Chapter 4
Introduction to Probability

- Random Experiments, Counting Rules, and Assigning Probabilities
- Events and Their Probability
- Some Basic Relationships of Probability
- Conditional Probability
- Bayes' Theorem

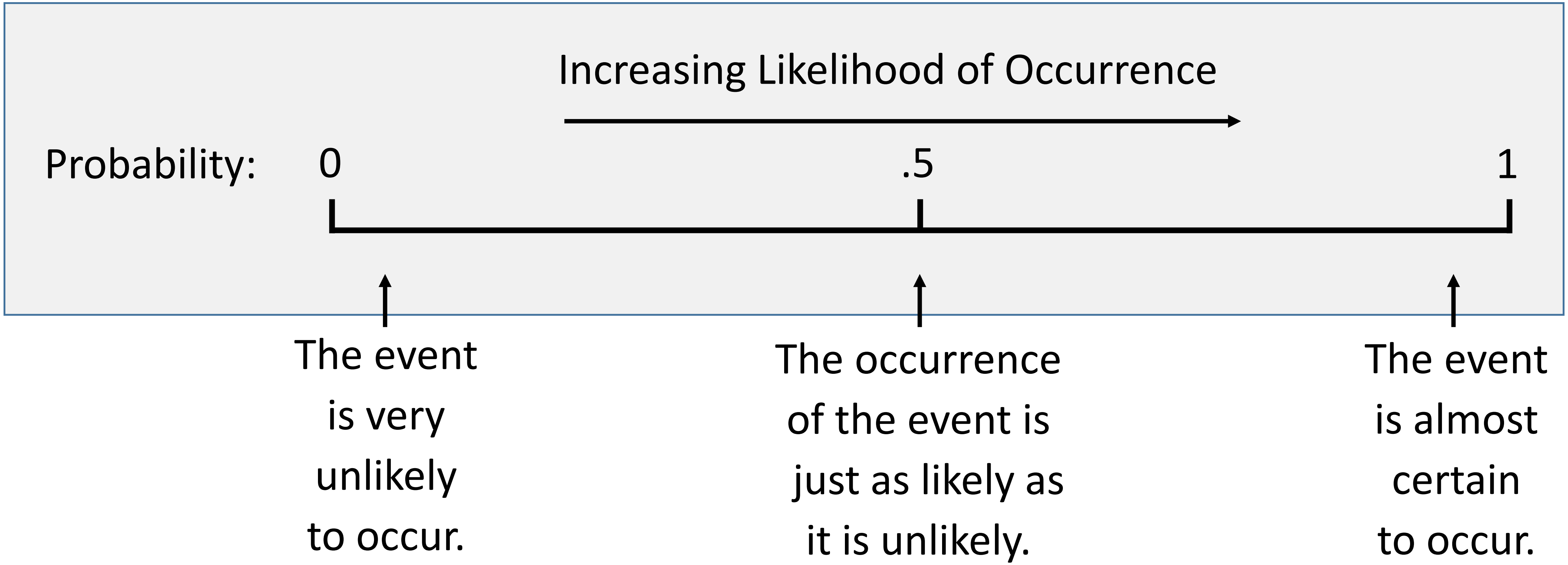
Uncertainties

- Managers often base their decisions on an analysis of uncertainties such as the following:
  - What are the *chances* that sales will decrease if we increase prices?
  - What is the *likelihood* a new assembly method will increase productivity?
  - What are the *odds* that a new investment will be profitable?

Probability

- Probability is a numerical measure of the likelihood that an event will occur.
- Probability values are always assigned on a scale from 0 to 1.
- A probability near zero indicates an event is quite unlikely to occur.
- A probability near one indicates an event is almost certain to occur.

Probability as a Numerical Measure of the Likelihood of Occurrence



Statistical Experiments

- In statistics, the notion of an experiment differs somewhat from that of an experiment in the physical sciences.
- In statistical experiments, probability determines outcomes.
- Even though the experiment is repeated in exactly the same way, an entirely different outcome may occur.
- For this reason, statistical experiments are sometimes called *random experiments (隨機實驗)*.

Random Experiment and Its Sample Space

- A random experiment (隨機實驗) is a process that generates well-defined experimental outcomes.
- The sample space (樣本空間) for an experiment is the set of all experimental outcomes.
- An experimental outcome is also called a sample point (樣本點).

Random Experiment and Its Sample Space

<u>Experiment (實驗)</u>	<u>Experiment Outcomes (實驗結果)</u>
Toss a coin	Head, tail
Inspect a part	Defective, non-defective
Conduct a sales call	Purchase, no purchase
Roll a die	1, 2, 3, 4, 5, 6
Play a football game	Win, lose, tie

Random Experiment and Its Sample Space

- Example: Bradley Investments

Bradley has invested in two stocks, Markley Oil and Collins Mining. Bradley has determined that the possible outcomes of these investments three months from now are as follows:

Investment Gain or Loss in 3 Months (in $1000s)	
Markley Oil	Collins Mining
10	8
5	-2
0	
-20	

A Counting Rule for Multiple-Step Experiments

- If an experiment consists of a sequence of k steps in which there are n_1 possible results for the first step, n_2 possible results for the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2) \ldots (n_k)$.
- A helpful graphical representation of a multiple-step experiment is a <u>tree diagram</u>.

A Counting Rule for Multiple-Step Experiments

- Example: Bradley Investments
  - Bradley Investments can be viewed as a two-step experiment. It involves two stocks, each with a set of experimental outcomes.

Markley Oil:	$n_1 = 4$
Collins Mining:	$n_2 = 2$
Total Number of Experimental Outcomes:	$n_1 n_2 = (4)(2) = 8$

