

Learning to Select Features

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Joint work with Anning Yang (SHU), Long Chen (SHU), Jianhao Li (SHU) and others

Outline

Introduction

Sparse Coding

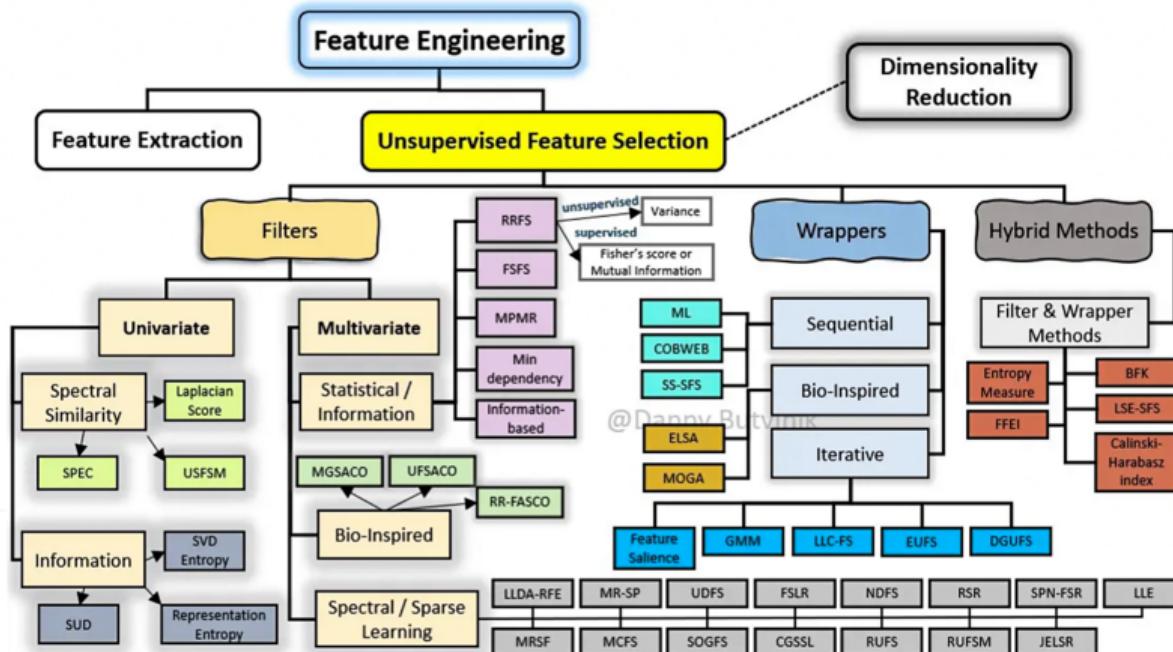
Contrastive Learning

Deep Unfolding Networks

Large Language Models

Future Work

- ▶ Unsupervised feature selection *vs.* Feature extraction
- ▶ Select a subset of input features without labels



PCA

- Given $X = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n) \in \mathbb{R}^{d \times n}$, principal component analysis (PCA) is

$$\min_{W \in \mathbb{R}^{d \times p}} \frac{1}{2} \|X - WW^\top X\|_F^2$$

$$\text{s.t. } W^\top W = I_p$$

\Updownarrow

$$\min_{W \in \mathbb{R}^{d \times p}} -\text{Tr}(W^\top X X^\top W)$$

$$\text{s.t. } W^\top W = I_p$$

- Unsupervised feature selection by sparse PCA

$$\min_{W \in \mathbb{R}^{d \times p}} -\text{Tr}(W^\top X X^\top W)$$

$$\text{s.t. } W^\top W = I_p, \|W\|_{2,0} \leq s$$

- The i -th feature can be measured by $\|\mathbf{w}^i\|$ since $\mathbf{z}_i = (\mathbf{w}^{1\top}, \mathbf{w}^{2\top}, \dots, \mathbf{w}^{d\top})\mathbf{x}_i$
- The dimension number is often omitted when it does not cause ambiguity

SOTA

- ▶ Li-Nie-Bian et al, Sparse PCA via $\ell_{2,p}$ -Norm Regularization for Unsupervised Feature Selection, IEEE TPAMI, 2023

$$\begin{aligned} \min_W \quad & -\text{Tr}(W^\top X X^\top W) + \lambda \|W\|_{2,p}^p \quad (0 < p < 1) \\ \text{s.t.} \quad & W^\top W = I \end{aligned}$$

- ▶ Li-Sun-Zhang, Unsupervised Feature Selection via Nonnegative Orthogonal Constrained Regularized Minimization, arXiv:2403.16966

$$\begin{aligned} \min_{W,Y} \quad & \text{Tr}(Y^\top LY) + \alpha \|Y - X^\top W\|_{2,1} + \beta \|W\|_{2,1} + \gamma \|W\|_F^2 \\ \text{s.t.} \quad & Y^\top Y = I, \quad Y \geq 0 \end{aligned}$$

- ▶ Hu-Wang-Zhang et al, Bi-Level Spectral Feature Selection, IEEE TNNLS, 2025
- ▶ Jiao-Xue-Zhang, Sparse Learning-Based Feature Selection in Classification: A Multi-Objective Perspective, IEEE TETCI, 2025
- ▶ Li-Yu-Yang et al, Exploring Feature Selection With Limited Labels: A Comprehensive Survey of Semi-Supervised and Unsupervised Approaches, IEEE TKDE, 2024

Contribution

- ▶ (Q1) How to learn feature structures ⇒ Sparse coding

Xiu-Yang-Huang et al, Enhancing Unsupervised Feature Selection via Double Sparsity Constrained Optimization, arXiv:2501.00726

- ▶ (Q2) How to learn data distributions ⇒ Contrastive learning

Xiu-Yang-Li, Sparse PCA Meets Contrastive Learning: A New Method for Unsupervised Feature Selection, April 2025

- ▶ (Q3) How to learn regularization parameters ⇒ Deep unfolding networks

Chen-Xiu, Tuning-Free Structured Sparse PCA via Deep Unfolding Networks, arXiv:2502.20837

- ▶ (Q4) How to learn feature selection ⇒ Large language models

Li-Xiu, LLM4FS: Leveraging Large Language Models for Feature Selection and How to Improve It, March 2025

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Contrastive Learning

Deep Unfolding Networks

Large Language Models

Future Work

Model

- (Q1) How to learn feature structures

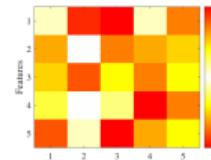
$$\min_W -\text{Tr}(W^\top X X^\top W)$$

$$\text{s.t. } W^\top W = I, \|W\|_{2,0} \leq s$$

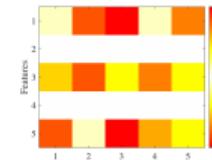


$$\min_W -\text{Tr}(W^\top X X^\top W)$$

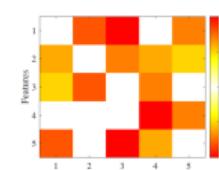
$$\text{s.t. } W^\top W = I, \|W\|_{2,0} \leq s_1, \|W\|_0 \leq s_2$$



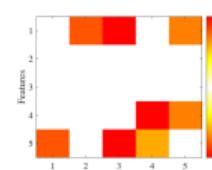
(a) Original



(b) Obtained by $\ell_{2,0}$ -norm



(c) Obtained by ℓ_0 -norm



(d) Obtained by double sparsity

- Double Sparsity Constrained Optimization for Feature Selection (DSCOFS)

- $\|W\|_{2,0} \leq s_1$: Global feature selection
- $\|W\|_0 \leq s_2$: Local feature selection

Algorithm

- ▶ Proximal alternating method (PAM)
- ▶ Model reformulation

$$\min_W - \text{Tr}(W^\top X X^\top W)$$

$$\text{s.t. } W^\top W = I, \|W\|_{2,0} \leq s_1, \|W\|_0 \leq s_2$$

↓

$$\min_{W,Y,Z} - \text{Tr}(W^\top X X^\top W)$$

$$\text{s.t. } W^\top W = I, \|Y\|_{2,0} \leq s_1, \|Z\|_0 \leq s_2$$

$$W = Y, W = Z$$

↓

$$\min_{W,Y,Z} - \text{Tr}(W^\top X X^\top W) + \mu_1 \|W - Y\|_F^2 + \mu_2 \|W - Z\|_F^2$$

$$\text{s.t. } W^\top W = I, \|Y\|_{2,0} \leq s_1, \|Z\|_0 \leq s_2$$

Algorithm

► Input: $X, \mu_1, \mu_2, s_1, s_2, \tau_1, \tau_2, \tau_3$

► Initialize: (W^0, Y^0, Z^0)

► While not converged do

► Update W^{k+1} by

$$\begin{aligned} \min_W \quad & -\text{Tr}(W^\top X X^\top W) + \mu_1 \|W - Y^k\|_{\text{F}}^2 + \mu_2 \|W - Z^k\|_{\text{F}}^2 + \tau_1 \|W - W^k\|_{\text{F}}^2 \\ \text{s.t.} \quad & W^\top W = I \end{aligned}$$

► Update Y^{k+1} by

$$\begin{aligned} \min_Y \quad & \|W^{k+1} - Y\|_{\text{F}}^2 + \tau_2 \|Y - Y^k\|_{\text{F}}^2 \\ \text{s.t.} \quad & \|Y\|_{2,0} \leq s_1 \end{aligned}$$

► Update Z^{k+1} by

$$\begin{aligned} \min_Z \quad & \|W^{k+1} - Z\|_{\text{F}}^2 + \tau_3 \|Z - Z^k\|_{\text{F}}^2 \\ \text{s.t.} \quad & \|Z\|_0 \leq s_2 \end{aligned}$$

Convergence

- ▶ Denote the objective function as

$$f(W, Y, Z) = -\text{Tr}(W^\top X X^\top W) + \mu_1 \|W - Y\|_F^2 + \mu_2 \|W - Z\|_F^2$$

