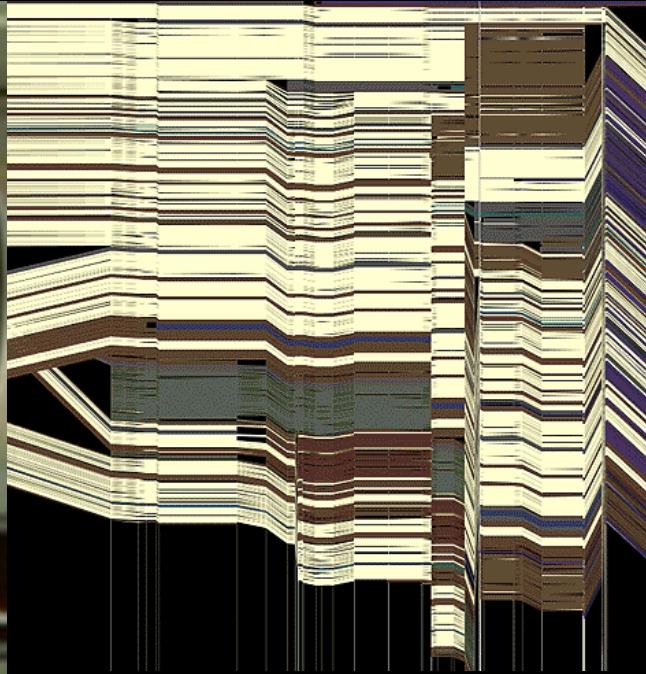
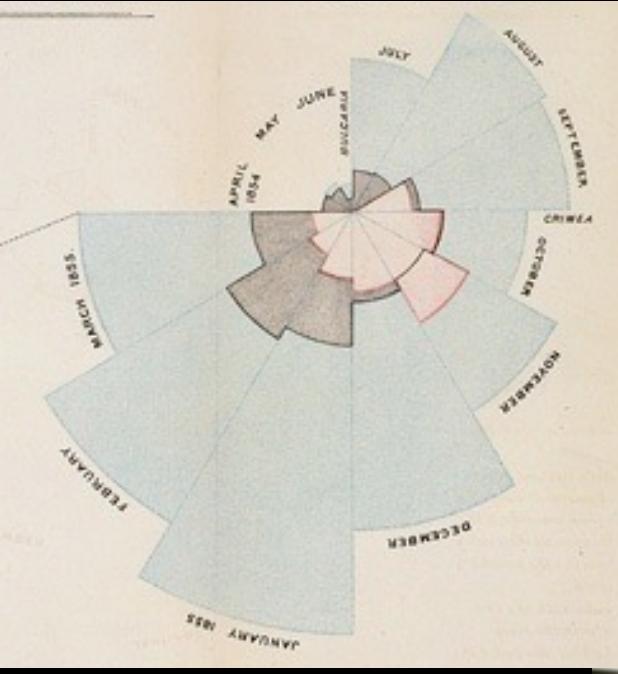


CSE 512 - Data Visualization

# Exploratory Data Analysis



Jeffrey Heer University of Washington

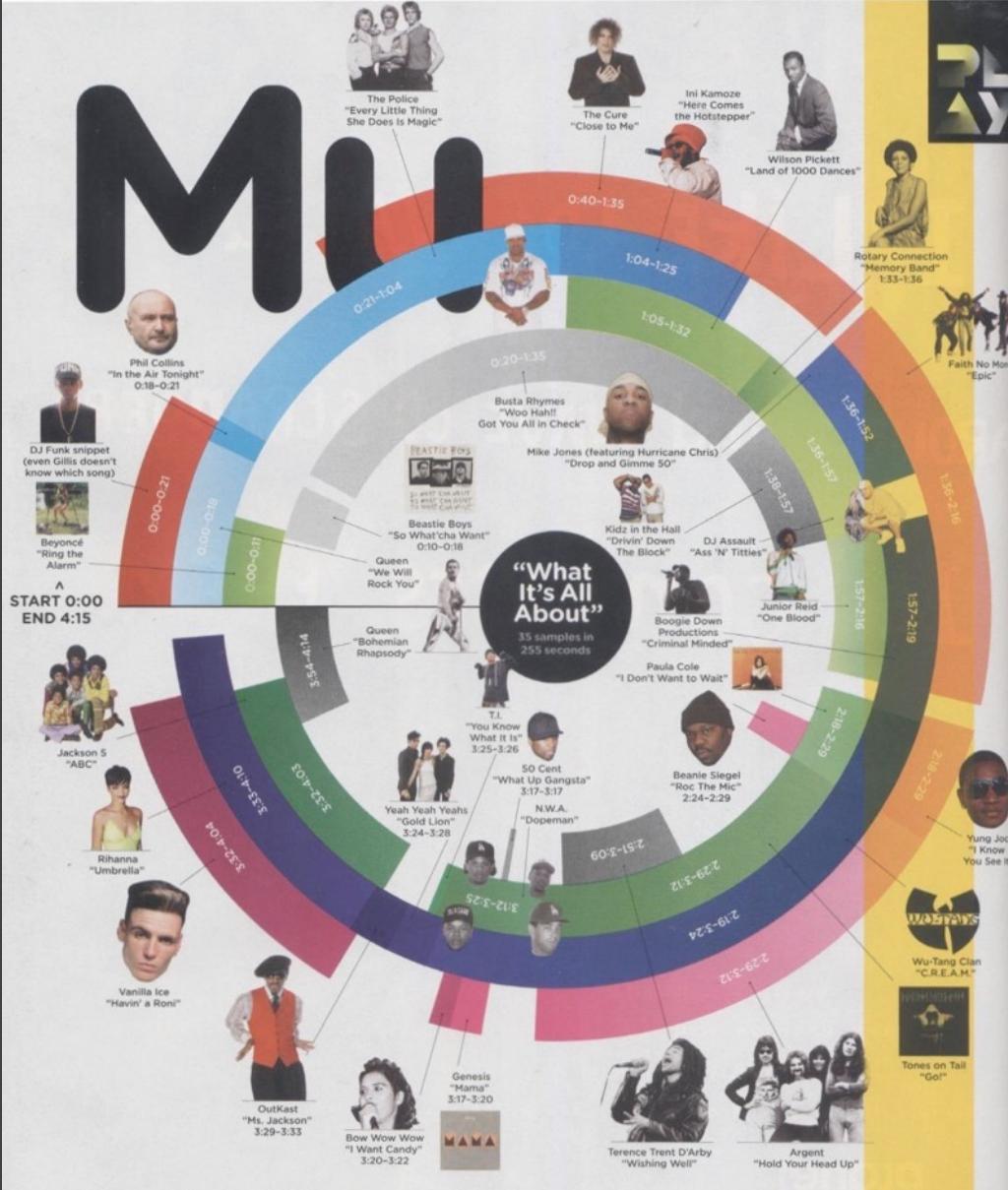
Last Time:  
Design & Re-Design

# TRIPPING POINT

Half the power plants planned over the last five years have still not come on steam due to slackness of the policy machinery



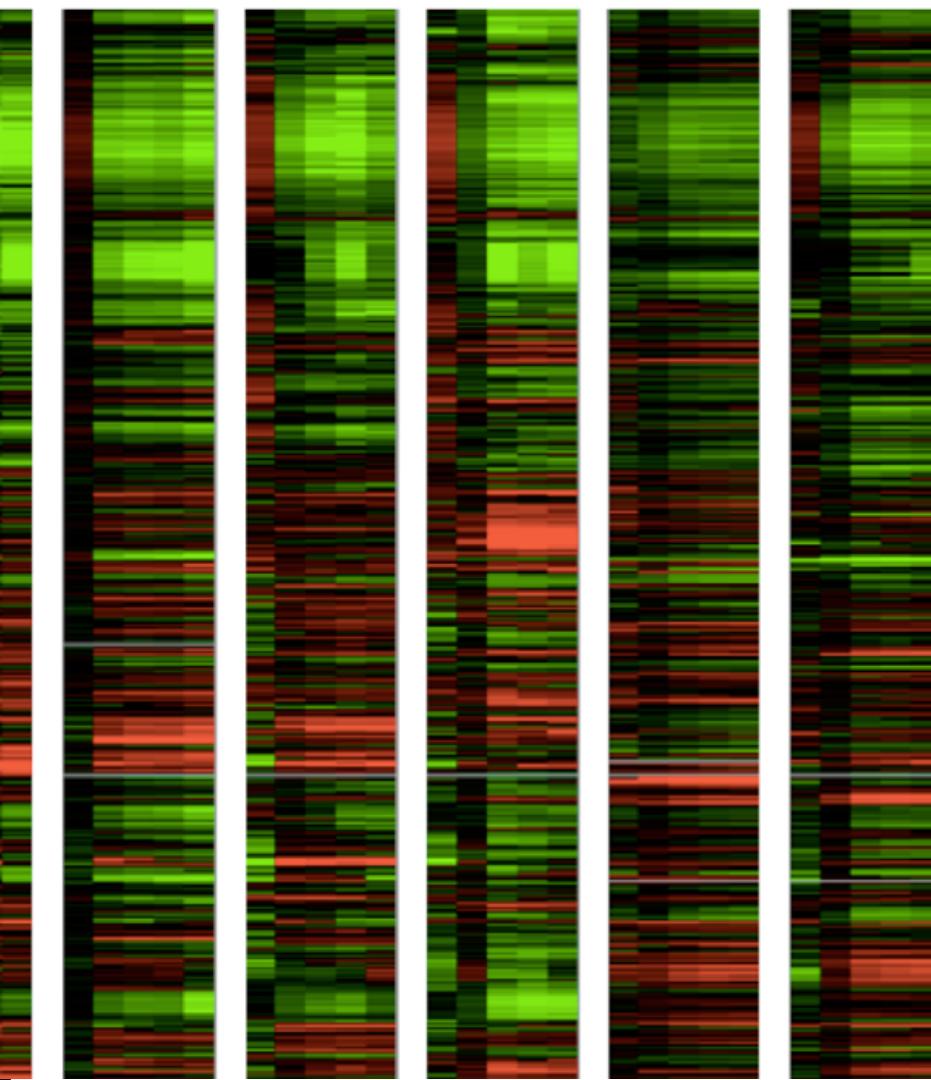
Source: *India Today*



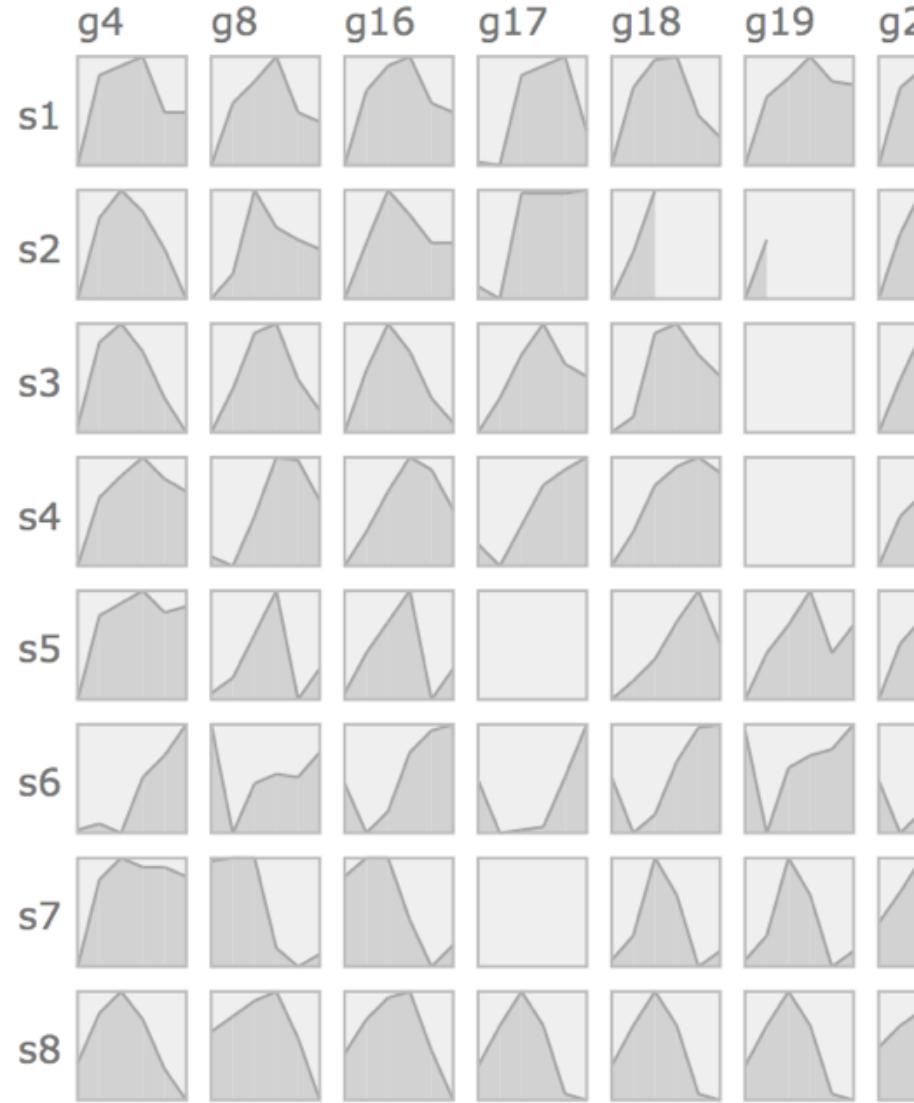
Source: *Wired Magazine*, September 2008 Edition  
Music: Super Cuts (page 92)