Tree Diagram

- Example: Bradley Investments

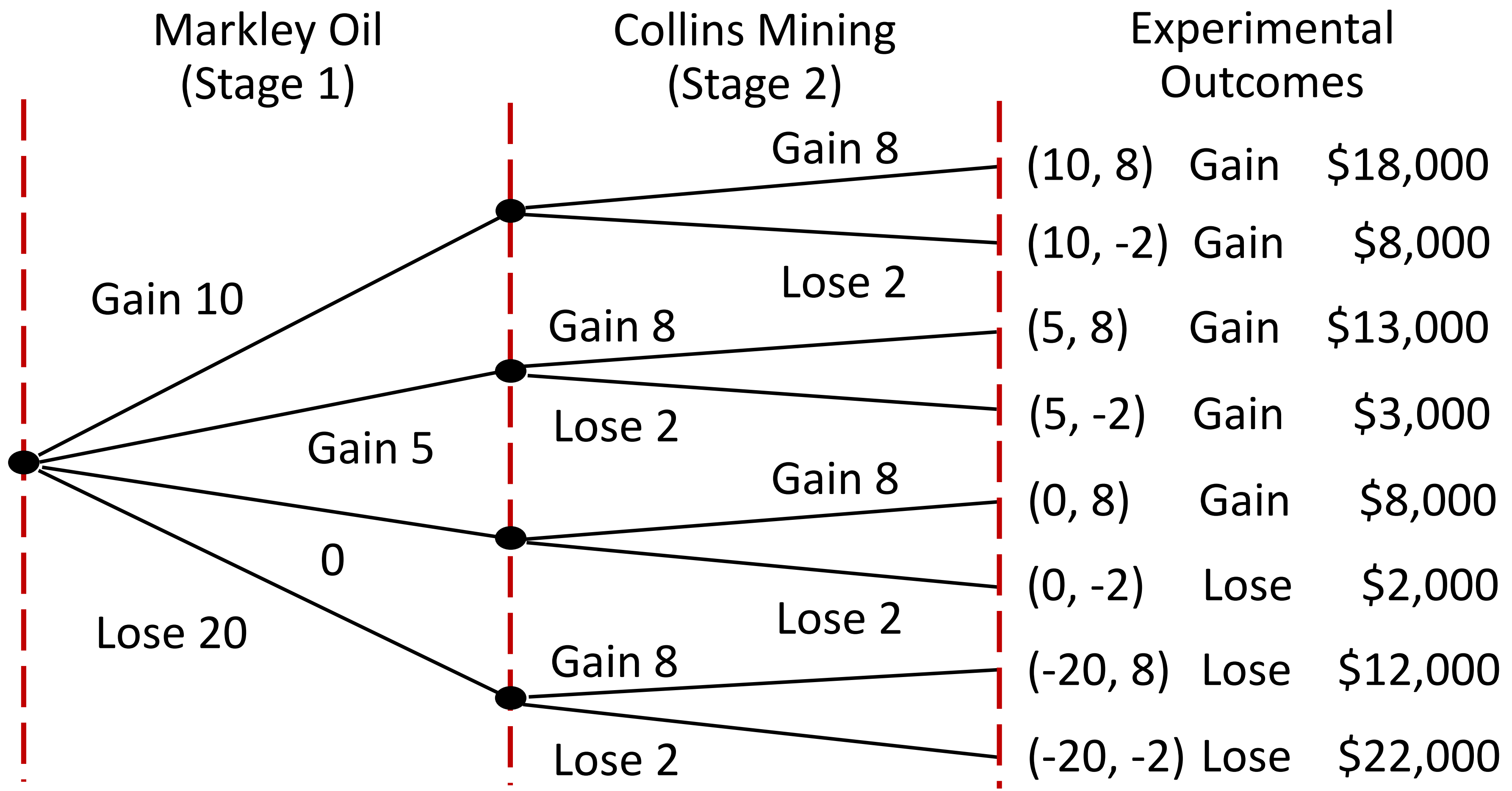


FIGURE 4.2 TREE DIAGRAM FOR THE EXPERIMENT OF TOSSING TWO COINS

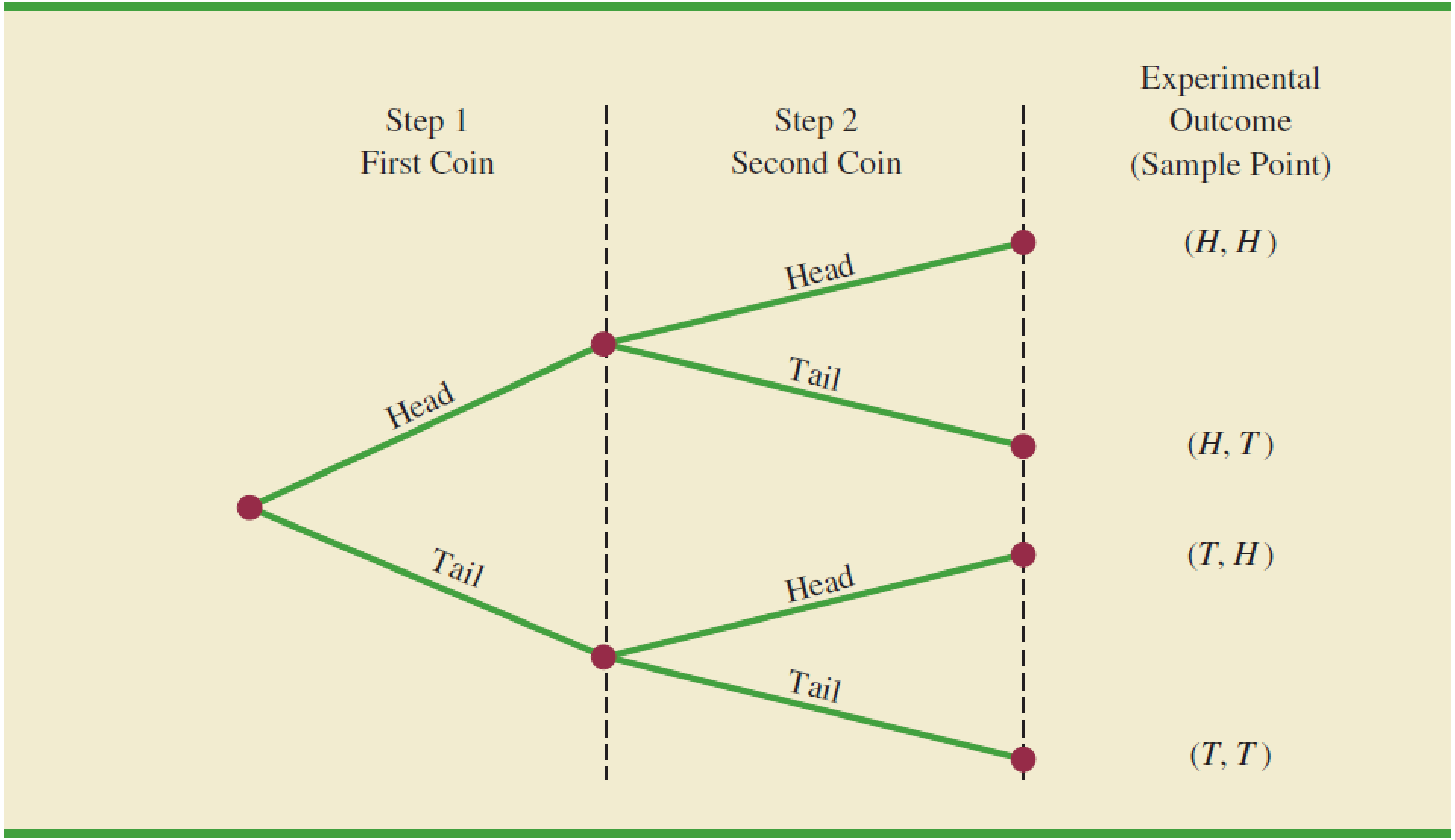
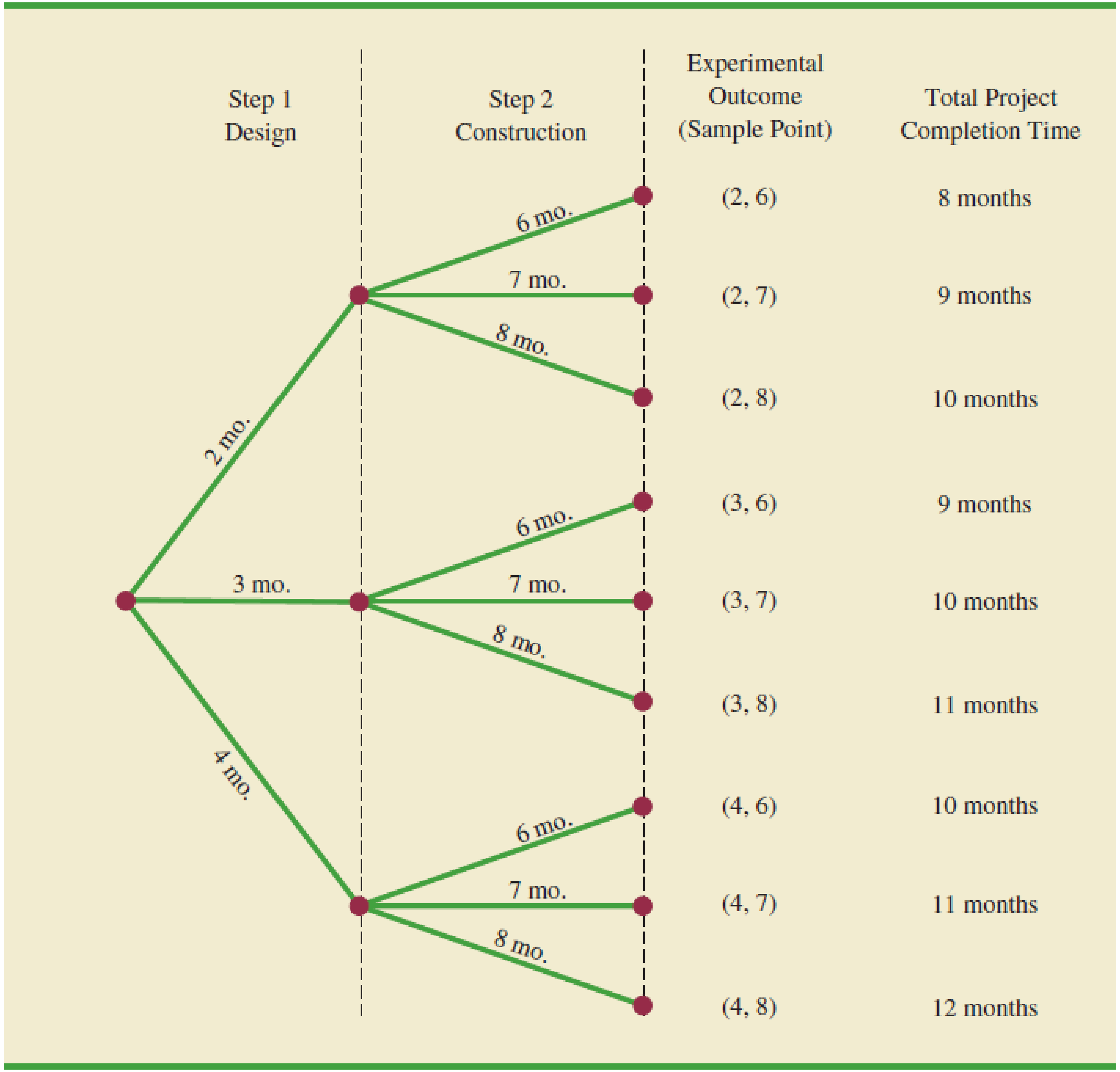


