CMP 模型公式

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1 不加磁场的公式

$$\begin{cases}\overline{G} = G_h \cup \Gamma_h = \{(x_j, t_k) : 0 \le j \le N; 0 \le k \le M\} \\ G_h = \{(x_j, t_k) : 0 < j < N; 0 < k \le M\} \\ \Gamma_h = \{(x_j, t_k) : j = 0, N; k = 1, \cdots, M\} \cup \{(x_j, t_0) : j = 0, \cdots, N\}\end{cases} \tag{1}$$

$$\nabla \cdot \boldsymbol{u} = 0 \tag{2}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} = 0 \tag{3}$$

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \eta \Delta \boldsymbol{u} + \rho \boldsymbol{F} \tag{4}$$

$$\begin{cases}\rho\left(\frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_1}{\partial y} + u_3\frac{\partial u_1}{\partial z}\right) = -\frac{\partial p}{\partial x} + \eta\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) \\ \rho\left(\frac{\partial u_2}{\partial t} + u_1\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_2}{\partial y} + u_3\frac{\partial u_2}{\partial z}\right) = -\frac{\partial p}{\partial y} + \eta\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial z^2}\right) \\ \rho\left(\frac{\partial u_3}{\partial t} + u_1\frac{\partial u_3}{\partial x} + u_2\frac{\partial u_3}{\partial y} + u_3\frac{\partial u_3}{\partial z}\right) = -\frac{\partial p}{\partial z} + \eta\left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} + \frac{\partial^2 u_3}{\partial z^2}\right)\end{cases} \tag{5}$$

$$\begin{cases}\frac{\partial p}{\partial x} + \rho u_1\frac{\partial u_1}{\partial x} + \rho u_2\frac{\partial u_1}{\partial y} = \eta\frac{\partial^2 u_1}{\partial z^2} \\ \frac{\partial p}{\partial y} + \rho u_1\frac{\partial u_2}{\partial x} + \rho u_2\frac{\partial u_2}{\partial y} = \eta\frac{\partial^2 u_2}{\partial z^2} \\ \frac{\partial p}{\partial z} = 0\end{cases} \tag{6}$$

$$\begin{cases}x = r\cos\theta \\ y = r\sin\theta \\ z = z\end{cases} \tag{7}$$

$$\begin{cases}\frac{\partial x}{\partial r} = \cos\theta \quad \frac{\partial x}{\partial \theta} = -r\cdot\sin\theta \\ \frac{\partial y}{\partial r} = \cos\theta \quad \frac{\partial y}{\partial \theta} = r\cdot\cos\theta\end{cases} \tag{8}$$

$$
\begin{cases}
\dfrac{\partial r}{\partial x} = \cos\theta \quad \dfrac{\partial \theta}{\partial x} = -\dfrac{\sin\theta}{r} \\
\dfrac{\partial r}{\partial y} = \cos\theta \quad \dfrac{\partial \theta}{\partial y} = \dfrac{\cos\theta}{r}
\end{cases} \tag{9}
$$

$$
\begin{cases}
u_1 = \omega \cdot \cos\theta - u \cdot \sin\theta \\
u_2 = \omega \cdot \sin\theta + u \cdot \cos\theta
\end{cases} \tag{10}
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2 加磁场后