- ▶ Suppose that $\beta \geq \max\{2(\lambda_0 + \lambda_1), 2m\lambda_2\}$
- ▶ **(Theorem)** Let $\{(W^k, Y^k, Z^k)\}$ be the generated sequence. Then the following properties hold:
 - ▶ $\{f(W^k, Y^k, Z^k)\}$ is strictly nonincreasing
 - ▶ The sequence $\{(W^k, Y^k, Z^k)\}$ is bounded
 - ▶ $\lim_{k \rightarrow \infty} \|(W^{k+1}, Y^{k+1}, Z^{k+1}) - (W^k, Y^k, Z^k)\|_F = 0$
 - ▶ Any accumulation point (W^*, Y^*, Z^*) of the sequence $\{(W^k, Y^k, Z^k)\}$ is a stationary point in the sense that

$$0 \in \nabla f(W^*, Y^*, Z^*) + N(W^*, Y^*, Z^*)$$

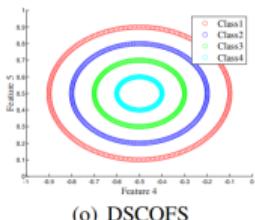
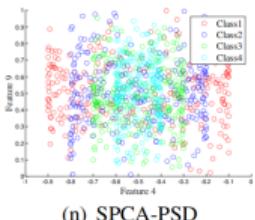
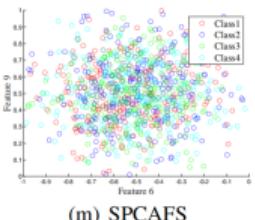
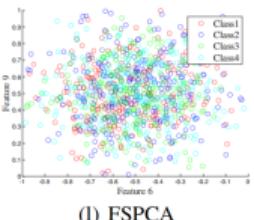
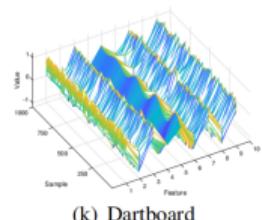
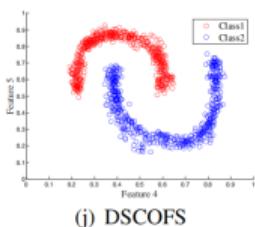
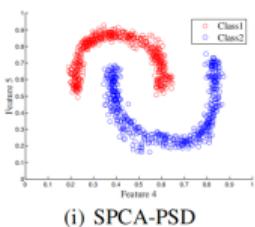
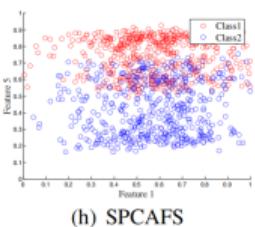
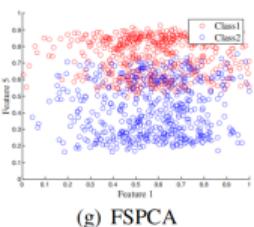
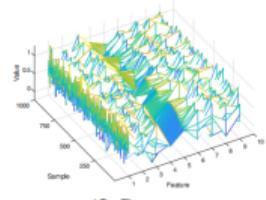
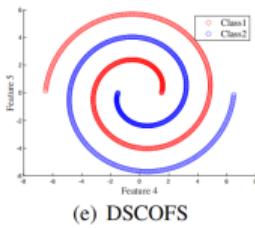
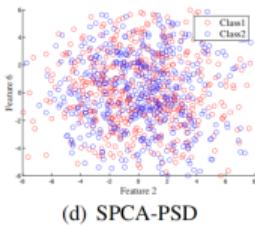
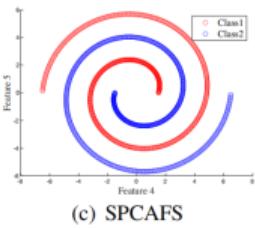
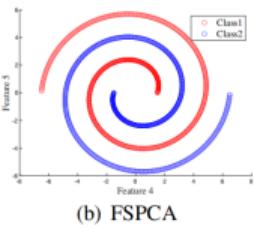
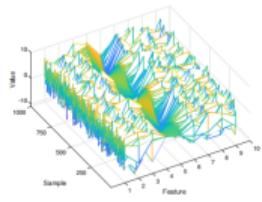
Experiments

- ▶ Compared methods
 - ▶ **LapScore**: He-Cai-Niyogi, NIPS, 2005
 - ▶ **UDFS**: Yang-Shen-Ma et al, IJCAI, 2011
 - ▶ **SOGFS**: Nie-Zhu-Li, IEEE TKDE, 2021
 - ▶ **RNE**: Liu-Ye-Li-Wang et al, KBS, 2020
 - ▶ **SPCAFS**: Li-Nie-Bian-Wu et al, IEEE TPAMI, 2023
 - ▶ **FSPCA**: Nie-Tian-Wang et al, IEEE TPAMI, 2023
 - ▶ **SPCA-PSD**: Zheng-Zhang-Liu et al, arXiv:2309.06202
- ▶ Implementation setups
 - ▶ **Initialization**: RandOrthhMat
 - ▶ **Sparsity level**: $s_1 \in \{10, 20, \dots, 100\}$, $s_2 \in \{0.1, 0.2, \dots, 0.9\}dp$
 - ▶ **Stopping criteria**:

$$\frac{|f(X^{k+1}, Y^{k+1}, Z^{k+1}) - f(X^k, Y^k, Z^k)|}{1 + |f(X^k, Y^k, Z^k)|} \leq 10^{-3}$$

Experiments

► Synthetic datasets



Experiments

- Real datasets: Accuracy (ACC) ↑

Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	FSPCA	SPCAFS	SPCA-PSD	DSCOFS
COIL20	57.74±4.93	54.82±3.91 (100)	58.71±3.47 (100)	49.66±4.81 (100)	55.84±4.41 (90)	50.15±4.70 (100)	54.39±3.67 (100)	56.57±3.05 (100)	60.51±4.63 (100)
USPS	65.12±4.95	62.02±4.09 (90)	59.52±2.97 (60)	55.58±3.07 (100)	46.04±2.69 (100)	67.38±4.36 (60)	67.34±4.49 (100)	65.38±4.26 (100)	69.67±4.97 (100)
lung_discrete	65.10±6.44	59.29±6.33 (70)	68.58±6.99 (100)	65.12±6.89 (100)	64.05±6.65 (100)	60.19±6.55 (40)	71.37±7.68 (100)	72.22±8.02 (80)	73.12±8.48 (100)
GLIOMA	56.84±5.24	58.88±3.96 (90)	56.80±4.85 (100)	57.44±6.16 (70)	58.32±7.31 (90)	47.92±4.61 (80)	50.60±5.02 (20)	59.28±5.01 (90)	60.88±6.31 (80)
UMIST	41.07±2.38	40.13±2.79 (100)	47.12±2.49 (40)	41.70±3.17 (100)	40.35±2.26 (90)	46.70±2.29 (100)	46.78±2.51 (90)	47.98±2.91 (90)	48.10±3.01 (70)
warpPIE10P	25.67±1.90	28.94±1.66 (100)	41.42±3.18 (20)	46.90±3.89 (20)	29.57±2.96 (90)	28.01±2.27 (50)	48.76±3.86 (50)	43.74±3.91 (70)	49.00±3.88 (40)
Isolet	57.89±3.82	52.21±2.76 (100)	41.95±2.07 (100)	49.31±2.32 (100)	47.12±2.06 (90)	53.62±2.36 (100)	53.04±2.33 (100)	51.91±2.15 (70)	59.67±3.46 (100)
MSTAR	77.04±7.98	67.87±3.49 (90)	78.15±5.80 (90)	73.74±5.89 (100)	69.16±6.03 (100)	75.52±6.22 (70)	80.80±5.95 (100)	79.70±6.43 (90)	82.59±7.41 (100)
Average	55.81±4.71	53.02±3.62	56.53±4.04	54.93±4.53	51.31±4.30	53.69±4.47	59.14±4.44	59.60±4.47	62.94±5.27

Experiments

- Real datasets: Normalized mutual information (NMI) ↑

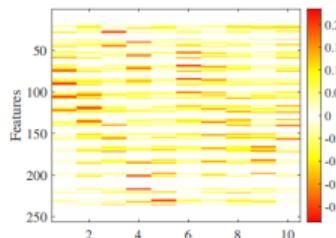
Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	FSPCA	SPCAFS	SPCA-PSD	DSCOFS
COIL20	75.37±1.96	69.59±1.48 (100)	73.54±1.76 (100)	68.92±1.84 (100)	70.43±1.92 (100)	68.50±1.56 (100)	69.98±1.45 (100)	69.85±1.41 (100)	76.25±1.71 (100)
USPS	61.12±2.01	59.46±1.80 (100)	54.69±2.11 (100)	52.96±1.54 (100)	45.36±1.93 (90)	62.00±1.87 (60)	60.98±2.37 (100)	60.90±2.02 (100)	64.06±2.58 (100)
lung_discrete	62.85±5.13	56.79±3.99 (100)	64.84±5.09 (100)	59.70±5.24 (100)	61.63±5.83 (70)	58.26±6.39 (40)	69.09±5.61 (100)	70.93±5.46 (80)	70.98±7.00 (100)
GLIOMA	48.86±5.72	51.03±2.48 (100)	47.22±3.53 (10)	48.67±10.98 (100)	48.62±6.32 (100)	21.94±5.28 (100)	24.14±6.97 (100)	51.44±5.62 (90)	51.06±6.19 (80)
UMIST	63.67±1.85	61.16±1.71 (100)	62.00±1.58 (100)	60.79±1.54 (100)	55.92±1.57 (70)	65.27±1.58 (100)	66.23±1.60 (90)	66.25±1.72 (100)	67.24±1.85 (100)
warpPIE10P	25.07±2.88	25.13±1.73 (90)	46.18±3.30 (20)	52.12±3.25 (20)	32.67±3.31 (90)	23.90±2.01 (50)	52.63±3.33 (50)	46.02±3.70 (70)	52.65±3.29 (50)
Isolet	75.72±1.70	69.77±1.20 (100)	56.29±1.11 (100)	67.40±1.44 (100)	64.27±0.95 (90)	70.79±1.12 (100)	67.71±1.33 (100)	69.69±0.80 (100)	75.01±1.35 (100)
MSTAR	82.42±3.31	74.10±1.76 (100)	76.45±2.47 (90)	76.39±1.70 (100)	66.87±1.99 (80)	78.39±2.17 (90)	80.33±2.50 (100)	79.17±2.77 (90)	81.14±3.13 (100)
Average	61.89±3.07	58.38±2.02	60.15±2.62	60.87±3.44	55.72±2.98	56.13±2.75	61.39±3.15	64.28±2.94	67.30±3.39