TABLE 4.1 EXPERIMENTAL OUTCOMES (SAMPLE POINTS) FOR THE KP&L PROJECT

Completion Time (months)			
Stage 1 Design	**Stage 2 Construction**	**Notation for Experimental Outcome**	**Total Project Completion Time (months)**
2	6	(2, 6)	8
2	7	(2, 7)	9
2	8	(2, 8)	10
3	6	(3, 6)	9
3	7	(3, 7)	10
3	8	(3, 8)	11
4	6	(4, 6)	10
4	7	(4, 7)	11
4	8	(4, 8)	12

FIGURE 4.3 TREE DIAGRAM FOR THE KP&L PROJECT



Counting Rule for Combinations

- Number of <u>Combinations</u> of *N* Objects Taken *n* at a Time
  - A second useful counting rule enables us to count the number of experimental outcomes when *n* objects are to be selected from a set of *N* objects.

$$C^N_n = \binom{N}{n} = \frac{N!}{n!(N-n)!}$$

where: $N! = N(N - 1)(N - 2) \ldots (2)(1)$

$n! = n(n - 1)(n - 2) \ldots (2)(1)$

$0! = 1$

Example: $C^5_2 = \binom{5}{2} = \frac{5!}{2!(5-2)!} = \frac{(5)(4)(3)(2)(1)}{(2)(1)(3)(2)(1)} = \frac{120}{12} = 10$

Counting Rule for Permutations

- Number of <u>Permutations</u> of *N* Objects Taken *n* at a Time
  - A third useful counting rule enables us to count the number of experimental outcomes when *n* objects are to be selected from a set of *N* objects, where the order of selection is important.

$$P_n^N = n!\binom{N}{n} = \frac{N!}{(N-n)!}$$

where: $N! = N(N-1)(N-2)\ldots(2)(1)$

$n! = n(n-1)(n-2)\ldots(2)(1)$

$0! = 1$

Example: $\mathrm{P}_2^5 = \frac{5!}{(5-2)!} = \frac{(5)(4)(3)(2)(1)}{(3)(2)(1)} = \frac{120}{6} = 20$

Assigning Probabilities

- Basic Requirements for Assigning Probabilities

  1. The probability assigned to each experimental outcome must be between 0 and 1, inclusively.

$$0 \leq P(E_i) \leq 1 \text{ for all } i$$

where: E_i is the i th experimental outcome and $P(E_i)$ is its probability

Assigning Probabilities

- Basic Requirements for Assigning Probabilities

  2. The sum of the probabilities for all experimental outcomes must equal 1.

$$P(E_1) + P(E_2) + . . . + P(E_n) = 1$$

where: n is the number of experimental outcomes

Assigning Probabilities

- Classical Method

  Assigning probabilities based on the assumption of equally likely outcomes

- Relative Frequency Method

  Assigning probabilities based on experimentation or historical data

- Subjective Method

  Assigning probabilities based on judgment

Classical Method

- Example: Rolling a Die

If an experiment has *n* possible outcomes, the classical method would assign a probability of 1/*n* to each outcome.

Experiment: Rolling a die

Sample Space: $S = \{1, 2, 3, 4, 5, 6\}$

Probabilities: Each sample point has a 1/6 chance of occurring

Relative Frequency Method

- Example: Lucas Tool Rental

Lucas Tool Rental would like to assign probabilities to the number of car polishers it rents each day. Office records show the following frequencies of daily rentals for the last 40 days.

Number of Polishers Rented	Number of Days
0	4
1	6
2	18
3	10
4	2

Relative Frequency Method

- Example: Lucas Tool Rental

  Each probability assignment is given by dividing the frequency (number of days) by the total frequency (total number of days).

Number of Polishers Rented	Number of Days	Probability
0	4	.10 = 4/40
1	6	.15
2	18	.45
3	10	.25
4	2	.05
	40	1.00

Subjective Method

- When economic conditions or a company’s circumstances change rapidly it might be inappropriate to assign probabilities based solely on historical data.
- We can use any data available as well as our experience and intuition, but ultimately a probability value should express our <u>degree of belief</u> that the experimental outcome will occur.
- The best probability estimates often are obtained by combining the estimates from the classical or relative frequency approach with the subjective estimate.

Subjective Method

- Example: Bradley Investments

  An analyst made the following probability estimates.

Experimental Outcome	Net Gain or Loss	Probability
(10, 8)	$18,000 Gain	.20
(10, -2)	$8,000 Gain	.08
(5, 8)	$13,000 Gain	.16
(5, -2)	$3,000 Gain	.26
(0, 8)	$8,000 Gain	.10
(0, -2)	$2,000 Loss	.12
(-20, 8)	$12,000 Loss	.02
(-20, -2)	$22,000 Loss	.06
		1.00

The following table shows the percentage of on-time arrivals, the number of mishandled baggage reports per 1000 passengers, and the number of customer complaints per 1000 passengers for 10 airlines (*Forbes* website, February, 2014).

Airline	On-Time Arrivals (%)	Mishandled Baggage per 1000 Passengers	Customer Complaints per 1000 Passengers
Virgin America	83.5	0.87	1.50
JetBlue	79.1	1.88	0.79
AirTran Airways	87.1	1.58	0.91
Delta Air Lines	86.5	2.10	0.73
Alaska Airlines	87.5	2.93	0.51
Frontier Airlines	77.9	2.22	1.05
Southwest Airlines	83.1	3.08	0.25
US Airways	85.9	2.14	1.74
American Airlines	76.9	2.92	1.80
United Airlines	77.4	3.87	4.24

a. If you randomly choose a Delta Air lines flight, what is the probability that this individual flight has an on-time arrival?

b. If you randomly choose one of the 10 airlines for a follow-up study on airline quality ratings, what is the probability that you will choose an airline with less than two mishandled baggage reports per 1000 passengers?

c. If you randomly choose 1 of the 10 airlines for a follow-up study on airline quality ratings, what is the probability that you will choose an airline with more than one customer complaint per 1000 passengers?

d. What is the probability that a randomly selected AirTran Airways flight will not arrive on time?

