

Plug-and-Play Tensor Low-Rank Approximation for Hyperspectral Anomaly Detection

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Joint work with [Jingjing Liu](#) (SHU), [Jiashun Jin](#) (SHU), [Manlong Feng](#) (SHU) and others

Outline

Introduction

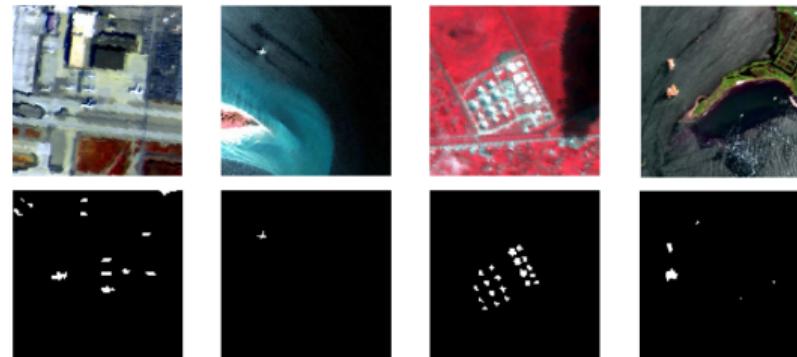
Plug-and-Play TLRA

Multidirectional Sparse TLRA

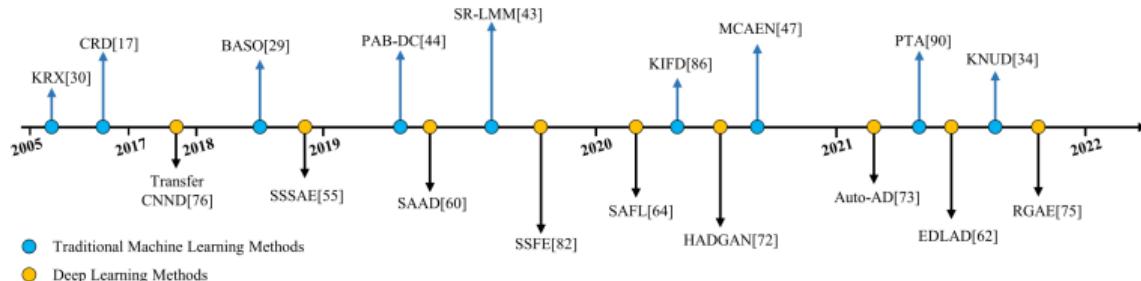
Future Work

HAD

- ▶ Hyperspectral anomaly detection (HAD): find abnormal targets, such as infected trees in forests, rare minerals in geosciences, and airplanes in defense



- ▶ Benchmark methods (2005-2022)



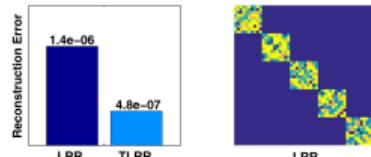
Tensor LRR

- ▶ From LRR to TLRR or TLRA

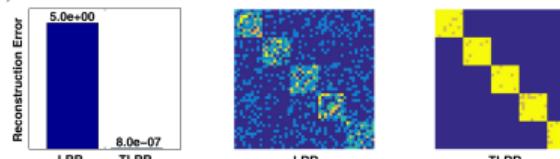
$$\begin{aligned} \min_{Z,E} \quad & \|Z\|_* + \lambda \|E\|_1 \\ \text{s.t. } & X = AZ + E \end{aligned}$$

⇒

$$\begin{aligned} \min_{\mathcal{Z},\mathcal{E}} \quad & \|\mathcal{Z}\|_{\text{TNN}} + \lambda \|\mathcal{E}\|_1 \\ \text{s.t. } & \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E} \end{aligned}$$



(a) Results of LRR and TLRR on the data with vector linear relations.



(b) Results of LRR and TLRR on the data with tensor linear relations.

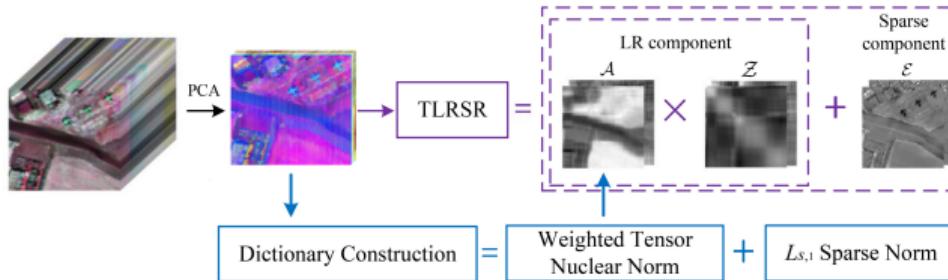
- ▶ What are the advantages of tensor modeling?

- ▶ Excellent data representation with (possibly) low computational complexity
- ▶ Different tensor decomposition including Tucker, CP, t-SVD, BT, TT, TR

Tensor LRR

- Wang-Wang-Hong-Roy-Chanussot, IEEE Transactions on Cybernetics, 2023

$$\begin{array}{ll} \min_{\mathcal{Z}, \mathcal{E}} & \|\mathcal{Z}\|_{\text{TNN}} + \lambda \|\mathcal{E}\|_1 \\ \text{s.t. } & \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E} \end{array} \Rightarrow \begin{array}{ll} \min_{\mathcal{Z}, \mathcal{E}} & \|\mathcal{Z}\|_{\text{WTNN}} + \lambda \|\mathcal{E}\|_1 \\ \text{s.t. } & \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E} \end{array}$$



- In this talk, we focus on the following questions
 - How to characterize sparsity?
 - How to integrate deep priors?

Outline

Introduction

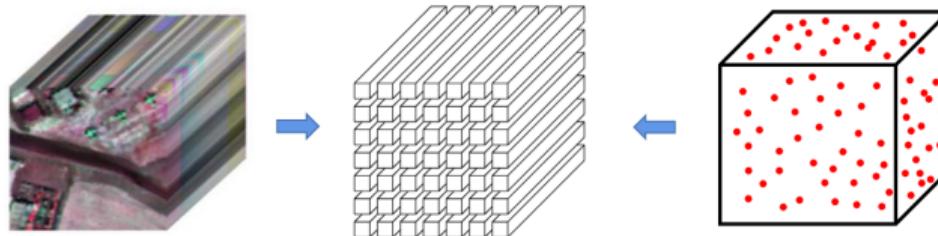
Plug-and-Play TLRA

Multidirectional Sparse TLRA

Future Work

Motivation

- ▶ How to characterize sparsity?



