



Machine Learning

Clustering

Unsupervised learning
introduction

Supervised learning



Training set: $\{(x^{(1)}, y^{(1)}), (x^{(2)}, y^{(2)}), (x^{(3)}, y^{(3)}), \dots, (x^{(m)}, y^{(m)})\}$

Unsupervised learning



Clustering algorithm

Training set: $\{\underline{x^{(1)}}, \underline{x^{(2)}}, x^{(3)}, \dots, \underline{x^{(m)}}\}$ ←

Applications of clustering



→ Market segmentation



→ Social network analysis



→ Organize computing clusters



→ Astronomical data analysis



Machine Learning

Clustering

K-means algorithm

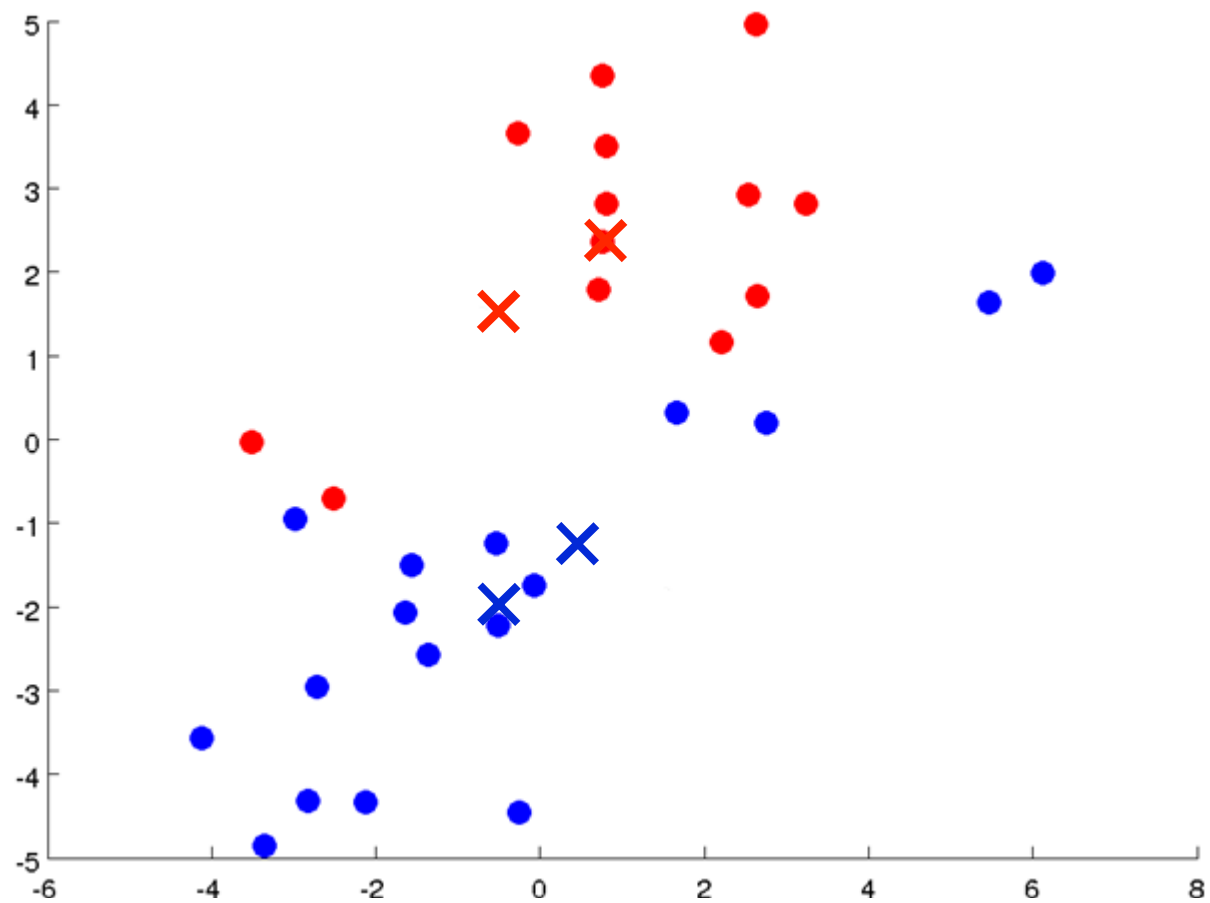


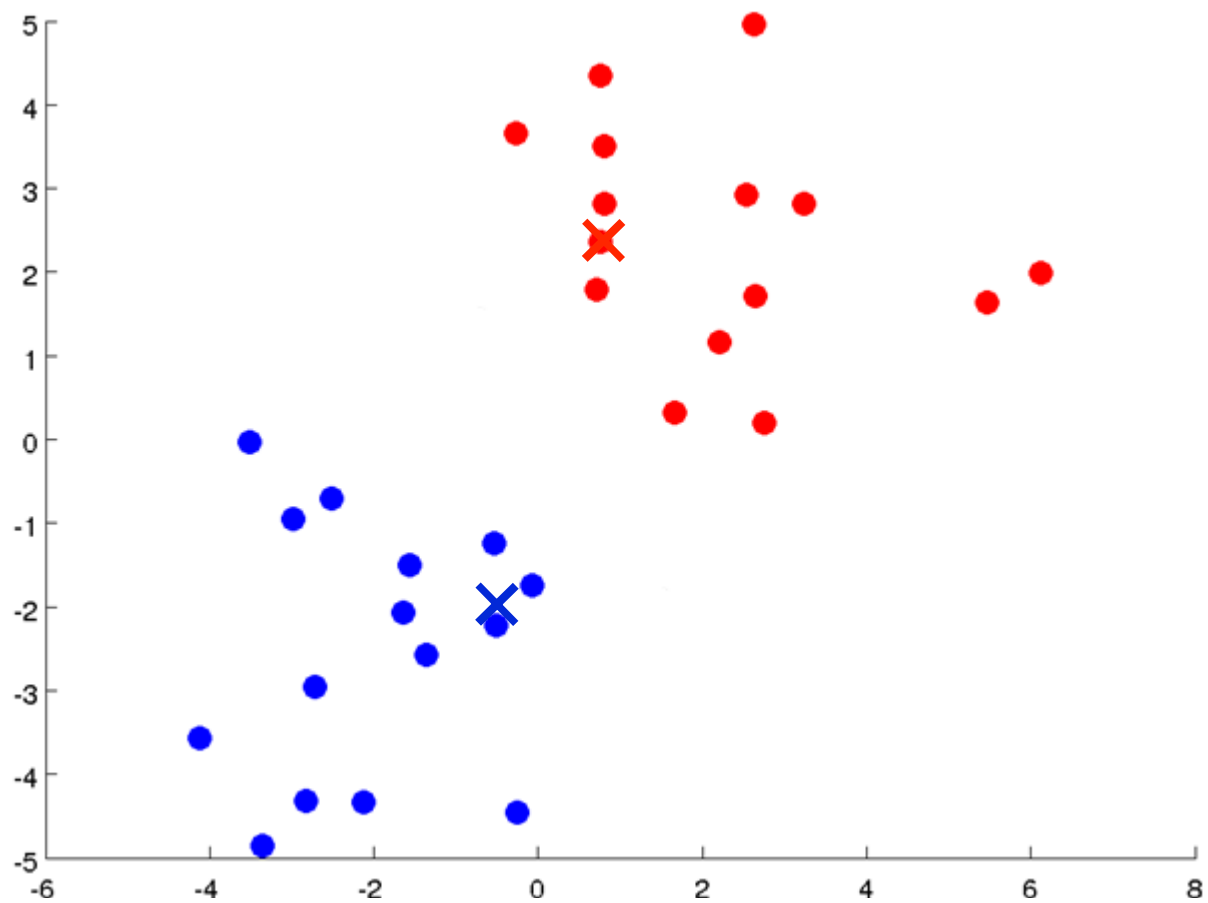


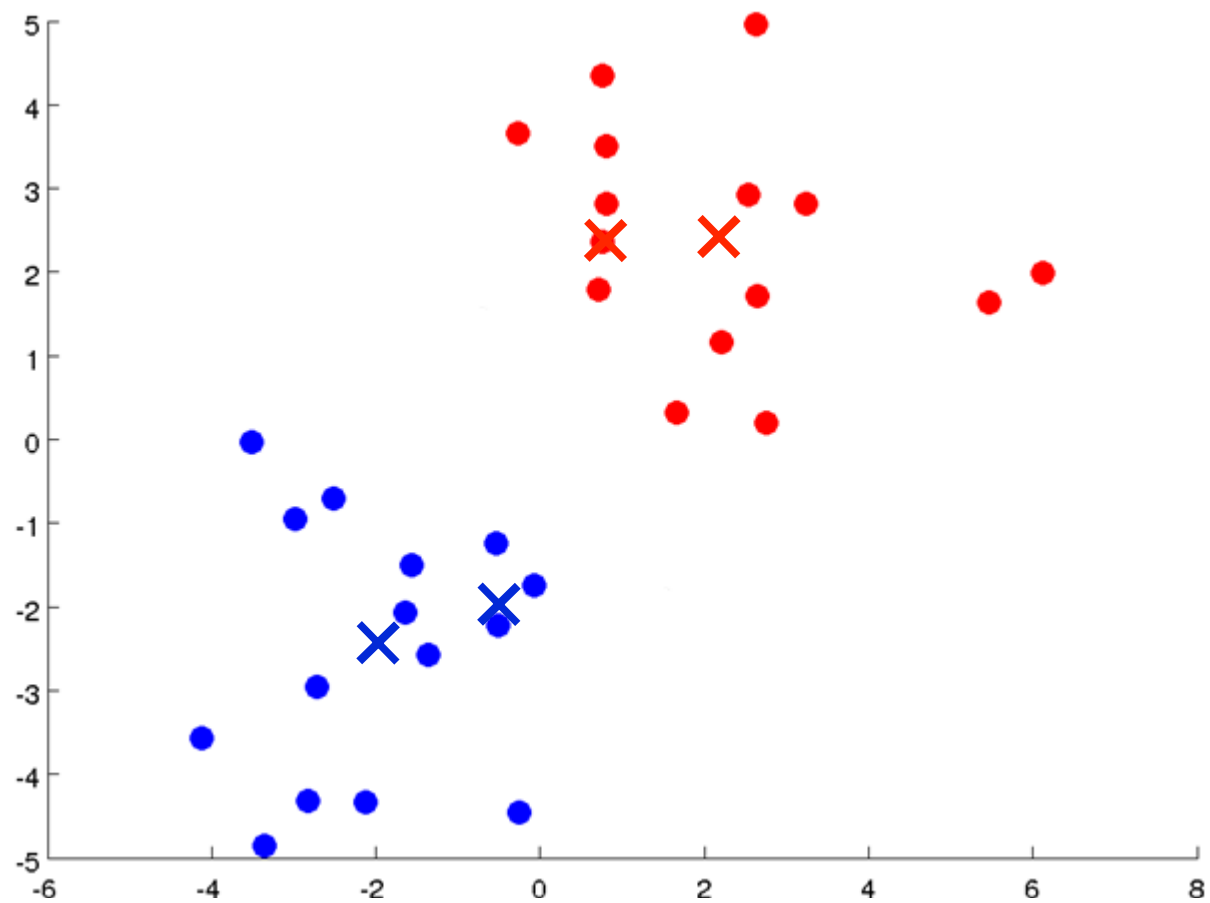














K-means algorithm

Input:

