Impact of Height and Downtilt of Base Station Antenna for Millimeter-Wave Communication Networks with 3D LoS Probability

Xiang Liu*†, Jing Xu‡, Ruijun Shu* and Yingrui Li*†

*Science and Technology on Microsystem Laboratory,
Shanghai Institute of Microsystem and Information Technology (SIMIT),
Chinese Academy of Sciences (CAS), Shanghai 200050, P. R. China

†University of Chinese Academy of Sciences, Beijing 100049, P.R.China

‡School of Information Science and Technology, East China Normal University, Shanghai 200241, P. R. China

E-mail: {xiang.liu,yingrui.li}@wico.sh, jxu@ce.ecnu.edu.cn, ruijun.shu@mail.sim.ac.cn

***Abstract*—Theoretical result for the system performance of the millimeter-Wave (mmWave) communication networks in 3D environment is still unclear. In this paper, the cell coverage and cell average data rate considering the 3D channel model including large-scale fading, 3D antenna gain and 3D line-of-sight probability function are studied in the paper. The Kullback-Leibler divergence based Gaussian approximation method is proposed, then the distribution of SNR (Signal-to-Noise-Ratio) is modeled as the Gaussian Mixture Model. The cell coverage is expressed as the weighted sum of error functions, which is shown to be mathematically tractable. Numerical investigation indicates the optimal height and downtilt of BS antenna in terms of the cell coverage and cell average data rate.**

I. Introduction

Millimeter-Wave (mmWave) has shown its great potential for providing large data rate due to its large bandwidth [1], [2]. Also, as compared with traditional microwave, the severer propagation attenuation for the higher frequency of mmWave can be mitigated by the deployment of high dimensional antenna arrays due to its small wave length [1], [3], [4].

Measurements at 28GHz and 73GHz [5] show that mmWave links are more sensitive to the blockages. Hence, for the mmWave communication networks, it is necessary to study the propagation attenuation model with the consideration of both LoS and NLoS transmission [6]. Based on the real-world measurements [2], the LoS probability for the 2D environment are modelled as the exponential function of the distance between the BS and UE. The positions and size of obstructions in 2D [7], [8] and 3D [9] environment are modeled as random variables based on the stochastic geometry, and then the LoS probability functions are presented.

For the analysis of system performance, the cell coverage and cell average data rate are studied in [10], [11] based on the stochastic geometry in 2D environment, where the BS antenna height is ignored. In [12], [13], the performance impact of BS antenna height is studied, in which the propagation attenuation model is well characterized for the 3D environment. However, the LoS probability function adopted in [12], [13] still follows from that of the 2D environment, where the 2D LoS probability decreases with the increase of the distance between the BS and UE. Impact of elevated of BS for the ultra-dense networks is analyzed in [14], where the height of BS in 3D LoS probability is considered. However, the 3D LoS probability adopted in [14] is obtained based on the curve fitting method in [15], the rationality for using the model has not been justified. Therefore, the system performance including the cell coverage and cell average data rate considering the 3D LoS probability and parameters of the environment have not been revealed.

In this paper, the cell coverage and cell average data rate in 3D environment are studied, and the impact of the BS antenna height and downtilt of the BS antenna is also investigated. The main contributions of our work are as follows:

- The distribution of SNR is well approximated by the weighted sum of Gaussian functions based on the proposed approximation method. Moreover, it is mathematically tractable for further analysis.
- The theoretical model of system performance including cell coverage and cell average data rate for the 3D environment is presented.

The paper is organized as follows. Section II briefly introduces the system model and problem formulation. The approximate method for the distribution of SNR and closed-form expression of the cell coverage are proposed in Section III. Numerical results for the performance of the proposed approximate method, the impact of the BS antenna height and downtilt of BS antenna are illustrated in Section IV. Conclusions are drawn in Section V.

II. System Model and Problem Formulation

The system model including the network geometry and channel model are introduced in the section.

A. Network Geometry

We consider a 3D network structure with downlink transmission. As shown in Fig. 1, the positions of the base station (BS) and user equipment (UE) are depicted. The height of the

BS antenna and UE are denoted as h'_B and h'_U, respectively. Let $h_B = h'_B - h'_U$ denote the height difference between the BS antenna and UE. The distance between the serving BS and UE is denoted as R. Using the polar coordinate system, the position of UE in the 2D plane is denoted as (R, Θ). Since user is random located in the cell, R and Θ are random variables.