- ▶ Entry-wise sparsity

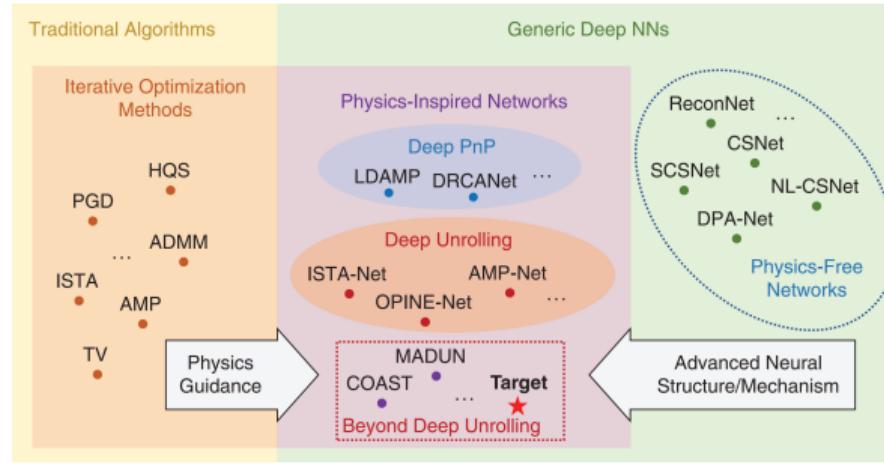
$$\|\mathcal{E}\|_1 = \sum_{i=1}^h \sum_{j=1}^v \sum_{k=1}^z |\mathcal{E}(i,j,k)| \quad \Rightarrow \quad \|\mathcal{E}\|_0 = \#\{(i,j,k) \mid |\mathcal{E}(i,j,k)| \neq 0\}$$

- ▶ Tube-wise sparsity

$$\|\mathcal{E}\|_{F,0} = \#\{(i,j) \mid \|\mathcal{E}(i,j,:)\|_2 \neq 0\}$$

Motivation

- ▶ How to integrate deep priors?



- ▶ What are the advantages of PnP modeling?
 - ▶ Implicit regularizer yet flexible
 - ▶ Learn priors by pre-trained neural networks

Formulation

- ▶ Plug-and-play tensor low-rank approximation (PnP-TLRA)

$$\min_{\mathcal{Z}, \mathcal{E}} \|\mathcal{Z}\|_{WTNN} + \lambda \|\mathcal{E}\|_1$$

$$\text{s.t. } \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E}$$

↓

$$\min_{\mathcal{Z}, \mathcal{E}, \mathcal{N}} \|\mathcal{Z}\|_{WTNN} + \lambda \|\mathcal{E}\|_{F,0} + \mu \|\mathcal{N}\|_F^2$$

$$\text{s.t. } \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E} + \mathcal{N}$$

↓

$$\min_{\mathcal{Z}, \mathcal{E}, \mathcal{N}} \phi(\mathcal{Z}) + \lambda \|\mathcal{E}\|_{F,0} + \mu \|\mathcal{N}\|_F^2$$

$$\text{s.t. } \mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E} + \mathcal{N}$$

- ▶ $\phi(\mathcal{Z})$: PnP prior ranging from WTNN, nonlinear TNN to DnCNN, FFDNet

Algorithm

- ▶ Alternating direction method of multipliers (ADMM)

$$\begin{aligned} L_\beta(\mathcal{Z}, \mathcal{E}, \mathcal{N}, \mathcal{Y}) = & \phi(\mathcal{Z}) + \lambda \|\mathcal{E}\|_{F,0} + \mu \|\mathcal{N}\|_F^2 \\ & - \langle \mathcal{Y}, \mathcal{X} - \mathcal{A} * \mathcal{Z} - \mathcal{E} - \mathcal{N} \rangle + \frac{\beta}{2} \|\mathcal{X} - \mathcal{A} * \mathcal{Z} - \mathcal{E} - \mathcal{N}\|_F^2 \end{aligned}$$

- ▶ The iterative scheme

- ▶ Update $\mathcal{Z}^{k+1} = \arg \min_{\mathcal{Z}} \phi(\mathcal{Z}) + \frac{\beta}{2} \|\mathcal{X} - \mathcal{A} * \mathcal{Z} - \mathcal{E}^k - \mathcal{N}^k - \mathcal{Y}^k / \beta\|_F^2$
- ▶ Update $\mathcal{E}^{k+1} = \arg \min_{\mathcal{E}} \lambda \|\mathcal{E}\|_{F,0} + \frac{\beta}{2} \|\mathcal{X} - \mathcal{A} * \mathcal{Z}^{k+1} - \mathcal{E} - \mathcal{N}^k - \mathcal{Y}^k / \beta\|_F^2$
- ▶ Update $\mathcal{N}^{k+1} = \arg \min_{\mathcal{N}} \mu \|\mathcal{N}\|_F^2 + \frac{\beta}{2} \|\mathcal{X} - \mathcal{A} * \mathcal{Z}^{k+1} - \mathcal{E}^{k+1} - \mathcal{N} - \mathcal{Y}^k / \beta\|_F^2$
- ▶ Update $\mathcal{Y}^{k+1} = \mathcal{Y}^k - \beta(\mathcal{X} - \mathcal{A} * \mathcal{Z}^{k+1} - \mathcal{E}^{k+1} - \mathcal{N}^{k+1})$

- ▶ Update \mathcal{Z} via solving

$$\min_{\mathcal{Z}} \phi(\mathcal{Z}) + \langle \nabla_{\mathcal{Z}} g(\mathcal{Z}^k), \mathcal{Z} - \mathcal{Z}^k \rangle + \frac{\eta}{2} \|\mathcal{Z} - \mathcal{Z}^k\|_F^2$$

- ▶ Consider $\phi(\mathcal{Z}) = \|\mathcal{Z}\|_{WTNN}$, then $\mathcal{Z}^{k+1} = \text{Shrink}_{\mathcal{W}^k}(\mathcal{Z}^k - \nabla_{\mathcal{Z}} g(\mathcal{Z}^k) / \eta, 1/\eta)$
- ▶ Consider $\phi(\mathcal{Z}) = \|\mathcal{Z}\|_{FFDNet}$, then $\mathcal{Z}^{k+1} = \text{FFDNet}(\mathcal{Z}^k - \nabla_{\mathcal{Z}} g(\mathcal{Z}^k) / \eta, 1/\eta)$

Convergence

Lemma (decreasing)

The generated sequence $\{L_\beta(\mathcal{Z}^k, \mathcal{E}^k, \mathcal{N}^k, \mathcal{Y}^k)\}$ is decreasing.