Experiments

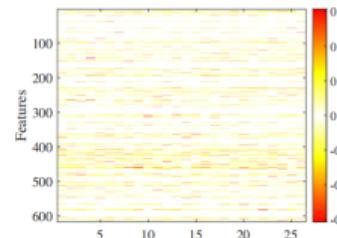
- ▶ Ablation studies: Feature similarity rate (FSR)

$$\text{FSR} = \frac{|\mathbb{T}_{\text{our}} \cap \mathbb{T}_{2,0}|}{n}$$

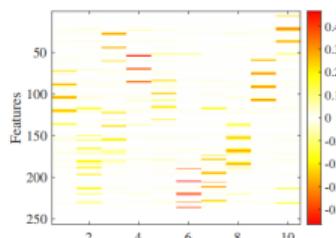
Datasets	$\ W\ _0 \leq s_2$	ACC \uparrow	NMI \uparrow	FSR
COIL20	✗	60.25 \pm 4.52	75.89 \pm 1.58	84
	✓	60.51 \pm 4.42	76.25 \pm 1.71	
USPS	✗	68.69 \pm 4.79	61.25 \pm 2.39	68
	✓	69.67 \pm 4.97	64.06 \pm 2.58	
lung_discrete	✗	71.42 \pm 7.95	69.74 \pm 6.11	92
	✓	73.12 \pm 8.48	70.98 \pm 7.00	
GLIOMA	✗	58.24 \pm 5.04	49.76 \pm 6.12	85
	✓	60.88 \pm 6.31	51.06 \pm 6.19	
UMIST	✗	47.33 \pm 3.05	67.44 \pm 1.88	95
	✓	48.10 \pm 3.01	67.24 \pm 1.85	
warpPIE10P	✗	47.91 \pm 4.99	51.19 \pm 3.79	89
	✓	49.00 \pm 3.88	52.65 \pm 3.29	
Isolet	✗	57.29 \pm 3.44	72.82 \pm 1.87	52
	✓	59.67 \pm 3.46	75.01 \pm 1.35	
MSTAR	✗	82.06 \pm 6.87	81.01 \pm 2.41	99
	✓	82.59 \pm 7.41	81.14 \pm 3.13	



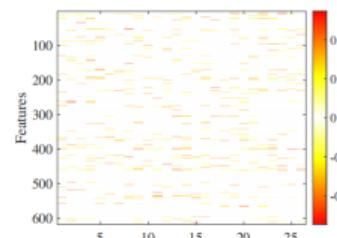
(a) USPS



(b) Isolet



(c) USPS



(d) Isolet

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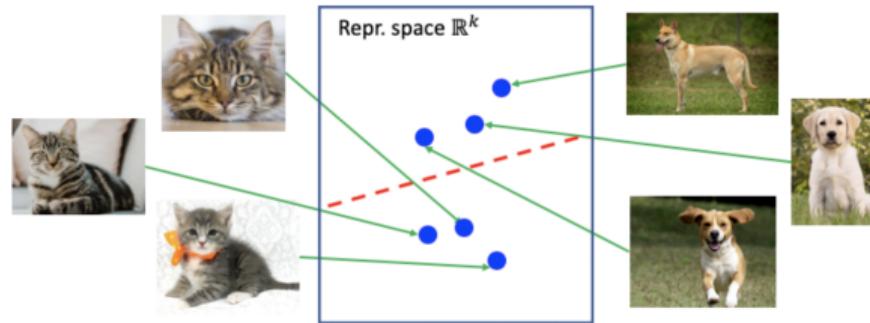
Future Work

Motivation

- (Q2) How to learn data distributions

$$\begin{aligned} \min_W \quad & \frac{1}{2} \|X - WW^\top X\|_F^2 \\ \text{s.t.} \quad & W^\top W = I, \|W\|_{2,0} \leq s_1, \|W\|_0 \leq s_2 \end{aligned}$$

- Convex loss: ℓ_1 -norm, $\ell_{2,1}$ -norm, quantile, Huber
- Nonconvex loss: ℓ_p -norm, $\ell_{2,p}$ -norm, SCAD, MCP, capped ℓ_1
- Contrastive learning: learn a discrimination model between positive and negative pairs



Motivation

- ▶ Let $X = (\mathbf{x}_1, \mathbf{x}_2, \dots, \mathbf{x}_n)$ and $Y = (\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n)$ be two different pairs, the contrastive loss is defined as

$$L_c(X, Y) = \frac{1}{2n} \sum_{i=1}^n (L_c(\mathbf{x}_i) + L_c(\mathbf{y}_i))$$

$$L_c(\mathbf{x}_i) = -\log \frac{\exp(s(\mathbf{x}_i, \mathbf{y}_i)/\tau)}{\sum_{j=1, j \neq i}^n \exp(s(\mathbf{x}_i, \mathbf{x}_j)/\tau) + \sum_{j=1}^n \exp(s(\mathbf{x}_i, \mathbf{y}_j)/\tau)}$$

$$L_c(\mathbf{y}_i) = -\log \frac{\exp(s(\mathbf{y}_i, \mathbf{x}_i)/\tau)}{\sum_{j=1, j \neq i}^n \exp(s(\mathbf{y}_i, \mathbf{y}_j)/\tau) + \sum_{j=1}^n \exp(s(\mathbf{y}_i, \mathbf{x}_j)/\tau)}$$

- ▶ $s(\mathbf{x}, \mathbf{y}) = \mathbf{x}^\top \mathbf{y}$ is the similarity metric, τ is the temperature parameter

A simple framework for **contrastive learning** of visual representations

[T Chen](#), [S Kornblith](#), [M Norouzi](#)... - ... on machine learning, 2020 - proceedings.mlr.press

... In our **contrastive learning**, as positive pairs are computed in the same device, the model can exploit the local information leakage to improve prediction accuracy without improving ...

☆ 保存 引用 被引用次数: 22684 相关文章 所有 24 个版本 »

Model

► DSCOFS with contrastive learning (DSCOFS-CL)

$$\begin{aligned} \min_W \quad & \frac{1}{2} \|X - WW^\top X\|_F^2 \\ \text{s.t.} \quad & W^\top W = I, \|W\|_{2,0} \leq s_1, \|W\|_0 \leq s_2 \\ & \Downarrow \end{aligned}$$

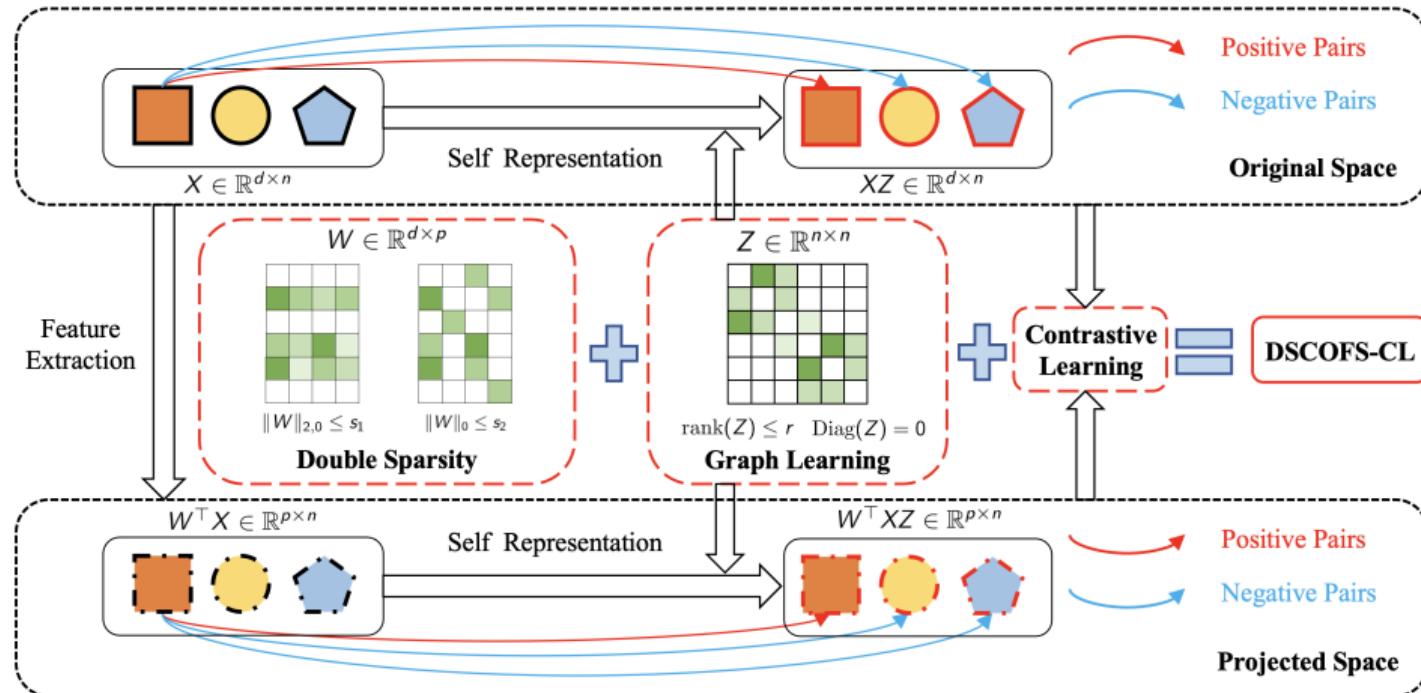
$$\begin{aligned} \min_W \quad & L_c(X, WW^\top X) \\ \text{s.t.} \quad & W^\top W = I, \|W\|_{2,0} \leq s_1, \|W\|_0 \leq s_2 \\ & \Downarrow \end{aligned}$$

$$\begin{aligned} \min_{W,Z} \quad & \lambda L_c(X, XZ) + (1 - \lambda) L_c(W^\top X, W^\top XZ) \\ \text{s.t.} \quad & W^\top W = I, \|W\|_{2,0} \leq s_1, \|W\|_0 \leq s_2 \\ & \text{rank}(Z) \leq r, \text{Diag}(Z) = 0 \end{aligned}$$