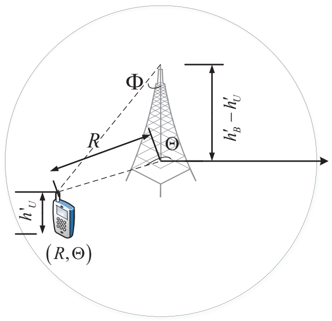


Fig. 1. Cell layout Geometry

It is assumed that UEs are uniformly distributed in the cell of the 2D plane, then the distribution of the 2D distance between the BS and UE is expressed as [16]

$$f_R\left(R = r\right) = \frac{2r}{R_c^2 - R_0^2}, R_0 \leqslant r \leqslant R_c, \tag{1}$$

$$f_\Theta\left(\Theta = \theta\right) = \frac{1}{2\pi}, 0 \leqslant \theta \leqslant 2\pi \tag{2}$$

where R_c and R_0 represent the radius of the cell and minimum distance between the UE and BS. Φ is the elevation angle from the BS to the UE.

B. Propagation Attenuation Model

In this paper, the large-scale fading including the distance-dependent path loss and shadowing are considered. For the mmWave communication links, the path loss models for the line-of-sight(LoS) and None-Line-of-sight(NLoS) states are greatly different. Therefore, the two-state path loss model is proposed in [14], and given the user position (R, Θ), the path loss L in dB is expressed as

$$L\left(R, \Theta\right) = \begin{cases} L_{LoS}\left(R\right), \text{with probability} P_{LoS}\left(R\right) \\ L_{NLoS}\left(R\right), \text{with probability} P_{NLoS}\left(R\right), \end{cases}$$

where $L_{LoS}\left(R\right)$ and $L_{NLoS}\left(R\right)$ represent the propagation attenuation including the distance-dependent path loss and shadowing for the line-of-sight (LoS) and none-line-of-sight (NLoS) transmission link, respectively. According to [6], [12], the propagation attenuation is expressed as the function of the transmission distance between the BS and UE, which is

$$L_K = \alpha_K + 0.5\xi^{-1}\beta_K \ln\left(R^2 + h_B^2\right) + S_K, \tag{3}$$

where α_K and β_K represent the distance-dependent path loss at the reference distance and the path loss exponent given the transmission state $K \in \{LoS, NLoS\}$, respectively, and $\xi = \frac{\ln 10}{10}$. $S_K \sim \mathcal{N}\left(0, \sigma_K^2\right)$ denotes the shadowing fading which is modeled as the zeor-mean Gaussian distributed random variable with variance σ_K^2 in dB domain. σ_K^2 is assumed to be independent of the distance between the BS and UE [6].

Moreover, $P_{LoS}\left(R\right)$ and $P_{NLoS}\left(R\right) = 1 - P_{LoS}\left(R\right)$ represent the probability of the LoS and NLoS transmission given the position of the UE, respectively. For the 3D environment, the height difference of the BS and UE accounts for the important factor for the LoS probability. Based on the stochastic geometry, the positions of buildings are modeled as the Poisson Point Process (PPP) in [7], [8], then the analytical 2D LoS probability is obtained. Leveraging the obstructions modeling method of random shape theory from [7], the 3D LoS probability has been obtained in [9]

$$P_{LoS}\left(R\right) = \exp\left(-a\frac{R}{h_B} - b\right), \tag{4}$$

where

$$\begin{cases} a &= \lambda_B \frac{\left(\bar{W} + \bar{L}\right)\left(h_H + h_L\right)}{\pi} \\ b &= \lambda_B \bar{W}\bar{L}, \end{cases} \tag{5}$$

λ_B, $\bar{W}$ and $\bar{L}$ represent the density, expected width and expected length of the random buildings, respectively. h_H and h_L are the maximum and minimum height difference between the random buildings and UE, respectively. Note that the increase of the BS antenna height may lead to the increase of the LoS probability as shown in (4), which is in good agreement with the real-world measurement [6].

Let N denote the transmission state, i.e., $N = 1$ and $N = 2$ denote the LoS and NLoS transmission state, respectively. Given the user position R, the probability for the transmission state N can be rewritten as

$$P\left(N = 1 | R\right) = P_{LoS}\left(R\right), \tag{6}$$

$$P\left(N = 2 | R\right) = P_{NLoS} = 1 - P_{LoS}\left(R\right), \tag{7}$$

then the distribution of N can be expressed as

$$f_{N|R}\left(N = n | R\right) = \sum_{i=1}^{2} \delta\left(n - i\right) P_N\left(N = i | R\right), \tag{8}$$

where $\delta\left(x\right)$ is a Dirac delta function.

