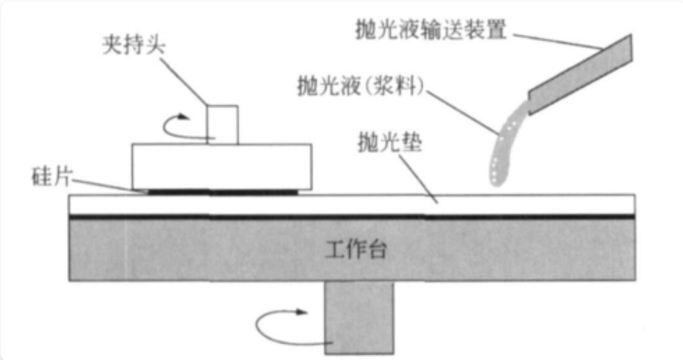


CMP工作示意图

具有对流效应的CMP润滑方程的推导

连续方程为:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

该方程等价于

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} = 0 \tag{2}$$

运动方程为:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \eta \Delta \boldsymbol{u} + \rho \boldsymbol{F} \tag{3}$$

当假设$\boldsymbol{F} = \boldsymbol{0}$时，该方程等价于

$$\begin{cases} \rho \left(\dfrac{\partial u_1}{\partial t} + u_1 \dfrac{\partial u_1}{\partial x} + u_2 \dfrac{\partial u_1}{\partial y} + u_3 \dfrac{\partial u_1}{\partial z} \right) = -\dfrac{\partial p}{\partial x} + \eta \left(\dfrac{\partial^2 u_1}{\partial x^2} + \dfrac{\partial^2 u_1}{\partial y^2} + \dfrac{\partial^2 u_1}{\partial z^2} \right) \\ \rho \left(\dfrac{\partial u_2}{\partial t} + u_1 \dfrac{\partial u_2}{\partial x} + u_2 \dfrac{\partial u_2}{\partial y} + u_3 \dfrac{\partial u_2}{\partial z} \right) = -\dfrac{\partial p}{\partial y} + \eta \left(\dfrac{\partial^2 u_2}{\partial x^2} + \dfrac{\partial^2 u_2}{\partial y^2} + \dfrac{\partial^2 u_2}{\partial z^2} \right) \\ \rho \left(\dfrac{\partial u_3}{\partial t} + u_1 \dfrac{\partial u_3}{\partial x} + u_2 \dfrac{\partial u_3}{\partial y} + u_3 \dfrac{\partial u_3}{\partial z} \right) = -\dfrac{\partial p}{\partial z} + \eta \left(\dfrac{\partial^2 u_3}{\partial x^2} + \dfrac{\partial^2 u_3}{\partial y^2} + \dfrac{\partial^2 u_3}{\partial z^2} \right) \end{cases}$$

假设条件

1. CMP过程中载荷完全由抛光液来承载，晶片由于抛光液的隔离完全与抛光垫分离，因为膜厚比较博，忽略质量力。
2. 假设抛光液是牛顿性流体，在CMP过程中是稳态不可压的，流体的黏性系数是常数η。由于稳态所以$\dfrac{\partial u_i}{\partial t} = 0$。
3. 满足无量纲润滑条件。即$\dfrac{\partial^2 u_i}{\partial x^2} + \dfrac{\partial^2 u_i}{\partial y^2}$比$\dfrac{\partial^2 u_i}{\partial z^2}$量级小，可以忽略。
4. 在z轴方向总压力梯度为0，z轴方向的速度忽略不计。即$u_3 = 0$

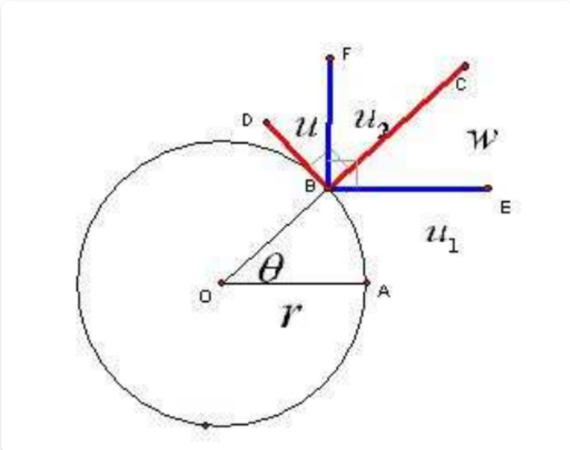
所以连续方程简化为

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} = 0 \tag{5}$$

运动方程简化为

$$\begin{cases} \frac{\partial p}{\partial x}+\rho u_1\frac{\partial u_1}{\partial x}+\rho u_2\frac{\partial u_1}{\partial y}=\eta\frac{\partial^2 u_1}{\partial z^2} \\ \frac{\partial p}{\partial y}+\rho u_1\frac{\partial u_2}{\partial x}+\rho u_2\frac{\partial u_2}{\partial y}=\eta\frac{\partial^2 u_2}{\partial z^2} \\ \frac{\partial p}{\partial z}=0 \end{cases} \tag{6}$$

