned}$$

where

$$c(\lambda,q) = (2\lambda(1-q))^{\frac{1}{2-q}} > 0$$

$$\kappa(\lambda,q) = (2-q)\lambda^{\frac{1}{2-q}}(2(1-q))^{\frac{q+1}{q-2}}$$

$$\varpi_q(a) \in \left\{x : x - a + \lambda q \,\text{sgn}(x) x^{q-1} = 0, x > 0\right\}$$

Update V

$$\min_{V} \ \lambda_1 \|V\|_{2,p}^p + \frac{\beta_2}{2}\|W^{k+1} - V\|_{\mathrm{F}}^2 + \frac{\tau_3}{2}\|V - V^k\|_{\mathrm{F}}^2$$

$$\Downarrow$$

$$\min_{V} \ \lambda_1 \|V\|_{2,p}^p + \frac{\beta_2 + \tau_3}{2}\|V - \frac{\beta_2}{\beta_2 + \tau_3}W^{k+1} + \frac{\tau_3}{\beta_2 + \tau_3}V^k\|_{\mathrm{F}}^2$$

$$\Downarrow$$

$$\min_{\mathbf{v}^i} \ \lambda_1 \sum_{i=1}^{d} \|\mathbf{v}^i\|_2^p + \frac{\beta_2 + \tau_3}{2}\|\mathbf{v}^i - \mathbf{z}^i\|_2^2$$

$$\Downarrow$$

$$\mathbf{v}^i = \mathrm{Prox}(\|\mathbf{z}^i\|_2, \lambda_1/(\beta_2 + \tau_3)) \cdot \frac{\mathbf{z}^i}{\|\mathbf{z}^i\|_2}$$

Experimental Details

- Compared methods
  - LapScore: He-Cai-Niyogi, NIPS, 2005
  - UDFS: Yang-Shen-Ma et al, IJCAI, 2011
  - SOGFS: Nie-Zhu-Li, IEEE TKDE, 2021
  - RNE: Liu-Ye-Li-Wang et al, KBS, 2020
  - FSPCA: Tian-Nie-Wang-Li et al, NIPS, 2020
  - SPCAFS: Li-Nie-Bian et al, IEEE TPAMI, 2023
  - SPCA-PSD: Zheng-Zhang-Liu et al, arXiv, 2023
  - FEN-PCAFS: Gao-Wu-Xu et al, IEEE TFS, 2024
- Implementation setups
  - Initialization: QR decomposition
  - Stopping criteria:

$$\frac{|f(W^{k+1}, U^{k+1}, V^{k+1}) - f(W^k, U^k, V^k)|}{\max\{1, |f(W^k, U^k, V^k)|\}} \leq 10^{-4}$$

Experimental Details

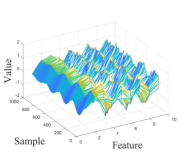
- Selected datasets

Type	Datasets	Features	Samples	Classes
Synthetic datasets	Dartboard1	9	1000	4
	Diamond9	9	3000	9
Real-world datasets	COIL20	1024	1440	20
	USPS	256	1000	10
	LUNG	325	73	7
	GLIOMA	4434	50	4
	UMIST	644	575	20
	pie	1024	1166	53
	Isolet	617	1560	26
	MSTAR	1024	2425	10

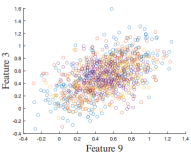
- Evaluation metrics
  - ACC: Accuracy
  - NMI: Normalized mutual information

