Unit Commitment Problem Based on Mixed-integer Linear Programming(MILP-UC)

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***ABSTRACT*--This paper presents a mixed-integer linear programming approach for unit commitment problem in economic dispatch. The model of the optimization includes objective function of fuel cost and start-up cost and several constraints. With a set of binary variables, we can precisely present quadratic fuel cost function, time-dependent start-up cost function and intertemporal constraints such as minup and mindown time by changing them into linear and convex expressions.Due to only one single set of binary variables required, this model allows the computing time to be shorter. Finally, a realistic power system example is used to test the performance and accuracy of the presented formulation on the platform of Matlab and Yalmip with Gurobi solver.**

***Keywords*--*Unit commitment; mixed-integer linear programming*; thermal generating units; Yalmip**

NOMENCLATURE

Constants

N	Number of power generators
T	Number of time periods
m	The m-th generating unit
h	The h-th time period
a_m,b_m,c_m	Coefficients of quadratic fuel cost function of unit m
MA_m	Marginal fuel cost of first part of unit m
MB_m	Marginal fuel cost of second part of unit m
$P\text{int}_m$	Intermediate output of unit m
$P^{min}(m)$	Minimum output of unit m
$P^{max}(m)$	Maximum output of unit m
$hc_m,cc_m,t_{cold}(m)$	Coefficients of the start-up cost function of unit m
K_m^t	Cost of the interval t of the stairwise start-up cost function of unit m
ND_m	Number of intervals of the stairwise start-up cost function of unit m

D_h	Load demand at time period h
R_h	Spinning reserve at time period h
ST_m	Minimum up-time of unit m
DT_m	Minimum down-time of unit m

Variables

Cost	Total cost of electricity generation
Fuelcost(m,h)	Fuel cost function of unit m at time period h
cu(m,h)	start-up cost of unit m at time period h
P(m,h)	Actual electricity generation of unit m at time period h
g_A(m,h)	Power generation between Pmin and Pint
g_B(m,h)	Power generation between Pmax and Pint
$t_m^{off}(h)$	Number of hours unit m has been offline prior to the start-up in time period h
z(m,h)	Indicator indicating whether unit m is committed or not at time period h,1:if committed,0:if not

I.Introduction of Economic Dispatch(ED) Problem

Economic dispatch plays an important role in traditional power system and is used to optimize the power generation to minimize the cost of generation in order to maximize the overall efficiency of the power system. Since the electricity market is developing fast, minimizing generation cost is no longer the only object to maximize the efficiency. ED problem is considered as a complicated issue for many aspects should be covered while doing the optimization. To meet the electricity demand of the consumers with a lower cost, it is necessary to schedule each generator during the dispatch process, which is called unit commitment(UC) optimization problem. The thermal unit commitment problem is traditionally solved in centralized power system to determine when to start-up or shutdown thermal generating units and how to dispatch those online generators to meet the load demand while satisfying some generators constraints like power balance, ramping limits and minimum up and down times and also some system constraints like spinning reserve requirements. To conclude, with unit commitment problem, the economic dispatch problem is now turned into a problem about how to dispatch each generator during each time period.

Mathematically, optimization problems often come across with polynomial objective functions, especially quadratic functions. Actually, it is a kind of large-scale, mixed-integer, combinarotial and nonlinear programming problem and we need efficient tools to give accurate resource scheduling results.

As a heat topic for decades, there are already some methods to solve this problem like heuristics [6]-[8], dynamic programming[9]-[11], mixed-integer linear programming(MILP)[12]-[13], simulated annealing [14]-[16], and evolution-inspired approaches [17]-[18].

In the aforementioned methods, MILP is a good way to solve UC problem. It has good convergence performance and there are also already some efficient solvers like Lpsolve and Gurobi and algorithms like branch-and-cut available to solve MILP problem.

In this paper, we present a mixed-integer linear formulation of the thermal unit commitment problem,which can be abbreviated as MILP-UC, with a single set of binary variables. Low number of binary variables in MILP-UC can lead to a reduction in the computing time. The main structure of this paper is as following:

1)In the section II, we give a brief introduction to what is MILP and build MILP-UC model;

2)In the section III, to prove that the presented model is efficient to solve MILP-UC, a realistic ten unit system is taken as an example. The system simulation is done on MATLAB platform with Yalmip and Gurobi solver;

3)Then during the remaining part, we give some conclusions and look forward to some further improvements. The data we used of the ten-unit system can be found in the appendix.