- K (number of clusters) 
- Training set $\{x^{(1)}, x^{(2)}, \dots, x^{(m)}\}$ 

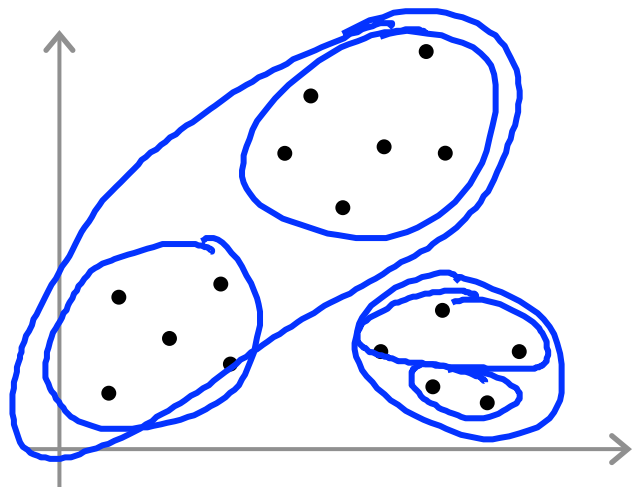
$x^{(i)} \in \mathbb{R}^n$ (drop $x_0 = 1$ convention)

K-means algorithm

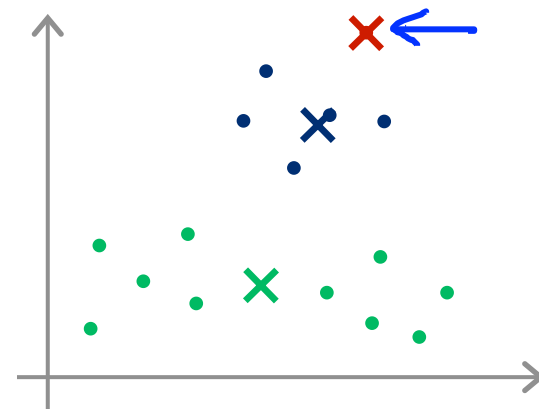
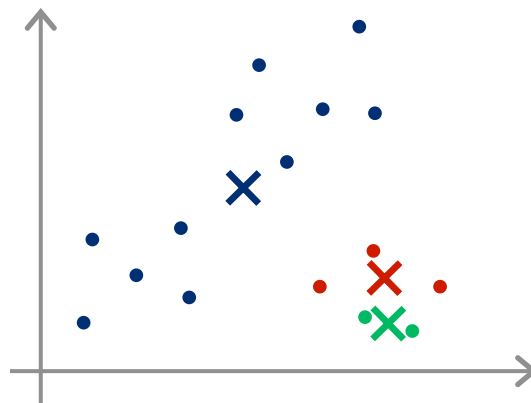
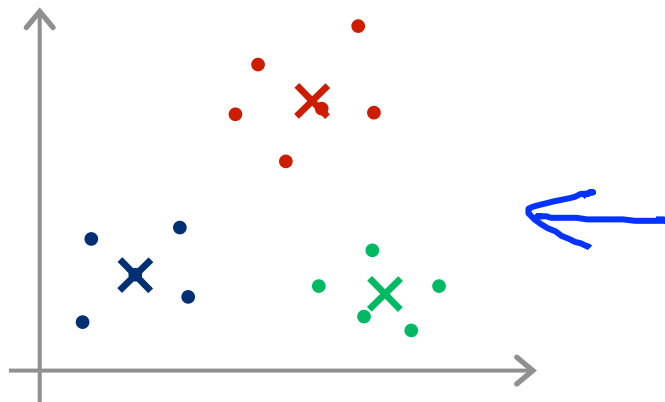
Randomly initialize K cluster centroids $\mu_1, \mu_2, \dots, \mu_K \in \mathbb{R}^n$

Repeat {
 for $i = 1$ to m
 $c^{(i)} :=$ index (from 1 to K) of cluster centroid
 closest to $x^{(i)}$
 for $k = 1$ to K
 $\mu_k :=$ average (mean) of points assigned to cluster k
}

Local optima



$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$$



Random initialization

For $i = 1$ to 100 {

Randomly initialize K-means.

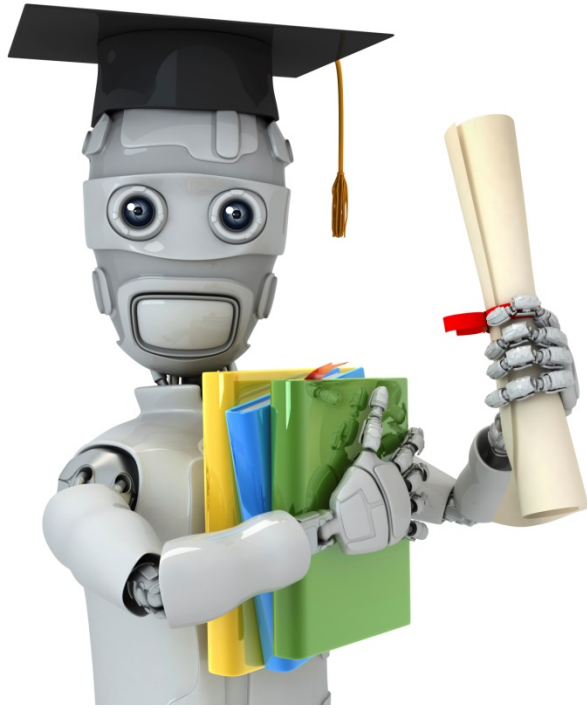
Run K-means. Get $c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K$.