# Gene Expression Time-Series [Meyer et al 11]

Color Encoding



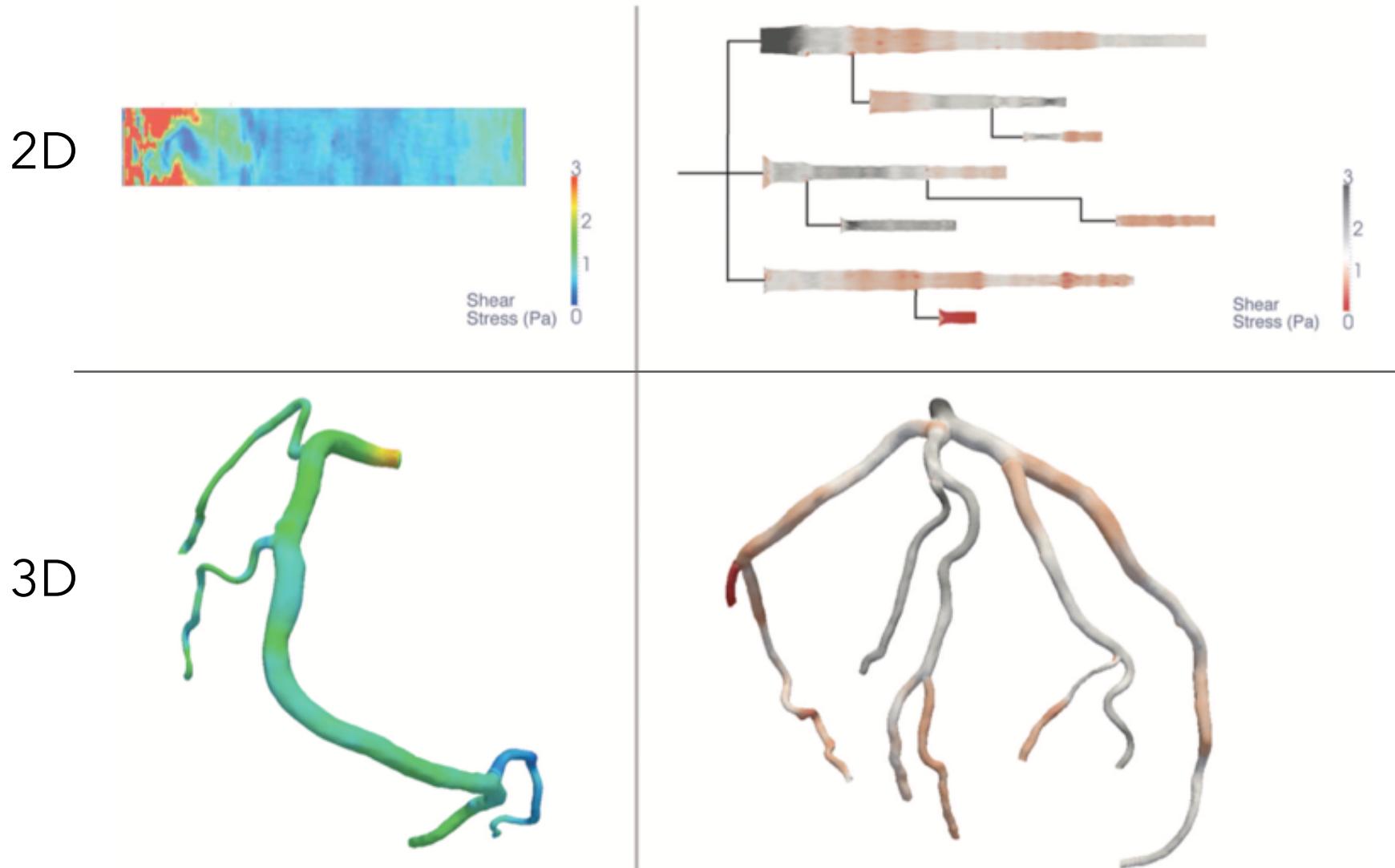
Position Encoding



# Artery Visualization

[Borkin et al 11]

Rainbow Palette

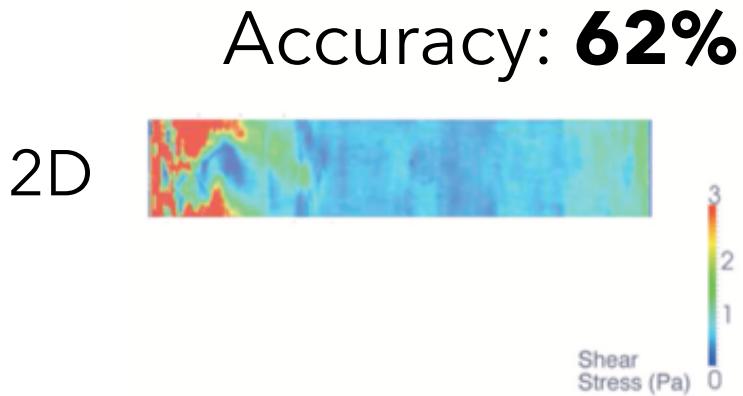


Diverging Palette

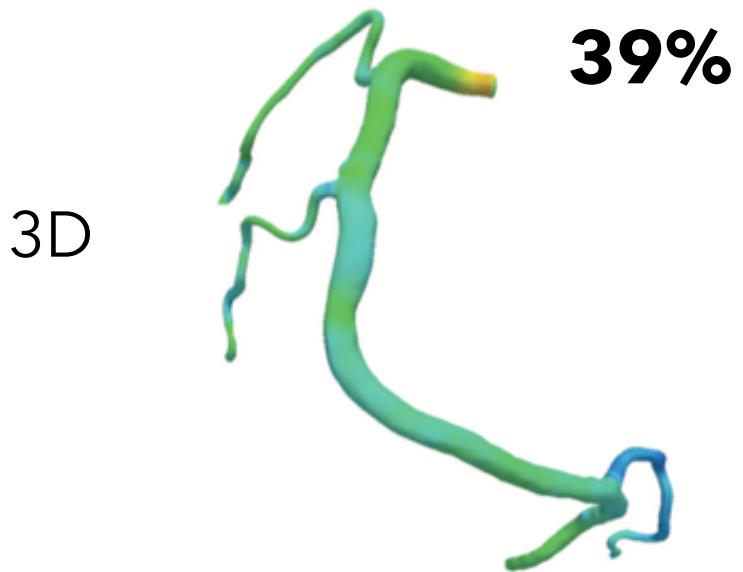
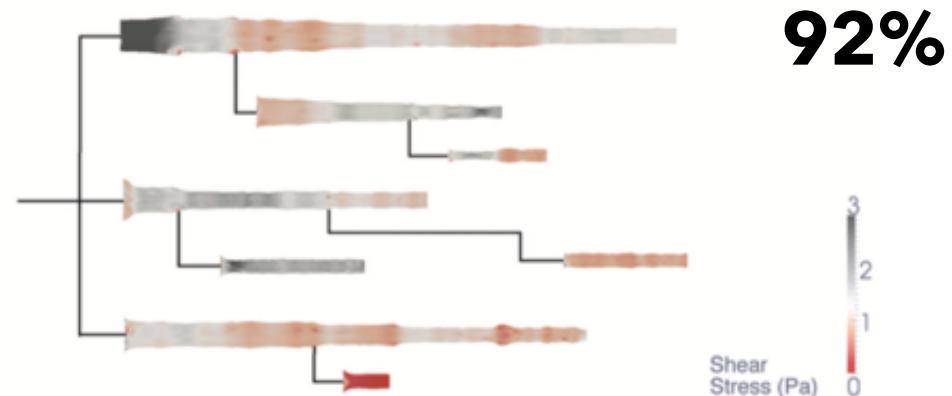
# Artery Visualization

[Borkin et al 11]

Rainbow Palette



Diverging Palette



What was the first  
data visualization?

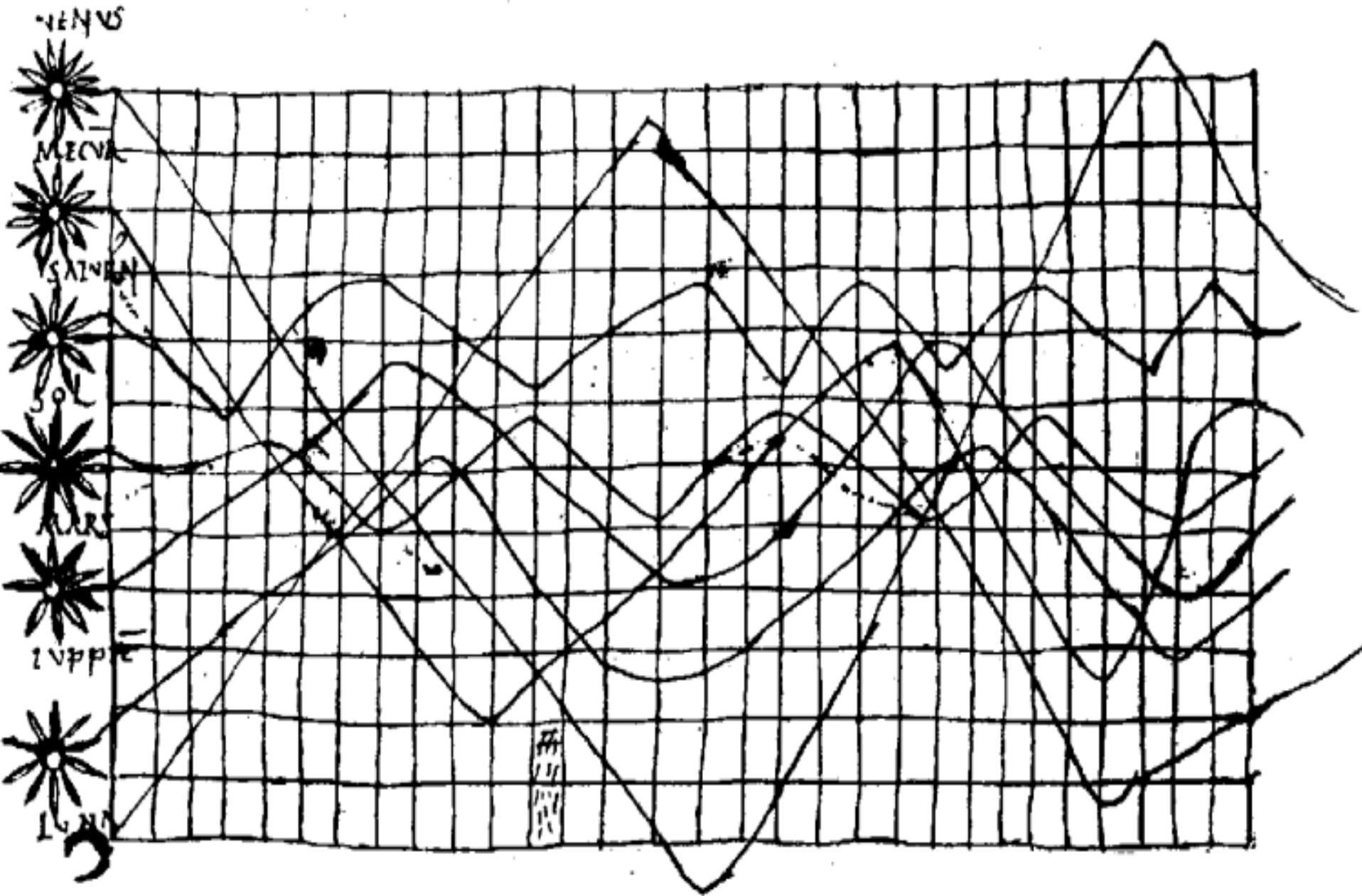
0 BC





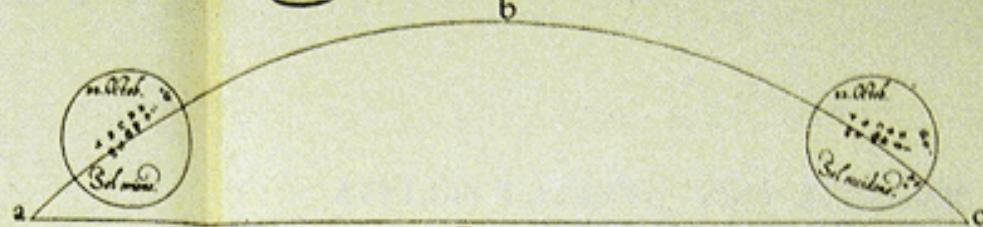
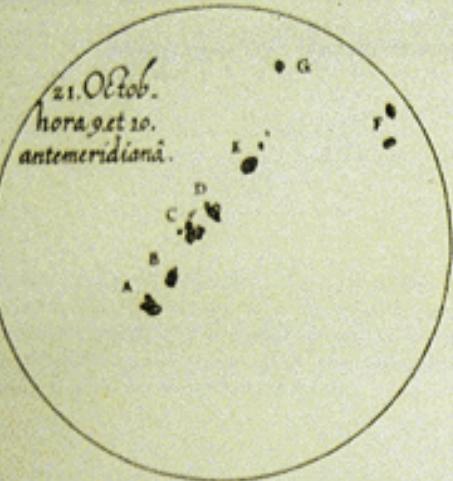
~6200 BC Town Map of Catal Hüyük, Konya Plain, Turkey

0 BC



~950 AD Position of Sun, Moon and Planets

MACVLAE IN SOLE APPARENTES, OBSERVATAE  
anno 1611. ad latitudinem grad. 48. min. 40.