- $\text{rank}(Z) \leq r$ represents the global structure
- $\text{Diag}(Z) = 0$ avoids the case where $Z = E$

Architecture

- DSCOFS-CL = Double Sparsity + Graph Learning + Contrastive Learning



Algorithm

- ▶ Proximal alternating method (PAM)

$$\min_{W,Z} \quad \lambda L_c(X, XZ) + (1 - \lambda) L_c(W^\top X, W^\top XZ)$$

$$\text{s.t.} \quad W^\top W = I, \quad \|W\|_{2,0} \leq s_1, \quad \|W\|_0 \leq s_2 \\ \text{rank}(Z) \leq r, \quad \text{Diag}(Z) = 0$$

↓

$$\min_{W,Z,Y,P,Q} \quad \lambda L_c(X, XZ) + (1 - \lambda) L_c(W^\top X, W^\top XZ)$$

$$\text{s.t.} \quad \|P\|_{2,0} \leq s_1, \quad \|Q\|_0 \leq s_2, \quad \text{rank}(Y) \leq r, \quad \text{Diag}(Z) = 0 \\ W^\top W = I, \quad Z = Y, \quad W = P, \quad W = Q$$

↓

$$\min_{W,Z,Y,P,Q} \quad \lambda L_c(X, XZ) + (1 - \lambda) L_c(W^\top X, W^\top XZ) + \mu \|W^\top W - I\|_F^2$$

$$+ \alpha \|Z - Y\|_F^2 + \beta \|W - P\|_F^2 + \gamma \|W - Q\|_F^2$$

$$\text{s.t.} \quad \|P\|_{2,0} \leq s_1, \quad \|Q\|_0 \leq s_2, \quad \text{rank}(Y) \leq r, \quad \text{Diag}(Z) = 0$$

Algorithm

- **Input:** $X, \lambda, \mu, \alpha, \beta, \gamma, s_1, s_2, r, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5$
- **Initialize:** $(W^0, Z^0, Y^0, P^0, Q^0)$
- **While** not converged **do**
 - Update W^{k+1} by

$$\begin{aligned} \min_W \quad & (1 - \lambda)L_c(W^\top X, W^\top XZ^k) + \mu\|W^\top W - I\|_F^2 \\ & + \beta\|W - P^k\|_F^2 + \gamma\|W - Q^k\|_F^2 + \tau_1\|W - W^k\|_F^2 \end{aligned}$$

- Update Z^{k+1} by

$$\begin{aligned} \min_Z \quad & \lambda L_c(X, XZ) + (1 - \lambda)L_c(W^{k+1,\top} X, W^{k+1,\top} XZ) \\ & + \alpha\|Z - Y^k\|_F^2 + \tau_2\|Z - Z^k\|_F^2 \\ \text{s.t.} \quad & \text{Diag}(Z) = 0 \end{aligned}$$

- Update Y^{k+1}
- Update P^{k+1}
- Update Q^{k+1}

Algorithm

- ▶ Define

$$\begin{aligned} f(W, Z, Y, P, Q) = & \lambda L_c(X, XZ) + (1 - \lambda)L_c(W^\top X, W^\top XZ) \\ & + \mu \|W^\top W - I\|_F^2 + \alpha \|Z - Y\|_F^2 + \beta \|W - P\|_F^2 + \gamma \|W - Q\|_F^2 \\ & + \delta(Z) + \delta(Y) + \delta(P) + \delta(Q) \end{aligned}$$

- ▶ We call (W, Z, Y, P, Q) is a critical point if $0 \in \partial f(W, Z, Y, P, Q)$
- ▶ (**Theorem**) For each k , the sequence $\{(W^k, Z^k, Y^k, P^k, Q^k)\}$ generated by our PAM algorithm converges and $0 \in \partial f(W^*, Z^*, Y^*, P^*, Q^*)$ with

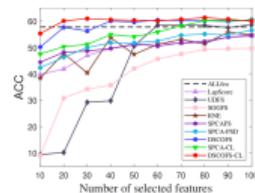
$$\lim_{k \rightarrow +\infty} (W^k, Z^k, Y^k, P^k, Q^k) = (W^*, Z^*, Y^*, P^*, Q^*)$$

- ▶ Sufficient decreasing
- ▶ Lower bounds for iterations
- ▶ Kurdyka-Łojasiewicz properties

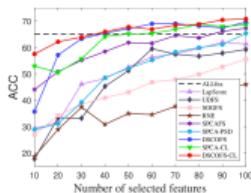
Experiments

► Real datasets: Accuracy (ACC) ↑

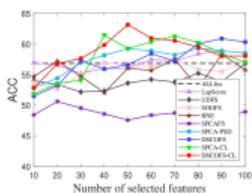
Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	SPCAFS	SPCA-PSD	DSCOFS	SPCA-CL	DSCOFS-CL
COIL20	57.74±4.93	54.82±3.91 (100)	58.71±3.47 (100)	49.66±4.81 (100)	55.84±4.41 (90)	54.39±3.67 (100)	56.57±3.05 (100)	60.51±4.63 (100)	60.31±3.49 (90)	61.32±5.18 (80)
USPS	65.12±4.95	62.02±4.09 (90)	59.52±2.97 (60)	55.58±3.07 (100)	46.04±2.69 (100)	67.34±4.49 (100)	65.38±4.26 (100)	69.67±4.97 (100)	68.88±4.05 (80)	70.82±4.77 (100)
GLIOMA	56.84±5.24	58.88±3.96 (90)	56.80±4.85 (100)	57.44±6.16 (70)	58.32±7.31 (90)	50.60±5.02 (20)	59.28±5.01 (90)	60.88±6.31 (80)	61.48±6.20 (40)	63.16±7.46 (50)
UMIST	41.07±2.38	40.13±2.79 (100)	47.12±2.49 (40)	41.70±3.17 (100)	40.35±2.26 (90)	46.78±2.51 (90)	47.98±2.91 (90)	48.10±3.01 (70)	49.55±3.00 (60)	50.95±3.15 (70)
Isolet	57.89±3.82	52.21±2.76 (100)	41.95±2.07 (100)	49.31±2.32 (100)	47.12±2.06 (90)	53.04±2.33 (100)	51.91±2.15 (70)	59.67±3.46 (100)	60.53±3.75 (90)	63.22±3.50 (90)
MSTAR	77.04±7.98	67.87±3.49 (90)	78.15±5.80 (90)	73.74±5.89 (100)	69.16±6.03 (100)	80.80±5.95 (100)	79.70±6.43 (90)	82.59±7.41 (100)	81.57±6.28 (100)	81.22±5.59 (100)
Average	59.28±4.88	55.99±3.50	57.04±3.69	54.57±4.24	52.81±4.13	58.83±4.00	60.14±3.97	63.57±4.96	63.72±4.46	65.12±4.94



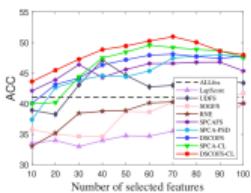
(a) COIL20



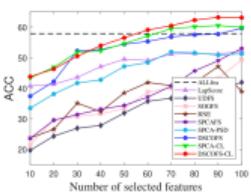
(b) USPS



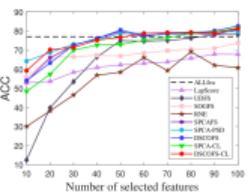
(c) GLIOMA



(d) UMIST



(e) Isolet

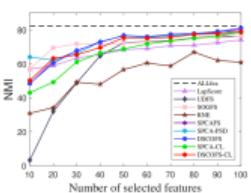
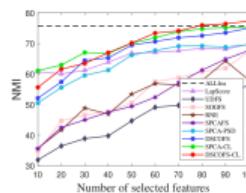
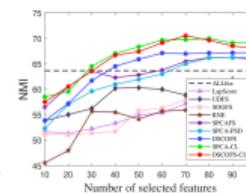
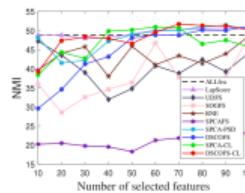
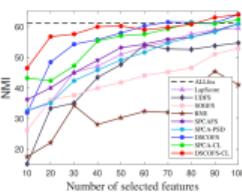
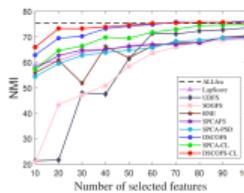