In (3), the propagation attenuation L given the position of the UE R and the transmission state N is modeled as the Gaussian distributed random variable, whose mean is the distance-dependent path loss and variance equals to the variance of the shadowing, then the distribution of L is expressed as

$$\begin{aligned} &f_{L|N,R}\left(l | N = n, R = r\right) \\ &= \frac{1}{\sqrt{2\pi\sigma_n^2}} \exp\left(-\frac{\left(l - \alpha_n - 0.5\xi^{-1}\beta_n \ln\left(r^2 + h_B^2\right)\right)^2}{2\sigma_n^2}\right), \end{aligned} \tag{9}$$

According to (1), (8) and (9), the distribution of R, distribution of N given R and distribution of L given R and N has been obtained.

C. Antenna Gain

According to [6], the antenna gain of BS in dB is expressed as

$$G_B(\Phi,\Theta) = G_h(\Theta) + G_v(\Phi), \tag{10}$$

where $G_h(\Theta)$ and $G_v(\Phi)$ represent the horizontal and vertical antenna gain, respectively. In this paper, horizontal antenna gain is assumed to be invariant with Θ, i.e., $G_h(\Theta) \equiv G_h$. The vertical antenna gain is expressed as

$$G_v(\Phi) = -\min\left(12\left(\frac{\Phi - \varphi_{tilt}}{\varphi_{3dB}}\right)^2, A_m\right), \tag{11}$$

where $\varphi_{tilt} = 65\pi/180$ represents the electric downtilt of the BS antenna, φ_{3dB} and $A_m = 30$ dB represent the 3dB beamwidth in radians and the maximum antenna attenuation in dB, respectively. Since $0 \le \Phi = \arctan\frac{R}{h_B} \le \pi/2$, we can write

$$G_v(\Phi) = G_v(R) = -12\left(\frac{\arctan\frac{R}{h_B} - \varphi_{tilt}}{\varphi_{3dB}}\right)^2. \tag{12}$$

Moreover, we assume that the UE is equipped with an isotropic antenna, and the antenna gain for the UE is denoted as G_U. Since the antenna gain is independent of transmission state N, the distribution of antenna gain G including the BS antenna and UE antenna given the user position R and transmission state N is expressed as

$$\begin{aligned} &f_{G|R,N}(g|r,n) \\ &= \delta\left(g - G_h + 12\left(\frac{\arctan\frac{r}{h_B} - \varphi_{tilt}}{\varphi_{3dB}}\right)^2 - G_U\right). \end{aligned} \tag{13}$$

D. Problem Formulation

The received power Y of the UE can be expressed as

$$Y = P_t + G - L, \tag{14}$$

where P_t represents the transmitter power of the BS. Give the system network geometry and channel model as described in Section II-A and Section II-B, the distribution for the received power given the transmission state is expressed as

$$\begin{aligned} &f_{Y|N}(y|\, N = n) \\ &= \frac{\int_{R_0}^{R_c} f_{Y|R,N}(y|r,n)\, f_{N|R}(n|r)\, f_R(r)\, dr}{\int_{R_0}^{R_c} f_{N|R}(n|r)\, f_R(r)\, dr}, \end{aligned} \tag{15}$$

where $f_{Y|R,N}(y|r,n)$ denotes the distribution of Y given user position R and transmission state N, it can be calculated as

$$\begin{aligned} &f_{Y|R,N}(y|r,n) \\ &= \int_{-\infty}^{+\infty} f_{G|R,N}(g|r,n)\, f_{L|R,N}(g - y + P_t|r,n)\, dg. \end{aligned} \tag{16}$$

Further, the distribution of Y is expressed as

$$f_Y(y) = P(1)\, f_{Y|N}(y|\, 1) + P(2)\, f_{Y|N}(y|\, 2), \tag{17}$$

where

$$\begin{aligned} &P(1) = P(N=1) = \int_{R_0}^{R_c} f_{N|R}(n|r)\, f_R(r)\, dr \\ &= \frac{2e^{-b}h_B^2 a^{-2}}{R_c^2 - R_0^2}\left(\left(\frac{aR_0}{h_B}+1\right)e^{-\frac{aR_0}{h_B}} - \left(\frac{aR_c}{h_B}+1\right)e^{-\frac{aR_c}{h_B}}\right), \\ &P(2) = P(N=2) = 1 - P(N=1). \end{aligned}$$

Since the distribution of Y as shown in (17) is too complicated for the further analysis, the approximate expression for the distribution of Y is provided in the next section. The analytical results for the system performance including the cell coverage and cell average data rate are also presented.

