股票现金流敏感性

一．论文题目：Contingent-Claim-Based Expected Stock Returns

二．论文内容简介：

1. 建立模型——contingent-claims-model

1.1 假设条件：产生现金流的资产在无套利市场上持续不断地被交易

i 公司 t 时刻的 observable operating rate of cash flows r_{it}^{X} 服从以下几何布朗运动：

$$r_{it}^{X} \equiv \frac{dX_{it}}{X_{it}} = \hat{\mu}_i dt + \sigma_i d\hat{W}_t,$$

股东股利：

$$D_{it} = \theta(X_{it} - C_i)(1 - \tau_{eff})$$

1.2 实物期权：The firm has an option to default, which leads to either immediate liquidation or debt renegotiation.

$$S_{it}(X_{iB}) = \eta(\alpha - \kappa)\frac{X_{iB}}{r - \mu_i},$$

$$\frac{\partial S_{it}}{\partial X_{it}}\Big|_{X_{it}=X_{iB}} = \eta(\alpha - \kappa)\frac{1}{r - \mu_i}.$$

1.3 命题1及其证明

Proposition 1 *For $X_{it} \geq X_{iB}$, the instantaneous stock return r_{it}^{M} of a firm, i, at time t is*

$$r_{it}^{M} = rdt + \epsilon_{it}(r_{it}^{X} - \mu_i dt), \tag{4}$$

where ϵ_{it} is the sensitivity of stock to cash flows

$$\begin{aligned}\epsilon_{it} =& \frac{X_{it}\partial S_{it}}{S_{it}\partial X_{it}} \\ =& 1 + \underbrace{\frac{C_i/r}{S_{it}}\theta(1-\tau_{eff})}_{Financial\ leverage\ (+)} - \underbrace{\frac{(1-\omega_i)}{S_{it}}\left[\frac{C_i}{r}\theta + \frac{X_{iB}}{r-\mu_i}(\eta(\alpha-\kappa)-\theta)\right](1-\tau_{eff})\pi_{it}}_{Option\ to\ go\ bankrupt\ (+)},\end{aligned} \tag{5}$$

证明如下：

Define a new Brownian motion

$$W_{it} = \hat{W}_{it} + \int_0^t \theta_i(s)ds, \tag{A1}$$

where $\theta_i = \lambda_i/\sigma_i$ is the price of risk. Girsanov's theorem states that, under a risk-neutral measure, the operating income X_{it} is governed by

$$\frac{dX_{it}}{X_{it}} = \mu_i dt + \sigma_i dW_{it}. \tag{A2}$$

$S = S(X,t)$下引入Ito公式和BS方程：

$$\frac{dS}{S} = \frac{1}{S}\left(\frac{\partial S}{\partial t} + \mu X\frac{\partial S}{\partial X} + \frac{\sigma^2}{2}X^2\frac{\partial^2 S}{\partial X^2}\right)dt + \frac{1}{S}X\sigma\frac{\partial S}{\partial X}dW$$

$$\frac{\partial S}{\partial t} + \mu X\frac{\partial S}{\partial X} + \frac{\sigma^2}{2}X^2\frac{\partial^2 S}{\partial X^2} - rS + D = 0.$$

带入化简：

$$\frac{dS + Ddt}{S} - rdt = \frac{X}{S}\frac{\partial S}{\partial X}\left(\frac{\partial X}{X} - \mu dt\right) = \epsilon\left(\frac{\partial X}{X} - \mu dt\right)$$

根据偏微分理论：

$$S(X) = \left(\frac{X}{r-\mu} - \frac{c}{r}\right)\theta(1-\tau_{eff}) + g_1 X^{\omega} + g_2 X^{\omega'} \tag{A9}$$

where $\omega < 0$ and $\omega' > 1$ are the roots of the following quadratic equation:

$$\frac{1}{2}\sigma^2\omega(\omega-1) + \mu\omega - r = 0. \tag{A10}$$

The standard no-bubble condition, $\lim_{X\to\infty} S(X)/X < \infty$, implies $g_2 = 0$. The value matching condition in equation (2) gives

$$g_1 = \left[\left(\frac{1}{X_B}\right)^{\omega}\left(\frac{c}{r}\theta + \frac{X_B}{r-\mu}(\eta(\alpha-\kappa)-\theta)\right)\right](1-\tau_{eff}). \tag{A11}$$