(f) MSTAR

Experiments

► Real datasets: Normalized mutual information (NMI) ↑

Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	SPCAFS	SPCA-PSD	DSCOFS	SPCA-CL	DSCOFS-CL
COIL20	75.37±1.96	69.59±1.48 (100)	73.54±1.76 (100)	68.92±1.84 (100)	70.43±1.92 (100)	69.98±1.45 (100)	69.85±1.41 (100)	76.25±1.71 (100)	74.79±1.48 (100)	75.76±1.76 (90)
USPS	61.12±2.01	59.46±1.80 (100)	54.69±2.11 (100)	52.96±1.54 (100)	45.36±1.93 (90)	60.98±2.37 (100)	60.90±2.02 (100)	64.06±2.58 (100)	62.29±2.40 (100)	63.95±2.67 (100)
GLIOMA	48.86±5.72	51.03±2.48 (100)	47.22±3.53 (10)	48.67±10.98 (100)	48.62±6.32 (100)	24.14±6.97 (100)	51.44±5.62 (90)	51.06±6.19 (80)	50.95±4.10 (60)	51.71±5.03 (70)
UMIST	63.67±1.85	61.16±1.71 (100)	62.00±1.58 (100)	60.79±1.54 (100)	55.92±1.57 (70)	66.23±1.60 (90)	66.25±1.72 (100)	67.24±1.85 (100)	69.98±1.84 (80)	70.54±1.70 (70)
Isolet	75.72±1.70	69.77±1.20 (100)	56.29±1.11 (100)	67.40±1.44 (100)	64.27±0.95 (90)	67.71±1.33 (100)	69.69±0.80 (100)	75.01±1.35 (100)	75.41±1.51 (100)	77.32±1.37 (100)
MSTAR	82.42±3.31	74.10±1.76 (100)	76.45±2.47 (90)	76.39±1.70 (100)	66.87±1.99 (80)	80.33±2.50 (100)	79.17±2.77 (90)	81.14±3.13 (100)	78.63±2.50 (100)	78.88±1.60 (100)
Average	67.86±2.76	64.19±1.74	61.70±2.09	62.52±3.17	58.58±2.45	61.56±2.70	66.22±2.39	69.13±2.80	68.68±2.31	69.69±2.36



(a) COIL20

(b) USPS

(c) GLIOMA

(d) UMIST

(e) Isolet

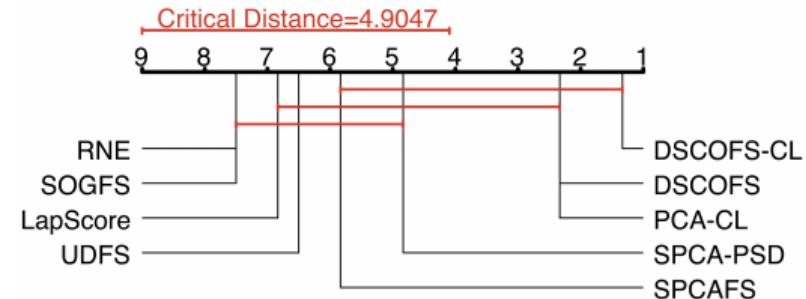
(f) MSTAR

Experiments

- ▶ Friedman tests (H_0 : There is no significant difference of compared methods)

Methods	Ranking	P-value	Hypothesis
LapScore	6.83	0.00001	Reject
UDFS	6.50		
SOGFS	7.50		
RNE	7.50		
SPCAFS	5.83		
SPCA-PSD	4.83		
DSCOFS	2.33		
SPCA-CL	2.33		
DSCOFS-CL	1.33		

- ▶ Post-hoc Nemenyi tests



Outline

Introduction

Sparse Coding

Contrastive Learning

Deep Unfolding Networks

Large Language Models

Future Work

Motivation

► (Q3) How to learn regularization parameters

$$\begin{aligned} \min_{W, Z, Y, P, Q} \quad & \lambda L_c(X, XZ) + (1 - \lambda)L_c(W^\top X, W^\top XZ) + \mu \|W^\top W - I\|_F^2 \\ & + \alpha \|Z - Y\|_F^2 + \beta \|W - P\|_F^2 + \gamma \|W - Q\|_F^2 \\ \text{s.t.} \quad & \|P\|_{2,0} \leq s_1, \|Q\|_0 \leq s_2, \text{rank}(Y) \leq r, \text{Diag}(Z) = 0 \end{aligned}$$

► $\mu, \alpha, \beta, \gamma \in \{10^{-6}, 10^{-4}, 10^{-2}, 10^0, 10^2, 10^4, 10^6\}$

► $s_1 \in \{10, 20, \dots, 100\}$

► $s_2 \in \{0.1, 0.2, \dots, 0.5\}dp$

► $r = 0.1d$

► $\lambda = 0.5$

► From iterative optimization to **deep unfolding networks**

► Gregor-LeCun, Learning Fast Approximations of Sparse Coding, ICML, 2010

► Chen-Liu-Yin, Learning to optimize: A Tutorial for Continuous and Mixed-Integer Optimization, SCCM, 2024

Model

- ▶ Consider structured sparse PCA

$$\begin{aligned} \min_W \quad & \frac{1}{2} \|X - WW^\top X\|_F^2 + \lambda \|W\|_{2,1} + \mu \|W\|_1 \\ \text{s.t.} \quad & W^\top W = I \end{aligned}$$

- ▶ Alternating direction method of multipliers (ADMM)

$$\begin{aligned} \min_W \quad & \frac{1}{2} \|X - WW^\top X\|_F^2 + \lambda \|Y\|_{2,1} + \mu \|Z\|_1 \\ \text{s.t.} \quad & W^\top W = I, \quad W = Y, \quad W = Z \end{aligned}$$

↓

$$\begin{aligned} \mathcal{L}(W, Y, Z, \Lambda, \Pi) = & \frac{1}{2} \|X - WW^\top X\|_F^2 + \lambda \|Y\|_{2,1} + \mu \|Z\|_1 \\ & + \langle \Lambda, W - Y \rangle + \frac{\alpha}{2} \|W - Y\|_F^2 + \langle \Pi, W - Z \rangle + \frac{\beta}{2} \|W - Z\|_F^2 \end{aligned}$$

SPCA-Net

- ▶ Update W -block

$$\min_W \quad f(W) := \frac{1}{2} \|X - WW^\top X\|_F^2 + \frac{\alpha}{2} \|W - Y^k + \Lambda^k/\alpha\|_F^2 + \frac{\beta}{2} \|W - Z^k + \Pi^k/\beta\|_F^2$$

$$\text{s.t.} \quad W^\top W = I$$

↓

$$\min_W \quad f(W^k) + \langle \nabla f(W^k), W - W^k \rangle + \frac{1}{2\eta} \|W - W^k\|_F^2$$

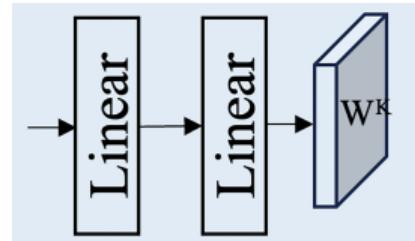
$$\text{s.t.} \quad W^\top W = I$$

↓

$$W^{k+1} = UV^\top$$

↓

$$W^{k+1} = \text{LargNet}(U, V^\top)$$



SPCA-Net

- ▶ Update Y -block

$$\min_Y \lambda \|Y\|_{2,1} + \frac{\alpha}{2} \|X^{k+1} - Y + \Lambda^k/\alpha\|_F^2$$

\Downarrow

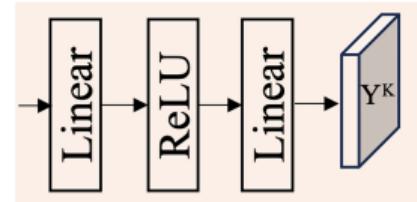
$$Y^{k+1} = \text{sign}(\|X^{k+1} + \Lambda^k/\alpha\|_2) \circ \max(\|X^{k+1} + \Lambda^k/\alpha\|_2 - \lambda/\alpha, 0)$$

\Downarrow

$$Y^{k+1} = \frac{X^{k+1} + \Lambda^k/\alpha}{\|X^{k+1} + \Lambda^k/\alpha\|_2} \text{ReLU}(\|X^{k+1} + \Lambda^k/\alpha\|_2 - \lambda/\alpha)$$

\Downarrow

$$Y^{k+1} = \text{GSoftNet}(X^{k+1} + \Lambda^k/\alpha, \lambda/\alpha)$$



SPCA-Net

- ▶ Update Z -block

$$\min_Z \mu \|Z\|_1 + \frac{\beta}{2} \|X^{k+1} - Z + \Pi^k / \beta\|_F^2$$

↓

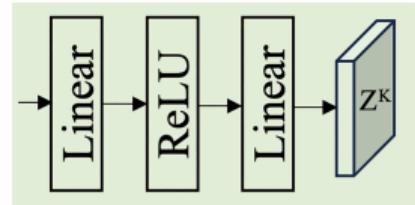
$$Z^{k+1} = \text{sign}(X^{k+1} + \Pi^k / \beta) \circ \max(|X^{k+1} + \Pi^k / \beta| - \mu / \beta, 0)$$

↓

$$Z^{k+1} = \frac{X^{k+1} + \Pi^k / \beta}{|X^{k+1} + \Pi^k / \beta|} \text{ReLU}(|X^{k+1} + \Pi^k / \beta| - \mu / \beta)$$