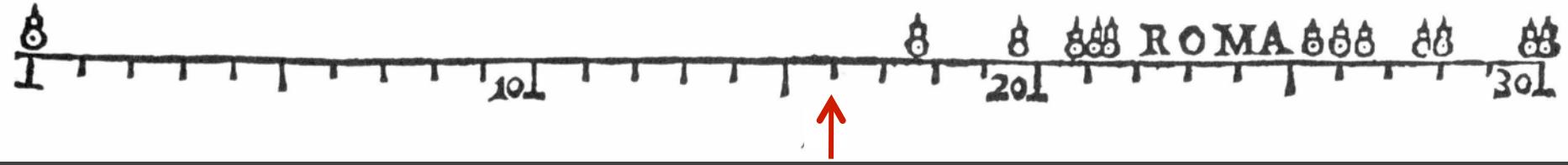
*b*

*a c, horizon. a b c, arcus solis diurnus. Sol oriens ex parte a, maculas exhibet quas vides, occidens vero c, easdem ratione primi motus, non nihil inuertit. Et hanc matutinam vespertinam mutationem, omnes maculae quotidie subeunt Quod semel exhibuisse et monuisse, sufficiat.*

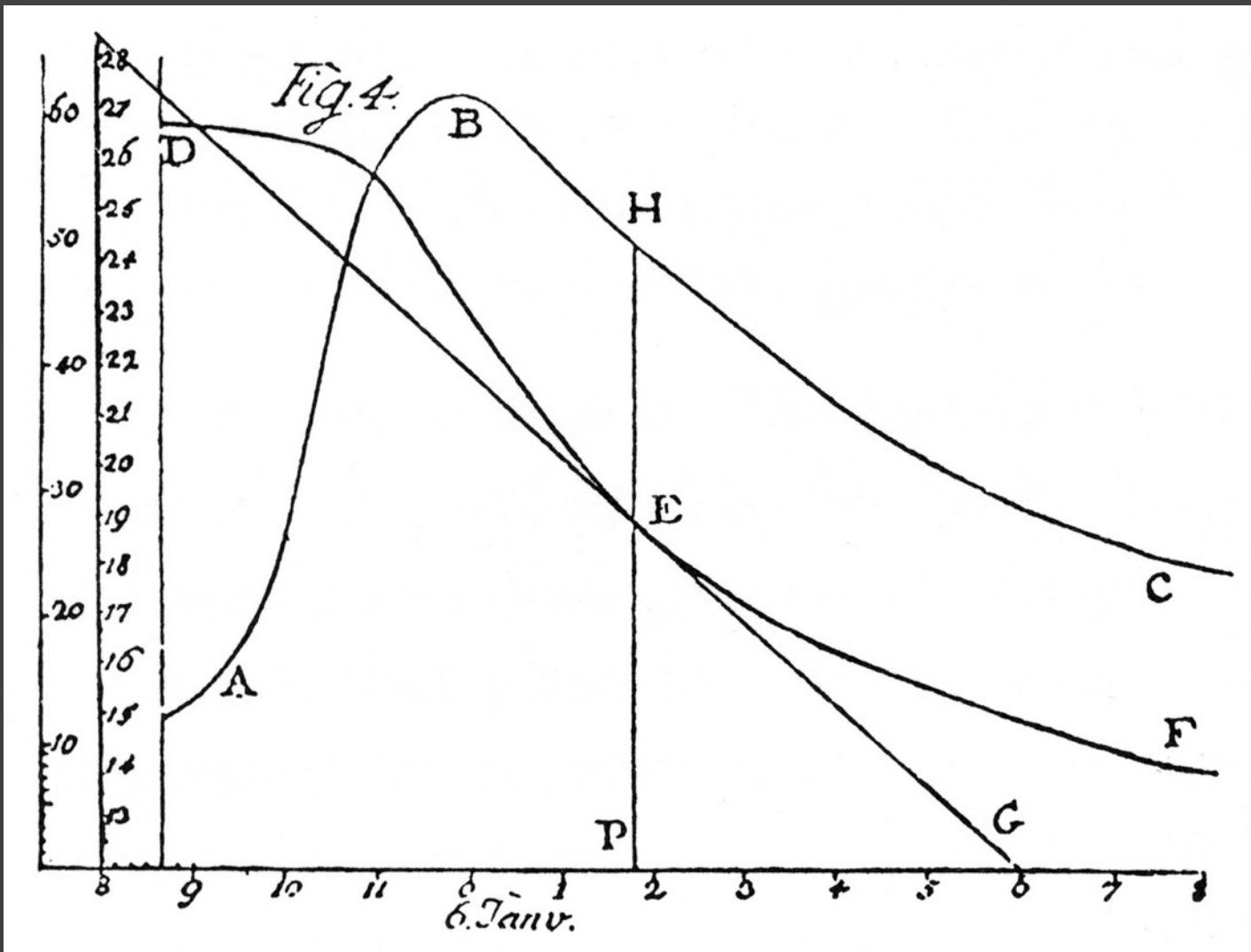


TOLEDO.

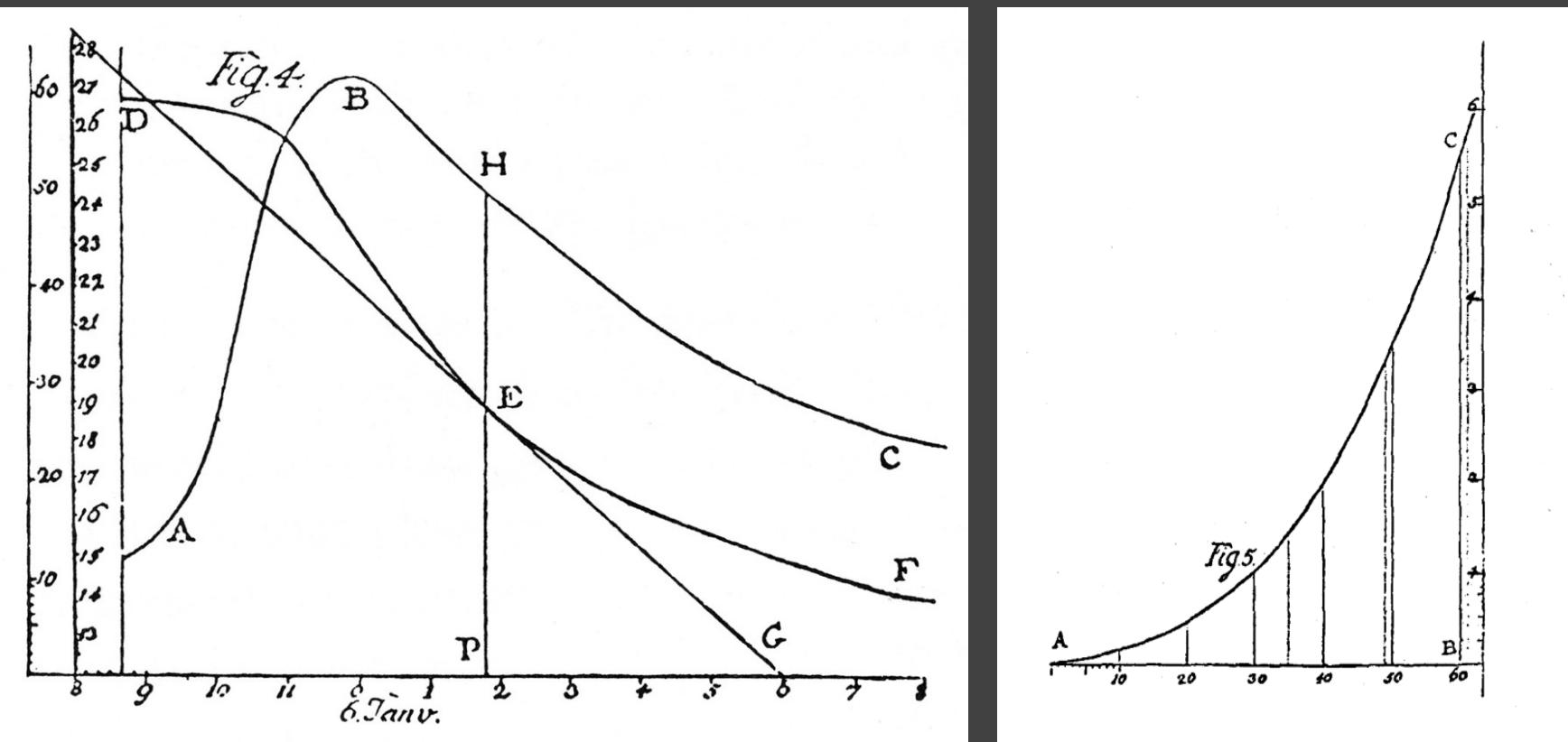
## GRADOS DE LA LONGITUD.