化简后，解为：

$$S=\left[\left(\frac{X}{r-\mu}-\frac{c}{r}\right)\theta+\left(\frac{c}{r}\theta+\frac{X_B}{r-\mu}(\eta(\alpha-\kappa)-\theta)\right)\left(\frac{X}{X_B}\right)^{\omega}\right](1-\tau_{eff}).$$

同时，考虑the optimal bankruptcy threshold：（X_B）

$$X_B=\frac{\theta\omega(C/r)}{(\omega-1)}\frac{r-\mu}{\theta-\eta(\alpha-\kappa)}.$$

X_B与θ负相关，与η正相关

The sensitivity of stocks to operating cash flows X is

$$\begin{aligned}\epsilon=&\frac{X\partial S}{S\partial X}\\=&\frac{1}{S}\left[\frac{\theta X}{\mu}(1-\tau_{eff})+g_1\omega X^{\omega}\right]\\=&\frac{1}{S}\left[S+\frac{c}{r}\theta(1-\tau_{eff})-g_1X^{\omega}+g_1\omega X^{\omega}\right]\\=&1+\frac{c/r}{S}\theta(1-\tau_{eff})+\frac{(\omega-1)}{S}g_1X^{\omega}\\=&1+\frac{c/r}{S}\theta(1-\tau_{eff})-\frac{(1-\omega)}{S}\left[\frac{c}{r}\theta+\frac{X_B}{r-\mu}(\eta(\alpha-\kappa)-\theta)\right](1-\tau_{eff})\left(\frac{X}{X_B}\right)^{\omega}\end{aligned}$$

证毕

2. 模型算法

信息集：

$$\Theta_{it}=(X_{it},S_{it},C_{it},\sigma^S_{it},r,\alpha,\kappa,\tau_{eff})$$

解方程：

$$\sigma^S_{it}=\mathbb{E}_t[\sigma_{it+1}\epsilon_{it+1}]\equiv\sigma_{it+1}\epsilon_{it+1};\tag{10}$$

$$S_{it}=\left[\left(\frac{X_{it}}{r-\mu_{it+1}}-\frac{C_{it}}{r}\right)\theta+\left(\frac{C_{it}}{r}\theta+\frac{X_{iB}}{r-\mu_{it+1}}(\eta(\alpha-\kappa)-\theta)\right)\left(\frac{X_{it}}{X_{iB}}\right)^{\omega_{it+1}}\right](1-\tau_{eff}).\tag{11}$$

其中有两个未知变量：

$$\epsilon_{it+1}=1+\frac{C_{it}/r}{S_{it}}\theta(1-\tau_{eff})+\frac{(\omega_{it+1}-1)}{S_{it}}\left[\frac{C_{it}}{r}\theta+\frac{X_{iB}}{r-\mu_{it+1}}(\eta(\alpha-\kappa)-\theta)\right](1-\tau_{eff})\left(\frac{X_{it}}{X_{iB}}\right)^{\omega_{it+1}}$$

ω_{it+1} is the negative root of the following equation:

$$\frac{1}{2}(\sigma_{it+1}^X)^2\omega_{it+1}(\omega_{it+1}-1)+\mu_{it+1}\omega_{it+1}-r=0.$$

预测收益：

$$r_{it+1}^M = r\Delta t + \epsilon_{it+1}\left(\frac{\Delta X_{it+1}}{X_{it}} - \mu_{it+1}\Delta t\right)$$

IS-GMM Framework：（因为θ和η无法得到数据）

1. A trial set of $\mathrm{b}_0 \equiv [\theta, \eta]'$ is initialized.

2. Given the initial values of b_0 and information set of Θ_{it}, the expected μ_{it+1} and σ_{it+1} are solved from the system of equations (10) and (11) for each portfolio-year observation.

3. Given b_0 and Θ_{it} as well as the implied $\mu_{it+1}(\mathrm{b}_0, \Theta_{it})$ and $\sigma_{it+1}(\mathrm{b}_0, \Theta_{it})$, ϵ_{it+1} and r_{it+1}^M are calculated based on equations (12) and (14) respectively.

4. The pricing error e_i^M for each portfolio is obtained from (17) and the objective value J_T in equation (18) across all the portfolios is calculated.