III. Gaussian Mixture Distribution for Receiving Power

As shown in (17), the closed-form expression for the distribution of Y cannot be obtained directly. In the section, the approximate method is introduced, then the distribution of Y is approximated by the weighted sum of two Gaussian functions.

A. Distribution of received power

The distribution of Y given the transmission state $N = 1$ is expressed as

$$\begin{aligned} &f_{Y|N}(y|\, N=1) \\ &= \int_{R_0}^{R_c} \frac{2re^{-\frac{ar}{h_B}-b}\left(R_c^2-R_0^2\right)^{-1}}{\left(2\pi\sigma_1^2\right)^{0.5}P(1)} \exp\left(-\frac{\left(y-P_{y_1}\right)^2}{2\sigma_1^2}\right)dr, \end{aligned} \tag{18}$$

where

$$\begin{aligned} &P_{y_1} = G_v(r) + G_h + G_U - \alpha_1 - 0.5\beta_1\xi^{-1}\ln\left(r^2 + h_B^2\right) \\ &= G_h - \frac{12}{\varphi_{3dB}^2}\left(\left(\arctan\frac{r}{h_B}\right)^2 - 2\varphi_{tilt}\arctan\frac{r}{h_B} + \varphi_{tilt}^2\right) \\ &+ G_U - 0.5\xi^{-1}\beta_1\left(\ln\left(\frac{r^2}{h_B^2}+1\right) + 2\ln h_B\right) - \alpha_1. \end{aligned} \tag{19}$$

Furthermore, function in (19) can be approximated by the second-order series expansion based on the logarithm function

$$\begin{cases} f_0(x) = \arctan x & \approx c_{0,0} + c_{0,1}\ln x + c_{0,2}(\ln x)^2 \\ f_1(x) = (\arctan x)^2 & \approx c_{1,0} + c_{1,1}\ln x + c_{1,2}(\ln x)^2 \\ f_2(x) = \ln\left(x^2+1\right) & \approx c_{2,0} + c_{2,1}\ln x + c_{2,2}(\ln x)^2, \end{cases}$$

where the parameter $c_{i,j}$ can be obtained by solving the following optimization problem.

$$\begin{aligned} &(c_{i,0}, c_{i,1}, c_{i,2}) = \\ &\underset{c_{i,0},c_{i,1},c_{i,2}}{\arg\min} \int_0^{+\infty} \begin{pmatrix} f_i(x) - c_{i,0} - c_{i,1}\ln(x) \\ -c_{i,2}(\ln(x))^2 \end{pmatrix}^2 dx. \end{aligned} \tag{20}$$

Further, the distribution of the received power is expressed as

$$f_{Y|N}\left(y|\,1\right) \approx \tilde{f}_{Y|N}\left(y|\,1\right) = \int_{R_0}^{R_c} \frac{\exp\left(-\frac{\left(y-\tilde{P_{y_1}}\right)^2}{2\sigma_1^2}\right) e^{-\frac{ar}{h_B}-b} 2r}{\left(2\pi\sigma_1^2\right)^2 P\left(1\right)\left(R_c^2-R_0^2\right)} dr, \tag{21}$$

where

$$\begin{cases} \tilde{P_{y_1}} &= d_0 + d_1 \ln \frac{r}{h_B} + d_2 \left(\ln \frac{r}{h_B}\right)^2 \\ d_0 &= P_t + G_h + G_U - \frac{12}{\varphi_{3dB}^2}\left(c_{1,0} - 2\varphi_{tilt} c_{0,0} + \varphi_{tilt}^2\right) \\ & -0.5\xi^{-1}\beta_1\left(c_{2,0} + 2\ln h_B\right) \\ d_1 &= -\frac{12}{\varphi_{3dB}^2}\left(c_{1,1} - 2\varphi_{tilt} c_{0,1}\right) - 0.5\xi^{-1}\beta_1 c_{2,1} \\ d_2 &= -\frac{12}{\varphi_{3dB}^2}\left(c_{1,2} - 2\varphi_{tilt} c_{0,2}\right) - 0.5\xi^{-1}\beta_1 c_{2,2}. \end{cases}$$

Then the distribution of Y given the transmission state $N = 1$ can be approximated by the Gaussian function based on the following lemma.