Synthetic Experiments

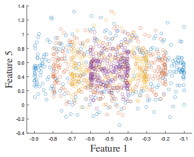
- Dartboard1



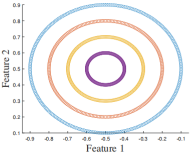
(a) Dartboard1



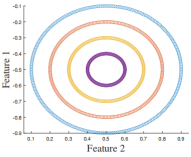
(b) LapScore



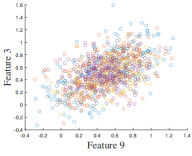
(c) SOGFS



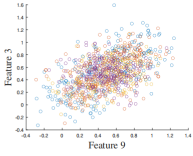
(d) RNE



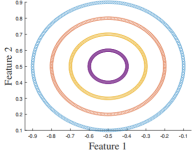
(e) UDFS



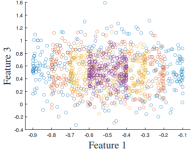
(f) SPCAFS



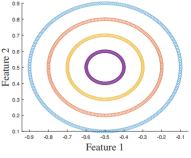
(g) FSPCA



(h) SPCA-PSD



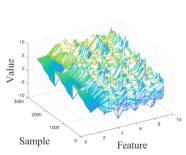
(i) FEN-PCAFS



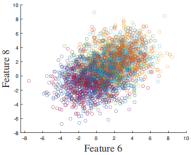
(j) BSUFS

Synthetic Experiments

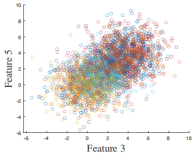
- Diamond9



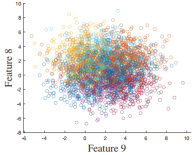
(a) Diamond9



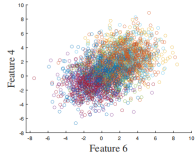
(b) LapScore



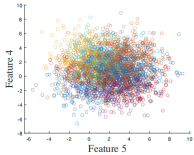
(c) SOGFS



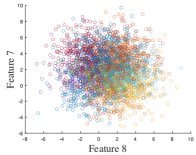
(d) RNE



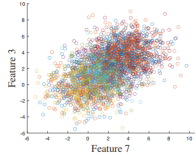
(e) UDFS



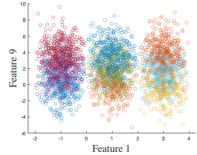
(f) SPCAFS



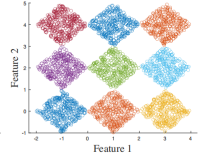
(g) FSPCA



(h) SPCA-PSD



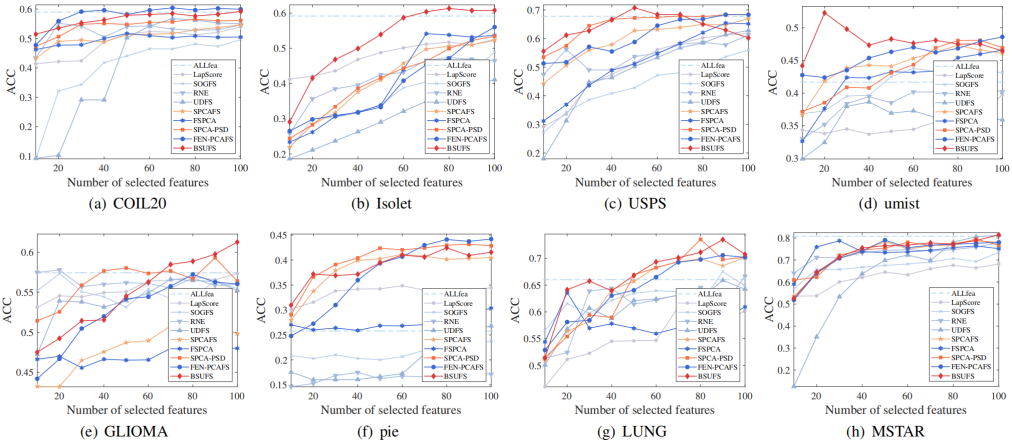
(i) FEN-PCAFS



(j) BSUFS

Real Experiments

- ACC comparisons



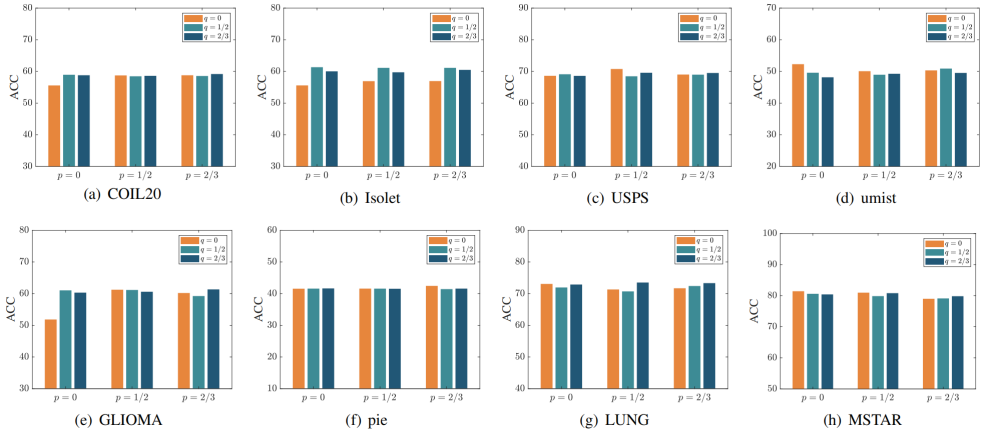
Real Experiments

- ACC comparisons

Datasets	ALLfea	LapScore	SOGFS	RNE	UDFS	SPCAFS	FSPCA	SPCA-PSD	FEN-PCAFS	BSUFS
COIL20	58.97±4.99 (10)	53.91±3.61 (100)	56.77±3.09 (70)	49.66±3.63 (100)	55.16±3.35 (20)	51.71±3.05 (50)	54.63±3.64 (100)	56.57±4.08 (80)	**60.41±4.41** (70)	**59.18±3.49** (100)
Isolet	59.18±3.19 (10)	52.55±2.83 (100)	41.11±1.71 (100)	48.93±2.69 (100)	47.39±2.91 (80)	54.15±2.69 (70)	52.26±2.81 (100)	53.45±2.82 (100)	**56.04±3.50** (100)	**61.34±3.33** (80)