Compute cost function (distortion)

$$J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$$

}

Pick clustering that gave lowest cost $J(c^{(1)}, \dots, c^{(m)}, \mu_1, \dots, \mu_K)$

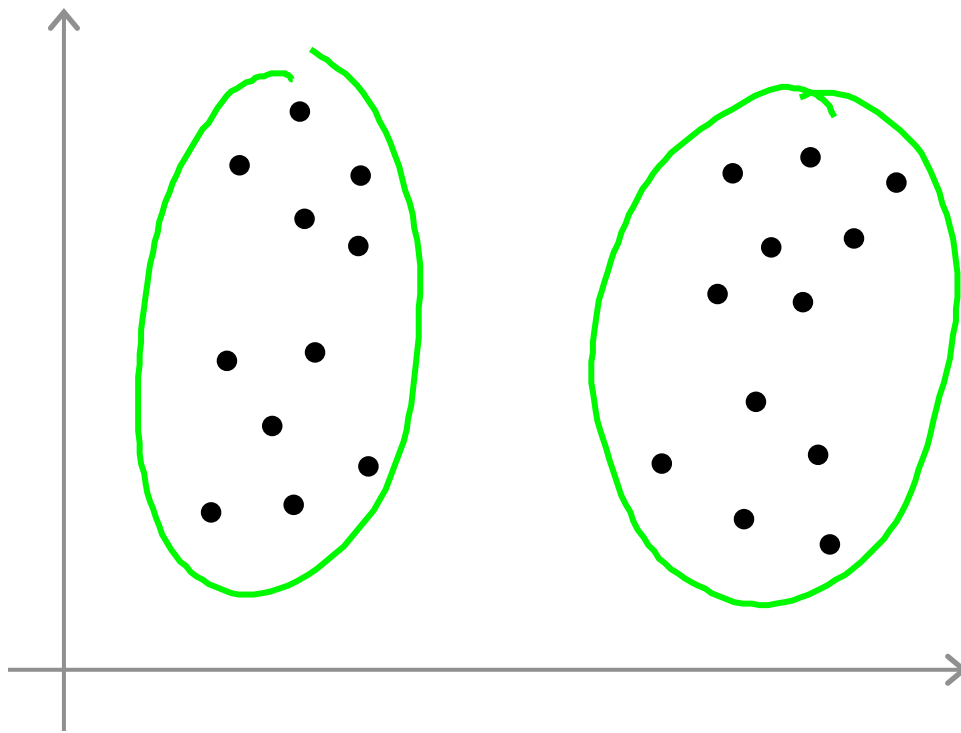


Machine Learning

Clustering

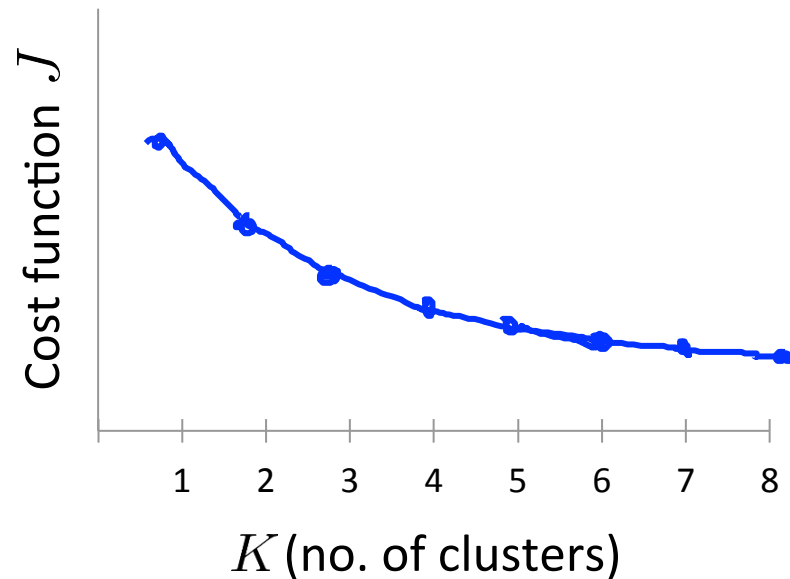
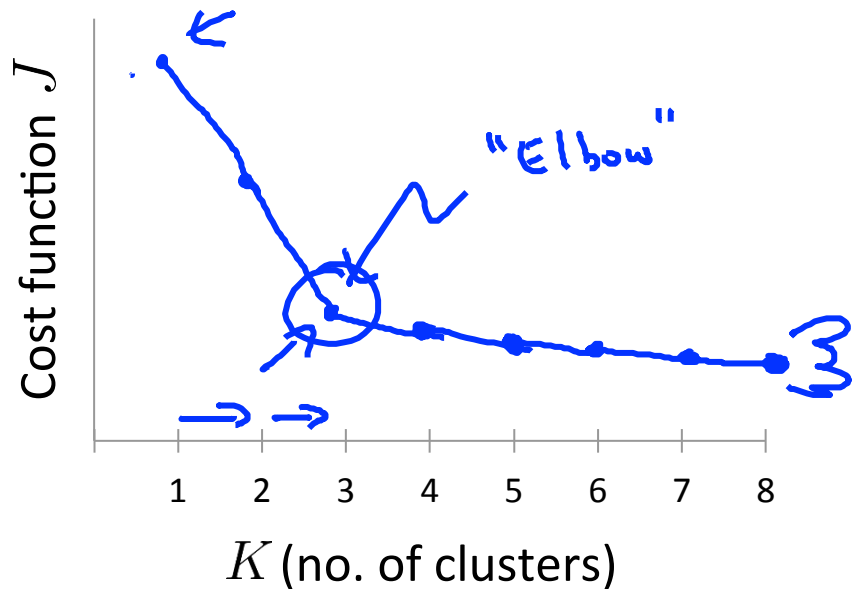
Choosing the
number of clusters

What is the right value of K?