5. Repeat from Step 1 until the optimal vector $\mathrm{b} \equiv [\theta, \eta]'$ is found that minimizes J_T.

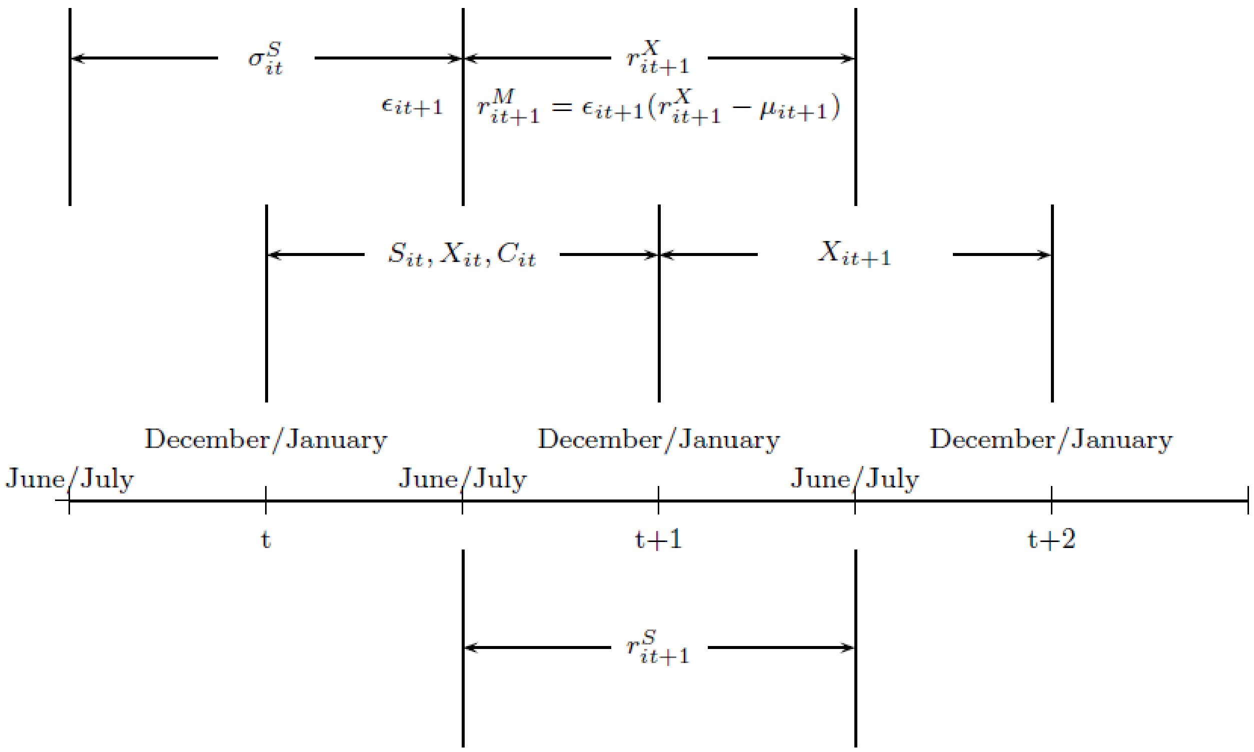
$$\begin{aligned} e_i^M &= \mathbb{E}[e_{it}^M(\mathbf{b}, \Theta_{it})] \\ &= \mathbb{E}[r_{it+1}^S - \mathbb{E}_t[r_{it+1}^M]] \\ &= \mathbb{E}[r_{it+1}^S - (r + \epsilon_{it+1}(r_{it+1}^X - \mu_{it+1}))] \end{aligned}$$