USPS	67.79±4.96 (10)	61.76±4.52 (100)	62.83±3.79 (100)	56.00±3.48 (100)	61.28±3.46 (100)	65.43±4.90 (90)	66.98±3.92 (100)	**68.38±3.85** (100)	68.36±4.62 (90)	**70.77±3.73** (50)
umist	41.68±2.46 (10)	39.71±3.28 (100)	38.64±1.61 (40)	43.81±2.98 (60)	41.01±2.25 (90)	46.58±2.34 (100)	47.32±3.48 (80)	48.08±3.06 (90)	**48.61±3.23** (100)	**52.29±3.61** (20)
GLIOMA	57.44±6.40 (10)	57.36±3.60 (100)	56.64±6.47 (70)	57.32±6.47 (20)	57.80±2.98 (20)	48.04±5.26 (90)	52.08±3.64 (80)	**59.32±6.27** (90)	57.24±8.16 (80)	**61.28±9.01** (100)
pie	25.79±1.39 (10)	34.86±1.43 (60)	26.82±1.32 (100)	23.78±1.19 (100)	17.49±0.76 (40)	30.39±1.43 (100)	41.16±2.46 (60)	**43.16±2.38** (90)	**44.21±2.03** (100)	42.45±1.74 (80)
LUNG	66.03±7.23 (10)	60.93±8.02 (70)	65.89±7.43 (90)	67.53±7.73 (90)	66.68±8.32 (100)	63.62±5.45 (20)	70.16±7.71 (100)	**73.53±8.91** (80)	70.58±6.88 (90)	**73.51±6.80** (90)
MSTAR	80.81±8.76 (10)	68.21±4.57 (100)	**81.25±7.48** (100)	73.46±5.61 (100)	77.82±6.16 (100)	78.74±5.20 (30)	78.63±8.68 (90)	79.53±6.75 (90)	79.03±6.02 (50)	**81.43±6.89** (100)
Average	57.21±4.92	53.66±3.98	53.74±4.11	52.56±4.22	53.08±3.77	54.83±3.79	57.90±4.54	60.25±4.76	**60.56±4.86**	**62.78±4.83**

Effects of p and q

- ACC comparisons

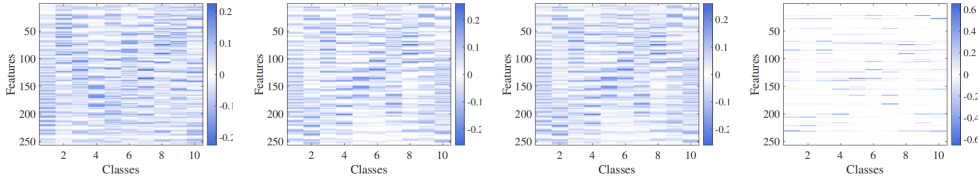


(a) COIL20 (b) Isolet (c) USPS (d) umist

(e) GLIOMA (f) pie (g) LUNG (h) MSTAR

Ablation Experiments

- Transformation matrix visualizations

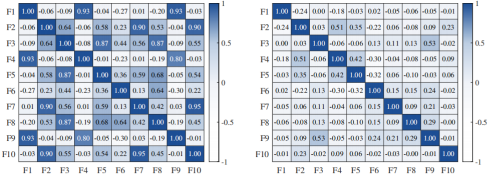


(a) USPS (Case I) (b) USPS (Case II) (c) USPS (Case III) (d) USPS (Case IV)