Choosing the value of K

Elbow method:

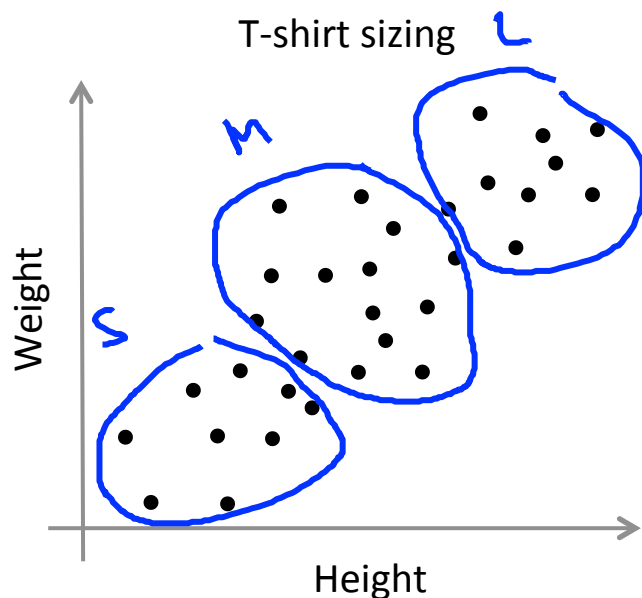


Choosing the value of K

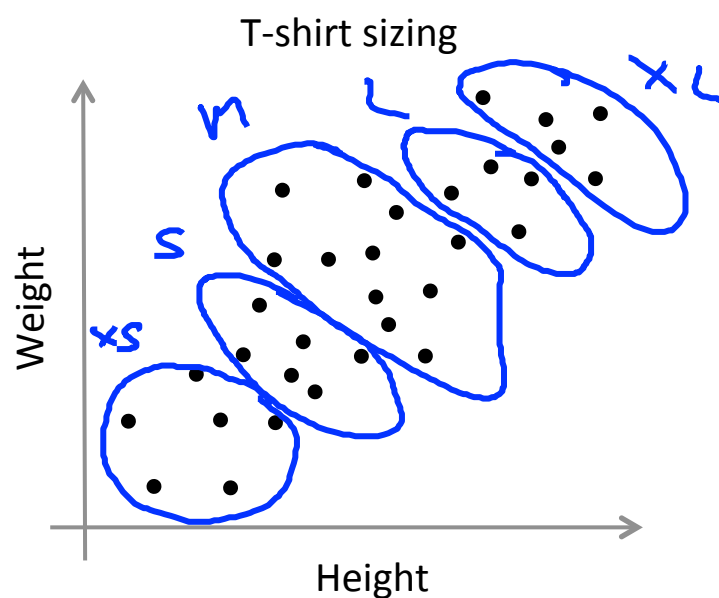
Sometimes, you're running K-means to get clusters to use for some later/downstream purpose. Evaluate K-means based on a metric for how well it performs for that later purpose.

$K=3$ S, M, L

E.g.



$K=5$ XS, S, M, L, XL



DATA MINING CLUSTERING

by Panayiotis Tsaparas

The k-means algorithm

Hierarchical Clustering

The DBSCAN algorithm

Evaluation

DBSCAN

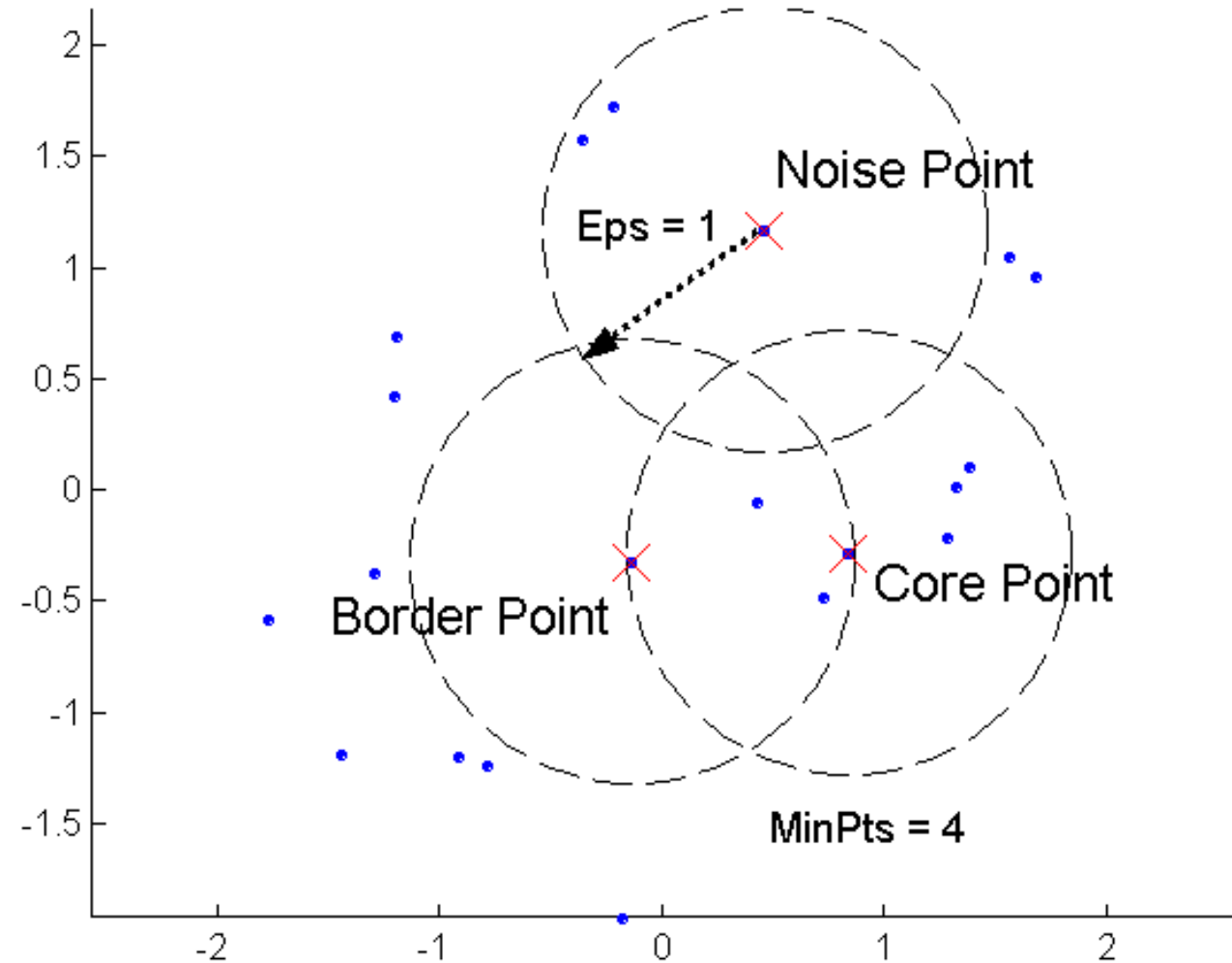
DBSCAN: Density-Based Clustering

- DBSCAN is a Density-Based Clustering algorithm
- Reminder: In density-based clustering we partition points into dense regions separated by not-so-dense regions.
- Important Questions:
 - How do we measure density?
 - What is a dense region?
- DBSCAN:
 - Density at point p : number of points within a circle of radius Eps
 - Dense Region: A circle of radius Eps that contains at least $MinPts$ points