Lemma (boundedness)

A generated sequence $\{(\mathcal{Z}^k, \mathcal{E}^k, \mathcal{N}^k, \mathcal{Y}^k)\}$ is bounded.

Theorem (subsequential convergence)

Any cluster point $\{\mathcal{Z}^, \mathcal{E}^*, \mathcal{N}^*, \mathcal{Y}^*\}$ converges to a stationary point of TLRA.*

Theorem (global convergence)

With the assumption of KL function, the sequence $\{(\mathcal{Z}^k, \mathcal{E}^k, \mathcal{N}^k, \mathcal{Y}^k)\}$ converges to a stationary point of TLRA.

Note that additional assumptions are required for PnP-TLRA!

Experiments

- ▶ Datasets: San Diego, HYDICE, Indian Pines, ABU (airport, beach, urban)
- ▶ Compared methods: matrix/tensor
 - ▶ RPCA-RX: Low-Rank and Sparse Matrix Decomposition-Based Anomaly Detection for Hyperspectral Imagery, 2019
 - ▶ LSMAD: A Low-Rank and Sparse Matrix Decomposition-Based Mahalanobis Distance Method for Hyperspectral Anomaly Detection, 2015
 - ▶ LRASR: Anomaly Detection in Hyperspectral Images Based on Low-Rank and Sparse Representation, 2016
 - ▶ GTVLRR: Graph and Total Variation Regularized Low-Rank Representation for Hyperspectral Anomaly Detection, 2020
 - ▶ ELRSF-SP: Hyperspectral Anomaly Detection via Enhanced Low-Rank and Smoothness Fusion Regularization Plus Saliency Prior, 2024
 - ▶ TPCA: A Preprocessing Method for Hyperspectral Target Detection Based on Tensor Principal Component Analysis, 2018
 - ▶ PTA: Prior-Based Tensor Approximation for Anomaly Detection in Hyperspectral Imagery, 2022
 - ▶ PCA-TLRSR: Learning Tensor Low-Rank Representation for Hyperspectral Anomaly Detection, 2023

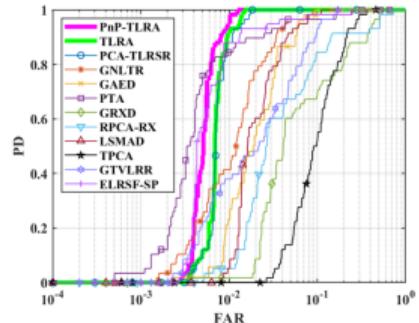
Experiments

► Area under the curve (AUC)

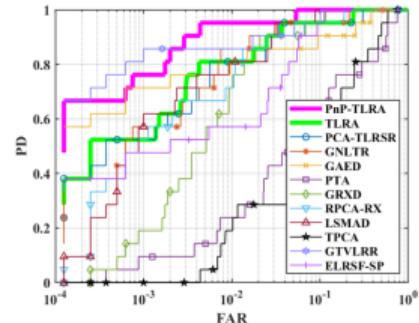
Dataset	GRXD	RPCA-RX	LSMAD	GTVLRR	TPCA	PTA	PCA-TLRSR	GAED	GNLTR	ELRSF-SP	TLRA	PnP-TLRA
SD	0.8886	0.9165	0.9457	0.9648	0.8849	0.9868	0.9923	0.9889	0.9838	0.9878	0.9927	0.9942
HYDICE	0.9857	0.9842	0.9906	<u>0.9918</u>	0.8242	0.8396	0.9802	0.9639	0.9859	0.9768	0.9840	0.9968
ABU-airport-1	0.8221	0.8089	0.8341	0.8957	0.8023	0.7698	0.9291	0.8106	0.9293	0.8221	0.9331	<u>0.9303</u>
ABU-airport-2	0.8403	0.8431	0.9192	0.8911	0.8891	0.8995	0.9345	0.9272	0.9242	0.8891	<u>0.9409</u>	0.9523
ABU-airport-3	0.9288	0.9275	0.9398	0.9287	0.9297	0.7369	0.9233	0.8638	0.9330	0.9209	<u>0.9379</u>	0.9380
ABU-airport-4	0.9526	0.9627	0.9864	0.9776	0.9432	0.9890	0.9914	0.9654	0.9606	0.9876	0.9932	<u>0.9918</u>
ABU-beach-1	0.9804	0.9761	0.9778	0.9703	0.9860	0.9682	0.9831	0.9223	0.9638	0.9780	<u>0.9868</u>	0.9895
ABU-beach-2	0.9106	0.9097	0.9056	0.9348	0.8061	0.8862	0.9331	0.5061	0.9472	0.9097	0.9580	<u>0.9533</u>
ABU-beach-3	0.9998	0.9995	0.9996	0.9866	0.9982	0.9483	0.9994	0.9891	0.9928	0.9945	0.9994	<u>0.9996</u>
ABU-beach-4	0.9538	0.9599	0.9349	0.9803	0.9338	0.9000	0.9755	0.8514	0.9044	0.9698	<u>0.9899</u>	0.9902
ABU-urban-1	0.9907	0.9922	0.9818	0.8742	0.9390	0.8940	0.9923	0.9409	0.9325	0.9907	<u>0.9934</u>	0.9951
ABU-urban-2	0.9946	0.9957	0.9849	0.8628	0.9409	0.9701	0.9928	0.9958	0.9874	0.9946	<u>0.9983</u>	0.9994
ABU-urban-3	0.9513	0.9577	0.9633	0.9365	0.8224	0.9090	0.9832	0.9725	0.9667	0.9513	0.9888	<u>0.9855</u>
ABU-urban-4	0.9887	0.9871	0.9809	0.9205	0.9835	<u>0.9937</u>	0.9857	0.9556	0.9931	0.9909	0.9883	0.9950
ABU-urban-5	0.9690	0.9658	0.9610	0.9347	0.9370	0.8693	0.9811	0.9148	0.9297	0.9658	<u>0.9821</u>	0.9837
Average	0.9438	0.9458	0.9537	0.9367	0.9080	0.9040	0.9718	0.9046	0.9556	0.9553	<u>0.9778</u>	0.9796

Experiments

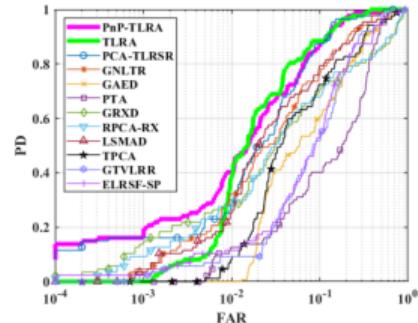
► Receiver operating characteristic (ROC)