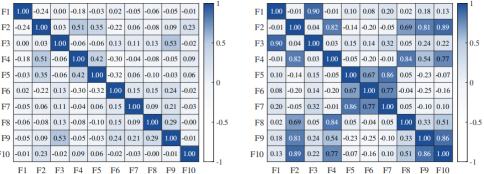
(e) umist (Case I) (f) umist (Case II) (g) umist (Case III) (h) umist (Case IV)

Feature Correlation

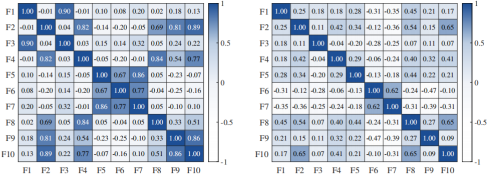
- Heatmap visualizations



(a) COIL20 (SPCAFS)

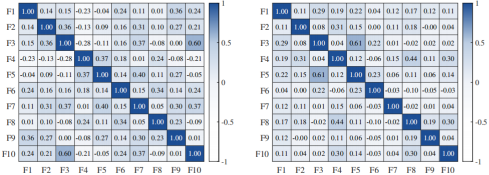


(b) Isolet (SPCAFS)

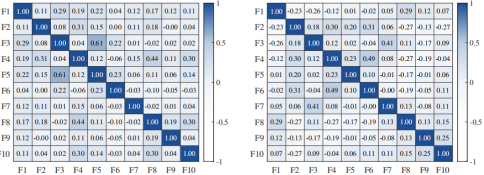


(c) USPS (SPCAFS)

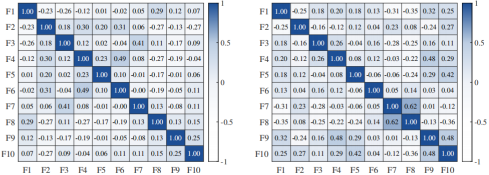
(d) LUNG (SPCAFS)



(e) COIL20 (BSUFS)



(f) Isolet (BSUFS)

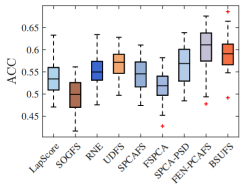


(g) USPS (BSUFS)

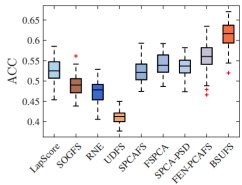
(h) LUNG (BSUFS)

Model Stability

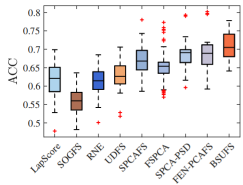
- Box-plots visualizations



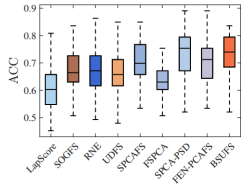
(a) COIL20 (ACC)



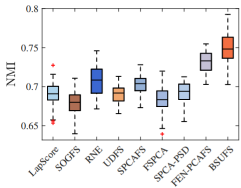
(b) Isolet (ACC)



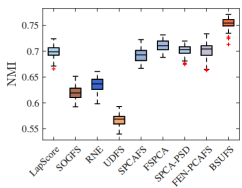
(c) USPS (ACC)



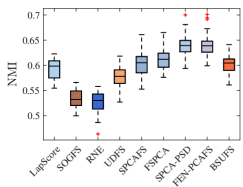
(d) LUNG (ACC)



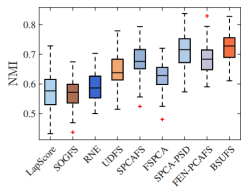
(e) COIL20 (NMI)



(f) Isolet (NMI)



(g) USPS (NMI)



(h) LUNG (NMI)