↓

$$Z^{k+1} = \text{SoftNet}(X^{k+1} + \Pi^k / \beta, \mu / \beta)$$



SPCA-Net

- ▶ **Input:** $X, \lambda, \mu, \alpha, \beta$
- ▶ **Initialize:** $(W^0, Y^0, Z^0, \Lambda^0, \Pi^0)$
- ▶ **While** $k = 1, \dots, K$ **do**
 - ▶ Update W^{k+1} by

$$W^{k+1} = \text{LargNet}(U, V^\top)$$

- ▶ Update Y^{k+1} by

$$Y^{k+1} = \text{GSoftNet}(X^{k+1} + \Lambda^k / \alpha, \lambda / \alpha)$$

- ▶ Update Z^{k+1} by

$$Z^{k+1} = \text{SoftNet}(X^{k+1} + \Pi^k / \beta, \mu / \beta)$$

- ▶ Update Λ^{k+1}, Π^{k+1} by

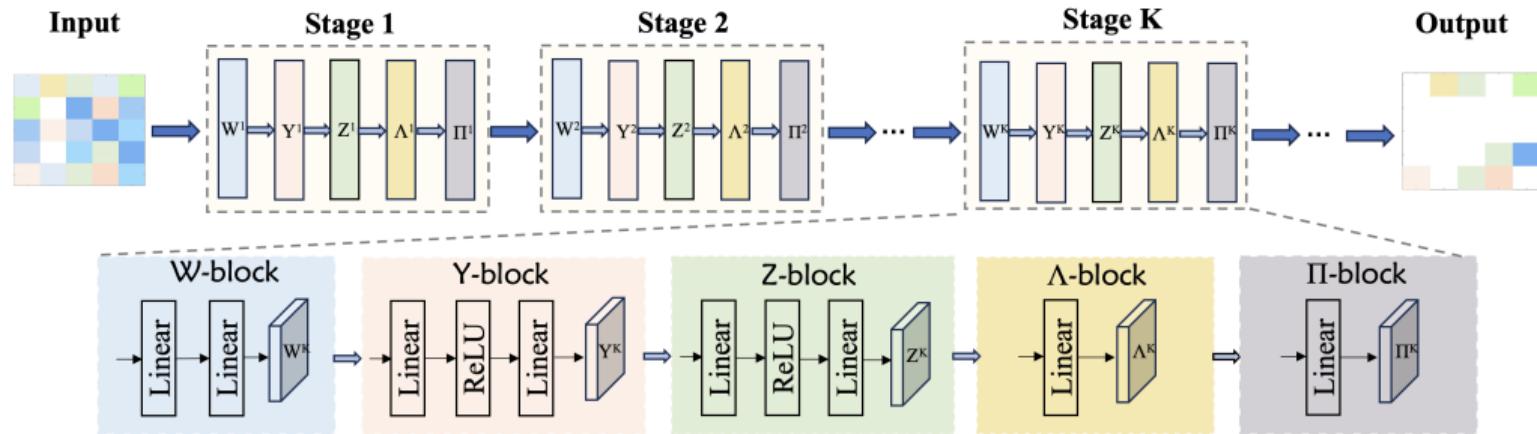
$$\Lambda^{k+1} = \text{Linear}(W^{k+1}, Y^{k+1}, \Lambda^k, \alpha), \quad \Pi^{k+1} = \text{Linear}(W^{k+1}, Z^{k+1}, \Pi^k, \beta)$$

- ▶ **Output:** Trained W

Architecture

- ▶ All parameters $(\lambda, \mu, \alpha, \beta)$ are trained in an end-to-end manner
- ▶ The loss is defined as

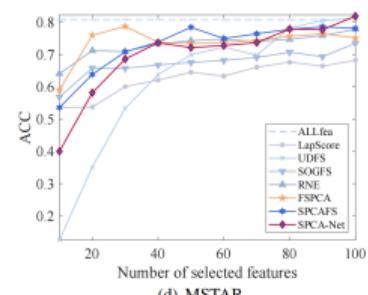
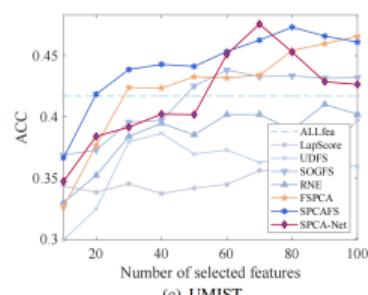
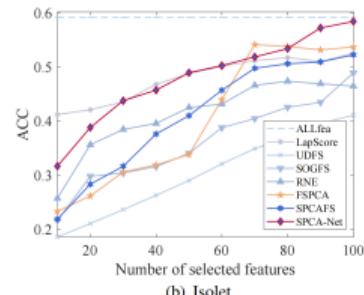
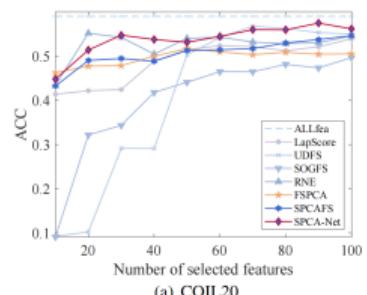
$$\text{Loss} = \frac{1}{2} \|X - \bar{W}\bar{W}^T X\|_F^2 + \lambda\|\bar{W}\|_{2,1} + \mu\|\bar{W}\|_1$$



Experiments

► Real datasets: Accuracy (ACC) ↑

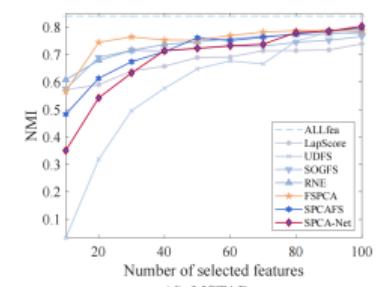
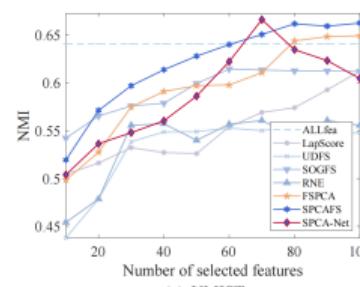
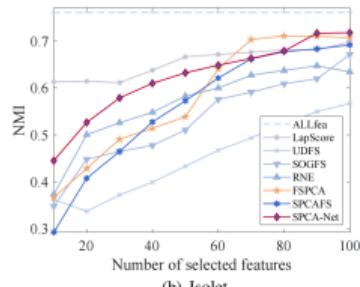
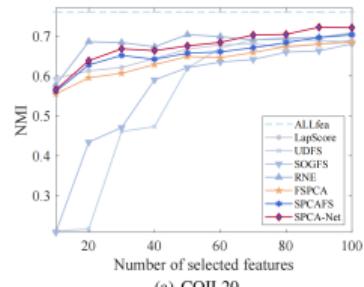
Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	FSPCA	SPCAFS	SPCA-Net
COIL20	58.97±4.99 (10)	53.91±3.61 (100)	56.70±3.09 (70)	49.66±3.63 (100)	55.16±3.35 (20)	51.71±3.05 (50)	54.63±3.64 (100)	57.46±2.76 (90)
Isolet	59.18±3.19 (10)	52.55±2.83 (100)	41.11±1.71 (100)	48.93±2.69 (100)	47.39±2.91 (80)	54.15±2.69 (70)	52.26±2.81 (100)	58.43±4.31 (100)
UMIST	41.68±2.46 (10)	39.71±3.28 (100)	38.64±1.61 (40)	43.81±2.98 (80)	41.01±2.25 (90)	46.58±2.34 (100)	47.32±3.48 (80)	47.58±4.97 (70)
MSTAR	80.81±8.76 (10)	68.21±4.57 (100)	81.25±7.48 (100)	73.46±5.61 (100)	77.82±6.16 (100)	78.74±5.20 (30)	78.63±8.68 (90)	81.90±6.87 (100)



Experiments

- Real datasets: Normalized mutual information (NMI) ↑

Datasets	ALLfea	LapScore	UDFS	SOGFS	RNE	FSPCA	SPCAFS	SPCA-Net
COIL20	76.04±1.69 (10)	69.01±1.53 (100)	69.12±1.17 (80)	68.03±1.59 (100)	70.76±2.07 (100)	68.41±1.60 (100)	70.29±1.31 (100)	72.21±2.68 (90)
Isolet	76.09±1.77 (10)	69.86±1.26 (100)	56.73±1.05 (100)	67.15±1.45 (100)	64.74±1.28 (90)	71.12±1.11 (80)	69.18±1.33 (100)	71.80±1.59 (100)
UMIST	64.07±1.76 (10)	61.23±2.15 (100)	55.43±1.50 (80)	61.46±2.03 (70)	56.08±1.80 (60)	64.94±1.65 (100)	66.26±1.74 (100)	66.62±7.52 (70)
MSTAR	83.96±3.14 (10)	73.90±1.62 (100)	78.18±3.64 (90)	76.56±1.54 (100)	78.26±2.51 (100)	78.87±2.52 (90)	79.62±2.30 (100)	80.67±3.47 (90)



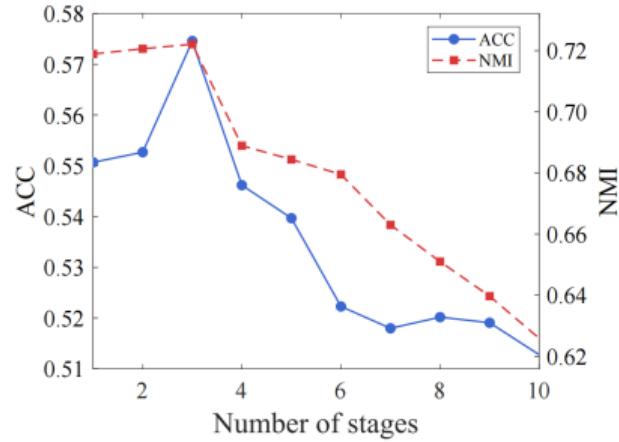
Experiments

► Ablation studies

Datasets	Network	ACC ↑	NMI ↑
COIL20	✗	55.12±2.67	70.44±1.37
	✓	57.46±2.76	72.21±2.68
Isolet	✗	51.84±2.82	67.02±1.43
	✓	58.43±4.31	71.80±1.59
UMIST	✗	40.65±2.29	55.88±1.62
	✓	47.58±4.97	66.62±7.52
MSTAR	✗	80.65±6.47	80.53±2.41
	✓	81.90±6.87	80.67±3.47