DBSCAN

- Characterization of points
 - A point is a **core point** if it has more than a specified number of points (**MinPts**) within **Eps**
 - These points belong in a **dense region** and are at the **interior** of a cluster
 - A **border point** has fewer than **MinPts** within **Eps**, but is in the neighborhood of a **core** point.
 - A **noise point** is any point that is not a core point or a border point.

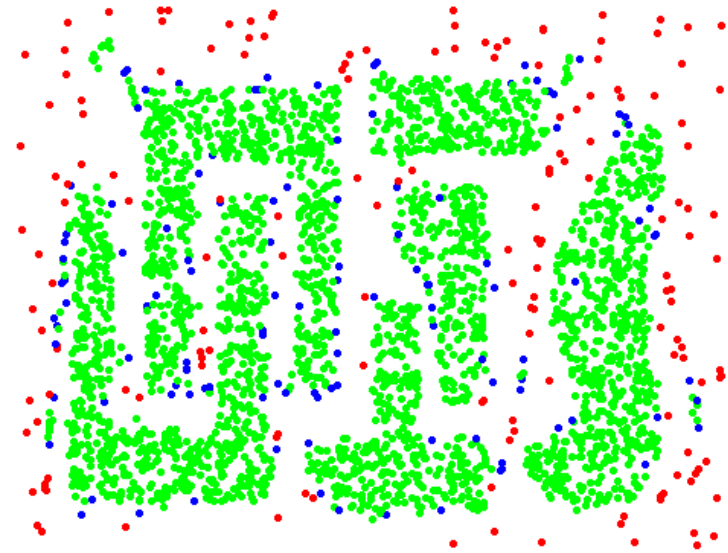
DBSCAN: Core, Border, and Noise Points



DBSCAN: Core, Border and Noise Points



Original Points



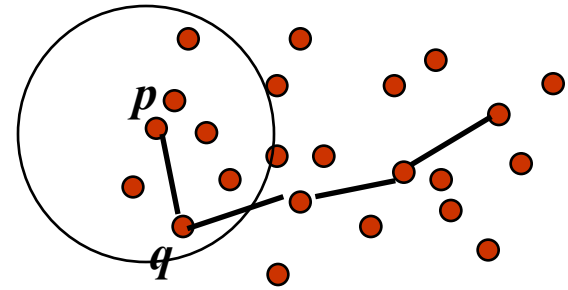
Point types: **core**, **border** and **noise**

Eps = 10, MinPts = 4

Density-Connected points

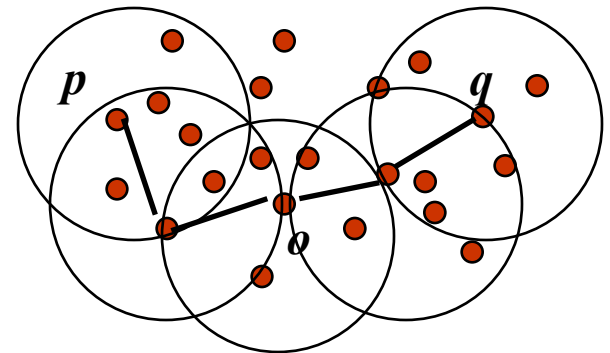
- Density edge

- We place an **edge** between two core points **q** and **p** if they are within distance **Eps**.



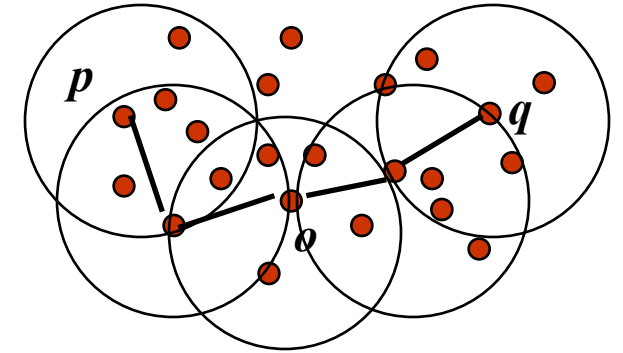
- Density-connected

- A point **p** is **density-connected** to a point **q** if there is a **path of edges** from **p** to **q**



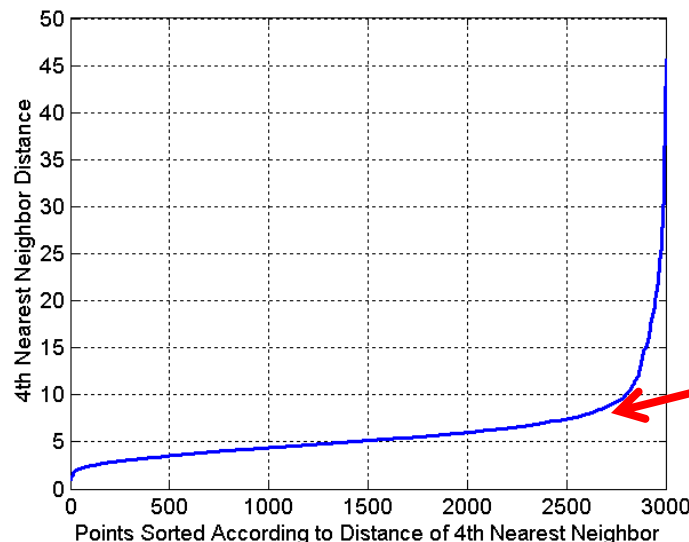
DBSCAN Algorithm

- Label points as **core**, **border** and **noise**
- Eliminate **noise** points
- For every **core** point **q** that has not been assigned to a cluster
 - Create a new cluster with the point **q** and all the points that are **density-connected** to **q**.
- Assign **border** points to the cluster of the closest core point.