II. Mathematical Model of MILP-UC

A.Basic Idea of Mixed-integer Linear Programming Problem

According to relevant papers, the definition of mixed integer linear programming is to minimize the function $f(x)$ with following constraints:

$$\begin{cases} \text{x\{intcon\} are integers} \\ A * x \le b \\ A_{eq} * x = b_{eq} \\ lb \le x \le ub \end{cases} \tag{1}$$

B. MILP-UC Model

In unit commitment problem, some variables is required in MILP to model the on/off-line states of generators. Dealing with this, in this paper, we formulate a programming approach, MILP-UC, with a single set of binary variables representing each unit at each period. By using a single variable set based on time, we can get an accurate model of MILP-UC problem and the number of constraints is reduced in order to save computing time.

The formulation of unit commitment problem is described as a sum of fuel cost and start-up cost over all the power generators over several time periods respectively.

$$\text{Cost}=\sum_{\text{m,h}}\text{Fuelcost(m,h)}+\sum_{\text{m,h}}\text{Startupcost(m,h)} \tag{2}$$

(1) Objective Function

1) Fuel Cost Function: Given the fuel cost function's simplest form, the cost function is a typical quadratic one of power output of each unit over those on-line time periods :

$$F(P(m,h)) = a_m + b_m P(m,h) + c_m (P(m,h))^2 \tag{3}$$

To build MILP-UC model, we use the method mentioned in [5] to linearize the above quadratic function. The accuracy of this linearization method has been proved in the reference.

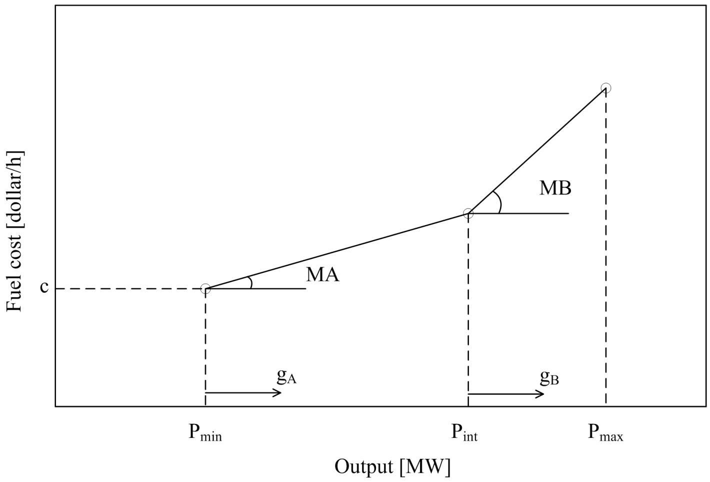

Figure 1. Stepwise cost function of power plant

As we can see in figure 1, to take fuel costs into account correctly, the typical quadratic cost curve of a power plant is approximated by a piecewise linear function,i.e., plants are divided in parts of equal marginal cost.

A binary variable set z(m,h) whose values are limited to be 1 or 0 is used to represent the on/off state of each unit at each time period.

To present the fuel cost function correctly, several constraints must be applied as (4)-(5):

$$\forall m \in N, \forall h \in T: P(m,h) = g_A(m,h) + g_B(m,h) + P\min_m * z(m,h) \tag{4}$$

$$\begin{aligned} &\forall m \in N, \forall h \in T: g_A(m,h) \le [P\mathrm{int}_m - P\min_m] * z(m,h) \\ &\forall m \in N, \forall h \in T: g_B(m,h) \le [P\max_m - P\mathrm{int}_m] * z(m,h) \end{aligned} \tag{5}$$

Then the fuel cost function can be presented as following:

$$\forall m \in N, \forall h \in T, \mathrm{Fuelcost}(m,h) = c_m \times z(m,h) + MA_m \times g_A(m,h) + MB_m \times g_B(m,h) \tag{6}$$