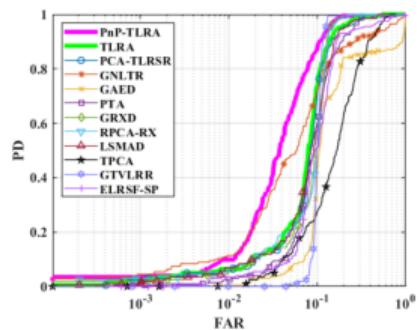
(a)



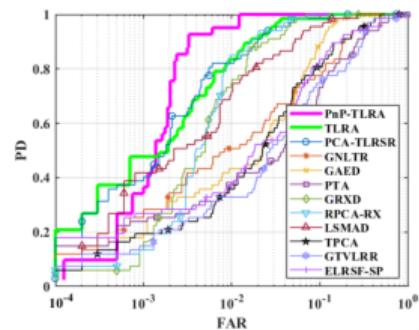
(b)



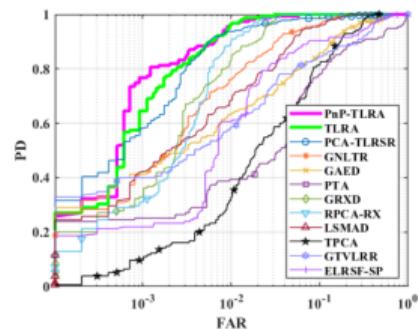
(c)



(d)



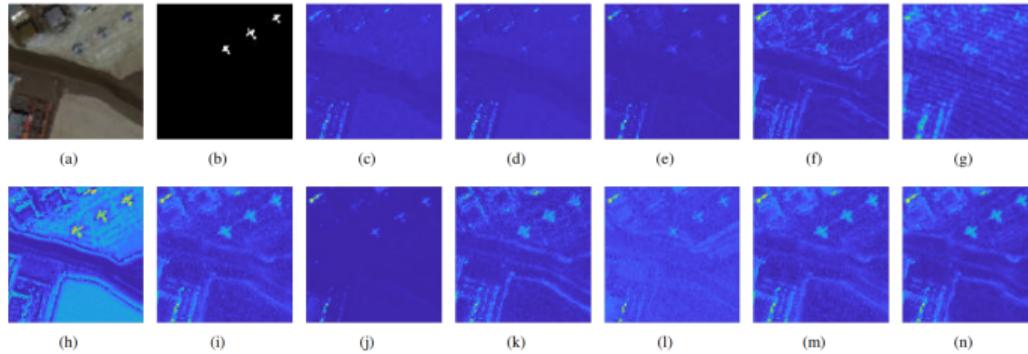
(e)



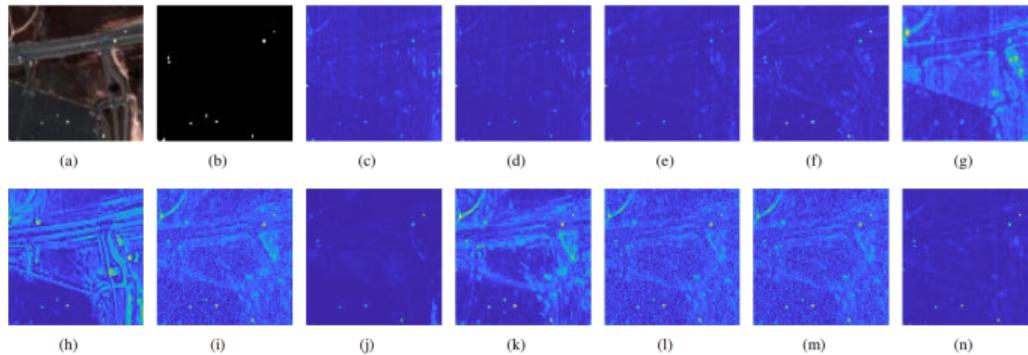
(f)

Experiments

► Results on the San Diego dataset

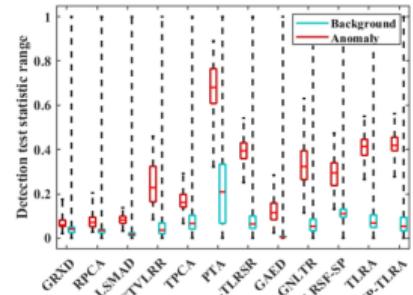


► Results on the HYDICE dataset

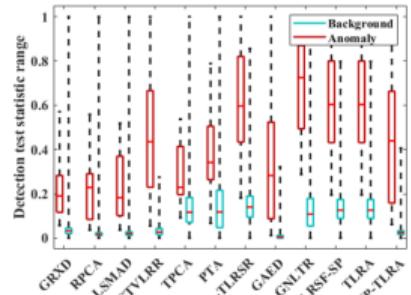


Experiments

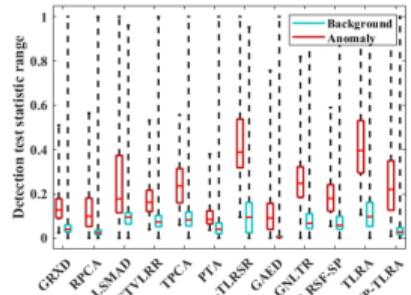
► Statistical separability analysis



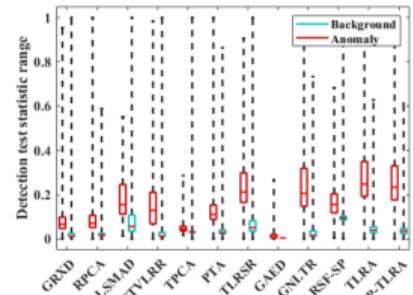
(a)



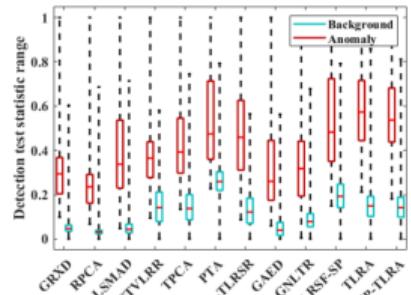
(b)



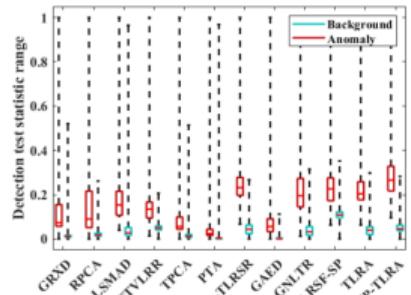
(c)