DBSCAN: Determining Eps and MinPts

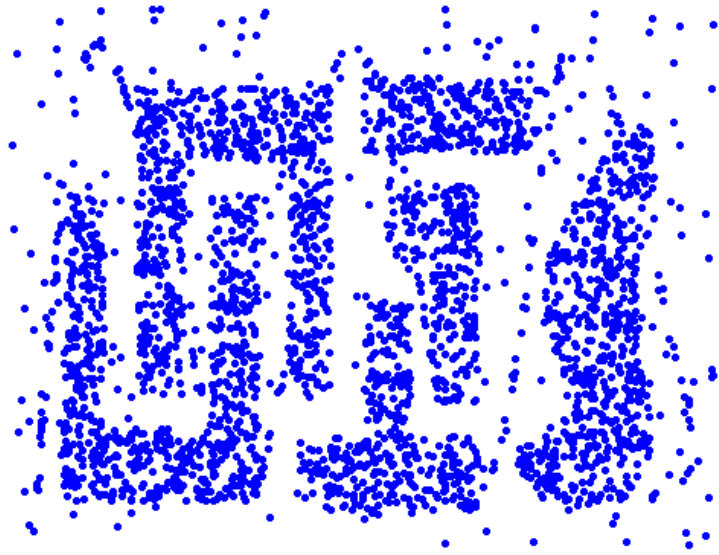
- Try different minPts = k
- So, plot sorted distance of every point to its k^{th} nearest neighbor
- Find the distance d where there is a “knee” in the curve
 - $\text{Eps} = d$, $\text{MinPts} = k$



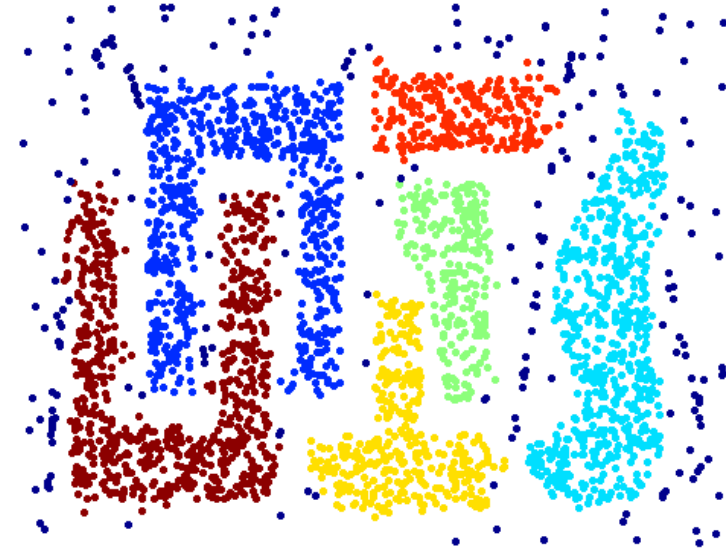
Noise points have the k^{th} nearest neighbor at farther distance

Eps ~ 7-10
MinPts = 4

When DBSCAN Works Well



Original Points



Clusters

- Resistant to Noise
- Can handle clusters of different shapes and sizes

DBSCAN: Sensitive to Parameters

Figure 8. DBScan results for DS1 with MinPts at 4 and Eps at (a) 0.5 and (b) 0.4.