Lemma 1: The optimal Gaussian approximation for the distribution $f_{Y|N}\left(y|\,N=1\right)$ based on Kullback-Leibler (K-L) divergence [17] is

$$f_{Y|N}\left(y|\,N=1\right) \approx \mathcal{N}\left(x; \mu_{Y_1}, \sigma_{Y_1}^2\right), \tag{22}$$

where

$$\begin{cases} \mu_{Y_1} &= d_0 + \frac{2h_B^2 e^{-b}\left(d_1 F_1\left(R_c, R_0\right) + d_2 F_2\left(R_c, R_0\right)\right)}{\left(R_c^2 - R_0^2\right) P(1)} \\ \sigma_{Y_1}^2 &= d_0^2 + \sigma_1^2 + 2d_0 d_1 F_1\left(R_c, R_0\right) \\ & + \left(d_1^2 + 2d_0 d_2\right) F_2\left(R_c, R_0\right) \\ & +2d_1 d_2 F_3\left(R_c, R_0\right) + d_2^2 F_4\left(R_c, R_0\right) \\ F_i\left(R_c, R_0\right) &= F_i\left(\frac{R_c}{h_B}\right) - F_i\left(\frac{R_0}{h_B}\right), \end{cases} \tag{23}$$

where $F_i\left(x\right) = \int_0^x \left(\ln t\right)^i \exp\left(-at\right) t dt$, and the closed-form expression for $F_i\left(x\right)$ can be obtained according to [18].

Proof: See Appendix A. ∎

Based on the Lemma 1, the distribution of received power given the transmission state $N = 1$ can be approximated by the Gaussian function. Further, following the same procedure, the distribution of received power given the transmission state $N = 2$ can also be approximated by the Gaussian function with mean μ_{Y_2} and variance $\sigma_{Y_2}^2$. Then the distribution of received power as shown in (17) can be approximated by the weighted sum of Gaussian functions

$$f_Y\left(y\right) = P\left(1\right)\mathcal{N}\left(y; \mu_{Y_1}, \sigma_{Y_1}^2\right) + P\left(2\right)\mathcal{N}\left(y; \mu_{Y_2}, \sigma_{Y_2}^2\right). \tag{24}$$

Let N_0 denote the noise power, then the distribution of SNR is expressed as

$$f_{\text{SNR}}\left(w\right) = f_Y\left(y - N_0\right) = P\left(1\right)\mathcal{N}\left(w; \mu_{Y_1} - N_0, \sigma_{Y_1}^2\right) + P\left(2\right)\mathcal{N}\left(w; \mu_{Y_2} - N_0, \sigma_{Y_2}^2\right). \tag{25}$$

B. System Performance

The cell coverage is defined as the probability that the SNR is greater than the threshold T', i.e., $P_c\left(T'\right) = P\left(\text{SNR} > \text{T}'\right)$. Let $T = T' + T_0$. Based on the approximate distribution of the received power, the cell coverage can be approximated as

$$P_c\left(T'\right) = \int_{-\infty}^{T} f_Y\left(y\right) dy \approx \frac{P\left(1\right)}{2}\left(1 + \text{erf}\left(\frac{T - \mu_{Y_1}}{\sigma_{Y_1}}\right)\right) + \frac{P\left(2\right)}{2}\left(1 + \text{erf}\left(\frac{T - \mu_{Y_2}}{\sigma_{Y_2}}\right)\right). \tag{26}$$

Based on the Shannon's Formula, the approximate cell average data rate Z can be calculated as

$$Z = \mathbb{E}\left(\ln\left(1 + \exp\left(\xi Y - \xi N_0\right)\right)\right) = \int_{-\infty}^{+\infty} \ln\left(1 + \exp\left(\xi Y - \xi N_0\right)\right) f_Y\left(y\right) dy. \tag{27}$$

By replacing $f_Y\left(y\right)$ with the weighted sum of Gaussian functions as shown in (24), the approximate cell average data rate is obtained.

IV. Numerical Results

In the section, the numerical results are illustrated to verify the accuracy of the proposed approximation method. The impact of the BS antenna height and downtilt angle of the BS vertical antenna are also investigated. The system parameters [5] are given in Table I.