Outline

Model

- Chen-Xiu, Tuning-Free Structured Sparse PCA via Deep Unfolding Networks, CCC, 2025

$$\begin{aligned} \min_{W} \quad & \frac{1}{2}\|X - WW^\top X\|_{\mathrm{F}}^2 + \lambda\|W\|_{2,1} + \mu\|W\|_1 \\ \text{s.t.} \quad & W^\top W = I \end{aligned}$$

- Alternating direction method of multipliers (ADMM)

$$\begin{aligned} \min_{W} \quad & \frac{1}{2}\|X - WW^\top X\|_{\mathrm{F}}^2 + \lambda\|Y\|_{2,1} + \mu\|Z\|_1 \\ \text{s.t.} \quad & W^\top W = I,\ W = Y,\ W = Z \end{aligned}$$

$$\Downarrow$$

$$\begin{aligned} \mathcal{L}(W, Y, Z, \Lambda, \Pi) = & \frac{1}{2}\|X - WW^\top X\|_{\mathrm{F}}^2 + \lambda\|Y\|_{2,1} + \mu\|Z\|_1 \\ & + \langle \Lambda, W - Y\rangle + \frac{\alpha}{2}\|W - Y\|_{\mathrm{F}}^2 + \langle \Pi, W - Z\rangle + \frac{\beta}{2}\|W - Z\|_{\mathrm{F}}^2 \end{aligned}$$

SPCA-Net

- Update W-block

$$\begin{aligned} \min_{W} \quad & f(W) := \frac{1}{2}\|X - WW^\top X\|_{\mathrm{F}}^2 + \frac{\alpha}{2}\|W - Y^k + \Lambda^k/\alpha\|_{\mathrm{F}}^2 + \frac{\beta}{2}\|W - Z^k + \Pi^k/\beta\|_{\mathrm{F}}^2 \\ \text{s.t.} \quad & W^\top W = I \end{aligned}$$

$$\Downarrow$$

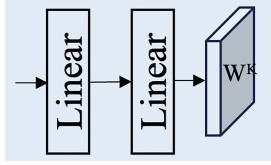
$$\begin{aligned} \min_{W} \quad & f(W^k) + \langle \nabla f(W^k), W - W^k \rangle + \frac{1}{2\eta}\|W - W^k\|_{\mathrm{F}}^2 \\ \text{s.t.} \quad & W^\top W = I \end{aligned}$$

$$\Downarrow$$

$$W^{k+1} = UV^\top$$

$$\Downarrow$$

$$W^{k+1} = \textbf{LargNet}(U, V^\top)$$



SPCA-Net

- Update Y-block

$$\min_{Y} \ \lambda\|Y\|_{2,1} + \frac{\alpha}{2}\|X^{k+1} - Y + \Lambda^k/\alpha\|_{\mathrm{F}}^2$$

$$\Downarrow$$

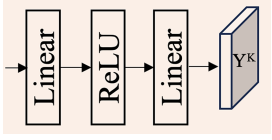
$$Y^{k+1} = \mathrm{sign}(\|X^{k+1} + \Lambda^k/\alpha\|_2) \circ \max(\|X^{k+1} + \Lambda^k/\alpha\|_2 - \lambda/\alpha, 0)$$

$$\Downarrow$$

$$Y^{k+1} = \frac{X^{k+1} + \Lambda^k/\alpha}{\|X^{k+1} + \Lambda^k/\alpha\|_2}\mathrm{ReLU}(\|X^{k+1} + \Lambda^k/\alpha\|_2 - \lambda/\alpha)$$

$$\Downarrow$$

$$Y^{k+1} = \mathbf{GSoftNet}(X^{k+1} + \Lambda^k/\alpha, \lambda/\alpha)$$



SPCA-Net

- Update Z-block

$$\min_{Z} \ \mu\|Z\|_1 + \frac{\beta}{2}\|X^{k+1} - Z + \Pi^k/\beta\|_{\mathrm{F}}^2$$

$$\Downarrow$$

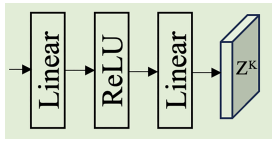
$$Z^{k+1} = \mathrm{sign}(X^{k+1} + \Pi^k/\beta) \circ \max(|X^{k+1} + \Pi^k/\beta| - \mu/\beta, 0)$$

$$\Downarrow$$

$$Z^{k+1} = \frac{X^{k+1} + \Pi^k/\beta}{|X^{k+1} + \Pi^k/\beta|}\mathrm{ReLU}(|X^{k+1} + \Pi^k/\beta| - \mu/\beta)$$