上式的坐标系是直角坐标系，将其转化为柱坐标，则速度转换关系如下图



CMP速度关系式

变换关系式为:

$$\begin{cases} x=r\cos\theta \\ y=r\sin\theta \\ z=z \end{cases} \tag{7}$$

$$\begin{cases} \frac{\partial x}{\partial r}=\cos\theta \quad \frac{\partial x}{\partial \theta}=-r\cdot\sin\theta \\ \frac{\partial y}{\partial r}=\sin\theta \quad \frac{\partial y}{\partial \theta}=r\cdot\cos\theta \end{cases} \tag{8}$$

$$\begin{cases} \frac{\partial r}{\partial x}=\cos\theta \quad \frac{\partial \theta}{\partial x}=-\frac{\sin\theta}{r} \\ \frac{\partial r}{\partial y}=\sin\theta \quad \frac{\partial \theta}{\partial y}=\frac{\cos\theta}{r} \end{cases} \tag{9}$$

速度关系式

$$\begin{cases} u_1=\omega\cdot\cos\theta-u\cdot\sin\theta \\ u_2=\omega\cdot\sin\theta+u\cdot\cos\theta \end{cases} \tag{10}$$

于是可得(5)与(6)式的柱坐标形式如下:

$$
\begin{aligned}
&\frac{\partial rw}{\partial r}+\frac{\partial u}{\partial \theta}=0\\
&\frac{\partial p}{\partial r}+\rho w\frac{\partial w}{\partial r}+\rho\frac{u}{r}\frac{\partial w}{\partial \theta}-\rho\frac{u^2}{r}=\eta\frac{\partial^2 w}{\partial z^2}\\
&\frac{1}{r}\frac{\partial p}{\partial \theta}+\rho w\frac{\partial u}{\partial r}+\rho\frac{u}{r}\frac{\partial u}{\partial \theta}+\rho\frac{wu}{r}=\eta\frac{\partial^2 u}{\partial z^2}\\
&\frac{\partial p}{\partial z}=0
\end{aligned}
$$

(10)

p只与x,y相关，即p=p(r,θ)，记

$$
\begin{aligned}
F(w,u)&=\rho w\frac{\partial w}{\partial r}+\rho\frac{u}{r}\frac{\partial w}{\partial \theta}-\rho\frac{u^2}{r}\\
G(w,u)&=\rho w\frac{\partial u}{\partial r}+\rho\frac{u}{r}\frac{\partial u}{\partial \theta}+\rho\frac{wu}{r}
\end{aligned}
$$

(11)

对(10)的中间两项在[0,z]上进行两次积分可得

$$
\left.
\begin{aligned}
w&=\frac{z^2}{2\eta}\frac{\partial p}{\partial r}+\frac{1}{\eta}\int_0^z\int_0^z F(w,u)\mathrm{d}z\mathrm{d}z+\\
&\quad\left.\frac{\partial w}{\partial z}\right|_{z=0}z+w_0\\
u&=\frac{z^2}{2r\eta}\frac{\partial p}{\partial \theta}+\frac{1}{\eta}\int_0^z\int_0^z G(w,u)\mathrm{d}z\mathrm{d}z+\\
&\quad\left.\frac{\partial u}{\partial z}\right|_{z=0}z+u_0
\end{aligned}
\right\}
$$

(12)

对一阶对z的偏导数进行泰勒展式:

$$
\left.
\begin{aligned}
\left.\frac{\partial w}{\partial z}\right|_{z=0}&\approx\frac{w_h-w_0}{h}-\frac{h}{2}\left.\frac{\partial^2 w}{\partial z^2}\right|_{z=0}\\
\left.\frac{\partial u}{\partial z}\right|_{z=0}&\approx\frac{u_h-u_0}{h}-\frac{h}{2}\left.\frac{\partial^2 u}{\partial z^2}\right|_{z=0}
\end{aligned}
\right\}
$$

(13)

将(10)中间两项代入(13)，(13)代入(12)，然后再将(12)代入到(10)的第一个公式中，则有

$$
\begin{aligned}
&\frac{\partial}{\partial r}\Big[r\Big(\frac{z^2-hz}{2\eta}\frac{\partial p}{\partial r}+\frac{w_h-w_0}{h}z+w_0+\\
&\quad\frac{1}{\eta}\int_0^z\int_0^z F(w,u)\mathrm{d}z\mathrm{d}z-\frac{hz}{2\eta}F(w_0,u_0)\Big)\Big]+\\
&\quad\frac{\partial}{\partial \theta}\Big[\frac{z^2-hz}{2r\eta}\frac{\partial p}{\partial r}+\frac{u_h-u_0}{h}z+u_0+\\
&\quad\frac{1}{\eta}\int_0^z\int_0^z G(w,u)\mathrm{d}z\mathrm{d}z-\frac{hz}{2\eta}G(w_0,u_0)\Big]=0
\end{aligned}
$$