TABLE I
SETTING OF SYSTEM PARAMETERS

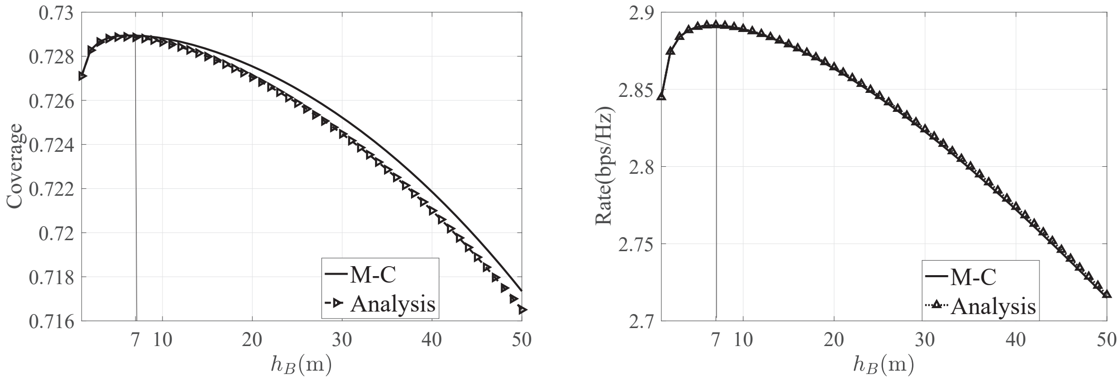
System Parameter	Value
Cell radius	500m
Minimum 2D distance between UEs and the BS	10m
Height of the BS	1m $\sim$ 50m
Path loss at the reference distance for LoS / NLoS	61.4dB / 72dB
Attenuation factor for LoS / NLoS	2 / 2.92
Standard deviation for shadowing of LoS / NLoS	5.8dB / 8.7dB
3dB beam-width in radius	$\varphi_{3dB} = 65\pi/180$
Maximum antenna attenuation	$A_m = 30$ dB

A. Performance of Approximation

The performance for the proposed approximate method is illustrated as shown in Fig. 2. For the different β_1, the approximate distribution perfect match with the Monte Carlo (M-C) simulations. Further, the K-L divergence for the different parameters are as given in Table II, and the performance of the approximation is presented.

B. Impact of BS height

The impact of the BS height with BS antenna donwtilt angle $\varphi_{tilt} = \pi/2$ is illustrated in this subsection, the cell coverage with SNR threshold $T = -10$ dB and cell average data rate for the different height differences between the BS and UE are depicted as shown in Fig. 3(a) and Fig. 3(b), respectively. In Fig. 3(a), the coverage first increases with h_B, and then decreases with h_B.



(a) Cell coverage.

(b) Cell average data rate.

Fig. 3. System Performance versus the height different between the BS and UE.

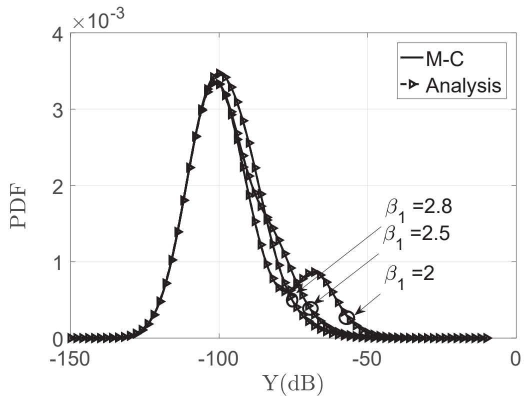


Fig. 2. PDF of Y versus β_1

TABLE II
KL-DIVERGENCE FOR THE DIFFERENT PATH LOSS EXPONENT

β_1	KL-divergence
2	1.14e-6
2.2	1.10e-6
2.4	9.19e-7
2.6	8.05e-7
2.8	9.70e-7

According to the analysis of 3D LoS probability in [9], the increase of h_B may lead to the increase of LoS probability. For the small h_B, with the increase of h_B, the NLoS transmission links may transform into the LoS transmission. This leads to the increases of the cell coverage. However, the increase of h_B may also lead to the increase of transmission distance. This leads to the increase of the propagation loss. Hence, for the large h_B, the impact of the propagation attenuation increase dominate the tendency of system performance, then the cell coverage decreases. Based on the simulation, the best deployment of the BS height can be obtained for the cell coverage.

According to the Fig. 3, we can the draw same conclusions as for the cell coverage. Further, the approximate results perfect match with the M-C simulations. Hence, according to Fig. 3(a) and Fig. 3 The optimal height for the BS antenna can be obtained in terms of the cell coverage and cell average data rate.