(d)



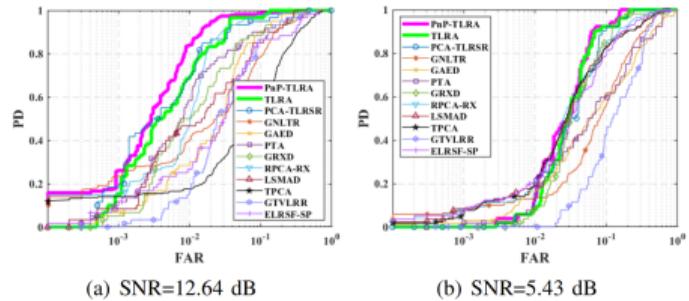
(e)



(f)

Experiments

► Noise robustness



SNR (dB)	GRXD	RPCA-RX	LSMAD	GTVLRR	TPCA	PTA
Original	0.9907	0.9922	0.9817	0.8742	0.9390	0.8940
12.64	0.9431	0.9474	0.9378	0.8548	0.9097	0.8548
5.43	0.9089	0.9044	0.8991	0.8221	0.8608	0.8364

SNR (dB)	PCA-TLRSR	GAED	GNLTR	ELRSF-SP	TLRA	PnP-TLRA
Original	0.9923	0.9409	0.9325	0.9907	<u>0.9934</u>	0.9951
12.64	0.9505	0.9209	0.9059	0.9389	<u>0.9513</u>	0.9577
5.43	0.9106	0.8840	0.8721	0.9027	<u>0.9171</u>	0.9199

► Runtime comparisons

Dataset	TPCA	PTA	PCA-TLRSR	TLRA	PnP-TLRA
San Diego	34.66	26.37	10.99	5.89	<u>10.63</u>
HYDICE	25.39	22.76	<u>9.31</u>	4.89	24.49
ABU-airport	41.63	29.21	11.32	6.12	<u>9.71</u>
ABU-beach	29.04	38.24	15.61	10.86	<u>12.31</u>
ABU-urban	36.95	28.01	17.35	8.38	<u>11.82</u>

Outline

Introduction

Plug-and-Play TLRA

Multidirectional Sparse TLRA

Future Work

Motivation

- Weighted multidirectional sparsity (WMS)

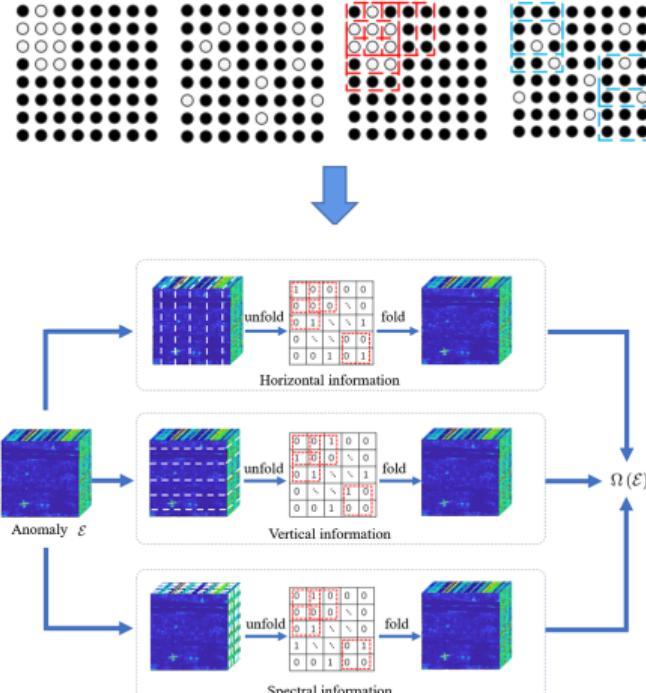
$$\|E\|_1 = \sum_{i=1}^m \sum_{j=1}^n |E(i,j)|$$



$$\Omega(E) = \sum_{j=1}^n \sum_{g \in \mathcal{G}} \|e_g^j\|_\infty$$



$$\Omega(\mathcal{E}) = \sum_{i=1}^3 w_i \text{fold}(\Omega(\text{unfold}(\mathcal{E}, i)), i)$$



Formulation

- ▶ WMS regularized low-rank tensor representation (WMS-LRTR)

$$\min_{\mathcal{Z}, \mathcal{E}} \|\mathcal{Z}\|_{\text{WTNN}} + \lambda \|\mathcal{E}\|_1$$

s.t. $\mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E}$

↓

$$\min_{\mathcal{Z}, \mathcal{E}} \|\mathcal{Z}\|_{\text{WTNN}} + \lambda \Omega(\mathcal{E})$$

s.t. $\mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E}$

↓

$$\min_{\mathcal{Z}, \mathcal{E}} \|\mathcal{Z}\|_{\text{WTNN}} + \lambda \Omega(\mathcal{E}) + \mu \phi(\mathcal{E})$$

s.t. $\mathcal{X} = \mathcal{A} * \mathcal{Z} + \mathcal{E}$

- ▶ $\phi(\mathcal{E})$: PnP denoiser including BM4D, WNNM, FFDNet, SwinIR

Algorithm

- ▶ Alternating direction method of multipliers (ADMM)

$$\begin{aligned} & \min_{\mathcal{Z}, \mathcal{E}, \mathcal{Y}, \mathcal{W}} \|\mathcal{Z}\|_{\text{WTNN}} + \lambda \Omega(\mathcal{E}) + \mu \phi(\mathcal{Y}) \\ \text{s.t. } & \mathcal{X} = \mathcal{A} * \mathcal{W} + \mathcal{E}, \quad \mathcal{Z} = \mathcal{W}, \quad \mathcal{Y} = \mathcal{E} \\ & \Downarrow \end{aligned}$$

$$\begin{aligned} L_\beta(\mathcal{Z}, \mathcal{E}, \mathcal{Y}, \mathcal{W}, \mathcal{Q}_1, \mathcal{Q}_2, \mathcal{Q}_3) = & \|\mathcal{Z}\|_{\text{WTNN}} + \lambda \Omega(\mathcal{E}) + \mu \phi(\mathcal{Y}) \\ & + \langle \mathcal{Q}_1, \mathcal{X} - \mathcal{A} * \mathcal{W} - \mathcal{E} \rangle + \frac{\beta}{2} \|\mathcal{X} - \mathcal{A} * \mathcal{W} - \mathcal{E}\|_{\text{F}}^2 \\ & + \langle \mathcal{Q}_2, \mathcal{Z} - \mathcal{W} \rangle + \frac{\beta}{2} \|\mathcal{Z} - \mathcal{W}\|_{\text{F}}^2 + \langle \mathcal{Q}_3, \mathcal{Y} - \mathcal{E} \rangle + \frac{\beta}{2} \|\mathcal{Y} - \mathcal{E}\|_{\text{F}}^2 \end{aligned}$$