Longitudinal distance between Toledo and Rome, van Langren 1644



The Rate of Water Evaporation, Lambert 1765

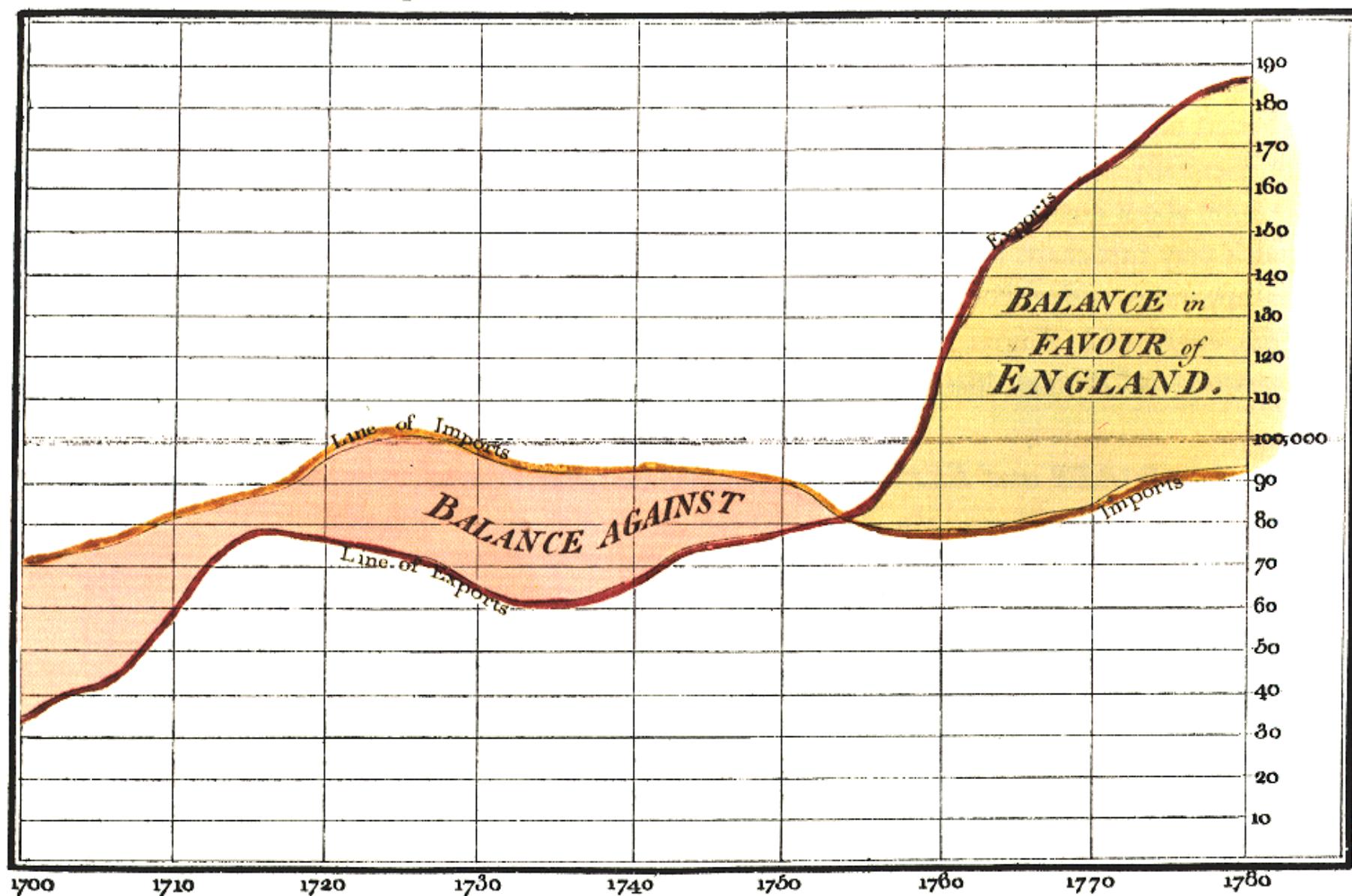


The Rate of Water Evaporation, Lambert 1765

# The Golden Age of Data Visualization

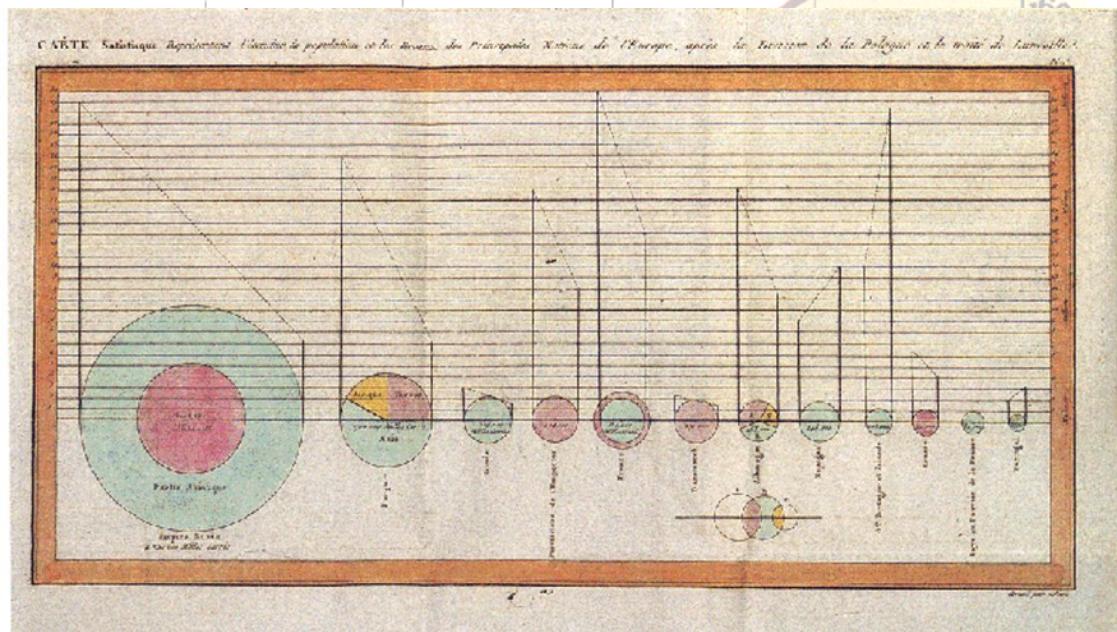
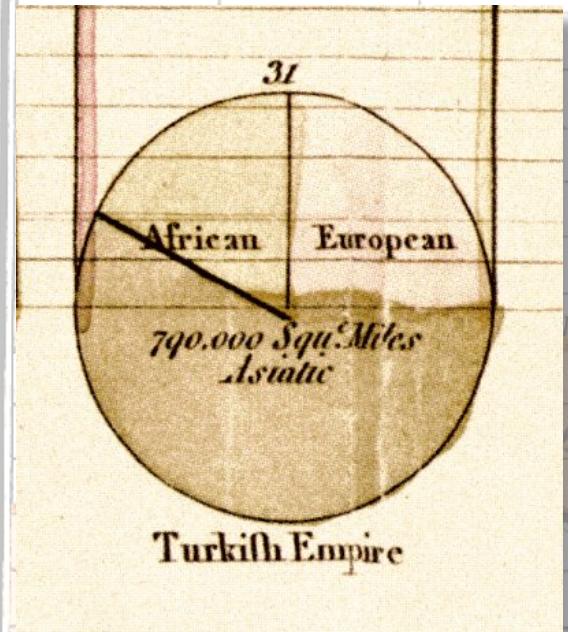
1786 1900

Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780.



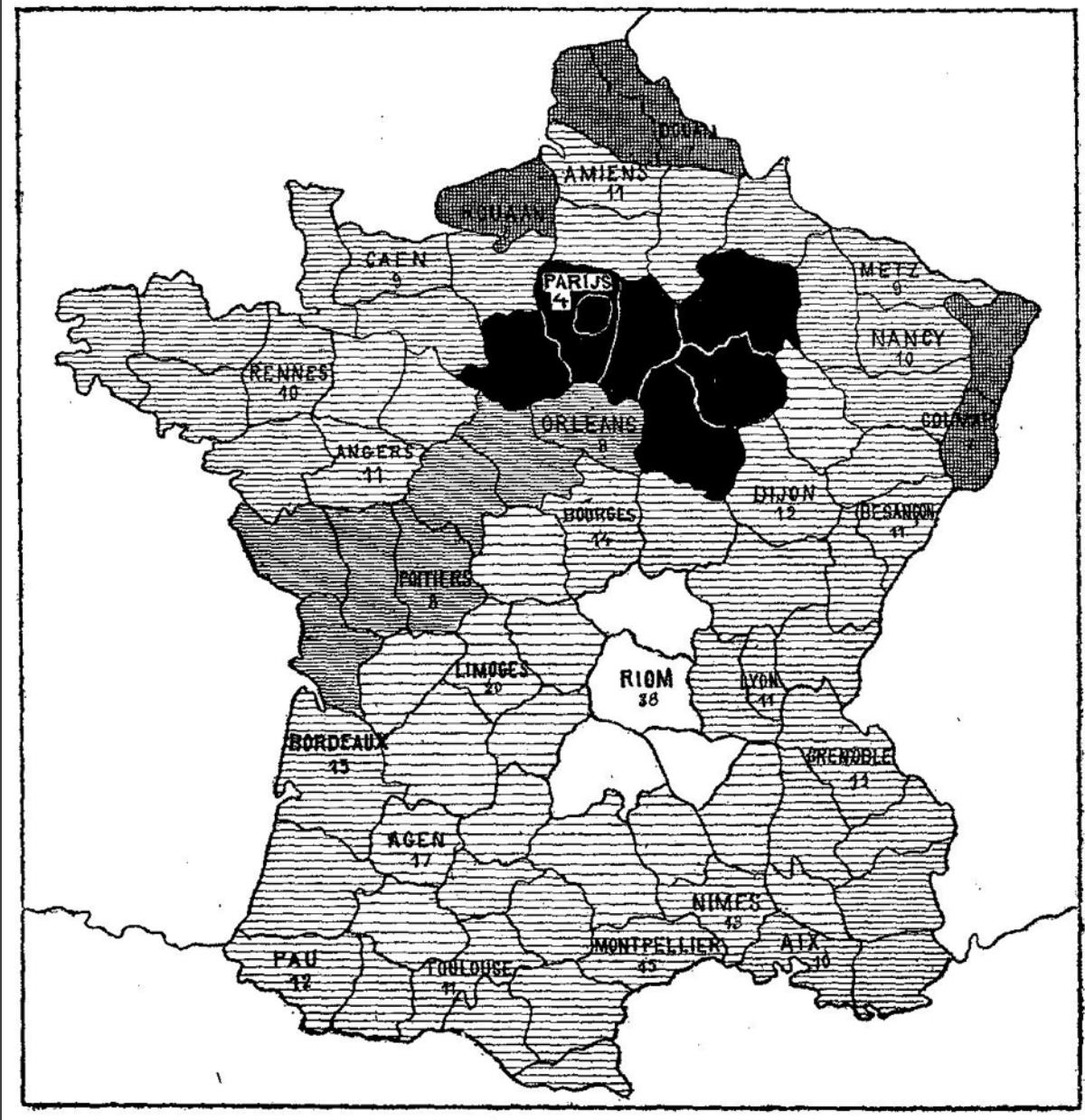
The Commercial and Political Atlas, William Playfair 1786

Exports and Imports to and from DENMARK & NORWAY from 1700 to 1780.



1700      1710      1720      1730      1740      1750      1760      1770      1780

Statistical Breviary, William Playfair 1801



1786

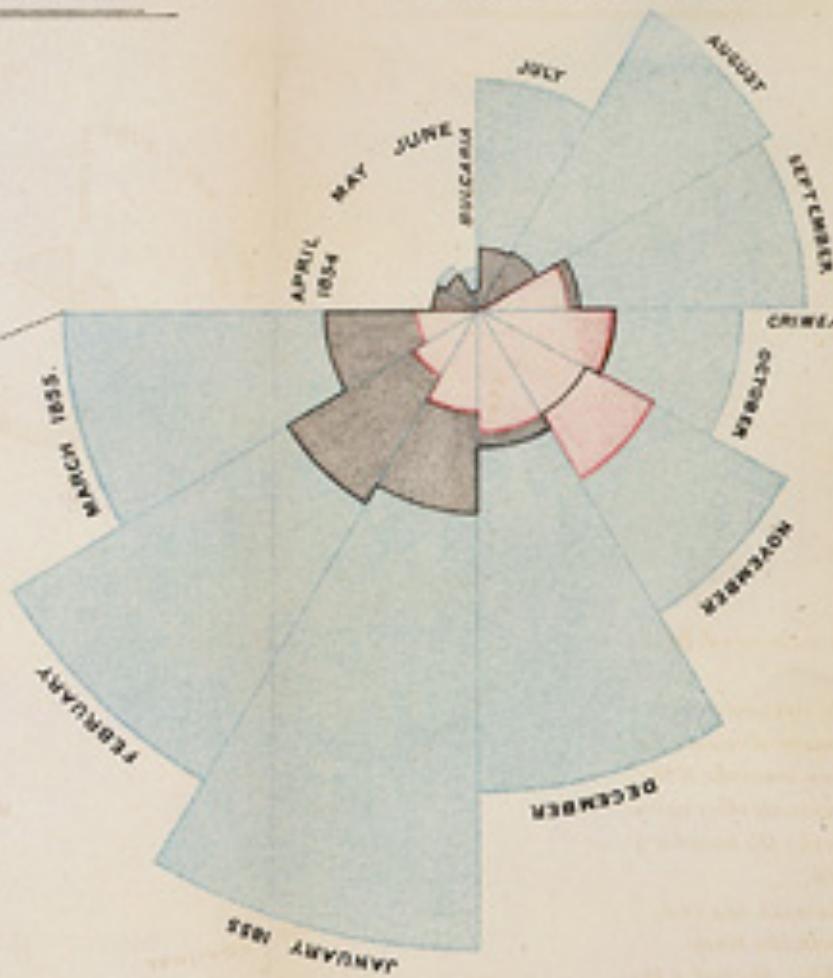
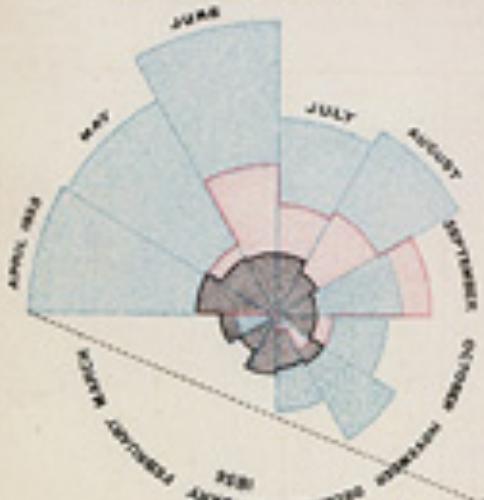
1826(?) Illiteracy in France, Pierre Charles Dupin



2.  
APRIL 1855 TO MARCH 1856.

DIAGRAM OF THE CAUSES OF MORTALITY  
IN THE ARMY IN THE EAST.