$$\Downarrow$$

$$Z^{k+1} = \mathbf{SoftNet}(X^{k+1} + \Pi^k/\beta, \mu/\beta)$$



SPCA-Net

- Input: X, $\lambda, \mu, \alpha, \beta$
- Initialize: $(W^0,\ Y^0,\ Z^0,\ \Lambda^0,\ \Pi^0)$
- While $k = 1, \ldots, K$ do
  - Update W^{k+1} by

$$W^{k+1} = \textbf{LargNet}(U, V^\top)$$

  - Update Y^{k+1} by

$$Y^{k+1} = \textbf{GSoftNet}(X^{k+1} + \Lambda^k/\alpha, \lambda/\alpha)$$

  - Update Z^{k+1} by

$$Z^{k+1} = \textbf{SoftNet}(X^{k+1} + \Pi^k/\beta, \mu/\beta)$$

  - Update Λ^{k+1}, Π^{k+1} by

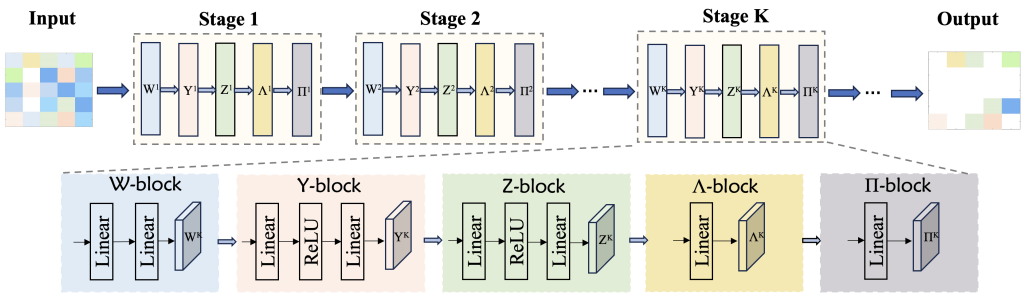
$$\Lambda^{k+1} = \textbf{Linear}(W^{k+1}, Y^{k+1}, \Lambda^k, \alpha),\ \Pi^{k+1} = \textbf{Linear}(W^{k+1}, Z^{k+1}, \Pi^k, \beta)$$

- Output: Trained W

Architecture

- All papameters $(\lambda, \mu, \alpha, \beta)$ are trained in an end-to-end manner
- The loss is defined as

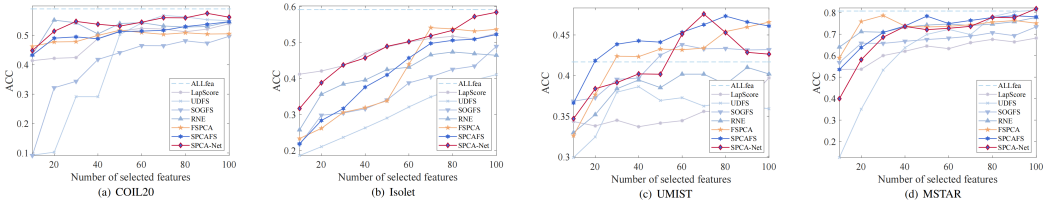
$$\text{Loss} = \frac{1}{2}\|X - \bar{W}\bar{W}^\top X\|_F^2 + \lambda\|\bar{W}\|_{2,1} + \mu\|\bar{W}\|_1$$



Real Experiments

- Accuracy (ACC) ↑

Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	FSPCA	SPCAFS	SPCA-Net
COIL20	58.97±4.99 (10)	53.91±3.61 (100)	**56.70±3.09 (70)**	49.66±3.63 (100)	55.16±3.35 (20)	51.71±3.05 (50)	54.63±3.64 (100)	**57.46±2.76 (90)**
Isolet	59.18±3.19 (10)	52.55±2.83 (100)	41.11±1.71 (100)	48.93±2.69 (100)	47.39±2.91 (80)	**54.15±2.69 (70)**	52.26±2.81 (100)	**58.43±4.31 (100)**
UMIST	41.68±2.46 (10)	39.71±3.28 (100)	38.64±1.61 (40)	43.81±2.98 (80)	41.01±2.25 (90)	46.58±2.34 (100)	**47.32±3.48 (80)**	**47.58±4.97 (70)**
MSTAR	80.81±8.76 (10)	68.21±4.57 (100)	**81.25±7.48 (100)**	73.46±5.61 (100)	77.82±6.16 (100)	78.74±5.20 (30)	78.63±8.68 (90)	**81.90±6.87 (100)**