- ▶ Two issues should be considered
 - ▶ How to update \mathcal{E} ⇒ Quadratic min-cost flow
 - ▶ How to construct dictionary ⇒ PCA

$$\min_{\mathcal{L}, \mathcal{S}} \|\mathcal{L}\|_{\text{WTNN}} + \alpha \Omega(\mathcal{S}) \quad \text{s.t. } \mathcal{X} = \mathcal{L} + \mathcal{S}$$

Experiments

- ▶ Datasets: ABU (airport, beach, urban)
- ▶ Compared methods: machine learning/deep learning
 - ▶ LRASR: Anomaly Detection in Hyperspectral Images Based on Low-Rank and Sparse Representation, 2016
 - ▶ GTVLRR: Graph and Total Variation Regularized Low-Rank Representation for Hyperspectral Anomaly Detection, 2020
 - ▶ PTA: Prior-Based Tensor Approximation for Anomaly Detection in Hyperspectral Imagery, 2022
 - ▶ PCA-TLRSR: Learning Tensor Low-Rank Representation for Hyperspectral Anomaly Detection, 2023
 - ▶ LARTVAD: Hyperspectral Anomaly Detection With Tensor Average Rank and Piecewise Smoothness Constraints, 2023
 - ▶ DeCNN-AD: Hyperspectral Anomaly Detection via Deep Plug-and-Play Denoising CNN Regularization, 2021
 - ▶ Auto-AD: Autonomous Hyperspectral Anomaly Detection Network Based on Fully Convolutional Autoencoder, 2022
 - ▶ RGAE: Hyperspectral Anomaly Detection With Robust Graph Autoencoders, 2022

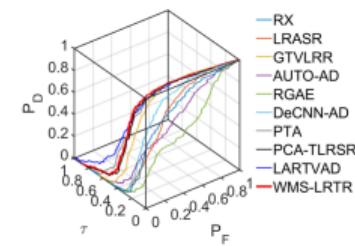
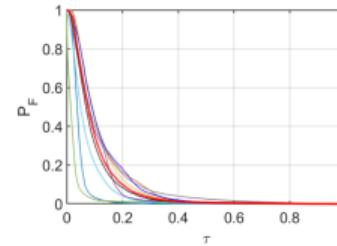
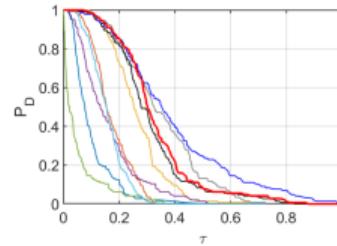
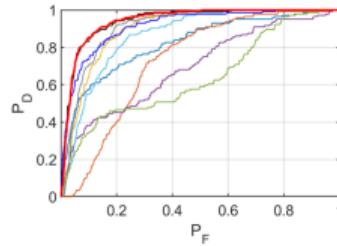
Experiments

► AUC values on the Airport scenes

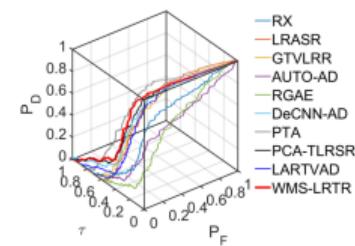
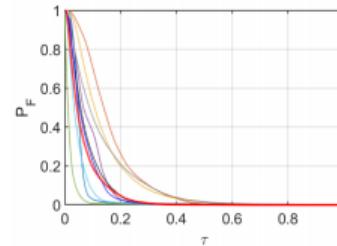
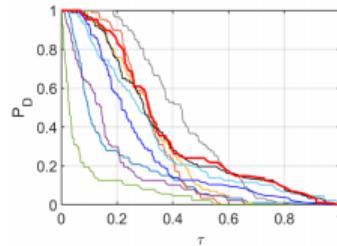
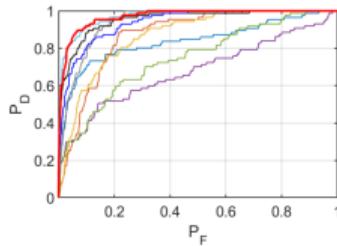
Dataset	AUC	RX	LRASR	GTVLRR	AUTO-AD	RGAE	DeCNN-AD	PTA	PCA-TLRSR	LARTVAD	WMS-LRTR
Airport-1	AUC _(D,F) ↑	0.8221	0.7284	0.8997	0.6941	0.6387	0.8662	0.9109	<u>0.9420</u>	0.9202	0.9435
	AUC _(D,τ) ↑	0.0987	0.1711	0.2665	0.1595	0.0506	0.1562	0.3471	0.3088	0.2540	<u>0.3284</u>
	AUC _(F,τ) ↓	0.0424	0.1209	0.1153	0.0991	0.0255	<u>0.0689</u>	0.1191	0.0918	0.0816	0.1001
	AUC _{TD} ↑	0.9208	0.8996	1.1647	0.8536	0.6889	1.0224	1.2580	<u>1.2508</u>	1.1742	1.2718
	AUC _{BS} ↑	0.7797	0.6075	0.7844	0.5950	0.6128	0.7974	0.7918	0.8502	0.8386	<u>0.8433</u>
	AUC _{TDBS} ↑	0.0563	0.0502	0.1496	0.0603	0.0252	0.0873	<u>0.2279</u>	0.2170	0.1724	0.2282
	AUC _{ODP} ↑	0.8784	0.7786	1.0493	0.7544	0.6635	0.9536	1.1388	<u>1.1590</u>	1.0927	1.1717
Airport-2	AUC _(D,F) ↑	0.8403	0.8707	0.8670	0.6764	0.7470	<u>0.9656</u>	0.9411	0.9543	0.9387	0.9704
	AUC _(D,τ) ↑	0.1841	0.3156	0.3175	0.1976	0.0770	0.3257	0.4334	0.3705	0.2845	<u>0.3807</u>
	AUC _(F,τ) ↓	0.0516	0.1613	0.1379	0.0862	0.0196	<u>0.0476</u>	0.1292	0.0753	0.0692	0.0652
	AUC _{TD} ↑	1.0245	1.1863	1.1845	0.8439	0.8239	1.2913	1.3745	1.3248	1.2233	<u>1.3511</u>
	AUC _{BS} ↑	0.7888	0.7094	0.7291	0.5902	0.7274	0.9180	0.8119	0.8790	0.8696	<u>0.9052</u>
	AUC _{TDBS} ↑	0.1325	0.1542	0.1797	0.0814	0.0574	0.2781	<u>0.3042</u>	0.2952	0.2154	0.3155
	AUC _{ODP} ↑	0.9709	1.0249	1.0467	0.7578	0.8044	1.2437	1.2453	<u>1.2495</u>	1.1541	1.2859
Airport-3	AUC _(D,F) ↑	0.9288	0.9234	0.9231	0.9210	0.8873	0.9235	0.9247	<u>0.9540</u>	0.8877	0.9579
	AUC _(D,τ) ↑	0.0660	0.0562	0.0695	0.1278	0.0511	0.0676	0.1665	<u>0.1398</u>	0.1203	0.1333
	AUC _(F,τ) ↓	0.0145	0.0126	0.0155	0.0395	0.0057	<u>0.0123</u>	0.0416	0.0279	0.0326	0.0194
	AUC _{TD} ↑	0.9948	0.9796	0.9927	1.0488	0.9384	0.9916	1.0911	1.0945	1.0080	<u>1.0912</u>
	AUC _{BS} ↑	0.9144	0.9108	0.9077	0.8815	0.8816	0.9117	0.8831	<u>0.9268</u>	0.8551	0.9385
	AUC _{TDBS} ↑	0.0516	0.0436	0.0540	0.0883	0.0454	0.0553	0.1249	0.1119	0.0877	<u>0.1139</u>
	AUC _{ODP} ↑	0.9804	0.9670	0.9772	1.0094	0.9327	0.9793	1.0496	<u>1.0666</u>	0.9754	1.0718
Airport-4	AUC _(D,F) ↑	0.9526	0.9566	0.9836	0.9840	0.7508	0.9239	0.9841	<u>0.9933</u>	0.9173	0.9961
	AUC _(D,τ) ↑	0.0736	0.3747	0.4437	0.4071	0.1172	0.4229	0.6476	0.4350	0.0931	<u>0.5110</u>
	AUC _(F,τ) ↓	0.0248	0.1053	0.0942	<u>0.0267</u>	0.0749	0.1646	0.1044	0.0924	0.0311	0.0427
	AUC _{TD} ↑	1.0262	1.3313	1.4273	1.3911	0.8679	1.3467	1.6317	1.4283	1.0104	<u>1.5071</u>
	AUC _{BS} ↑	0.9278	0.8513	0.8894	0.9573	0.6759	0.7593	0.8798	0.9008	0.8862	<u>0.9534</u>
	AUC _{TDBS} ↑	0.0489	0.2693	0.3496	0.3804	0.0423	0.2583	0.5432	0.3426	0.0620	<u>0.4684</u>
	AUC _{ODP} ↑	1.0015	1.2260	1.3332	1.3644	0.7930	1.1822	1.5273	1.3358	0.9793	<u>1.4645</u>