1.  
APRIL 1854 TO MARCH 1855.

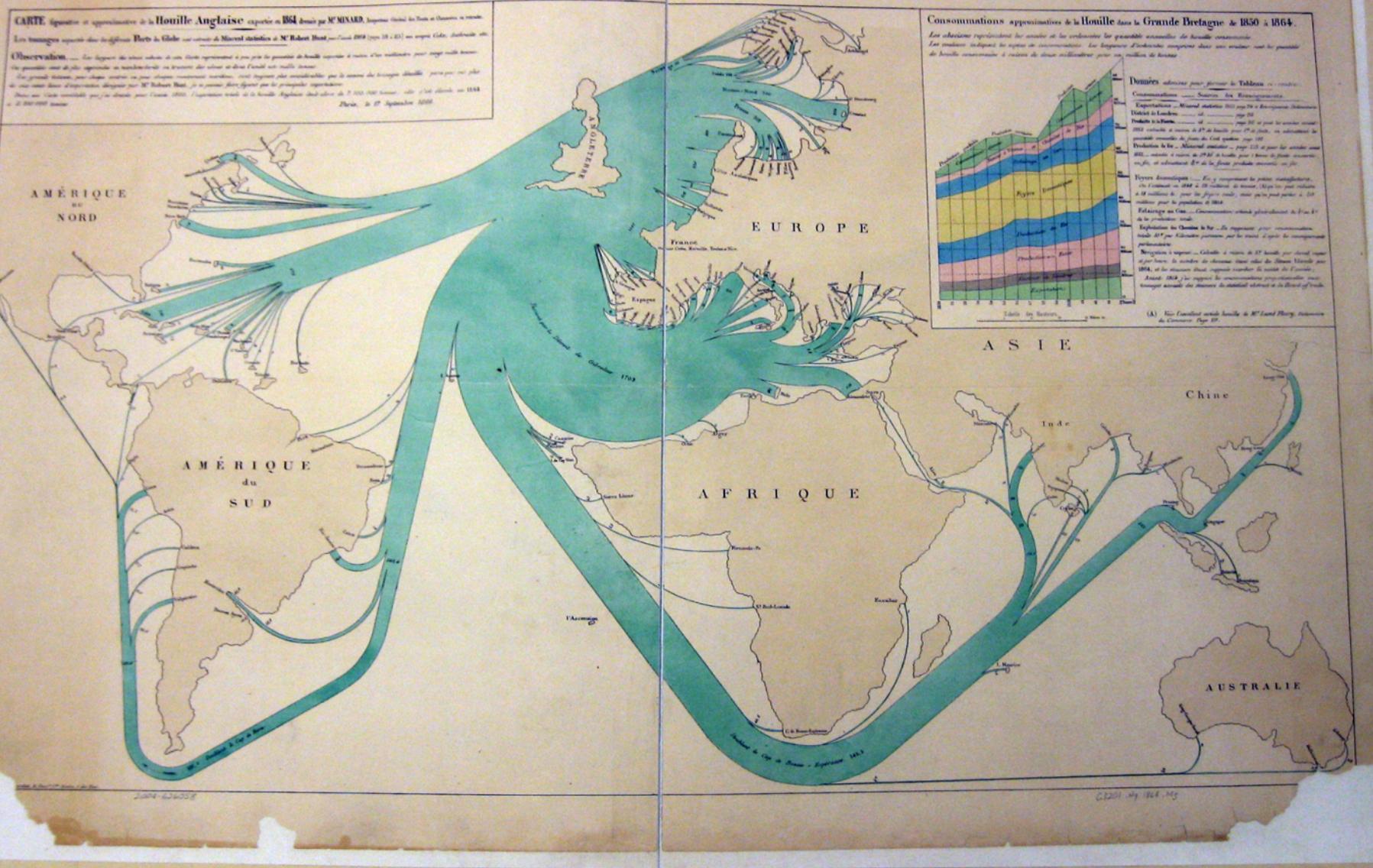


"to affect thro' the Eyes  
what we fail to convey to  
the public through their  
word-proof ears"

1786

1856 "Coxcomb" of Crimean War Deaths, Florence Nightingale



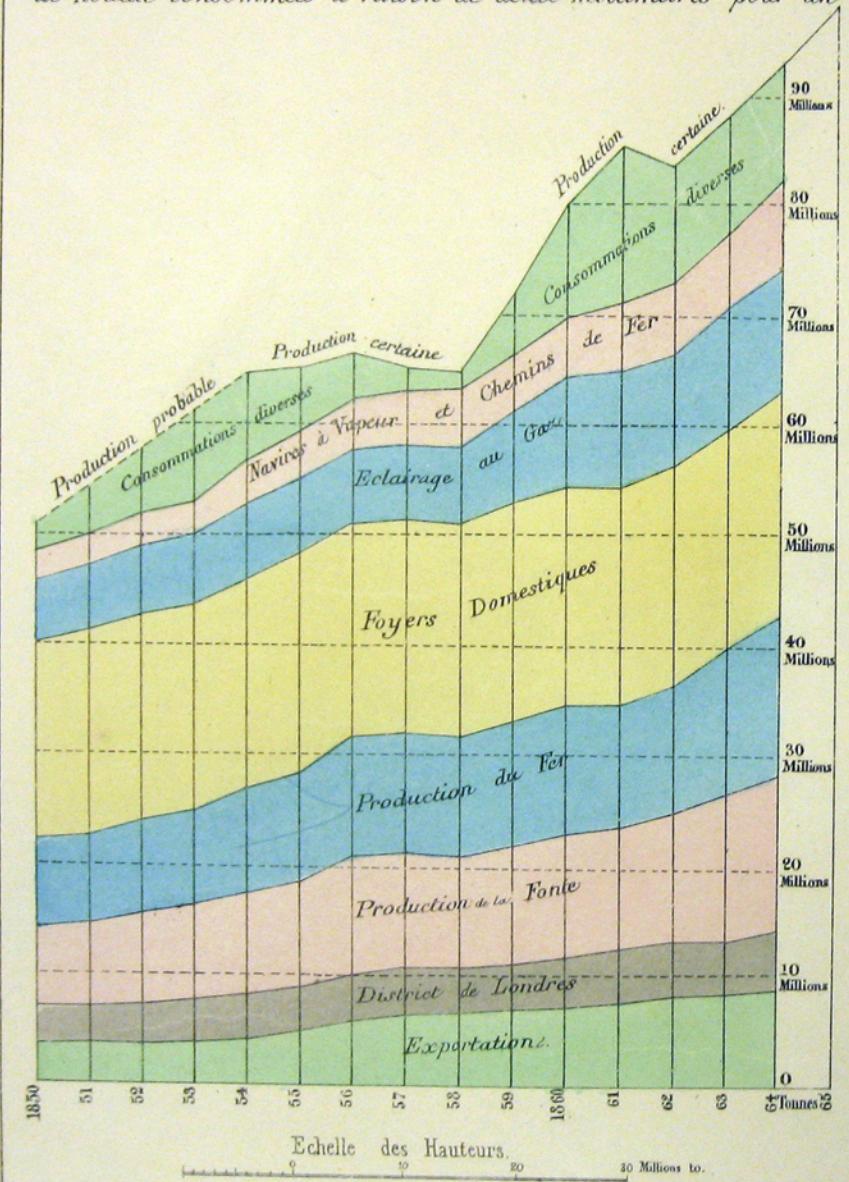


1786

# Consommations approximatives de la Houille dans la Grande Bretagne de 1850 à 1864.

Les abscisses représentent les années et les ordonnées les quantités annuelles de houille consommée.

Les couleurs indiquent les espèces de consommations. Les longueurs d'ordonnées comprises dans une couleur sont les quantités de houille consommées à raison de deux millimètres pour un million de tonnes.



Données admises pour former le Tableau ci-contre.

Consommations. — Sources des Renseignements.

Exportations. — *Mineral statistics 1865 page 214 et Renseignements Parlementaires*  
District de Londres. — *id. page 213*

Produits de la Fonte. — *id. page 215* et pour les années avant  
1855 calculée à raison de 3<sup>10</sup> de houille pour 1<sup>10</sup> de fonte, en admettant les  
quantités annuelles de fonte du Coal question page 192.

Production du fer. — *Mineral statistics — page 215* et pour les années avant  
1855 — calculée à raison de 3<sup>10</sup>.35 de houille pour 1 tonne de fonte convertie  
en fer; et admettant  $\frac{9}{10}$  de la fonte produite convertis en fer.

Foyers domestiques: — En y comprenant les petites manufactures.

On l'estimait en 1848 à 19 millions de tonnes, (A) qu'on peut réduire  
à 18 millions to. pour les foyers seuls, mais qu'on peut porter à 20  
millions pour la population de 1864.

Eclairage au Gaz. — Consommation estimée généralement du  $\frac{3}{4}$  au  $\frac{5}{6}$   
de la production totale.

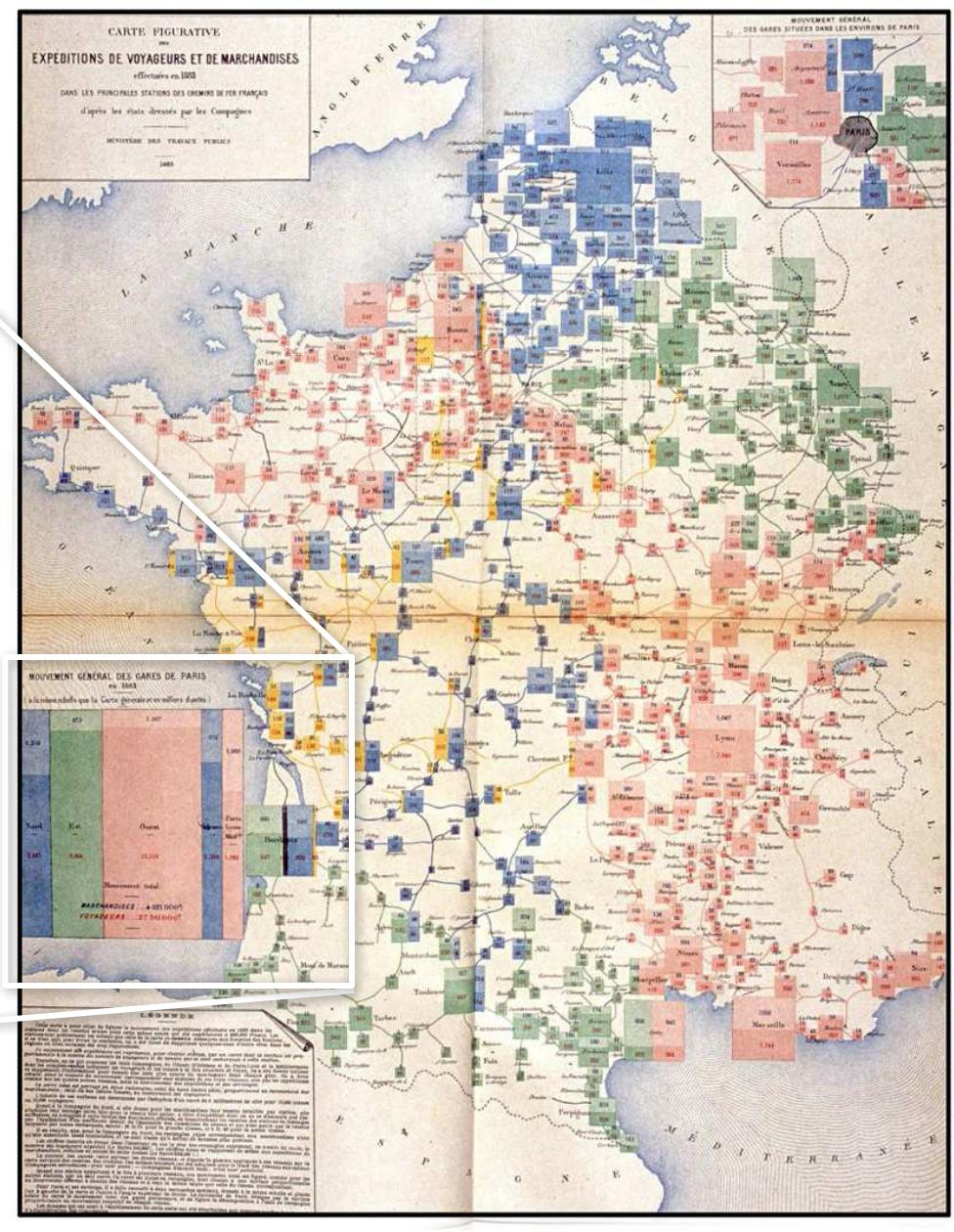
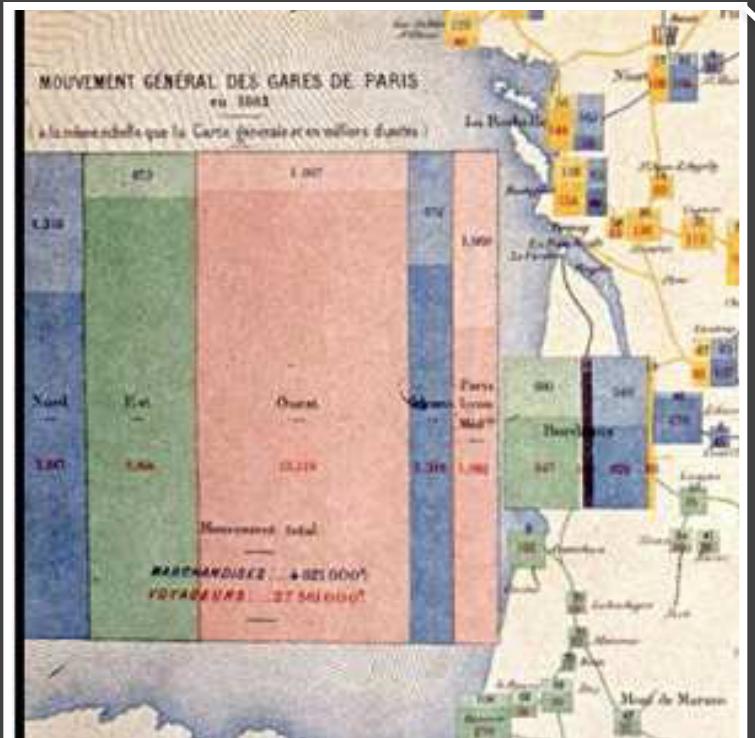
Exploitation des Chemins de Fer. — En supposant pour consommation  
totale 10<sup>8</sup> par Kilomètre parcouru par les trains d'après les renseignements  
parlementaires.