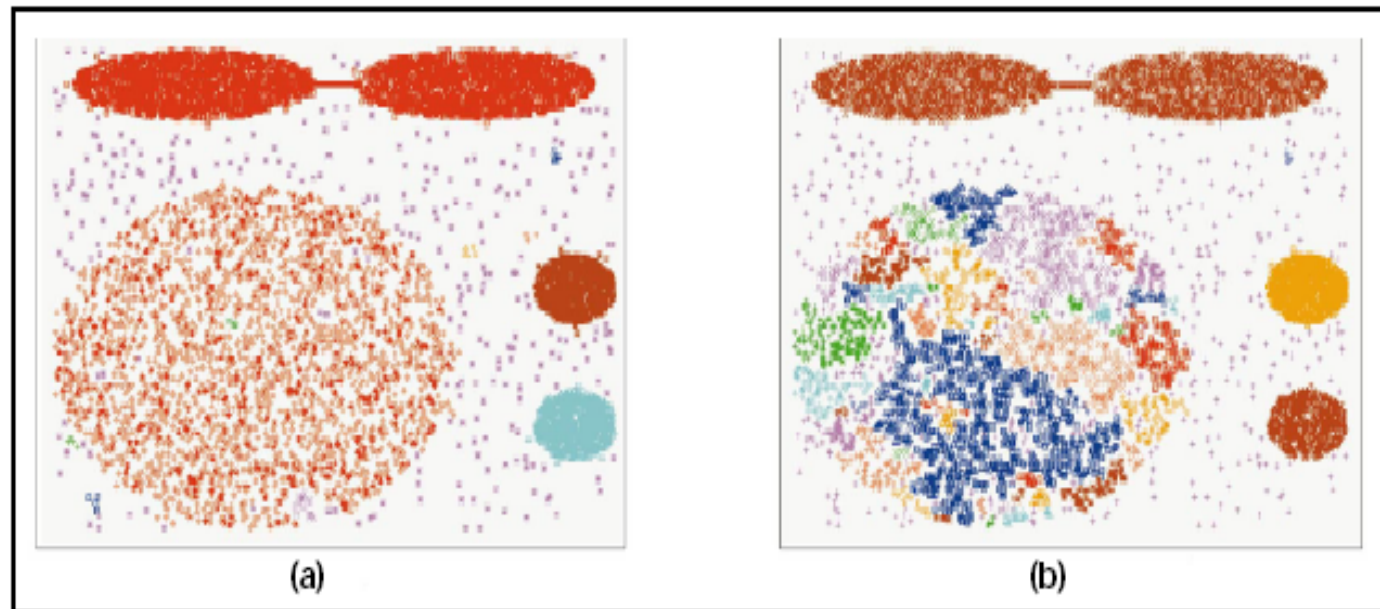
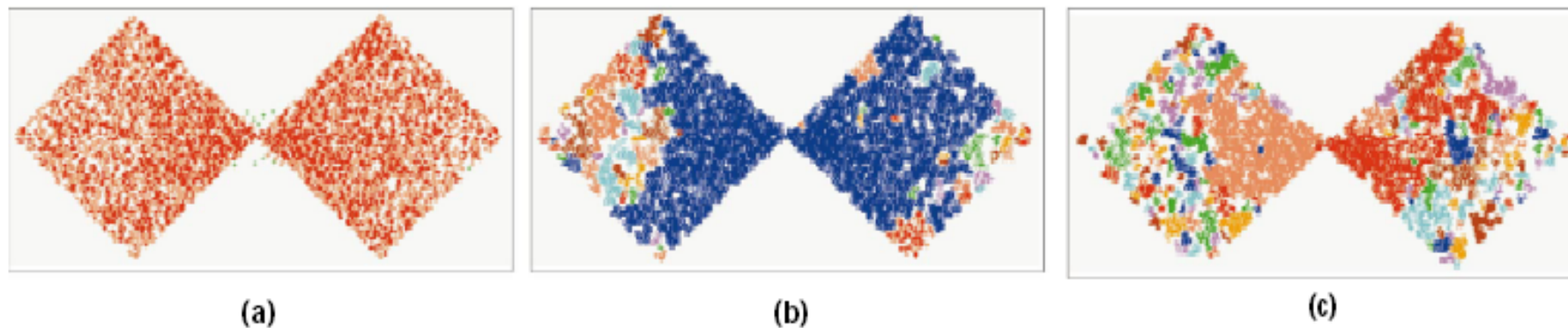
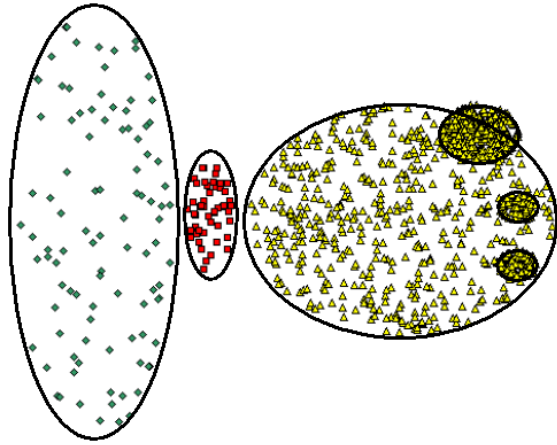


Figure 9. DBScan results for DS2 with MinPts at 4 and Eps at (a) 5.0, (b) 3.5, and (c) 3.0.



When DBSCAN Does NOT Work Well



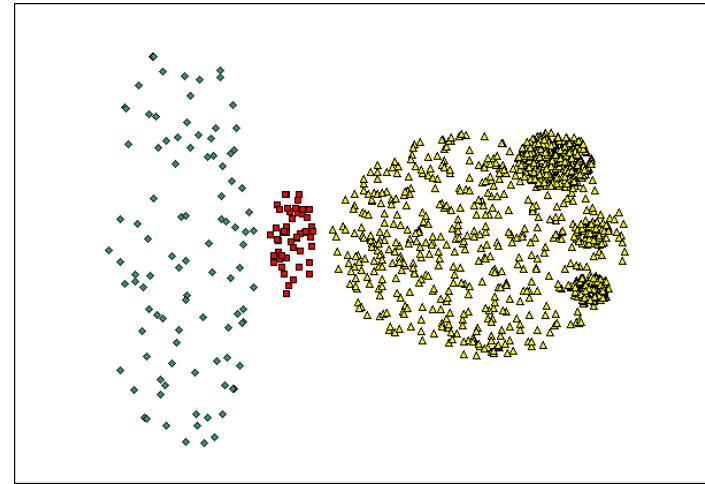
Original Points

- Varying densities

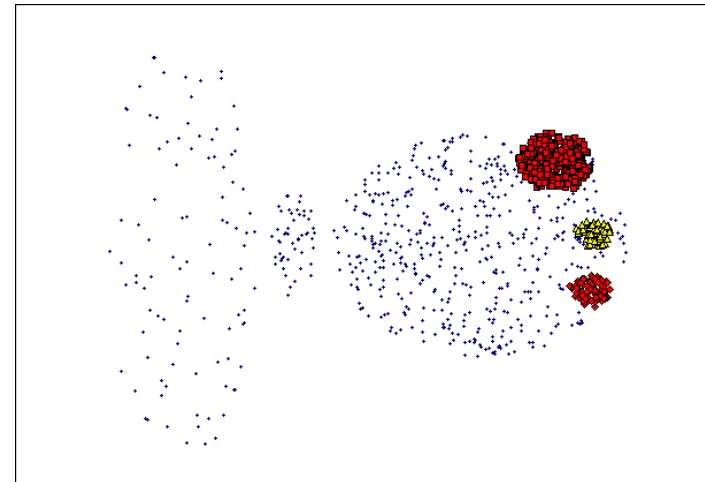
- large differences in densities, since the minPts-Eps combination cannot then be chosen appropriately for all clusters.

- High-dimensional data

- difficult to find an appropriate value for Eps



(MinPts=4, Eps=9.75).



(MinPts=4, Eps=9.92)