Experiments

► ROC curves on the Airport-1 dataset

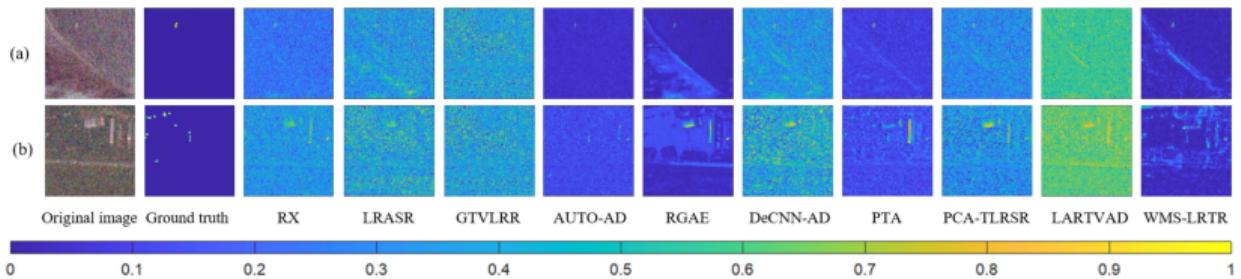


► ROC curves on the Airport-2 dataset



Experiments

► Noise robustness



Dataset	AUC	RX	LRASR	GTVLRR	AUTO-AD	RGAE	DeCNN-AD	PTA	PCA-TLRSR	LARTVAD	WMS-LRTR
Noisy Beach-3	$AUC_{(D,F)} \uparrow$	0.9267	0.7543	0.6208	0.7905	0.8067	0.7806	0.9740	0.8109	0.8602	0.9755
	$AUC_{(D,\tau)} \uparrow$	0.5386	0.5523	0.5122	0.2924	0.3427	0.5665	0.5110	<u>0.5785</u>	0.7330	0.4833
	$AUC_{(F,\tau)} \downarrow$	0.2453	0.3667	0.4267	<u>0.0607</u>	0.0630	0.3506	0.1056	0.3105	0.5712	0.0491
	$AUC_{TD} \uparrow$	1.4652	1.3066	1.1330	1.0829	1.1494	1.3470	<u>1.4850</u>	1.3894	1.5932	1.4589
	$AUC_{BS} \uparrow$	0.6814	0.3876	0.1941	0.7297	0.7437	0.4300	<u>0.8684</u>	0.5004	0.2889	0.9264
	$AUC_{TDBS} \uparrow$	0.2933	0.1856	0.0854	0.2316	0.2797	0.2158	<u>0.4054</u>	0.2680	0.1618	0.4342
	$AUC_{ODP} \uparrow$	1.2200	0.9400	0.7063	1.0221	1.0864	0.9964	<u>1.3794</u>	1.0790	1.0220	1.4097
Noisy Urban-3	$AUC_{(D,F)} \uparrow$	0.6873	0.5919	0.5124	0.6794	0.4518	0.5492	0.9085	0.6180	0.6893	0.9281
	$AUC_{(D,\tau)} \uparrow$	0.4440	<u>0.4611</u>	0.4061	0.2357	0.0781	0.4445	0.3519	0.4470	0.6803	0.2803
	$AUC_{(F,\tau)} \downarrow$	0.3539	0.4127	0.3971	0.1358	<u>0.0955</u>	0.4177	0.1543	0.3956	0.6219	0.0714
	$AUC_{TD} \uparrow$	1.1312	1.0530	0.9185	0.9151	0.5299	0.9937	<u>1.2604</u>	1.0650	1.3696	1.2084
	$AUC_{BS} \uparrow$	0.3334	0.1791	0.1153	0.5436	0.3563	0.1315	<u>0.7541</u>	0.2225	0.0674	0.8567
	$AUC_{TDBS} \uparrow$	0.0901	0.0484	0.0090	0.0998	0.0175	0.0268	<u>0.1976</u>	0.0514	0.0583	0.2089
	$AUC_{ODP} \uparrow$	0.7774	0.6403	0.5214	0.7792	0.4344	0.5760	<u>1.1061</u>	0.6695	0.7477	1.1370

