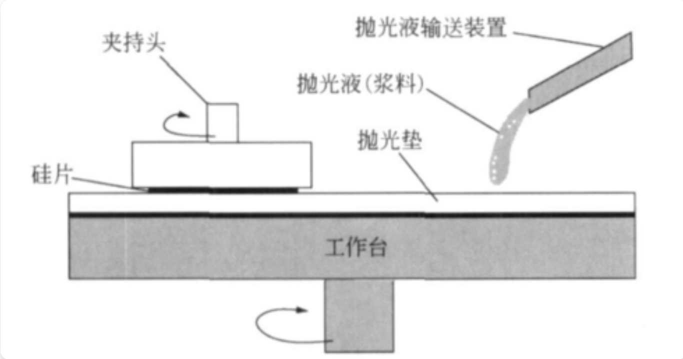


CMP工作示意图

具有对流效应的CMP润滑方程的推导

连续方程为:

$$\nabla \cdot \boldsymbol{u} = 0 \tag{1}$$

该方程等价于

$$\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{\partial y} + \frac{\partial u_3}{\partial z} = 0 \tag{2}$$

运动方程为:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + \eta \Delta \boldsymbol{u} + \rho \boldsymbol{F} \tag{3}$$

当假设$\boldsymbol{F} = \boldsymbol{0}$时，该方程等价于

$$\begin{cases} \rho\left(\frac{\partial u_1}{\partial t} + u_1\frac{\partial u_1}{\partial x} + u_2\frac{\partial u_1}{\partial y} + u_3\frac{\partial u_1}{\partial z}\right) = -\frac{\partial p}{\partial x} + \eta\left(\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2}\right) \\ \rho\left(\frac{\partial u_2}{\partial t} + u_1\frac{\partial u_2}{\partial x} + u_2\frac{\partial u_2}{\partial y} + u_3\frac{\partial u_2}{\partial z}\right) = -\frac{\partial p}{\partial y} + \eta\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} + \frac{\partial^2 u_2}{\partial z^2}\right) \\ \rho\left(\frac{\partial u_3}{\partial t} + u_1\frac{\partial u_3}{\partial x} + u_2\frac{\partial u_3}{\partial y} + u_3\frac{\partial u_3}{\partial z}\right) = -\frac{\partial p}{\partial z} + \eta\left(\frac{\partial^2 u_3}{\partial x^2} + \frac{\partial^2 u_3}{\partial y^2} + \frac{\partial^2 u_3}{\partial z^2}\right) \end{cases}$$

假设条件

1. CMP过程中载荷完全由抛光液来承载，晶片由于抛光液的隔离完全与抛光垫分离，因为膜厚比较博，忽略质量力。
2. 假设抛光液是牛顿性流体，在CMP过程中是稳态不可压的，流体的黏性系数是常数η。由于稳态所以$\frac{\partial u_i}{\partial t} = 0$。
3. 满足无纲量润滑条件。即$\frac{\partial^2 u_i}{\partial x^2} + \frac{\partial^2 u_i}{\partial y^2}$比$\frac{\partial^2 u_i}{\partial z^2}$量级小，可以忽略。
4. 在z轴方向总压力梯度为0，z轴方向的速度忽略不计。即$u_3 = 0$

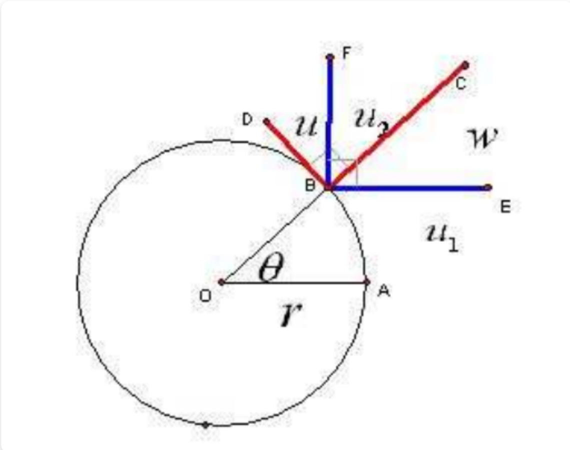
所以连续方程简化为

$$\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}=0 \tag{5}$$

运动方程简化为

$$\begin{cases}\dfrac{\partial p}{\partial x}+\rho u_1\dfrac{\partial u_1}{\partial x}+\rho u_2\dfrac{\partial u_1}{\partial y}=\eta\dfrac{\partial^2 u_1}{\partial z^2}\\ \dfrac{\partial p}{\partial y}+\rho u_1\dfrac{\partial u_2}{\partial x}+\rho u_2\dfrac{\partial u_2}{\partial y}=\eta\dfrac{\partial^2 u_2}{\partial z^2}\\ \dfrac{\partial p}{\partial z}=0\end{cases} \tag{6}$$

上式的坐标系是直角坐标系，将其转化为柱坐标，则速度转换关系如下图



CMP速度关系式

变换关系式为:

$$\begin{cases}\dfrac{\partial x}{\partial r}=\cos\theta \quad \dfrac{\partial x}{\partial \theta}=-r\cdot\sin\theta\\ \dfrac{\partial y}{\partial r}=\cos\theta \quad \dfrac{\partial y}{\partial \theta}=r\cdot\cos\theta\end{cases} \tag{7}$$

$$\begin{cases}\dfrac{\partial r}{\partial x}=\cos\theta \quad \dfrac{\partial \theta}{\partial x}=-\dfrac{\sin\theta}{r}\\ \dfrac{\partial r}{\partial y}=\cos\theta \quad \dfrac{\partial \theta}{\partial y}=\dfrac{\cos\theta}{r}\end{cases} \tag{8}$$