Events and Their Probabilities

- An event is a collection of sample points.
- The probability of any event is equal to the sum of the probabilities of the sample points in the event.
- If we can identify all the sample points of an experiment and assign a probability to each, we can compute the probability of an event.

Events and Their Probabilities

- Example: Bradley Investments

Event M = Markley Oil Profitable

$$M = \{(10, 8), (10, -2), (5, 8), (5, -2)\}$$

$$P(M) = P(10, 8) + P(10, -2) + P(5, 8) + P(5, -2)$$

$$= .20 + .08 + .16 + .26$$

$$= .70$$

Events and Their Probabilities

- Example: Bradley Investments

  Event C = Collins Mining Profitable

$$C = \{(10, 8), (5, 8), (0, 8), (-20, 8)\}$$

$$P(C) = P(10, 8) + P(5, 8) + P(0, 8) + P(-20, 8)$$

$$= .20 + .16 + .10 + .02$$

$$= .48$$

Some Basic Relationships of Probability

- There are some <u>basic probability relationships</u> that can be used to compute the probability of an event without knowledge of all the sample point probabilities.

Complement (餘集) of an Event

Union (聯集) of Two Events

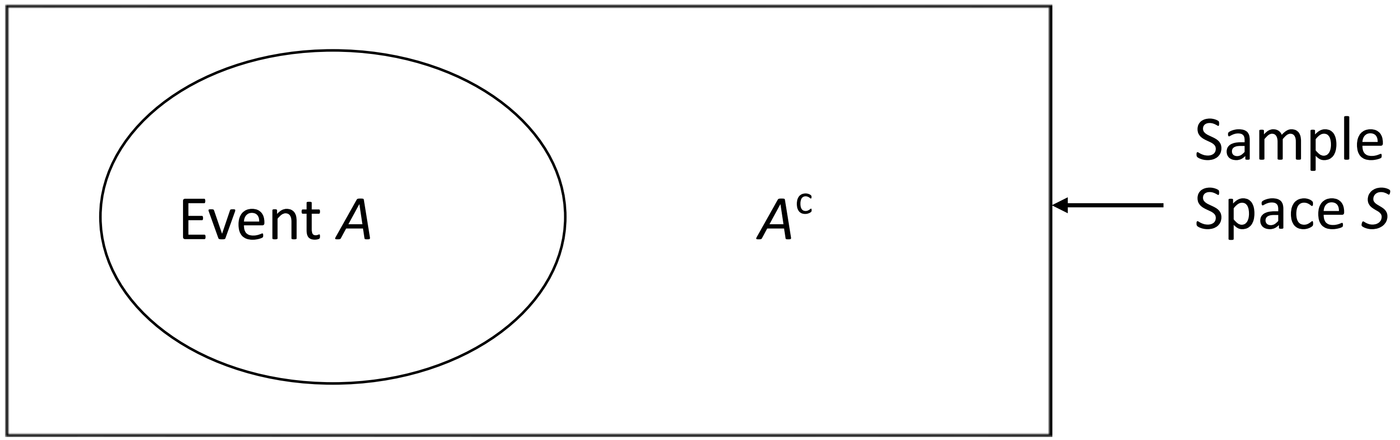
Intersection (交集) of Two Events

Mutually Exclusive (互斥) Events

Complement of an Event

- The complement of event *A* is defined to be the event consisting of all sample points that are *not* in *A.*

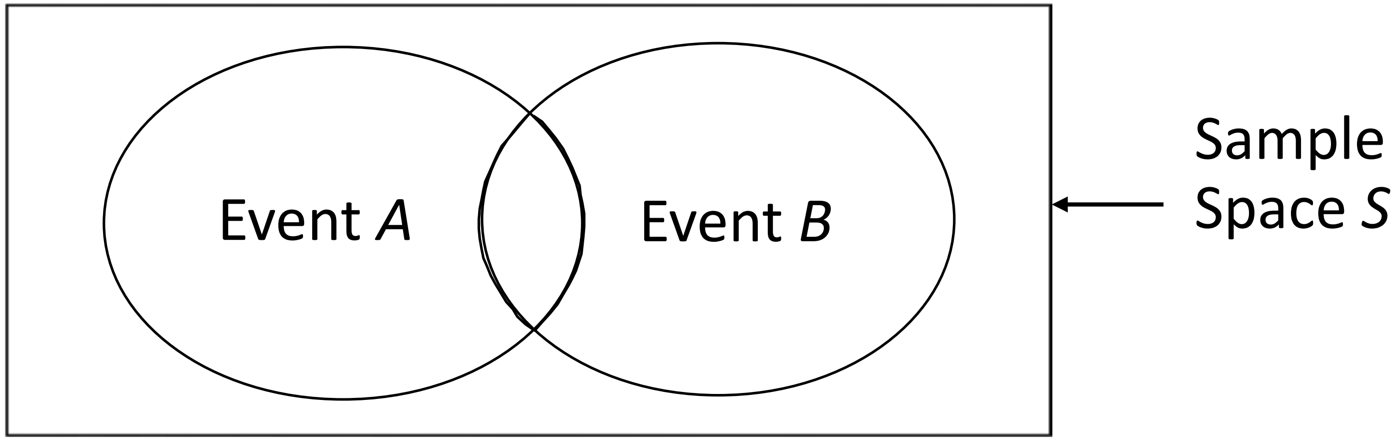
- The complement of *A* is denoted by A^c.



Venn Diagram

Union of Two Events

- The union of events *A* and *B* is the event containing all sample points that are in *A* or *B* or both.
- The union of events *A* and *B* is denoted by $A \cup B$.



Venn Diagram

Union of Two Events

- Example: Bradley Investments

Event M = Markley Oil Profitable

Event C = Collins Mining Profitable

$M \cup C$ = Markley Oil Profitable
or Collins Mining Profitable (or both)

$M \cup C = \{(10, 8), (10, -2), (5, 8), (5, -2), (0, 8), (-20, 8)\}$

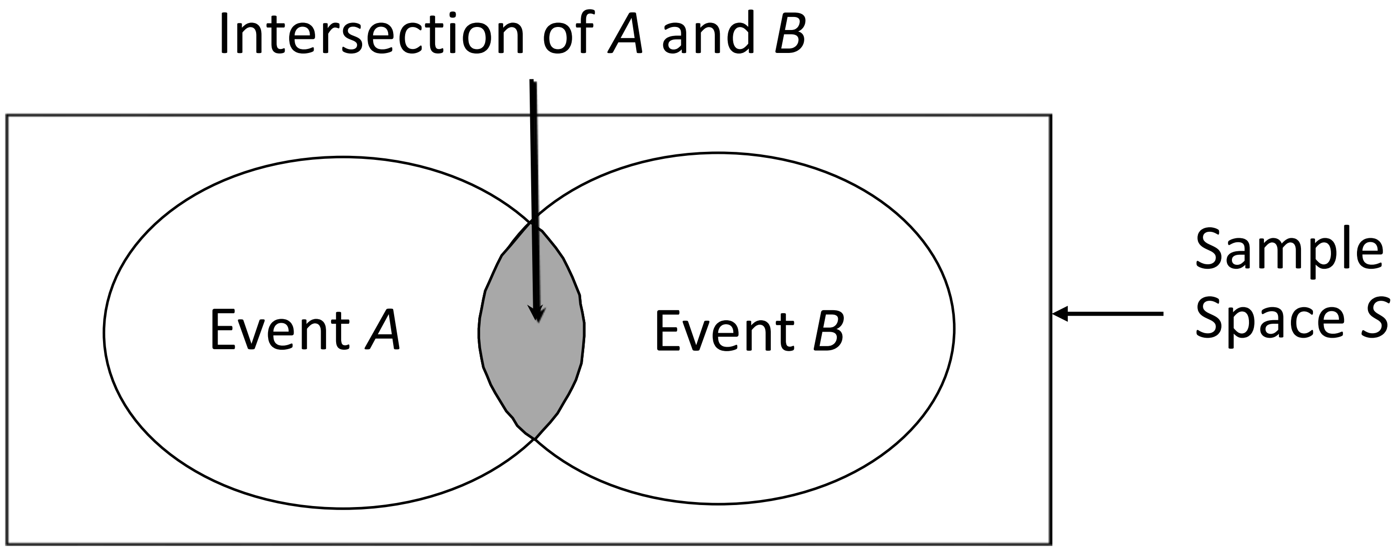
$P(M \cup C) = P(10, 8) + P(10, -2) + P(5, 8) + P(5, -2) + P(0, 8) + P(-20, 8)$