$$J_T = \mathbf{g}_T' \mathbf{W} \mathbf{g}_T,$$

$$\text{s.t. } 0 < \theta \leq 1,$$

$$0 < \eta \leq 1,$$

模型时间序列：

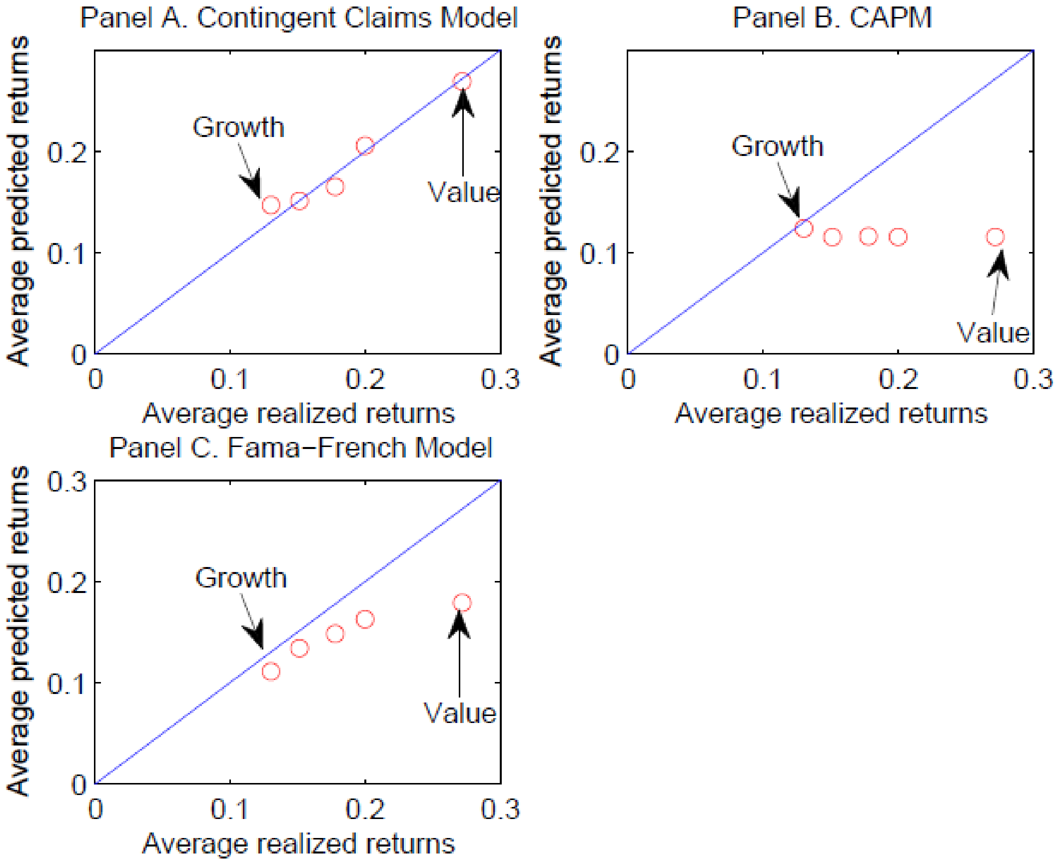


3. 模型效果及结论

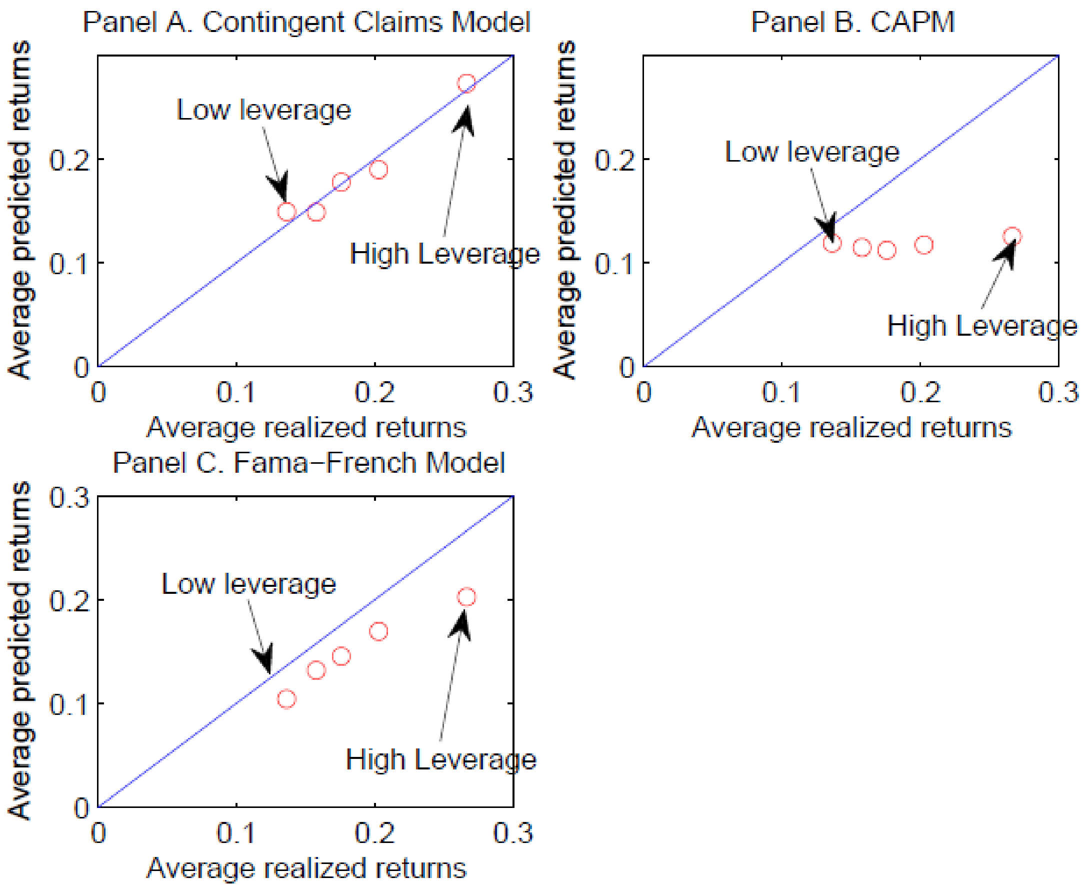
3.1 回归效果

将 contingent-claims-model 模型回归并与 CAPM ，Fama-French 比较（以 market leverage, book-to-market equity, assetgrowth rate, and equity size 分为 5 组）

以 market leverage, book-to-market equity 为例：



Panel A. Contingent Claims Model
Growth
Value
Average predicted returns
Average realized returns
Panel B. CAPM
Growth
Value
Average predicted returns
Average realized returns
Panel C. Fama−French Model
Growth
Value
Average predicted returns
Average realized returns



Panel A. Contingent Claims Model
Low leverage
High Leverage
Average predicted returns
Average realized returns
Panel B. CAPM
Low leverage
High Leverage
Average predicted returns
Average realized returns