Datasets	Dynamic	ACC ↑	NMI ↑
COIL20	✗	56.71±3.83	71.49±3.67
	✓	57.46±2.76	72.21±2.68
Isolet	✗	52.06±3.71	68.91±2.36
	✓	58.43±4.31	71.80±1.59
UMIST	✗	42.63±2.78	60.12±1.69
	✓	47.58±4.97	66.62±7.52
MSTAR	✗	80.74±5.28	80.59±3.67
	✓	81.90±6.87	80.67±3.47

► Effect of deep unfolding stages



Outline

Introduction

Sparse Coding

Contrastive Learning

Deep Unfolding Networks

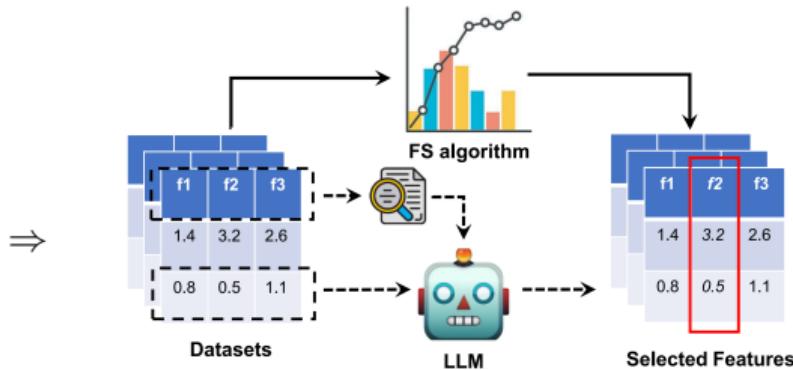
Large Language Models

Future Work

Motivation

- ▶ (Q4) How to learn feature selection

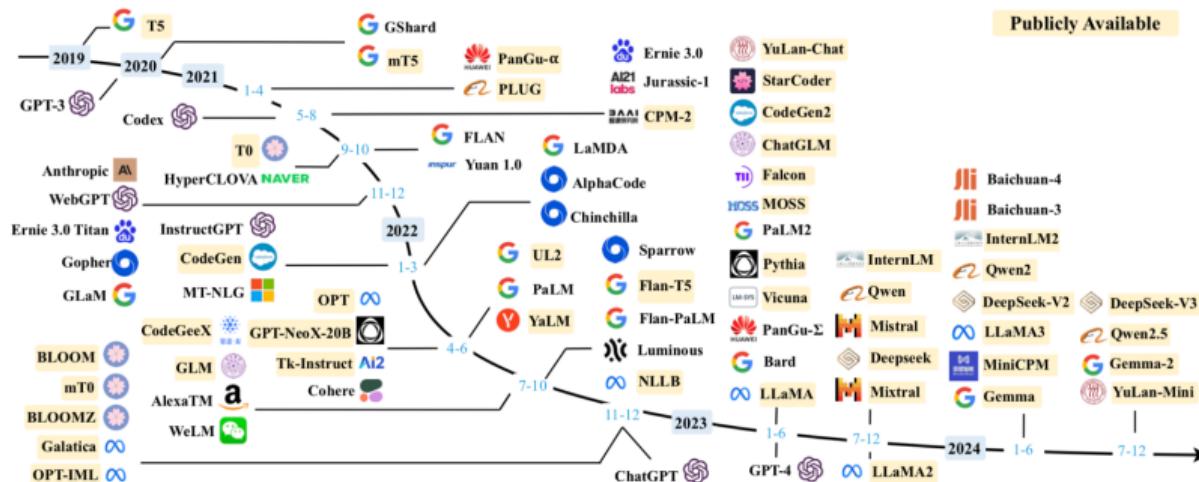
$$\begin{aligned} \min_W \quad & -\text{Tr}(W^\top X X^\top W) \\ \text{s.t.} \quad & W^\top W = I, \|W\|_{2,0} \leq s \end{aligned}$$



- ▶ From deep learning to **large language models (LLMs)**
 - ▶ Cho-Cund-Srivastava et al, LMPriors: Pre-Trained Language Models as Task-Specific Priors, NeurIPS, 2022
 - ▶ Han-Yoon-Arik et al, Large Language Models Can Automatically Engineer Features for Few-Shot Tabular Learning, ICML, 2024
 - ▶ Li-Tan-Liu, Exploring Large Language Models for Feature Selection: A Data-centric Perspective, SIGKDD, 2025

DeepSeek

- ▶ Guo-Yang-Zhang et al, DeepSeek-R1: Incentivizing Reasoning Capability in LLMs via Reinforcement Learning, arXiv:2501.12948
- ▶ Arrieta-Ugarte-Valle et al, o3-mini vs DeepSeek-R1: Which One is Safer? arXiv:2501.18438
- ▶ Muennighoff-Yang-Shi et al, S1: Simple Test-time Scaling, arXiv:2501.19393
- ▶ Gao-Jin-Ke et al, A Comparison of DeepSeek and Other LLMs, arXiv:2502.03688



Method

► Dataset-specific Context

Using data collected via a telemarketing campaign at a Portuguese banking institution from 2008 to 2013, we wish to build a machine learning model that can predict whether a client will subscribe to a term deposit (target variable). The dataset contains a total of 16 features (e.g., age, marital status, whether the client has a housing loan). Prior to training the model, we first want to identify a subset of the 16 features that are most important for reliable prediction of the target variable.

► Main System Prompt

For each feature input by the user, your task is to provide a feature importance score (between `<0.0>` and `<1.0>`; larger value indicates greater importance) for predicting whether an individual will subscribe to a term deposit and a reasoning behind how the importance score was assigned. The results need to be written directly into a JSON file. Therefore, please do not include any extra text and return the results strictly in the given format. The scores for each feature should be different from one another.

► Output Format Instruction

Here is an example output: `"concept-1": "has credit in default ", "reasoning": "Clients with credits in default might be more hesitant to open new financial products due to their current financial situation and may be deemed a higher risk by the bank. Therefore, the score is 0.9.", "score": 0.9.`

► Main User Prompt

Provide a score and reasoning formatted according to the output schema above.

Method

- ▶ Dataset-specific Context

Same as above

- ▶ Main System Prompt

Please use the Random Forest (/ forward sequential selection / backward sequential selection / recursive feature elimination RFE / minimum redundancy maximum relevance selection MRMR / filtering by mutual information MI) model to directly analyze the dataset samples. This is a classification task, where "Class" represents the classification. Please analyze the importance scores of these features. The score range is [0.0, 1.0], and the score of each feature should be different. The output format is as follows, in JSON file format.

- ▶ Output Format Instruction

Same as above

- ▶ Main User Prompt

Same as above

Experiments

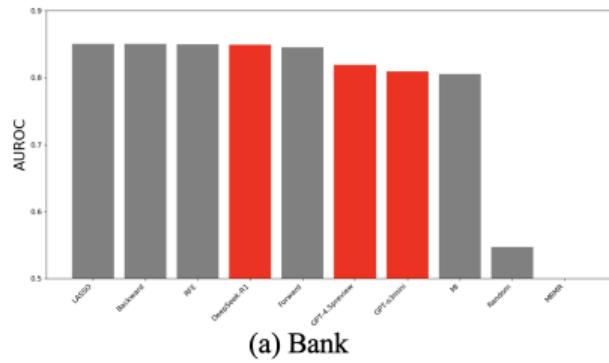
- ▶ Compared methods
 - ▶ DeepSeek-R1 (2025-01-20)
 - ▶ GPT-o3mini (2025-01-31)
 - ▶ GPT-4.5preview (2025-02-27)
 - ▶ LASSO
 - ▶ Forward sequential selection (Forward)
 - ▶ Backward sequential selection (Backward)
 - ▶ Recursive feature elimination (RFE)
 - ▶ Minimum redundancy maximum relevance selection (MRMR)
 - ▶ Mutual information (MI)
 - ▶ Random feature selection (Random)

- ▶ Statistics of datasets

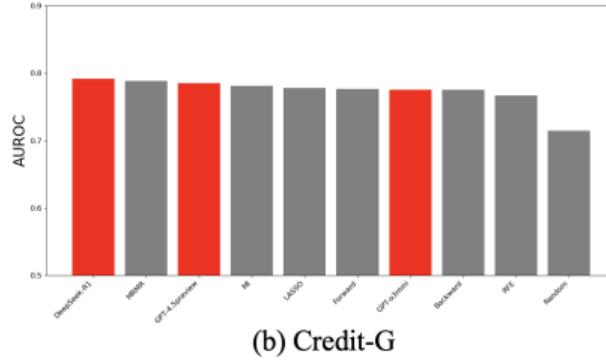
Datasets	Samples	Features
Bank	45211	16
Credit-G	1000	20
Pima Indians Diabetes	768	8
Give Me Some Credit	120269	10