Experiments

► Ablation study

Dataset	Without PCA		Without WTNN		Without WMS		Without PnP Prior		WMS-LRTR	
	AUC _(D,F) ↑	Time(s)	AUC _(D,F) ↑	Time(s)	AUC _(D,F) ↑	Time(s)	AUC _(D,F) ↑	Time(s)	AUC _(D,F) ↑	Time(s)
Airport-1	0.9076	14801.741	0.8294	226.314	0.9350	46.210	0.9240	195.416	0.9435	222.551
Airport-2	0.9322	12431.595	0.9585	96.175	0.9627	26.713	0.9441	62.733	0.9704	92.963
Airport-3	0.9274	15124.256	0.9529	271.938	0.9546	132.316	0.9533	179.844	0.9579	297.676
Airport-4	0.9779	13547.221	0.9914	138.498	0.9952	41.520	0.9906	96.701	0.9961	130.514
Average	0.9363	13976.203	0.9331	183.231	0.9619	61.690	0.9530	133.674	0.9670	185.926

► Runtime comparisons

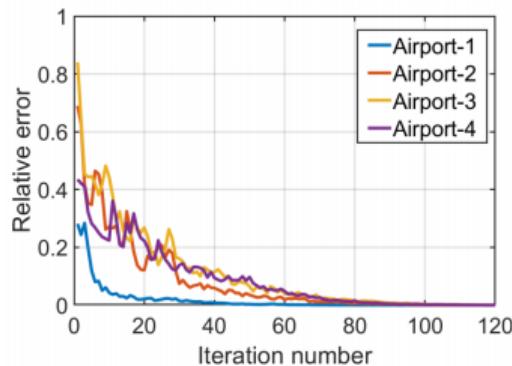
Dataset	RX	LRASR	GTVLRR	AUTO-AD	RGAE	DeCNN-AD	PTA	PCA-TLRSR	LARTVAD	WMS-LRTR
Airport-1	0.102	36.594	214.276	53.080	151.695	56.391	41.515	5.185	46.579	222.551
Airport-2	0.387	52.394	223.684	24.520	144.780	61.706	30.623	5.322	51.808	92.963
Airport-3	0.089	47.754	171.489	20.675	152.961	73.266	36.622	21.625	40.472	297.676
Airport-4	0.092	40.181	180.609	26.694	156.080	77.445	33.261	22.451	55.220	130.514
Average	0.168	44.231	197.515	31.242	151.379	67.202	35.505	13.646	48.520	185.926

Experiments

► Multidirectional analysis

Dataset	Index	H	V	S	M
Airport-1	$AUC_{(D,F)} \uparrow$	0.9428	0.9415	0.9412	0.9435
Airport-2	$AUC_{(D,F)} \uparrow$	0.9679	0.9686	0.9629	0.9704
Airport-3	$AUC_{(D,F)} \uparrow$	0.9547	0.9526	0.9577	0.9579
Airport-4	$AUC_{(D,F)} \uparrow$	0.9958	0.9956	0.9950	0.9961

► Convergence analysis



► Block-size analysis

Dataset	Index	2×2	3×3	4×4	5×5
Airport-1	$AUC_{(D,F)} \uparrow$ Time(s)	0.9435 222.551	0.9424 240.222	0.9379 313.199	0.9335 506.953
Airport-2	$AUC_{(D,F)} \uparrow$ Time(s)	0.9704 92.963	0.9663 111.301	0.9659 132.522	0.9612 204.909
Airport-3	$AUC_{(D,F)} \uparrow$ Time(s)	0.9579 297.676	0.9597 358.482	0.9587 419.583	0.9595 494.698
Airport-4	$AUC_{(D,F)} \uparrow$ Time(s)	0.9961 130.514	0.9960 150.457	0.9953 179.795	0.9955 278.522

Outline

Introduction

Plug-and-Play TLRA

Multidirectional Sparse TLRA

Future Work

Future Work

- ▶ Tuning-free parameter \Rightarrow deep unfolding networks
 - ▶ F. Wu, T. Zhang, L. Li, Y. Huang, Z. Peng, RPCANet: Deep Unfolding RPCA Based Infrared Small Target Detection, [IEEE/CVF Winter Conference on Applications of Computer Vision](#), 2024
 - ▶ J. Zhang, B. Ghanem, ISTA-Net: Interpretable Optimization-Inspired Deep Network for Image Compressive Sensing, [IEEE Conference on Computer Vision and Pattern Recognition](#), 2018
 - ▶ K. Gregor, Y. LeCun, Learning Fast Approximations of Sparse Coding, [International Conference on Machine Learning](#), 2010
- ▶ Low complexity \Rightarrow parallel and distributed computing
 - ▶ K. Yu, Z. Wu, J. Sun, Y. Zhang, Y. Xu, Z. Wei, Accelerating Hyperspectral Anomaly Detection With Enhanced Multivariate Gaussianization Based on FPGA, [IEEE Transactions on Geoscience and Remote Sensing](#), 2024
 - ▶ Q. Du, B. Tang, W. Xie, W. Li, Parallel and Distributed Computing for Anomaly Detection From Hyperspectral Remote Sensing Imagery, [Proceedings of the IEEE](#), 2021
 - ▶ J. Lei, G. Yang, W. Xie, Y. Li, X. Jia, A Low-Complexity Hyperspectral Anomaly Detection Algorithm and Its FPGA Implementation, [IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing](#), 2020

References

- ▶ J. Liu, J. Jin, X. Xiu, W. Liu, J. Zhang, Exploiting Weighted Multidirectional Sparsity for Prior Enhanced Anomaly Detection in Hyperspectral Images, [Remote Sensing](#), 2025
- ▶ J. Liu, M. Feng, X. Xiu, X. Zeng, J. Zhang, Tensor Low-Rank Approximation via Plug-and-Play Priors for Anomaly Detection in Remote Sensing Images, [Under Review](#)

Thank you for your attention!

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