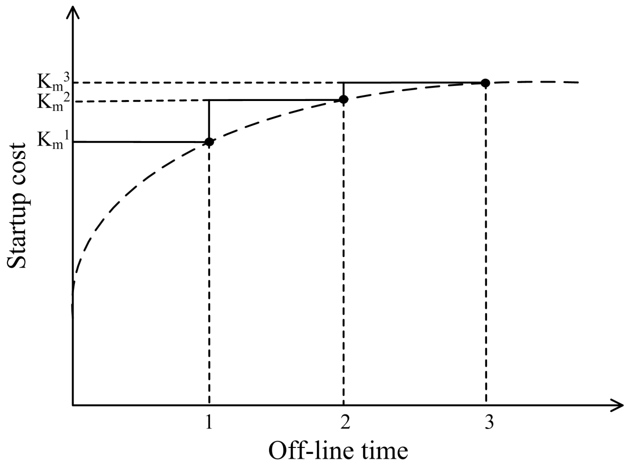

Figure 2. Exponential, discrete, and stairwise start-up cost functions

2)Start-up Cost: For start-up cost, it can be seen in figure 2 that the typical function is also a quadratic curve as the dashed line shows. In the UC problem, the time span is divided into hourly periods, which means that we discretize the start-up cost function into several intervals. And then these discretized curve can be asymptotically approximated by a stairwise line. Obviously, the more number of intervals, the more accurate the stairwise function is.

The definition of the stairwise function is as following:

$$cu(m,h)=\begin{cases} hc_m & if\ \mathrm{t}_{\mathrm{m}}^{\mathrm{off}}(h)\le t_{cold}(m)+DT_m \\ cc_m & if\ \mathrm{t}_{\mathrm{m}}^{\mathrm{off}}(h)> t_{cold}(m)+DT_m \end{cases} \quad \forall m\in N,\forall h\in T \tag{7}$$

With the binary variable z, the mixed-integer formulation of the start-up cost function is

$$\begin{gathered} \forall m\in N,\forall h\in T,\forall t=1,...,ND_m, cu(m,h)\ge K_m^t\left[z(m,h)-\sum_{n=1}^{t} z(m,h-n)\right] \\ \forall m\in N,\forall h\in T, cu(m,h)\ge 0 \end{gathered} \tag{8}$$

Where,

$$K_m^t=\begin{cases} hc_m & \text{if } \mathrm{t}=1,...,\mathrm{t}_{\mathrm{cold}}(m)+DT_m \\ cc_m & \text{if } \mathrm{t}=\mathrm{t}_{\mathrm{cold}}(m)+DT_m+1,...,ND_m \end{cases} \quad \forall m\in N \tag{9}$$

(3)Constraints

The presented objective function formulation should be restrained by some generator and system constraints which are described as following.

*1)Power Balance Constraints:*The amount of supply and demand should be constrained by the following equation:

$$\sum_{m=1}^{N} P(m,h)z(m,h)=D_h \tag{10}$$

2)Spinning Reserve Constraints:

$$\sum_{m=1}^{N} z(m,h)P^{\max}(m)\ge D_h+R_h \tag{11}$$

As usual, we assume that the spinning reserve requirement is 10% of load demand to ensure all the demand satisfied.

3)Generation Limits Constraints:

The power output of each generator over all time periods is limited by its minimum and maximum power output, which is obvious a nonnegative variable.

$$P^{\min}(m)*z(m,h)\le P(m,h)\le P^{\max}(m)z(m,h) \tag{12}$$

4)Minup and Mindown Constraints:

The minup and mindown time constraints are formulated as mixed-integer linear expressions relying on binary variables associated with on/off states of generators.

The expressions for the minup and mindown time constraints are then as follows:

$$\begin{cases} z(m,h)=1 & \text{for } \sum_{\mathrm{t=h\text{-}ST_m}}^{\mathrm{h\text{-}1}} z(m,t)\ge ST_m \\ z(m,h)=0 & \text{for } \sum_{\mathrm{t=h\text{-}DT_m}}^{\mathrm{h\text{-}1}} (1-\mathrm{z(m,t)})\ge \mathrm{DT_m} \end{cases} \tag{13}$$

Ⅲ.Numerical Result of the Example

Using a system with 10 units shown in fig.3 to test the proposed formulation.The data of the original ten-unit system [3] can be found in the Appendix.