Navigation à vapeur. — Calculée à raison de 5<sup>8</sup> houille par cheval vapeur  
et par heure, le nombre de chevaux étant celui du Steam Vessels pour  
1864, et les steamers étant supposés marcher la moitié de l'année;

Avant 1864 j'ai supposé les consommations proportionnelles aux  
tonnages annuels des steamers du statistical abstract et du Board of trade.

(A) Voir l'excellent article houille de M<sup>r</sup> Lamé Fleury, Dictionnaire  
du Commerce Page III.





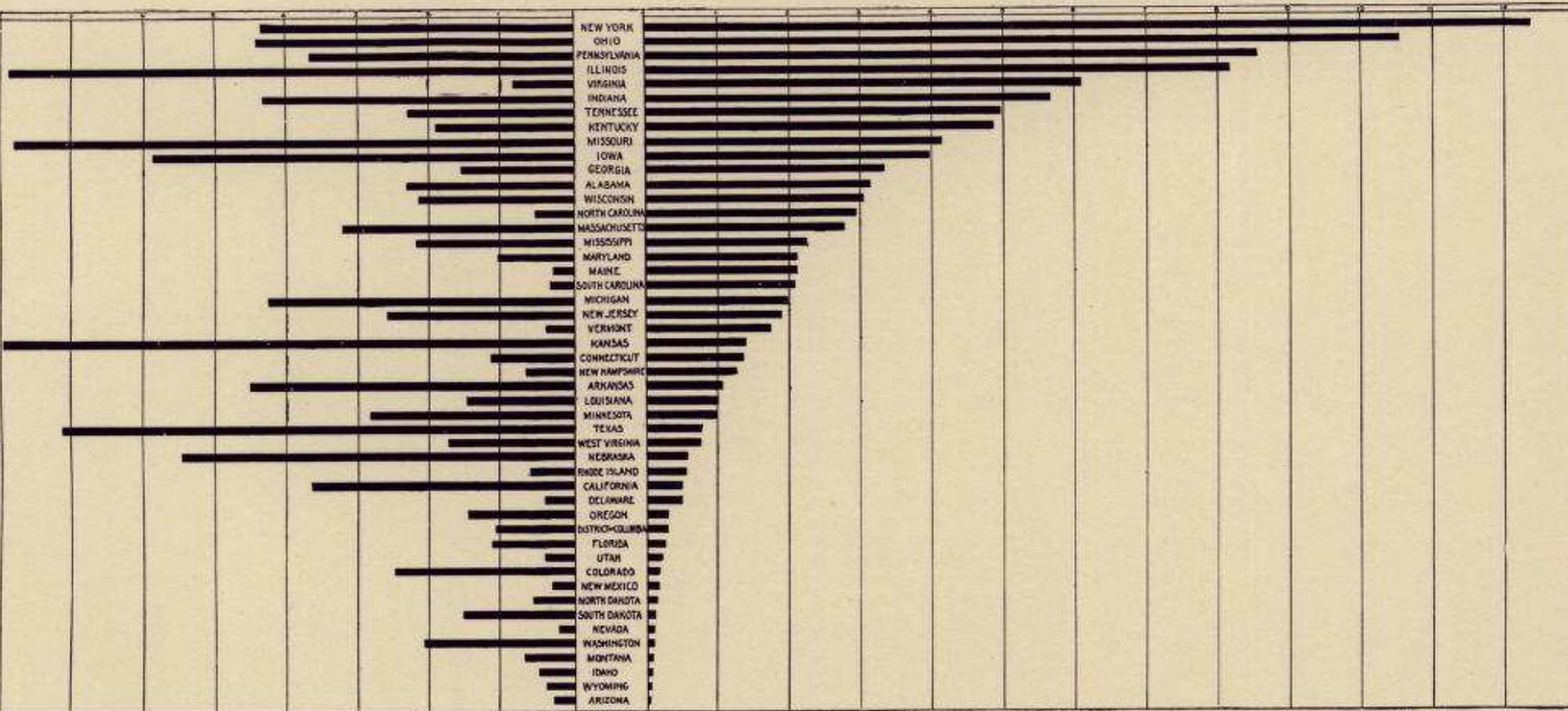
1786

66. INTERSTATE MIGRATION—NUMBER OF NATIVE IMMIGRANTS AND NATIVE EMIGRANTS, BY STATES AND TERRITORIES: 1890.

Native immigrants.

[Hundreds of thousands.]

Native emigrants.



# The Rise of Statistics

1786

1900

1950



Rise of **formal methods** in statistics and social science – Fisher, Pearson, ...

**Little innovation** in graphical methods

A period of **application and popularization**

Graphical methods enter textbooks, curricula, and **mainstream use**

1786

1900

1950





1786

Data Analysis & Statistics, Tukey 1962





Four major influences act on data analysis today:

1. The formal theories of statistics.
2. Accelerating developments in computers and display devices.
3. The challenge, in many fields, of more and larger bodies of data.
4. The emphasis on quantification in a wider variety of disciplines.



The last few decades have seen the rise of formal theories of statistics, "legitimizing" variation by confining it by assumption to random sampling, often assumed to involve tightly specified distributions, and restoring the appearance of security by emphasizing narrowly optimized techniques and claiming to make statements with "known" probabilities of error.



While some of the influences of statistical theory on data analysis have been helpful, others have not.



**Exposure**, the effective laying open of the data to display the unanticipated, is to us a major portion of data analysis. Formal statistics has given almost no guidance to exposure; indeed, it is not clear how the **informality** and **flexibility** appropriate to the **exploratory character of exposure** can be fitted into any of the structures of formal statistics so far proposed.



Nothing - not the careful logic of mathematics, not statistical models and theories, not the awesome arithmetic power of modern computers - nothing can substitute here for the **flexibility of the informed human mind.**

Accordingly, both approaches and techniques need to be structured so as to **facilitate human involvement and intervention.**

Set A

X	Y
10	8.04
8	6.95
13	7.58
9	8.81
11	8.33
14	9.96
6	7.24
4	4.26
12	10.84
7	4.82
5	5.68

Set B

X	Y
10	9.14
8	8.14
13	8.74
9	8.77
11	9.26
14	8.1
6	6.13
4	3.1
12	9.11
7	7.26
5	4.74

Set C

X	Y
10	7.46
8	6.77
13	12.74
9	7.11
11	7.81
14	8.84
6	6.08
4	5.39
12	8.15
7	6.42
5	5.73

Set D

X	Y
8	6.58
8	5.76
8	7.71
8	8.84
8	8.47
8	7.04
8	5.25
19	12.5
8	5.56
8	7.91
8	6.89

**Summary Statistics**

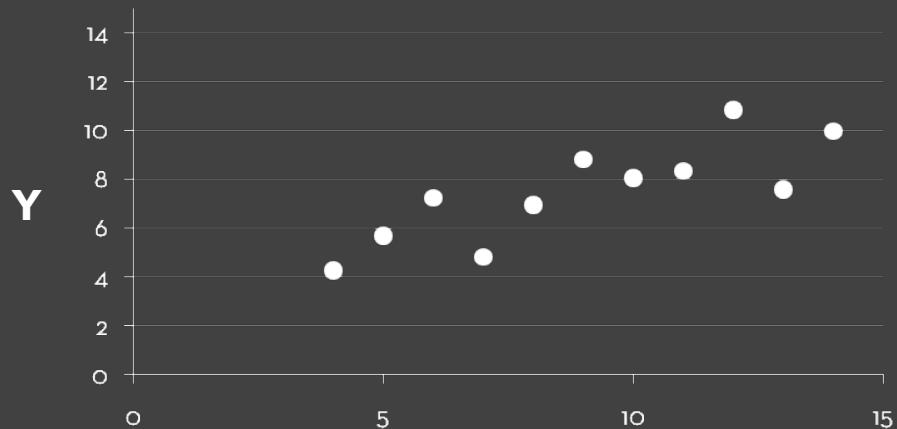
$$\begin{aligned} u_X &= 9.0 & \sigma_X &= 3.317 \\ u_Y &= 7.5 & \sigma_Y &= 2.03 \end{aligned}$$

**Linear Regression**

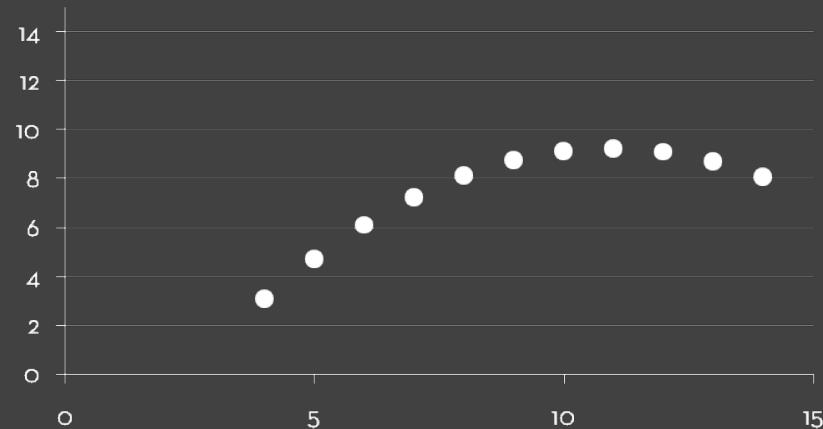
$$\begin{aligned} Y &= 3 + 0.5 X \\ R^2 &= 0.67 \end{aligned}$$

[Anscombe 1973]