Experiments

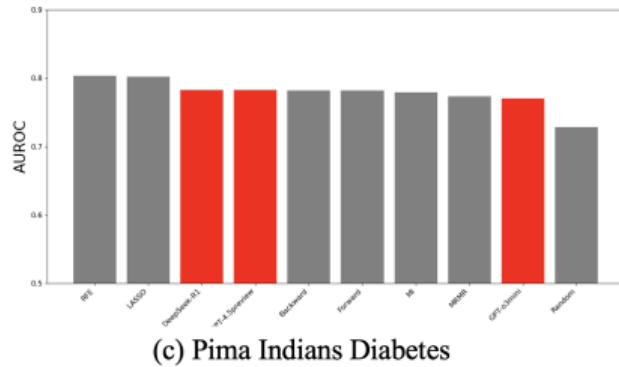
► LLMs vs. Data-driven methods



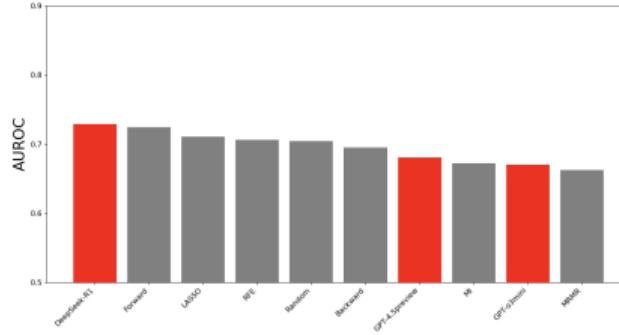
(a) Bank



(b) Credit-G



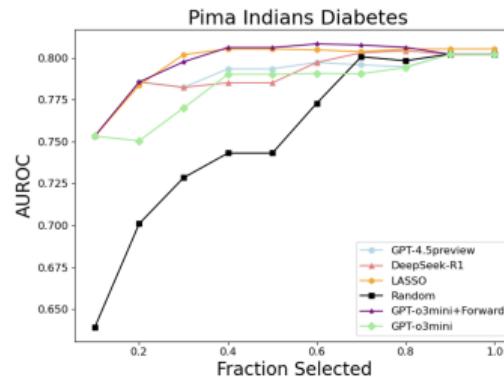
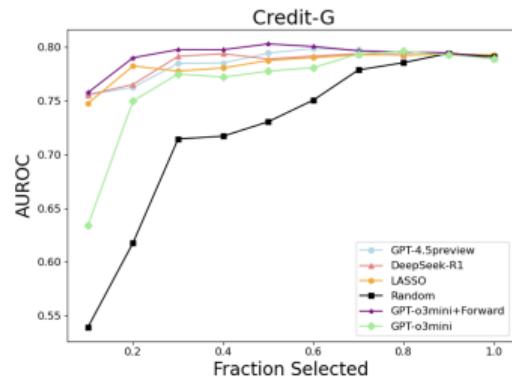
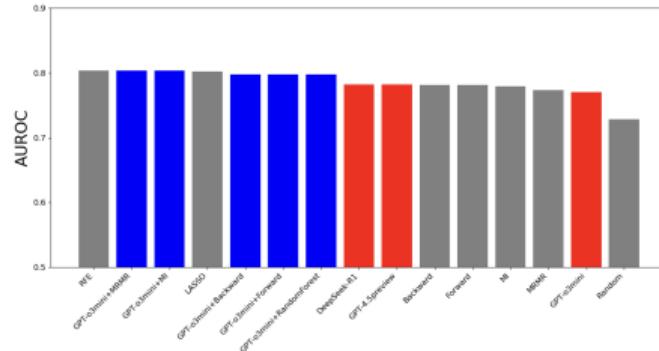
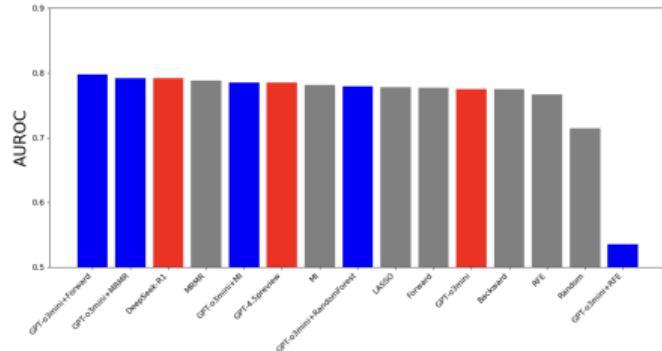
(c) Pima Indians Diabetes



(d) Give Me Some Credit

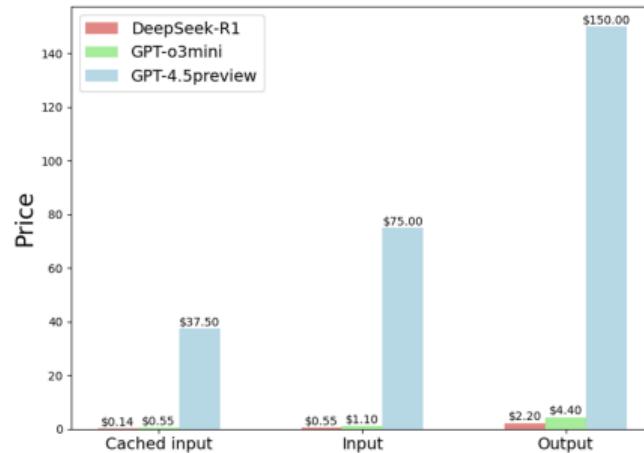
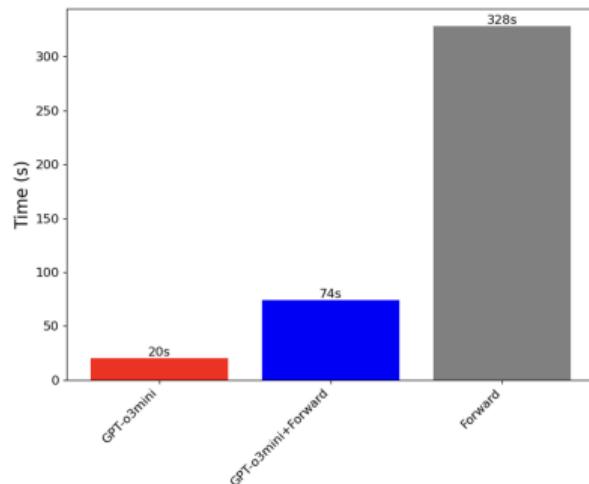
Experiments

► LLMs + Data-driven methods vs. LLMs vs. Data-driven methods



Experiments

- More interesting things should be investigated
 - Consider **large datasets** with more features, especially larger than thousands
 - Apply DeepSeek-R1 with **different parameters**, including 7B, 14B, 32B, 70B
 - Try **RAG** and **fine-tuning** to improve the stability and reliability
 - Expand to **regression tasks**, analyze feature correlation, etc



Outline

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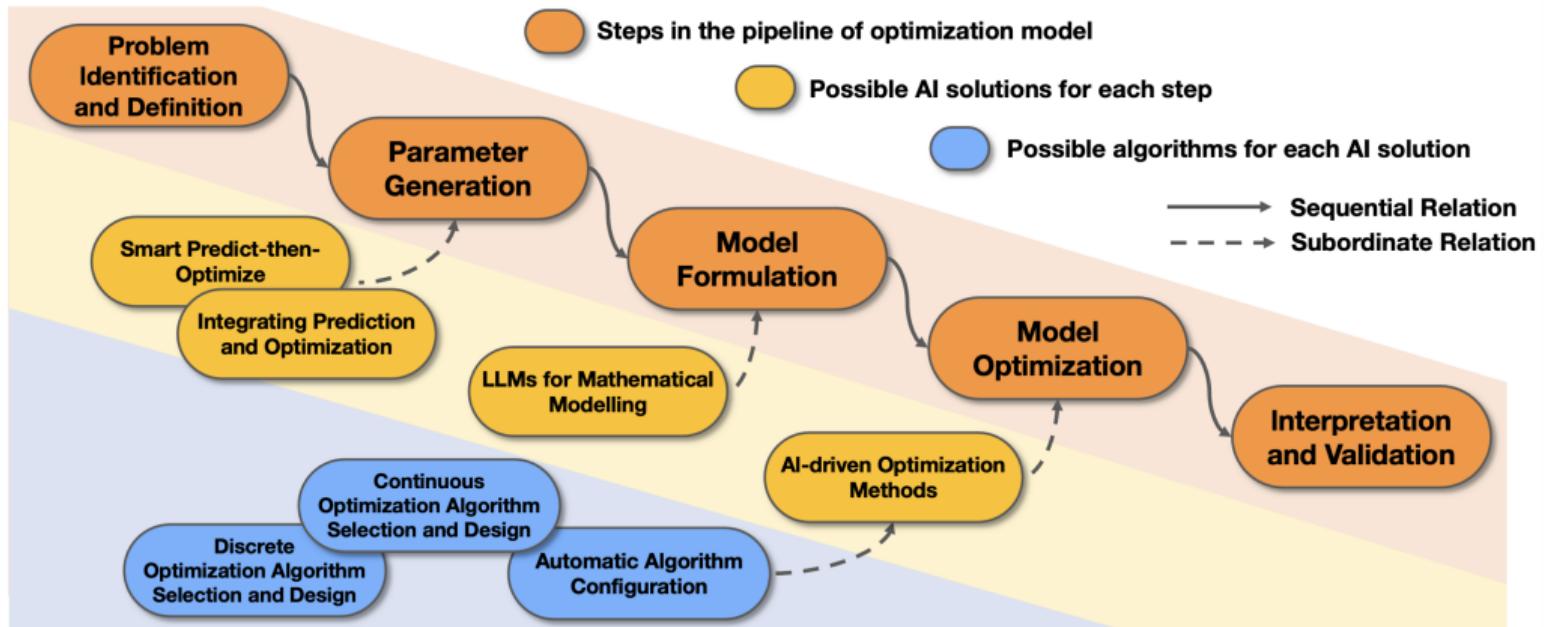
Deep Unfolding Networks

Large Language Models

Future Work

Future Work

► AI for optimization



Future Work

- ▶ Ramamonjison-Yu-Li et al, NL4Opt Competition: Formulating Optimization Problems Based on Their Natural Language Descriptions, NeurIPS, 2022
- ▶ Yang-Wang-Lu et al, Large Language Models as Optimizers, ICLR, 2024
- ▶ AhmadiTeshnizi-Gao-Udell, OptiMUS: Scalable Optimization Modeling with (MI)LP Solvers and Large Language Models, ICML, 2024
- ▶ Gao-Jiang-Cai et al, StrategyLLM: Large Language Models as Strategy Generators, Executors, Optimizers, and Evaluators for Problem Solving, NeurIPS, 2024
- ▶ Romera-Paredes-Barekatain et al, Mathematical Discoveries from Program Search with Large Language Models, Nature, 2024
- ▶ Jiang-Shu-Qian et al, LLMOPT: Learning to Define and Solve General Optimization Problems from Scratch, ICLR, 2025

Thank you for your attention

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