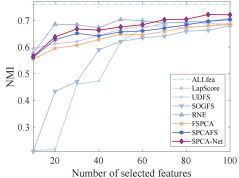


(a) COIL20 (b) Isolet (c) UMIST (d) MSTAR

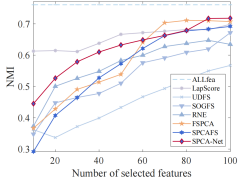
Real Experiments

- Normalized mutual information (NMI) ↑

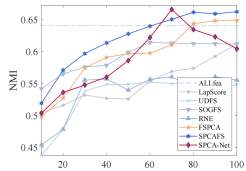
Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	FSPCA	SPCAFS	SPCA-Net
COIL20	76.04±1.69 (10)	69.01±1.53 (100)	69.12±1.17 (80)	68.03±1.59 (100)	**70.76±2.07 (100)**	68.41±1.60 (100)	70.29±1.31 (100)	**72.21±2.68 (90)**
Isolet	76.09±1.77 (10)	69.86±1.26 (100)	56.73±1.05 (100)	67.15±1.45 (100)	64.74±1.28 (90)	**71.12±1.11 (80)**	69.18±1.33 (100)	**71.80±1.59 (100)**
UMIST	64.07±1.76 (10)	61.23±2.15 (100)	55.43±1.50 (80)	61.46±2.03 (70)	56.08±1.80 (60)	64.94±1.65 (100)	**66.26±1.74 (100)**	**66.62±7.52 (70)**
MSTAR	83.96±3.14 (10)	73.90±1.62 (100)	78.18±3.64 (90)	76.56±1.54 (100)	78.26±2.51 (100)	78.87±2.52 (90)	**79.62±2.30 (100)**	**80.67±3.47 (90)**



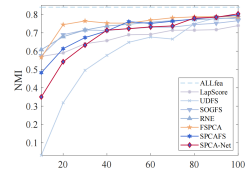
(a) COIL20



(b) Isolet



(c) UMIST



(d) MSTAR

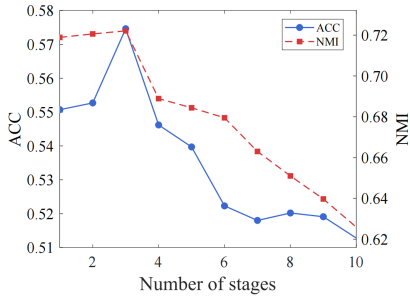
Discussions

- Ablation studies

Datasets	Network	ACC ↑	NMI ↑
COIL20	×	55.12±2.67	70.44±1.37
	√	57.46±2.76	72.21±2.68
Isolet	×	51.84±2.82	67.02±1.43
	√	58.43±4.31	71.80±1.59
UMIST	×	40.65±2.29	55.88±1.62
	√	47.58±4.97	66.62±7.52
MSTAR	×	80.65±6.47	80.53±2.41
	√	81.90±6.87	80.67±3.47

Datasets	Dynamic	ACC ↑	NMI ↑
COIL20	×	56.71±3.83	71.49±3.67
	√	57.46±2.76	72.21±2.68
Isolet	×	52.06±3.71	68.91±2.36
	√	58.43±4.31	71.80±1.59
UMIST	×	42.63±2.78	60.12±1.69
	√	47.58±4.97	66.62±7.52
MSTAR	×	80.74±5.28	80.59±3.67
	√	81.90±6.87	80.67±3.47