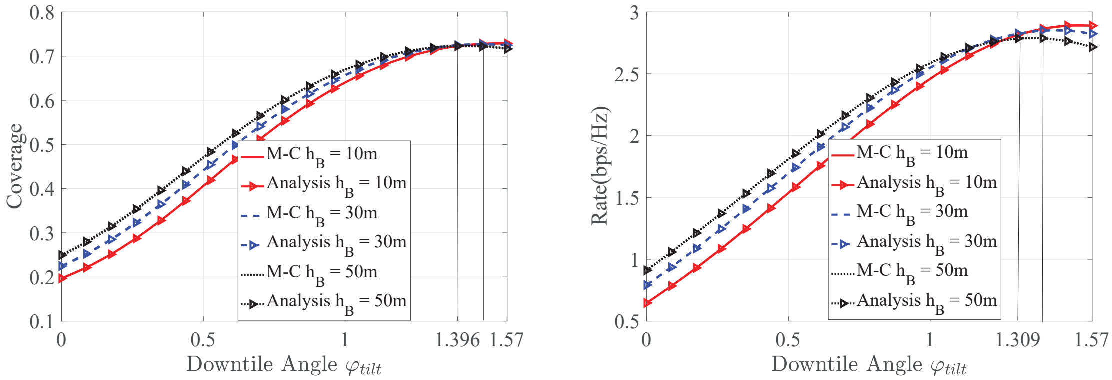
C. Impact of Downtilt of BS Antenna

The impact of the downtilt for the BS's vertical antenna is illustrated in this subsection. More specifically, the cell coverage and the cell average data rate versus the downtilt are depicted as shown in Fig. 4(a) and Fig. 4(b), respectively. The results of M-C simulation match well with the analysis.

As shown in Fig. 4, the optimal downtilt angle decreases with the increase of the BS antenna height. According to (12), with the increase of the BS antenna downtilt, the antenna gain may increase for cell edge users but decrease for the users near the BS. Since antenna gain can be calculated according to (12), that is, the increase of antenna height may result in the decrease of antenna downtilt angle, the decrease of antenna gain can be mitigated by the decrease of the BS antenna downtilt.

V. CONCLUSION

In this paper, the system performance for 3D environment including the cell coverage and cell average data rate are studied. The K-L divergence based Gaussian approximation method are proposed for the analysis of the received power, where the 3D LoS probability and antenna gain are considered. Based on the proposed method, the distribution of the received power is approximated by the sum of two Gaussian functions. Further, the numerical results verified the performance of the proposed method. Simulation results indicate the optimal height and downtilt of the BS antenna in terms of the cell coverage and cell average data rate.



(a) Cell coverage.

(b) Cell average data rate.

Fig. 4. System Performance versus the downtilt of BS's vertical antenna.

APPENDIX A
PROOF OF LEMMA 1

Let $g_Y(y) = \mathcal{N}(\mu, \sigma^2)$. The optimal Gaussian approximation of $\tilde{f}_{Y|N}(y|1)$ can be obtained by solving the optimization problem

$$\left(\mu_{Y_1}, \sigma_{Y_1}^2\right) = \arg\min_{\mu,\sigma^2} \left\{ KL\left(\tilde{f}_{Y|N}(y|1) \| g_Y(y)\right)\right\}. \quad (28)$$

According to the optimization theory, we have

$$\begin{cases} \left.\dfrac{\partial\left\{KL\left(\tilde{f}_{Y|N}(y|1)\|g_Y(y)\right)\right\}}{\partial\mu}\right|_{\mu=\mu_{Y_1}} = 0 \\ \left.\dfrac{\partial\left\{KL\left(\tilde{f}_{Y|N}(y|1)\|g_Y(y)\right)\right\}}{\partial\sigma^2}\right|_{\sigma^2=\sigma_{Y_1}^2} = 0. \end{cases}$$

We have $\mu_{Y_1} = \mu_{\tilde{f}_{Y_1}}, \sigma_{Y_1}^2 = \sigma_{\tilde{f}_{Y_1}}^2$. Then the mean and the variance can be calculated as shown in (23). Its Hessian matrix is positive definite

$$\begin{bmatrix} \frac{\partial^2 KL'}{\partial\mu^2} & \frac{\partial^2 KL'}{\partial\mu\partial\sigma^2} \\ \frac{\partial^2 KL'}{\partial\sigma^2\partial\mu} & \frac{\partial^2 KL'}{\partial(\sigma^2)^2} \end{bmatrix}_{\mu=\mu_{Y_1},\sigma^2=\sigma_{Y_1}^2} \succ 0,$$

where $KL' = KL\left(\tilde{f}_{Y|N}(y|1) \| g_Y(y)\right)$. Hence, the optimization problem (28) can be solved and the solution is shown in (23).