$= .20 + .08 + .16 + .26 + .10 + .02$

$= .82$

Intersection of Two Events

- The intersection of events *A* and *B* is the set of all sample points that are in both *A* and *B*.
- The intersection of events *A* and *B* is denoted by $A \cap B$.



Venn Diagram

Intersection of Two Events

- Example: Bradley Investments

Event M = Markley Oil Profitable

Event C = Collins Mining Profitable

$M \cap C$ = Markley Oil Profitable and Collins Mining Profitable

$M \cap C = \{(10, 8), (5, 8)\}$

$P(M \cap C) = P(10, 8) + P(5, 8)$

$= .20 + .16$

$= .36$

Addition Law

- The addition law provides a way to compute the probability of event *A,* or *B,* or both *A* and *B* occurring.
- The law is written as:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Addition Law (加法律)

- Example: Bradley Investments

Event M = Markley Oil Profitable

Event C = Collins Mining Profitable

$M \cup C$ = Markley Oil Profitable <u>or</u> Collins Mining Profitable

We know: $P(M) = .70$, $P(C) = .48$, $P(M \cap C) = .36$

Thus: $P(M \cup C) = P(M) + P(C) - P(M \cap C)$

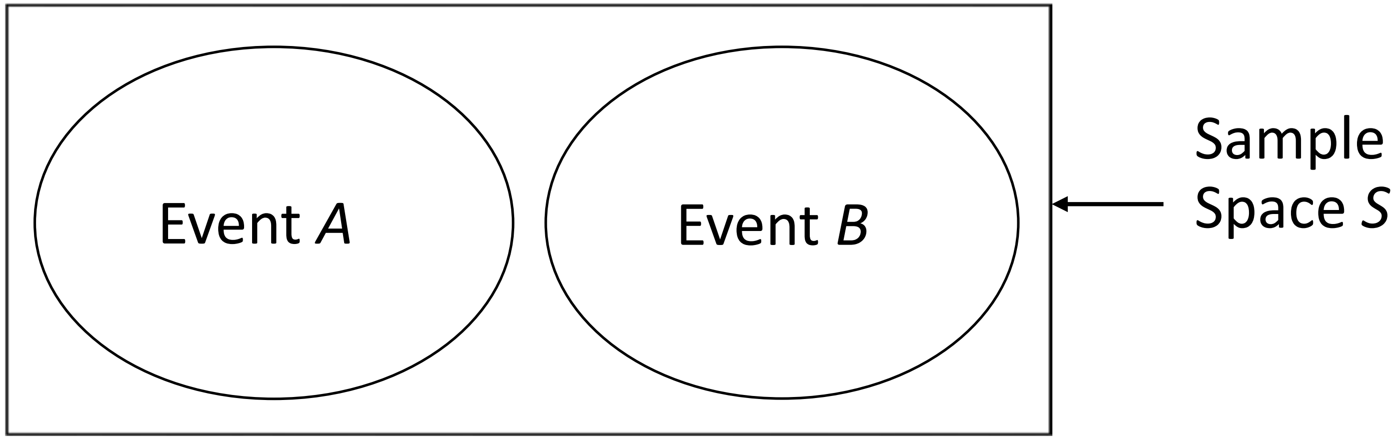
$$= .70 + .48 - .36$$

$$= .82$$

(This result is the same as that obtained earlier using the definition of the probability of an event.)

Mutually Exclusive Events

- Two events are said to be mutually exclusive if the events have no sample points in common.
- Two events are mutually exclusive if, when one event occurs, the other cannot occur.



Venn Diagram

Mutually Exclusive Events

- If events A and B are mutually exclusive, $P(A \cap B) = 0$.
- The addition law for mutually exclusive events is:

$$P(A \cup B) = P(A) + P(B)$$

The U.S. Census Bureau provides data on the number of young adults, ages 18–24, who are living in their parents' home. Let

M = the event a male young adult is living in his parents' home

F = the event a female young adult is living in her parents' home

If we randomly select a male young adult and a female young adult, the Census Bureau data enable us to conclude $P(M) = .56$ and $P(F) = .42$ (*The World Almanac,* 2006). The probability that both are living in their parents' home is .24.

a. What is the probability at least one of the two young adults selected is living in his or her parents' home?

b. What is the probability both young adults selected are living on their own (neither is living in their parents' home)?

A financial manager made two new investments— one in the oil industry and one in municipal bonds. After a one-year period, each of the investments will be classified as either successful or unsuccessful. Consider the making of the two investments as an experiment.

a. How many sample points exist for this experiment?
b. Show a tree diagram and list the sample points.
c. Let *O* = the event that the oil industry investment is successful and *M* = the event that the municipal bond investment is successful. List the sample points in *O* and in *M*.
d. List the sample points in the union of the events $(O \cup M)$.
e. List the sample points in the intersection of the events $(O \cap M)$.
f. Are events *O* and *M* mutually exclusive?

Conditional Probability (條件機率)

- The probability of an event given that another event has occurred is called a conditional probability.
- The conditional probability of *A* given *B* has already occurred is denoted by $P(A|B)$.
- A conditional probability is computed as follows :

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Conditional Probability

- Example: Bradley Investments

Event M = Markley Oil Profitable

Event C = Collins Mining Profitable

$P(C|M)$ = Collins Mining Profitable given Markley Oil Profitable

We know: $P(M \cap C) = .36$, $P(M) = .70$

Thus: $P(C|M) = \frac{P(C \cap M)}{P(M)} = \frac{.36}{.70} = .5143$

Multiplication Law (乘法律)

- The multiplication law provides a way to compute the probability of the intersection of two events.
- The law is written as:

$$P(A \cap B) = P(B)P(A|B)$$

or

$$P(A \cap B) = P(A)P(B|A)$$

Multiplication Law

- Example: Bradley Investments

Event M = Markley Oil Profitable

Event C = Collins Mining Profitable

$M \cap C$ = Markley Oil Profitable <u>and</u> Collins Mining Profitable

We know: $P(M) = .70$, $P(C|M) = .5143$

Thus: $P(M \cap C) = P(M)P(C|M)$

$= (.70)(.5143)$

$= .36$

(This result is the same as that obtained earlier using the definition of the probability of an event.)

Joint Probability Table (聯合機率表)

Markley Oil	Collins Mining Profitable (C)	 Not Profitable (C^c)	Total
Profitable (M)	.36	.34	.70
Not Profitable (M^c)	.12	.18	.30
Total	.48	.52	1.00

- Joint probabilities appear in the body of the table.
- Marginal probabilities (邊際機率) appear in the margins of the table.