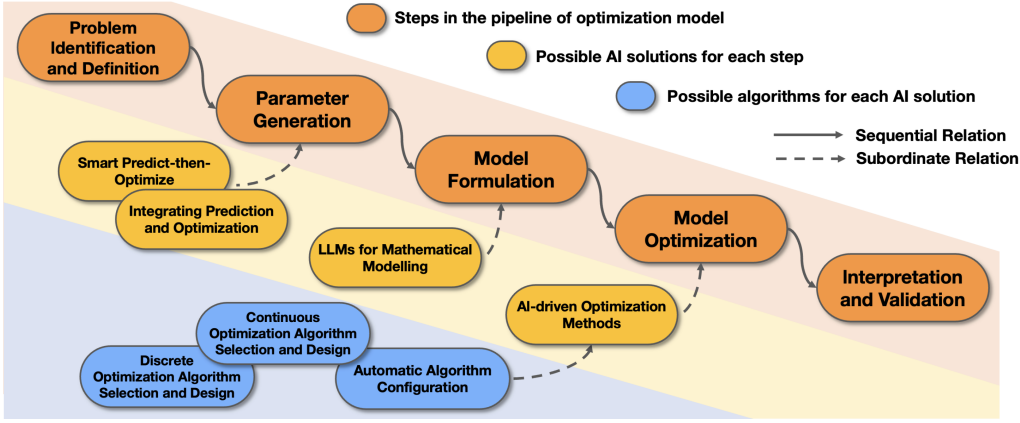
- Effect of deep unfolding stages



Outline

Future Work

- Large Language Models for Optimization

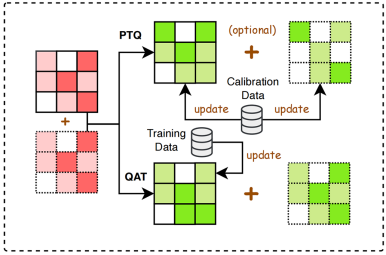


Future Work

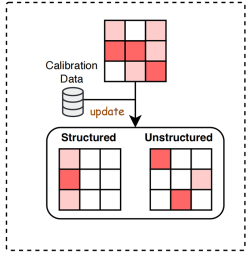
- Ramamonjison-Yu-Li et al, NL4Opt Competition: Formulating Optimization Problems Based on Their Natural Language Descriptions, NeurIPS, 2022
- Yang-Wang-Lu et al, Large Language Models as Optimizers, ICLR, 2024
- AhmadiTeshnizi-Gao-Udell, OptiMUS: Scalable Optimization Modeling with (MI)LP Solvers and Large Language Models, ICML, 2024
- Gao-Jiang-Cai et al, StrategyLLM: Large Language Models as Strategy Generators, Executors, Optimizers, and Evaluators for Problem Solving, NeurIPS, 2024
- Romera-Paredes-Barekatain et al, Mathematical Discoveries from Program Search with Large Language Models, Nature, 2024
- Jiang-Shu-Qian et al, LLMOPT: Learning to Define and Solve General Optimization Problems from Scratch, ICLR, 2025
- Huang-Tang-Hu et al, ORLM: A Customizable Framework in Training Large Models for Automated Optimization Modeling, Operations Research, 2025
- Chen-Flores-Mantri et al, OptiChat: Bridging Optimization Models and Practitioners with Large Language Models, INFORMS Journal on Data Science, 2025

Future Work

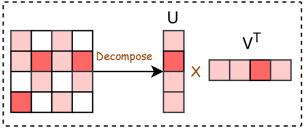
- Optimization for Large Language Models



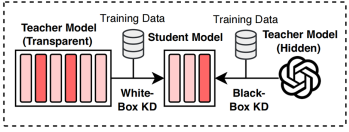
(a) Quantization



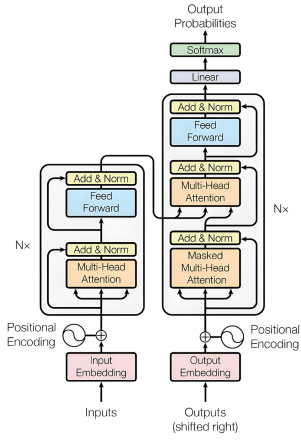
(b) Parameter Pruning



(c) Low-Rank Approximation



(d) Knowledge Distillation



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- Ma-Fang-Wang, LLM-Pruner: On the Structural Pruning of Large Language Models, NeurIPS, 2023
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- Deng-Jiao-Liu, et al, DRPruning: Efficient Large Language Model Pruning through Distributionally Robust Optimization, ACL, 2025
- Zhao-Hu-Li, et al, FISTAPruner: Layer-wise Post-training Pruning for Large Language Models, EMNLP, 2025
- Liu-Liu-Wang et al, ARMOR: High-Performance Semi-Structured Pruning via Adaptive Matrix Factorization, arXiv, 2025

Thank you for your attention!

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