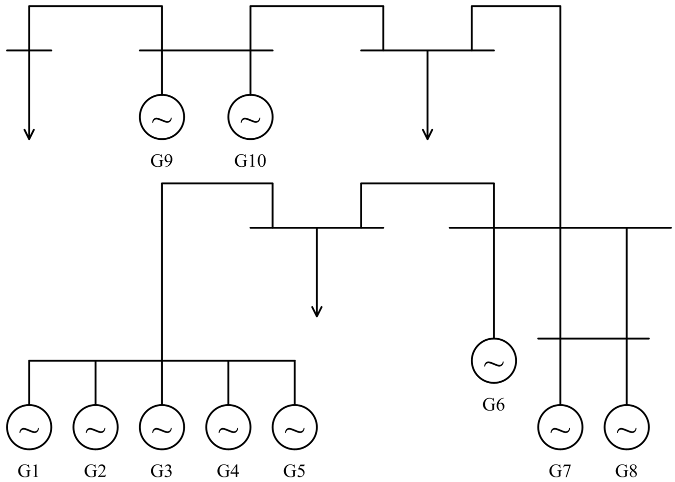

Figure.3 The 10-unit system

The system is simulated on Matlab using Yalmip and Gurobi solver. The results are summarized in the following table I and table II .

Table I

Production Schedule of time period 1-12(MW)

Units	Periods											
	1	2	3	4	5	6	7	8	9	10	11	12
1	455	455	455	455	455	455	455	455	455	455	455	455
2	245	295	370	455	390	360	410	455	455	455	455	455
3	0	0	0	0	0	130	130	130	130	130	130	130
4	0	0	0	0	130	130	130	130	130	130	130	130
5	0	0	25	40	25	25	25	30	85	162	162	162
6	0	0	0	0	0	0	0	0	20	33	73	80
7	0	0	0	0	0	0	0	0	25	25	25	25
8	0	0	0	0	0	0	0	0	0	10	10	43
9	0	0	0	0	0	0	0	0	0	0	10	10
10	0	0	0	0	0	0	0	0	0	0	0	10

Table II

Production Schedule1 of time period 13-24(MW)

Units	Periods											
	13	14	15	16	17	18	19	20	21	22	23	24
1	455	455	455	455	455	455	455	455	455	455	455	455
2	455	455	455	310	260	360	455	455	455	455	270	0
3	130	130	130	130	130	130	130	130	130	0	0	0
4	130	130	130	130	130	130	130	130	130	130	130	130
5	162	85	30	25	25	25	30	162	100	40	25	162
6	33	20	0	0	0	0	0	38	20	20	20	43
7	25	25	0	0	0	0	0	0	0	0	0	0
8	10	0	0	0	0	0	0	10	10	0	0	10
9	0	0	0	0	0	0	0	10	0	0	0	0
10	0	0	0	0	0	0	0	10	0	0	0	0

We give an intuitive image about the power scheduling in Fig 4. according to the above data.