TABLE 4.4 PROMOTION STATUS OF POLICE OFFICERS OVER THE PAST TWO YEARS

	Men	Women	Total
Promoted	288	36	324
Not Promoted	672	204	876
Total	960	240	1200

M = event an officer is a man

W = event an officer is a woman

A = event an officer is promoted

A^c = event an officer is not promoted

$$P(A|\, M) = \frac{288}{960} = \frac{288/1200}{960/1200} = .30 \quad P(A|\, W) = \frac{36}{240} = \frac{36/1200}{240/1200} = .15$$

$P(M \cap A) = 288/1200 = .24$ probability that a randomly selected officer is a man *and* is promoted

$P(M \cap A^c) = 672/1200 = .56$ probability that a randomly selected officer is a man *and* is not promoted

TABLE 4.5 JOINT PROBABILITY TABLE FOR PROMOTIONS

Joint probabilities appear in the body of the table.

	Men (M)	Women (W)	Total
Promoted (A)	.24	.03	.27
Not Promoted (A^c)	.56	.17	.73
Total	.80	.20	1.00

Marginal probabilities appear in the margins of the table.

$$P(A|M) = \frac{P(A \cap M)}{P(M)} = \frac{.24}{.80} = .30 \qquad P(A|W) = \frac{P(A \cap W)}{P(W)} = \frac{.03}{.20} = .15$$

Independent Events (獨立事件)

- If the probability of event *A* is not changed by the existence of event *B*, we would say that events *A* and *B* are <u>independent</u>.
- Two events *A* and *B* are independent if:

$$P(A|B) = P(A) \quad \text{or} \quad P(B|A) = P(B)$$

- The multiplication law is written as:

$$P(A \cap B) = P(B)\ P(A|B) \text{ or } P(A \cap B) = P(A)\ P(B|A)$$

- The multiplication law also can be used as a test to see if two events are independent.
- The law is written as:

$$P(A \cap B) = P(A)P(B)$$

Multiplication Law for Independent Events

- Example: Bradley Investments

Event M = Markley Oil Profitable

Event C = Collins Mining Profitable

Are events M and C independent?

Does $P(M \cap C) = P(M)P(C)$?

We know: $P(M \cap C) = .36$, $P(M) = .70$, $P(C) = .48$

But: $P(M)P(C) = (.70)(.48) = .34$, not .36

Hence: M and C are not independent.

Mutual Exclusiveness and Independence

- Do not confuse the notion of mutually exclusive events with that of independent events.
- Two events with nonzero probabilities cannot be both mutually exclusive and independent.
- If one mutually exclusive event is known to occur, the other cannot occur; thus, the probability of the other event occurring is reduced to zero (and they are therefore dependent (相依)).
- Two events that are not mutually exclusive, might or might not be independent.

Assume that we have two events, A and B, that are mutually exclusive. Assume further that we know $P(A) = .30$ and $P(B) = .40$.

a. What is $P(A \cap B)$?
b. What is $P(A|B)$?
c. A student in statistics argues that the concepts of mutually exclusive events and independent events are really the same, and that if events are mutually exclusive they must be independent. Do you agree with this statement? Use the probability information in this problem to justify your answer.
d. What general conclusion would you make about mutually exclusive and independent events given the results of this problem?

Students taking the Graduate Management Admissions Test (GMAT) were asked about their undergraduate major and intent to pursue their MBA as a full-time or part-time student. A summary of their responses follows.

		Undergraduate Major			
		Business	Engineering	Other	Totals
Intended Enrollment Status	Full-Time	352	197	251	800
	Part-Time	150	161	194	505
	Totals	502	358	445	1305

a. Develop a joint probability table for these data.

b. Use the marginal probabilities of undergraduate major (business, engineering, or other) to comment on which undergraduate major produces the most potential MBA students.

c. If a student intends to attend classes full-time in pursuit of an MBA degree, what is the probability that the student was an undergraduate engineering major?

d. If a student was an undergraduate business major, what is the probability that the student intends to attend classes full-time in pursuit of an MBA degree?

e. Let *A* denote the event that the student intends to attend classes full-time in pursuit of an MBA degree, and let *B* denote the event that the student was an undergraduate business major. Are events *A* and *B* independent?

Bayes’ Theorem (貝氏定理)

- Often we begin probability analysis with initial or <u>prior probabilities</u>.
- Then, from a sample, special report, or a product test we obtain some additional information.
- Given this information, we calculate revised or <u>posterior probabilities</u>.
- <u>Bayes’ theorem</u> provides the means for revising the prior probabilities.

Prior Probabilities (先驗機率) → New Information → Application of Bayes’ Theorem → Posterior Probabilities (事後機率)

Bayes' Theorem

- Example: L. S. Clothiers

A proposed shopping center will provide strong competition for downtown businesses like L. S. Clothiers. If the shopping center is built, the owner of L. S. Clothiers feels it would be best to relocate to the shopping center.

The shopping center cannot be built unless a zoning change is approved by the town council. The planning board must first make a recommendation, for or against the zoning change, to the council.

Prior Probabilities

- Example: L. S. Clothiers

  Let:

  A_1 = town council approves the zoning change

  A_2 = town council disapproves the change

  Using subjective judgment:

  $P(A_1) = .7$, $P(A_2) = .3$

New Information

- Example: L. S. Clothiers

The planning board has recommended against the zoning change. Let *B* denote the event of a negative recommendation by the planning board.

Given that *B* has occurred, should L. S. Clothiers revise the probabilities that the town council will approve or disapprove the zoning change?

Conditional Probabilities

- Example: L. S. Clothiers

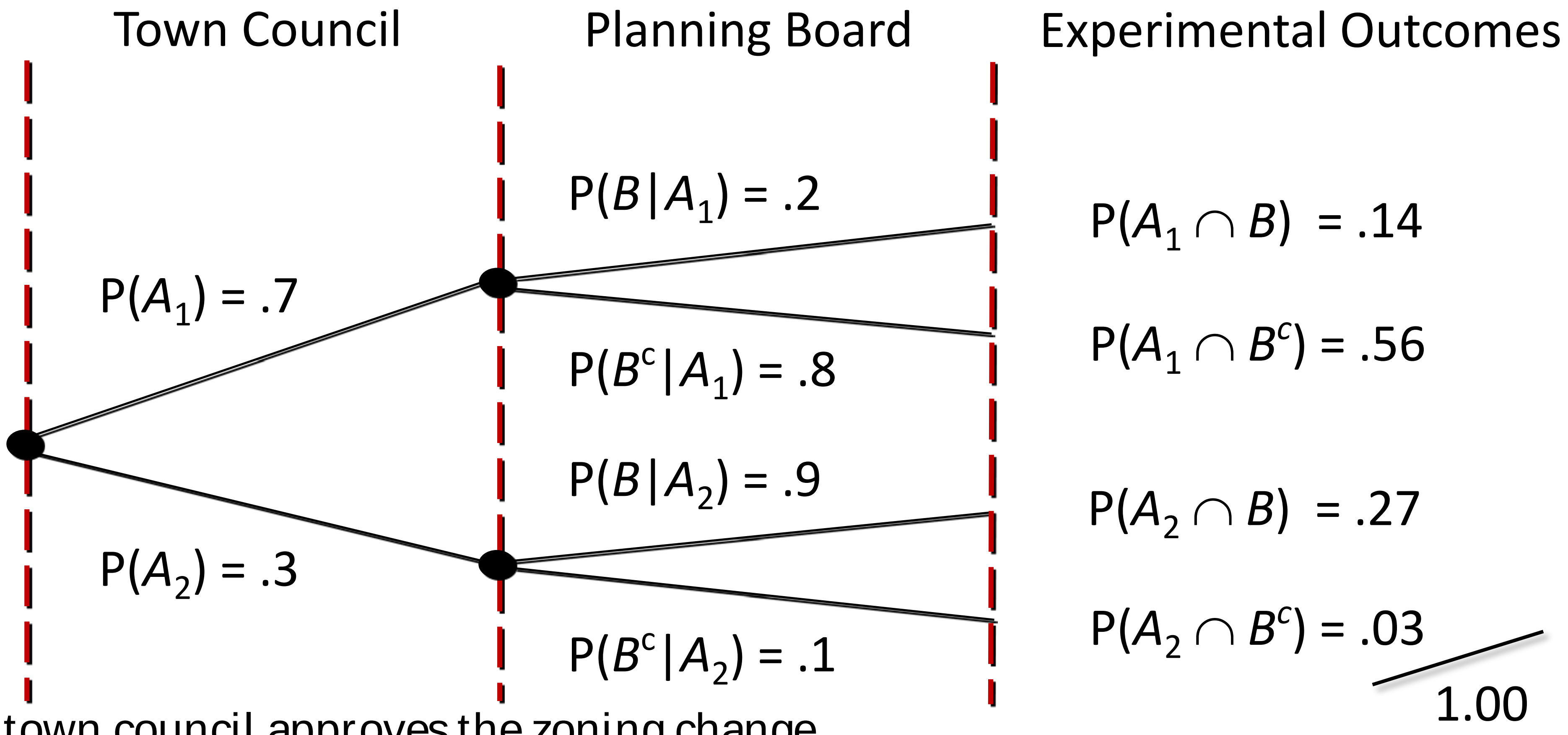
Past history with the planning board and the town council indicates the following:

$$P(B|A_1) = .2 \quad \text{and} \quad P(B|A_2) = .9$$

Hence: $$P(B^C|A_1) = .8 \quad \text{and} \quad P(B^C|A_2) = .1$$

Tree Diagram

- Example: L. S. Clothiers



A_1 = town council approves the zoning change

A_2 = town council disapproves the change

B = a negative recommendation by the planning board

Bayes' Theorem

- To find the posterior probability that event A_i will occur given that event B has occurred, we apply Bayes' theorem.

$$P(A_i|B) = \frac{P(A_i)P(B|A_i)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2) + \cdots + P(A_n)P(B|A_n)}$$

$$\mathrm{P}(B) = \mathrm{P}(A_1 \cap B) + \mathrm{P}(A_2 \cap B) + \ldots + \mathrm{P}(A_n \cap B)$$

- Bayes' theorem is applicable when the events for which we want to compute posterior probabilities are mutually exclusive and their union is the entire sample space.

Posterior Probabilities

- Example: L. S. Clothiers

  Given the planning board's recommendation not to approve the zoning change, we revise the prior probabilities as follows:

$$P(A_1|B) = \frac{P(A_1)P(B|A_1)}{P(A_1)P(B|A_1) + P(A_2)P(B|A_2)}$$

$$= \frac{(.7)(.2)}{(.7)(.2)+(.3)(.9)}$$

$$= .34$$

$$\mathrm{P}(B) = \mathrm{P}(A_1 \cap B) + \mathrm{P}(A_2 \cap B)$$

Posterior Probabilities

- Example: L. S. Clothiers

  The planning board’s recommendation is good news for L. S. Clothiers. The posterior probability of the town council approving the zoning change is .34 compared to a prior probability of .70.

Bayes' Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 1

    Prepare the following three columns:

    <u>Column 1</u> - The mutually exclusive events for which posterior probabilities are desired.

    <u>Column 2</u> - The prior probabilities for the events.

    <u>Column 3</u> - The conditional probabilities of the new information *given* each event.

Bayes' Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 1

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B\|A_i)$	(4)	(5)
A_1	.7	.2		
A_2	.3	.9		
	1.0			

Bayes' Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 2

    Prepare the fourth column

    <u>Column 4</u>

    Compute the joint probabilities for each event and the new information *B* by using the multiplication law.

    Multiply the prior probabilities in column 2 by the corresponding conditional probabilities in column 3. That is, $P(A_i \cap B) = P(A_i)\, P(B|A_i)$.

Bayes' Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 2

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B\|A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5)
A_1	.7	.2	.14 = (.7)(.2)	
A_2	.3	.9	.27	
	1.0			

Bayes’ Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 2 (continued)

    We see that there is a .14 probability of the town council approving the zoning change and a negative recommendation by the planning board.

    There is a .27 probability of the town council disapproving the zoning change and a negative recommendation by the planning board.

Bayes' Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 3

    Sum the joint probabilities in Column 4. The sum is the probability of the new information, $P(B)$. The sum .14 + .27 shows an overall probability of .41 of a negative recommendation by the planning board.

Bayes' Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 3

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B\|A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5)
A_1	.7	.2	.14	
A_2	.3	.9	.27	
	1.0		$P(B) =$ .41	

Bayes' Theorem: Tabular Approach

- Example: L. S. Clothiers
  - Step 4

    Prepare the fifth column:

    <u>Column 5</u>

    Compute the posterior probabilities using the basic relationship of conditional probability.

$$P(A_i|B) = \frac{P(A_i \cap B)}{P(B)}$$

The joint probabilities $P(A_i \text{ I } B)$ are in column 4 and the probability $P(B)$ is the sum of column 4.