ACKNOWLEDGEMENT

This work was supported by the National Key R&D Program of China (Grant No. 2017YFE0121500) and National Natural Science Foundation of China (Grant No. 61571303).

REFERENCES

[1] J. G. Andrews, S. Buzzi, W. Choi, S. V. Hanly, A. Lozano, A. C. K. Soong, and J. C. Zhang, "What will 5G be?" *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1065–1082, Jun. 2014.

[2] T. S. Rappaport, G. R. MacCartney, S. Sun, H. Yan, and S. Deng, "Small-Scale, local area, and transitional millimeter mave propagation for 5G communications," *IEEE Transactions on Antennas and Propagation*, vol. 65, no. 12, pp. 6474–6490, Dec. 2017.

[3] J. G. Andrews, T. Bai, M. N. Kulkarni, A. Alkhateeb, A. K. Gupta, and R. W. Heath, "Modeling and analyzing millimeter wave cellular systems," *IEEE Transactions on Communications*, vol. 65, no. 1, pp. 403–430, Jan. 2017.

[4] T. Bai and R. W. Heath, "Coverage and rate analysis for millimeter-wave cellular networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 2, pp. 1100–1114, Feb. 2015.

[5] M. R. Akdeniz, Y. Liu, M. K. Samimi, S. Sun, S. Rangan, T. S. Rappaport, and E. Erkip, "Millimeter wave channel modeling and cellular capacity evaluation," *IEEE Journal on Selected Areas in Communications*, vol. 32, no. 6, pp. 1164–1179, Jun. 2014.

[6] 3GPP TR36.873, "Study on 3D channel model for LTE, v12.2.0," Mar. 2014.

[7] T. Bai, R. Vaze, and R. W. Heath, "Analysis of blockage effects on urban cellular networks," *IEEE Transactions on Wireless Communications*, vol. 13, no. 9, pp. 5070–5083, Sep. 2014.

[8] I. Atzeni, J. Arnau, and M. Kountouris, "Downlink cellular network analysis with LoS/NLoS propagation and elevated base stations," *IEEE Transactions on Wireless Communications*, vol. 17, no. 1, pp. 142–156, Jan. 2018.

[9] X. Liu, J. Xu, and H. Tang, "Analysis of frequency-dependent line-of-sight probability in 3-d environment," *IEEE Communications Letters*, vol. 22, no. 8, pp. 1732–1735, Aug. 2018.

[10] M. Ding, P. Wang, D. Lpez-Prez, G. Mao, and Z. Lin, "Performance impact of LoS and NLoS transmissions in dense cellular networks," *IEEE Transactions on Wireless Communications*, vol. 15, no. 3, pp. 2365–2380, Mar. 2016.

[11] M. D. Renzo, "Stochastic geometry modeling and analysis of multi-tier millimeter wave cellular networks," *IEEE Transactions on Wireless Communications*, vol. 14, no. 9, pp. 5038–5057, Sep. 2015.

[12] M. Ding and D. Lpez-Prez, "Performance impact of base station antenna heights in dense cellular networks," *IEEE Transactions on Wireless Communications*, vol. 16, no. 12, pp. 8147–8161, Dec. 2017.

[13] H. Wu, N. Zhang, Z. Wei, S. Zhang, X. Tao, X. Shen, and P. Zhang, "Content-aware cooperative transmission in HetNets with consideration of base station height," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 7, pp. 6048–6062, Jul. 2018.

[14] H. Cho, C. Liu, J. Lee, T. Noh, and T. Q. S. Quek, "Impact of elevated base stations on the ultra-dense networks," *IEEE Communications Letters*, vol. 22, no. 6, pp. 1268–1271, Jun. 2018.

[15] A. Al-Hourani, S. Kandeepan, and S. Lardner, "Optimal lap altitude for maximum coverage," *IEEE Wireless Communications Letters*, vol. 3, no. 6, pp. 569–572, 2014.

[16] X. Liu, J. Xu, Y. Pei, and Y. Liang, "Gaussian mixture model for millimeter-wave cellular communication networks," *IEEE Transactions on Vehicular Technology*, vol. 68, no. 4, pp. 3174–3188, April 2019.

[17] S. Kullback and R. A. Leibler, "On information and sufficiency," *Annals of Mathematical Statistics*, vol. 22, no. 22, pp. 79–86, 1951.

[18] I. S. Gradshteyn and I. M. Ryzhik, *In Table of Integrals, Series, and Products*. ACADEMIC, 1980.