Panel C. Fama−French Model
Low leverage
High Leverage
Average predicted returns
Average realized returns

3.2 模型结论

（1）通过 stock cash flow sensitivity 对权益收益率的估计，捕获到了财务杠杆，账面权益与市值比，资产增长率等财务指标截面变化及整个商业周期中违约概率的变化。

（2）Value stocks/high-leverage/low-asset-growth/small size stocks 的 stock cash flow sensitivity 更高，特别是在经济衰退时期。

三. 因子测试

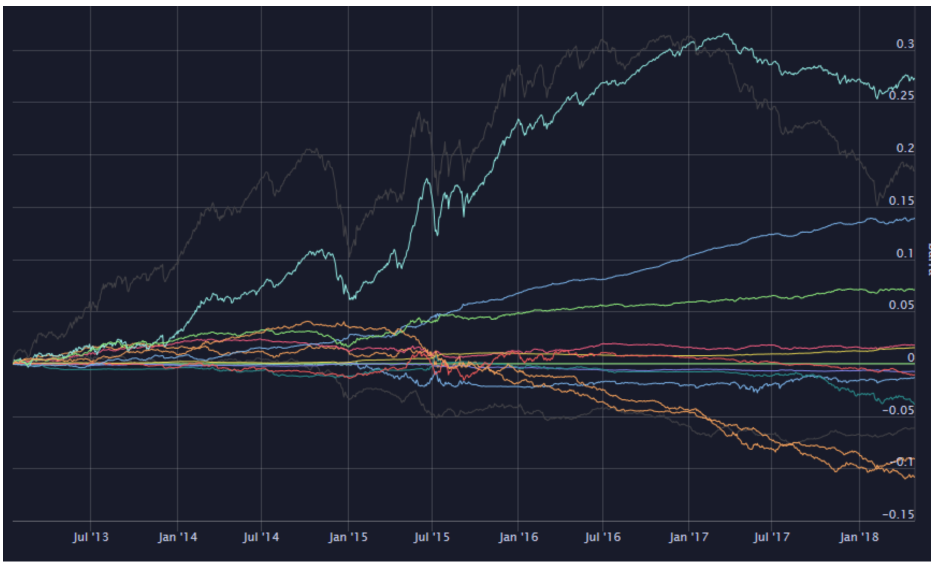
$$\text{Stock cash flow sensitivity}=\frac{X}{S}\frac{\partial S}{\partial X}$$

用差分近似：（pnl 图）

S——net operating cash flow

X——total equity

其他尝试：对 S 同比，S/X，${}^{\Delta X}/_{X\cdot S}$，将 X 更换为 mkt_cap_ard 数据等作为因子



尝试更换 S：（经营活动）stot_cash_inflows_oper_act（流入）

stot_cash_outflows_oper_act（流出）

（投资活动）stot_cash_inflows_inv_act(流入)

stot_cash_outflows_inv_act（流出）

net_cash_flows_inv_act（净流入）

（筹资活动）stot_cash_inflows_fnc_act（流入）

stot_cash_outflows_fnc_act（流出）

stot_cash_inflows_fnc_act（净流入）