Bayes' Theorem: Tabular Approach

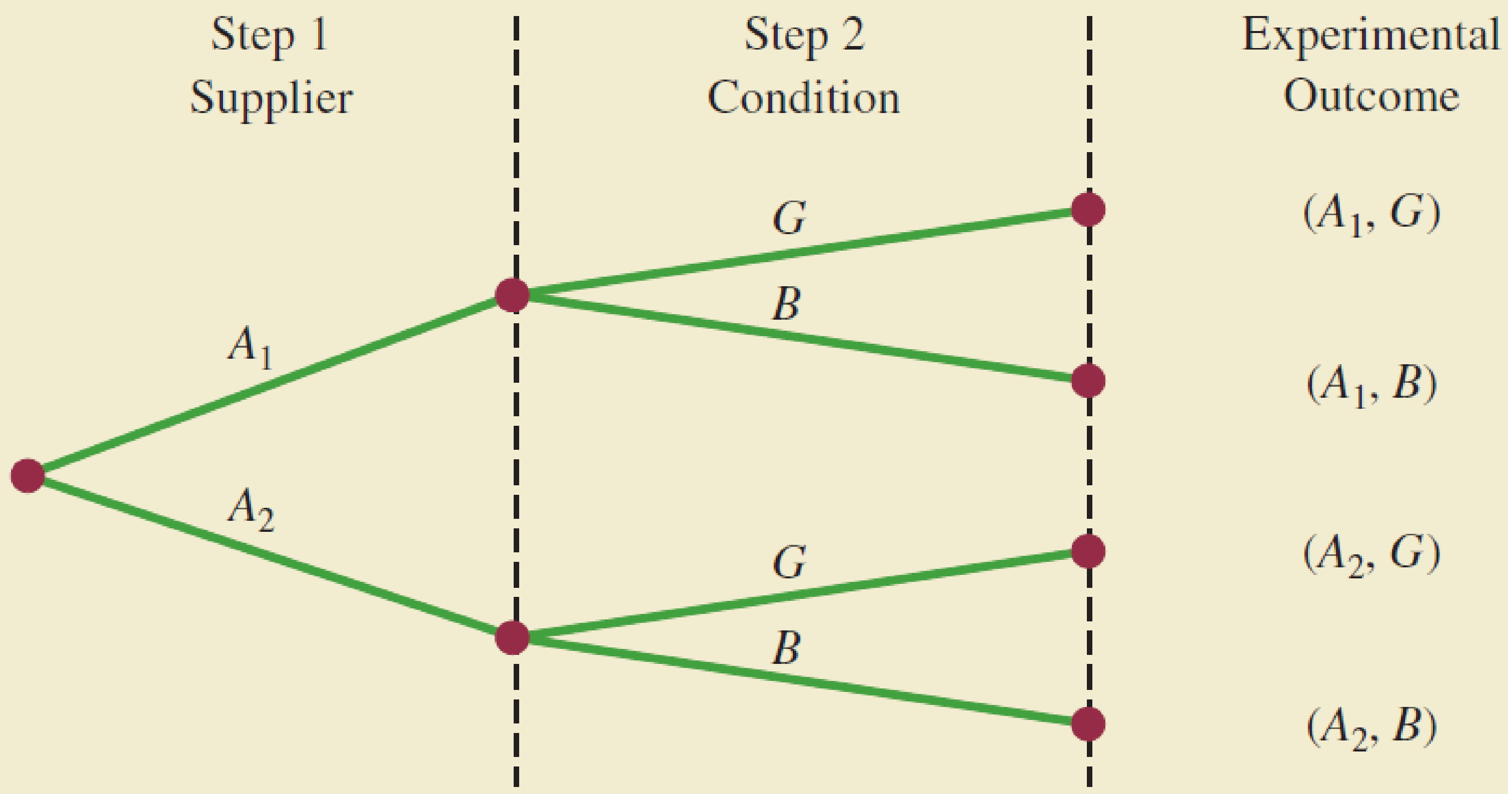
- Example: L. S. Clothiers
  - Step 4

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B\|A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5) Posterior Probabilities $P(A_i\|B)$
A_1	.7	.2	.14	.3415 = .14/.41
A_2	.3	.9	.27	.6585
	1.0		$P(B)$ = .41	1.0000

Historical quality levels of two suppliers

	Percentage Good Parts	Percentage Bad Parts
Supplier 1	98	2
Supplier 2	95	5

FIGURE 4.10 TREE DIAGRAM FOR TWO-SUPPLIER EXAMPLE



Note: Step 1 shows that the part comes from one of two suppliers, and step 2 shows whether the part is good or bad.

FIGURE 4.11 PROBABILITY TREE FOR TWO-SUPPLIER EXAMPLE

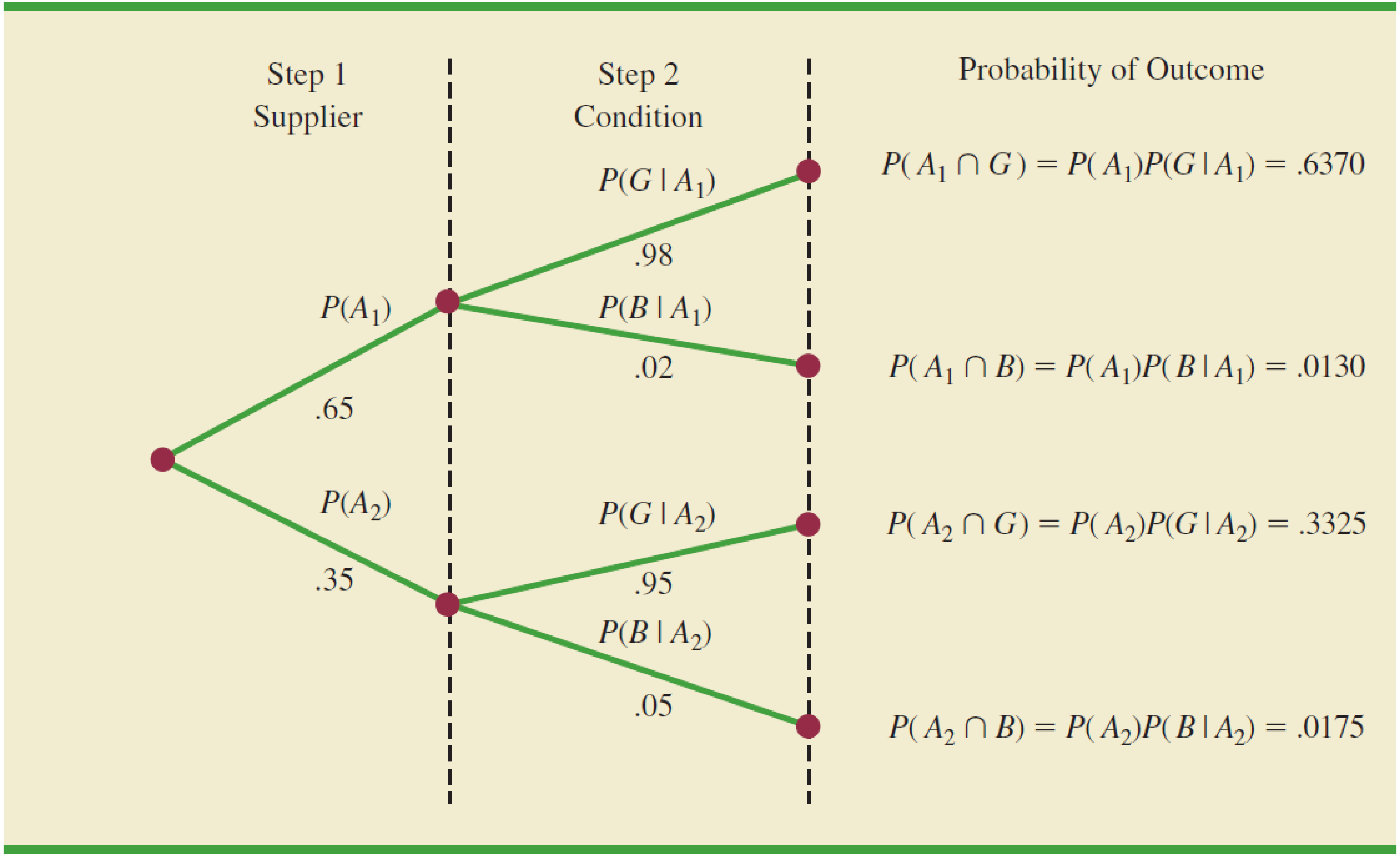


TABLE 4.7 TABULAR APPROACH TO BAYES' THEOREM CALCULATIONS FOR THE TWO-SUPPLIER PROBLEM

(1) Events A_i	(2) Prior Probabilities $P(A_i)$	(3) Conditional Probabilities $P(B \mid A_i)$	(4) Joint Probabilities $P(A_i \cap B)$	(5) Posterior Probabilities $P(A_i \mid B)$
A_1	.65	.02	.0130	.0130/.0305 = .4262
A_2	.35	.05	.0175	.0175/.0305 = .5738
	1.00		$P(B) = .0305$	1.0000

Exercise

The prior probabilities for events A_1 and A_2 are $P(A_1) = .40$ and $P(A_2) = .60$. It is also known that $P(A_1 \cap A_2) = 0$. Suppose $P(B \mid A_1) = .20$ and $P(B \mid A_2) = .05$.

a. Are A_1 and A_2 mutually exclusive?
b. Compute $P(A_1 \cap B)$ and $P(A_2 \cap B)$.
c. Compute $P(B)$.
d. Apply Bayes' theorem to compute $P(A_1 \mid B)$ and $P(A_2 \mid B)$.

In an article about investment alternatives, *Money* magazine reported that drug stocks provide a potential for long-term growth, with over 50% of the adult population of the United States taking prescription drugs on a regular basis. For adults age 65 and older, 82% take prescription drugs regularly. For adults age 18 to 64, 49% take prescription drugs regularly. The 18–64 age group accounts for 83.5% of the adult population (*Statistical Abstract of the United States*, 2008).

a. What is the probability that a randomly selected adult is 65 or older?
b. Given an adult takes prescription drugs regularly, what is the probability that the adult is 65 or older?