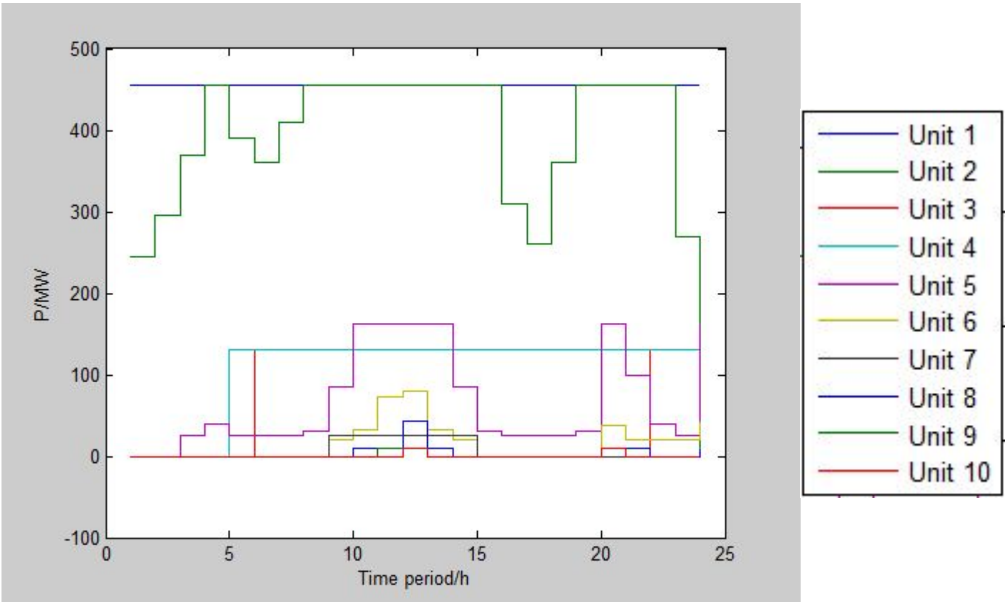

Figure 4. Stairstep graph of power scheduling

We can conclude from above figures that our mixed integer linear programming model for unit commitment problem can be successfully solved to give us an optimal solution.

The comparison of total cost among different computation methods is as following:

Table III

Comparison of Total cost of Ten Units

Approach	Total operation cost($)
LR[1]	565727
MILP-UC	562811
GA[1]	562743
EP[18]	562388
LRGA[19]	561312
PL[8]	560844
ELR[20]	560567

Compare the total cost obtained by different methods like the genetic algorithm, evolutionary programming and several other methods listed in this table, we can see that the mixed integer linear programming gives us a not bad result.

IV.Conclusion

A computationally efficient mixed-integer linear formulation has been constructed in this paper to solve the unit commitment problem of thermal units.In the presented model, we use a single type of binary variables not only to accurately express generators and system constraints like power balance,spinning reserve, and so on, but also to cause a reduction of computing time required. On the Matlab platform with Yalmip and Gurobi solver, the proposed model has been proved to successfully solve the MILP-UC problem.

V. Further Improvement

*A.*Though the good accuracy we use to linearize the fuel cost function has been proven in the reference[5], if we divide the curve into more segments, the results will be more accurate. Besides we can also consider other ways like linear interpolation to do linearization.

B.For a more practical UC problem, we should take more factors into account like the ramp rate limit constraint and shutdown costs;

C. When dealing with the UC problem, we can measure how the performance of MILP is by comparing it with other methods about convergence, the ability to solve larger systems, and so on.

Appendix

Table A

System Data

	Unit1	Unit2	Unit3	Unit4	Unit5
Pmax(MW)	455	455	130	130	162
Pmin(MW)	150	150	20	20	25
a($/h)	1000	970	700	680	450
b($/MWh)	16.19	17.26	16.60	16.50	17.70
c($/MW2-h)	0.00048	0.00031	0.002	0.00211	0.00398
min up(h)	8	8	5	5	6
min dn(h)	8	8	5	5	6
hot start cost($)	4500	5000	550	560	900
cold start cost($)	9000	10000	1100	1120	1800
cold start hrs(h)	5	5	4	4	4
initial status(h)	8	8	-5	-5	-6
	Unit6	Unit7	Unit8	Unit9	Unit10
Pmax(MW)	80	85	55	55	55
Pmin(MW)	20	25	10	10	10
a($/h)	370	480	660	665	670
b($/MWh)	22.26	27.74	25.92	27.27	27.79
c($/MW2-h)	0.00712	0.0079	0.00413	0.00222	0.00173
min up(h)	3	3	1	1	1
min dn(h)	3	3	1	1	1
hot start cost($)	170	260	30	30	30
cold start cost($)	340	520	60	60	60
cold start hrs(h)	2	2	0	0	0
initial status(h)	-3	-3	-1	-1	-1

Table B

Load Demand

Hour	P_{load}	Hour	P_{load}
1	700	13	1400
2	750	14	1300
3	850	15	1200
4	950	16	1050
5	1000	17	1000
6	1100	18	1100
7	1150	19	1200
8	1200	20	1400
9	1300	21	1300
10	1400	22	1100
11	1450	23	900
12	1500	24	800

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