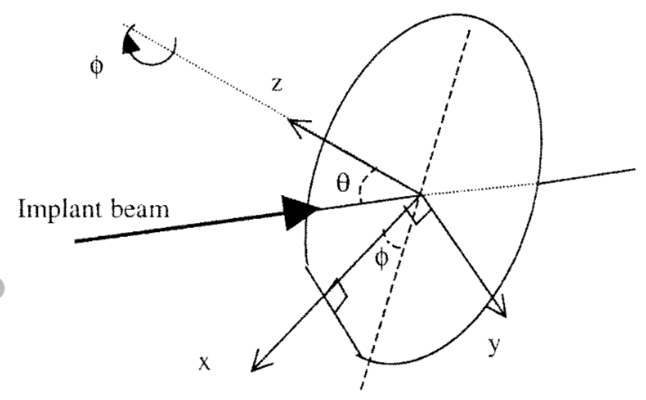
(14)

将(14)沿z轴[0,h]积分，积分过程中采用中矩形公式近似，于是得到了CMP润滑方程

$$
\begin{aligned}
&\frac{\partial}{\partial r}\left(rh^3\frac{\partial p}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial\theta}\left(h^3\frac{\partial p}{\partial\theta}\right)= \\
&\qquad 6\eta\frac{\partial}{\partial r}[(w_h+w_0)h]+6\eta\frac{\partial}{\partial\theta}[(u_h+u_0)h]+ \\
&\qquad \frac{\partial}{\partial r}[rh^3(F(w_h,u_h)-2F(w_0,u_0))]+ \\
&\qquad \frac{\partial}{\partial\theta}[h^3(G(w_h,u_h)-2G(w_0,u_0))]
\end{aligned}
$$

记晶片中心厚度为h_{piv}，晶片转角和倾角分别为$\alpha\quad\beta$，则在(r,θ)处膜厚h满足

$$h=h_{\mathrm{piv}}-r\sin\alpha\cos\theta-r\sin\beta\sin\theta$$



(b)

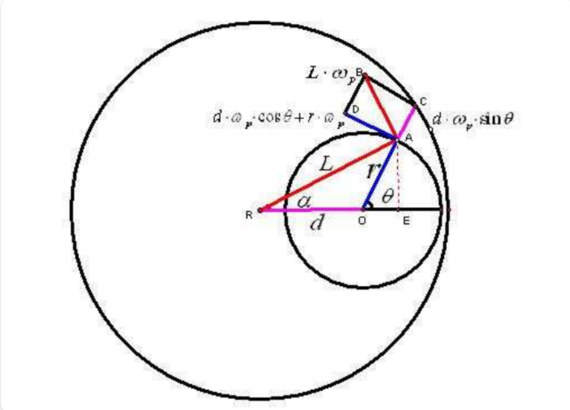
倾斜角和旋转角的定义。晶片首先倾斜 10 度，然后绕其法线方向旋转。这个过程导致倾斜和旋转角度的定义，如下所示。倾斜角定义为入射离子束方向与晶片法线方向（[100] 方向）之间的角度。旋转角定义为晶片入射离子束方向的投影与垂直于[110]（图中x方向）晶片平面的晶片表面上的矢量之间的夹角。该图显示了倾斜和旋转后的晶片 (a) 和 (b) 的配置。

转角倾角示意图

速度边界条件满足如下:

$$
\left.
\begin{aligned}
&u=(r+d\cos\theta)\omega_p \\
&w=d\sin\theta\omega_p\,,\quad z=0 \\
&u=r\omega_w \\
&w=0\,,\quad z=h
\end{aligned}
\right\}
$$

其中ω_ω为晶片的转速，ω_p为抛光垫的转速，d为抛光垫中心到晶片中心的距离。



速度边界关系图

无量纲

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将一个物理导出量用若干个基本量的乘方之积表示出来的表达式，称为该物理量的量纲式，简称量纲。 它是在选定了单位制之后，由基本物理量单位表达的式子。有量纲的物理量都可以进行无量纲化处理。

中文名	无量纲	意　思	没有单位的物理量
外文名	dimensionless	成　因	两个量纲相同的物理量的比值

无量纲百度解释

形成流体动力润滑的条件是

1.两工件之间的间隙必须有楔形间隙。

2.两工件表面之间必须连续充满润滑油或其它液体。

3.两工件表面必须有相对滑动速度。其运动方向必须保证润滑油从大截面流进，从小截面出来

润滑条件百度解释