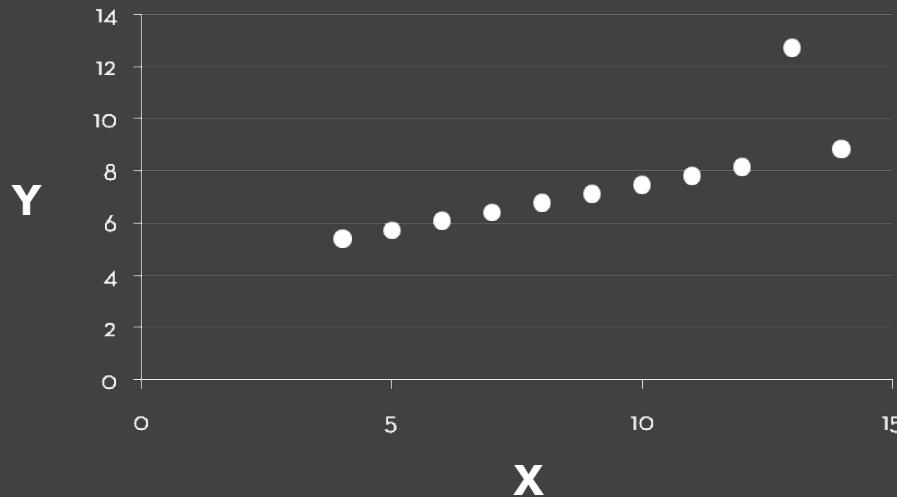
# Set A



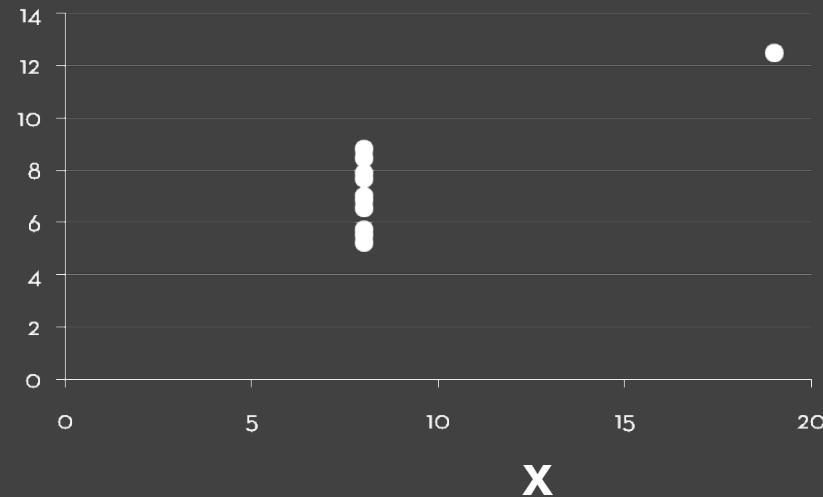
# Set B



# Set C



# Set D



# Topics

## Exploratory Data Analysis

Data Diagnostics

Graphical Methods

Data Transformation

## Incorporating Statistical Models

Data Space & Model Space

Graphical Inference

# Data Diagnostics

Reported crime in Alabama

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	4525375	4029.3	987	2732.4	309.9
2005	4548327	3900	955.8	2656	289
2006	4599030	3937	968.9	2645.1	322.9
2007	4627851	3974.9	980.2	2687	307.7
2008	4661900	4081.9	1080.7	2712.6	288.6

Reported crime in Alaska

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	657755	3370.9	573.6	2456.7	340.6
2005	663253	3615	622.8	2601	391
2006	670053	3582	615.2	2588.5	378.3
2007	683478	3373.9	538.9	2480	355.1
2008	686293	2928.3	470.9	2219.9	237.5

Reported crime in Arizona

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	5739879	5073.3	991	3118.7	963.5
2005	5953007	4827	946.2	2958	922
2006	6166318	4741.6	953	2874.1	914.4
2007	6338755	4502.6	935.4	2780.5	786.7
2008	6500180	4087.3	894.2	2605.3	587.8

Reported crime in Arkansas

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	2750000	4033.1	1096.4	2699.7	237
2005	2775708	4068	1085.1	2720	262
2006	2810872	4021.6	1154.4	2596.7	270.4
2007	2834797	3945.5	1124.4	2574.6	246.5
2008	2855390	3843.7	1182.7	2433.4	227.6

Reported crime in California

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	35842038	3423.9	686.1	2033.1	704.8
2005	36154147	3321	692.9	1915	712
2006	36457549	3175.2	676.9	1831.5	666.8
2007	36553215	3032.6	648.4	1784.1	600.2
2008	36756666	2940.3	646.8	1769.8	523.8

Reported crime in Colorado

Year	Population	Property crime rate	Burglary rate	Larceny-theft rate	Motor vehicle theft rate
2004	4601821	3918.5	717.3	2679.5	521.6

# Data Wrangling

One often needs to manipulate data prior to analysis. Tasks include reformatting, cleaning, quality assessment, and integration.

Approaches include:

Manual manipulation in spreadsheets

Writing custom code (dplyr in R, Pandas in Python)

Trifacta Wrangler <http://www.trifecta.com/products/wrangler/>

Open Refine <http://openrefine.org/>

# Data Quality

"The first sign that a visualization is good is that it shows you a problem in your data..."

...every successful visualization that I've been involved with has had this stage where you realize, "Oh my God, this data is not what I thought it would be!" So already, you've discovered something."

Martin Wattenberg



Age: 95  
Sex: Female  
Race: Caucasian  
County (Res): Prince Georges  
Zip Code (Res): 20770  
Received: 940706  
Complaint Sequence: 1  
Source: Citizen  
Reason: Delinquent  
Alleged Offense: HARAS  
Offense Level: 2 - Misdemeanor  
County (Off): Prince Georges  
Zip Code (Off): 20770  
Area: V  
Office: 71610  
Intake Decision Date: 940729  
Intake Decision: Closed  
Days to ID: 23  
Court Finding: NONE  
Disposition Date: 0  
Disposition:

Offenses  
Count  
Area:  
Office:  
Intake



Age

TC

Query Result: 4792 out of 4792 (100%)

# Graph Viewer

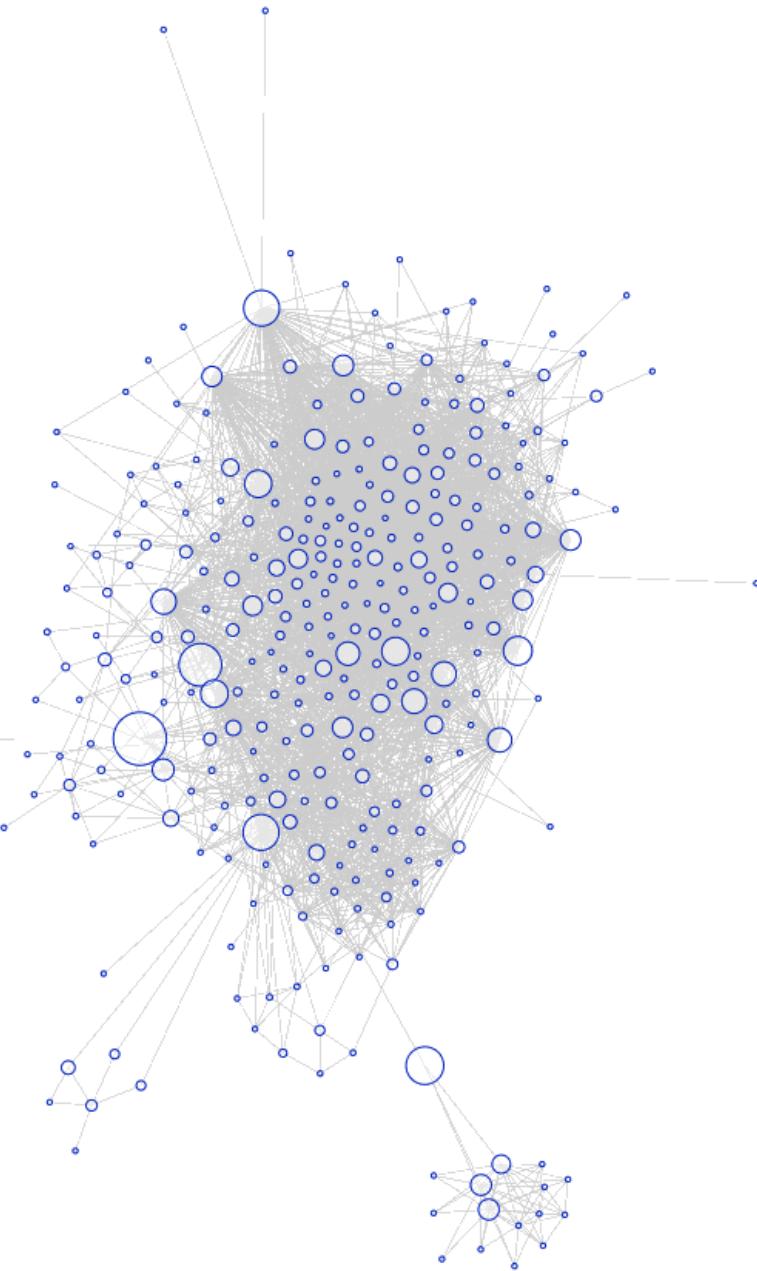
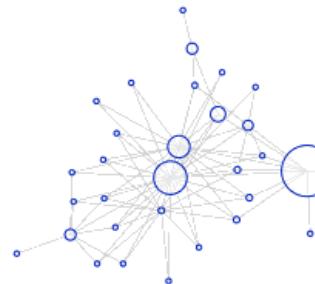
## Graph Viewer

Roll-up by:

Visualization:

Sort by:

Edge centrality filters:



Images

Animate

# Graph Viewer

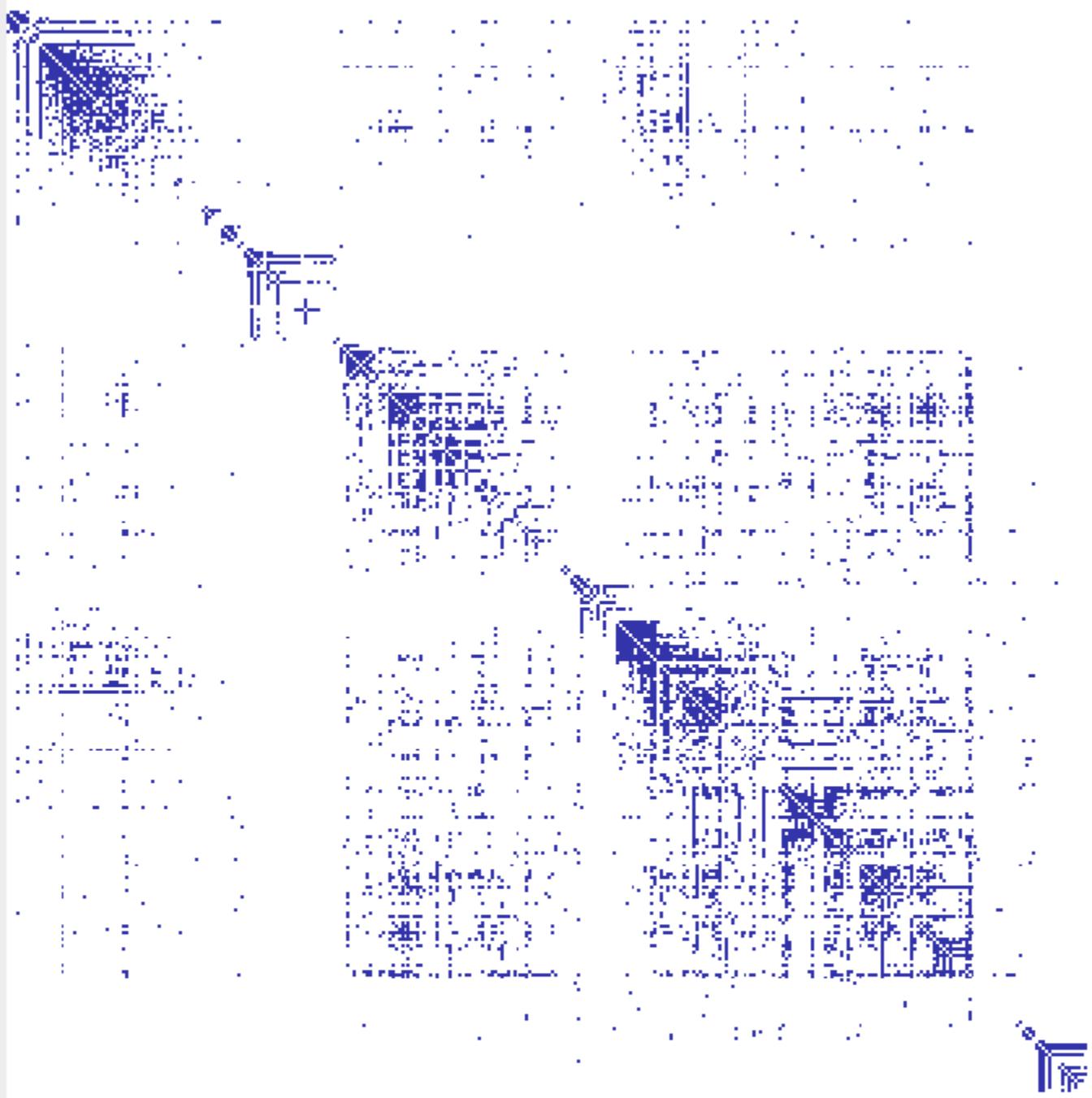
## Graph Viewer

Roll-up by:

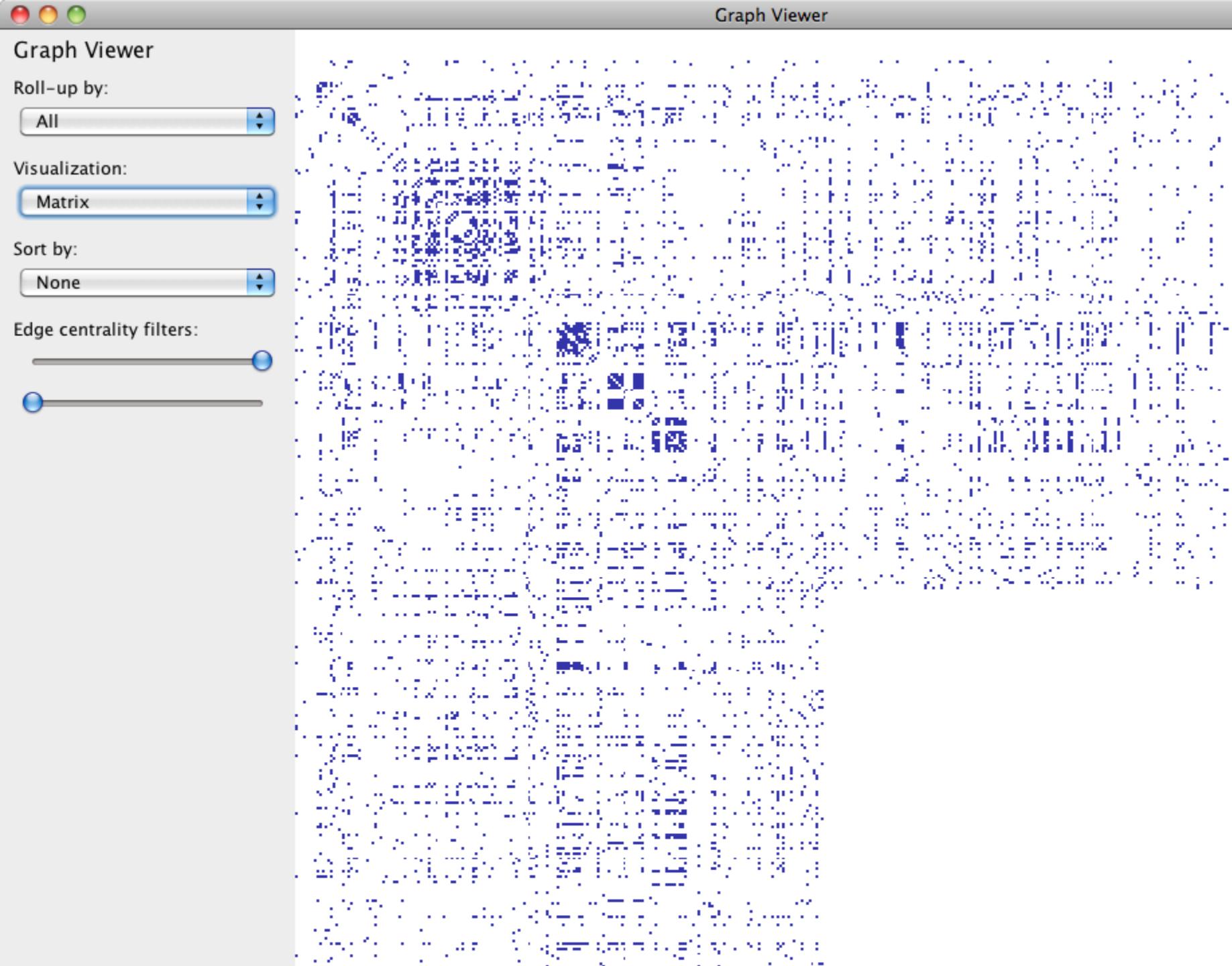
Visualization:

Sort by:

Edge centrality filters:



Graph Viewer



# Visualize Friends by School?

Berkeley



Cornell



Harvard



Harvard University



Stanford



Stanford University



UC Berkeley



UC Davis



University of California at Berkeley



University of California, Berkeley



University of California, Davis



# Data Quality Hurdles

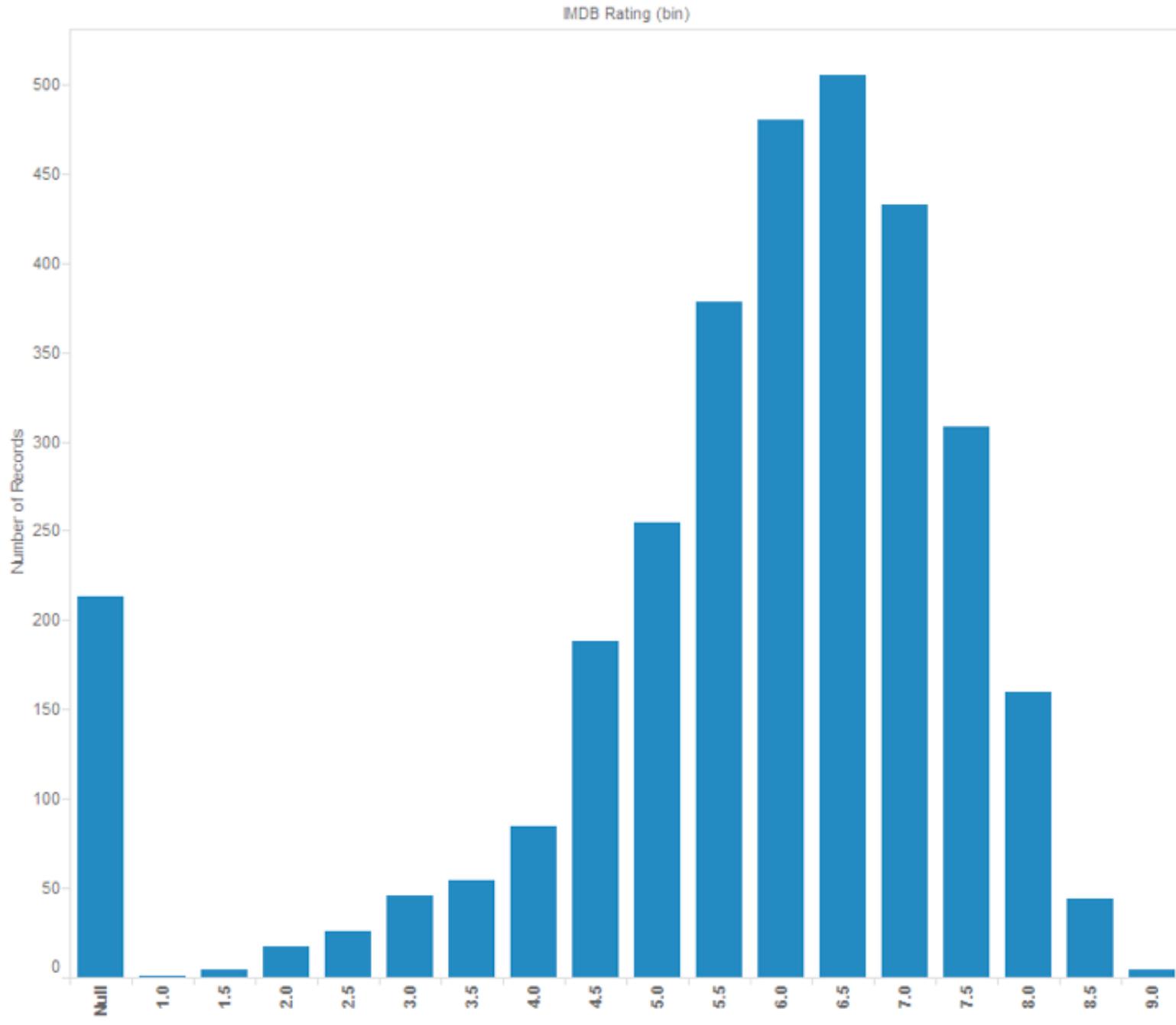
Missing Data	no measurements, redacted, ...?
Erroneous Values	misspelling, outliers, ...?
Type Conversion	e.g., zip code to lat-lon
Entity Resolution	diff. values for the same thing?
Data Integration	effort/errors when combining data

*LESSON:* Anticipate problems with your data.  
Many research problems around these issues!

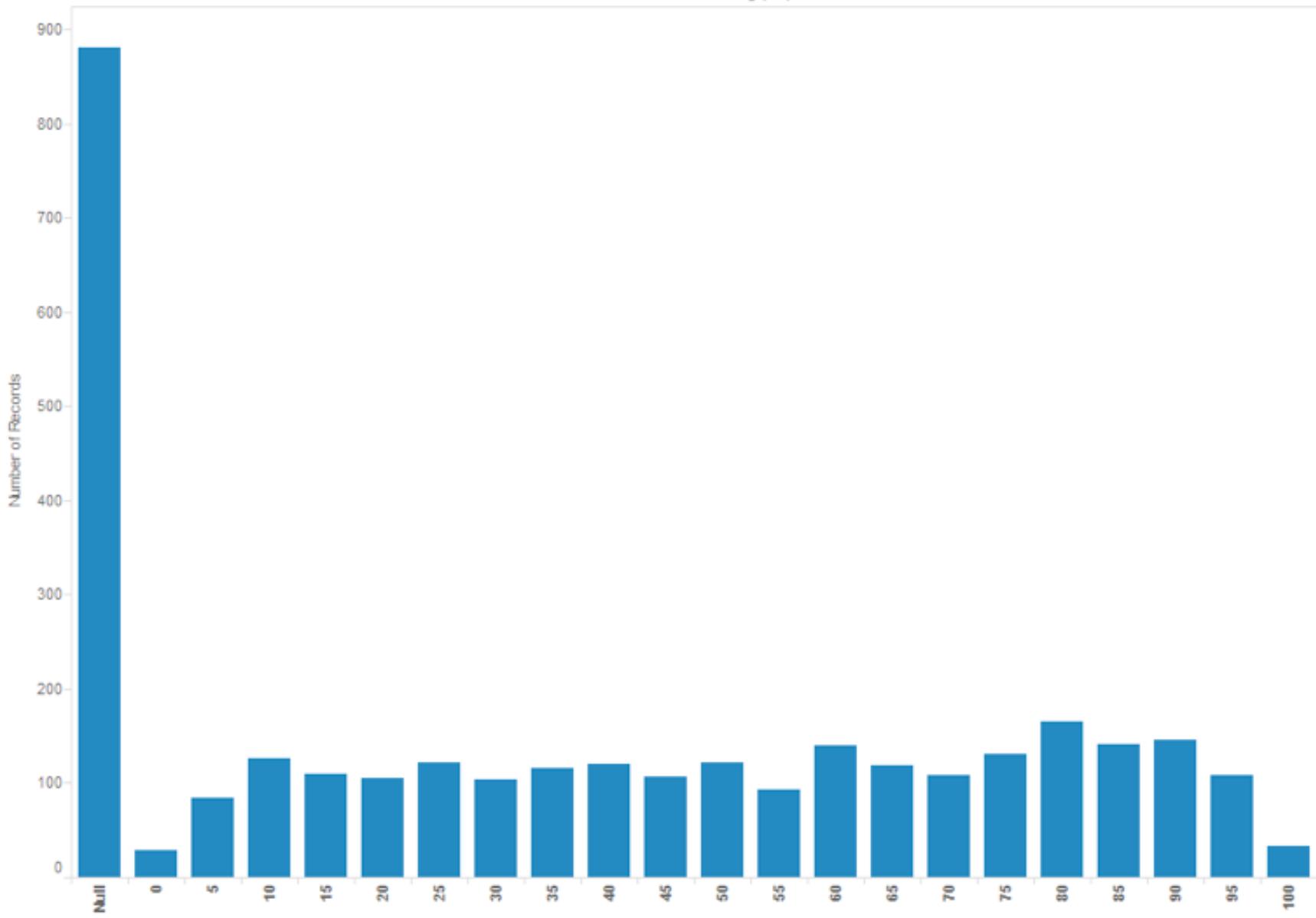
# Analysis Example: Motion Pictures Data

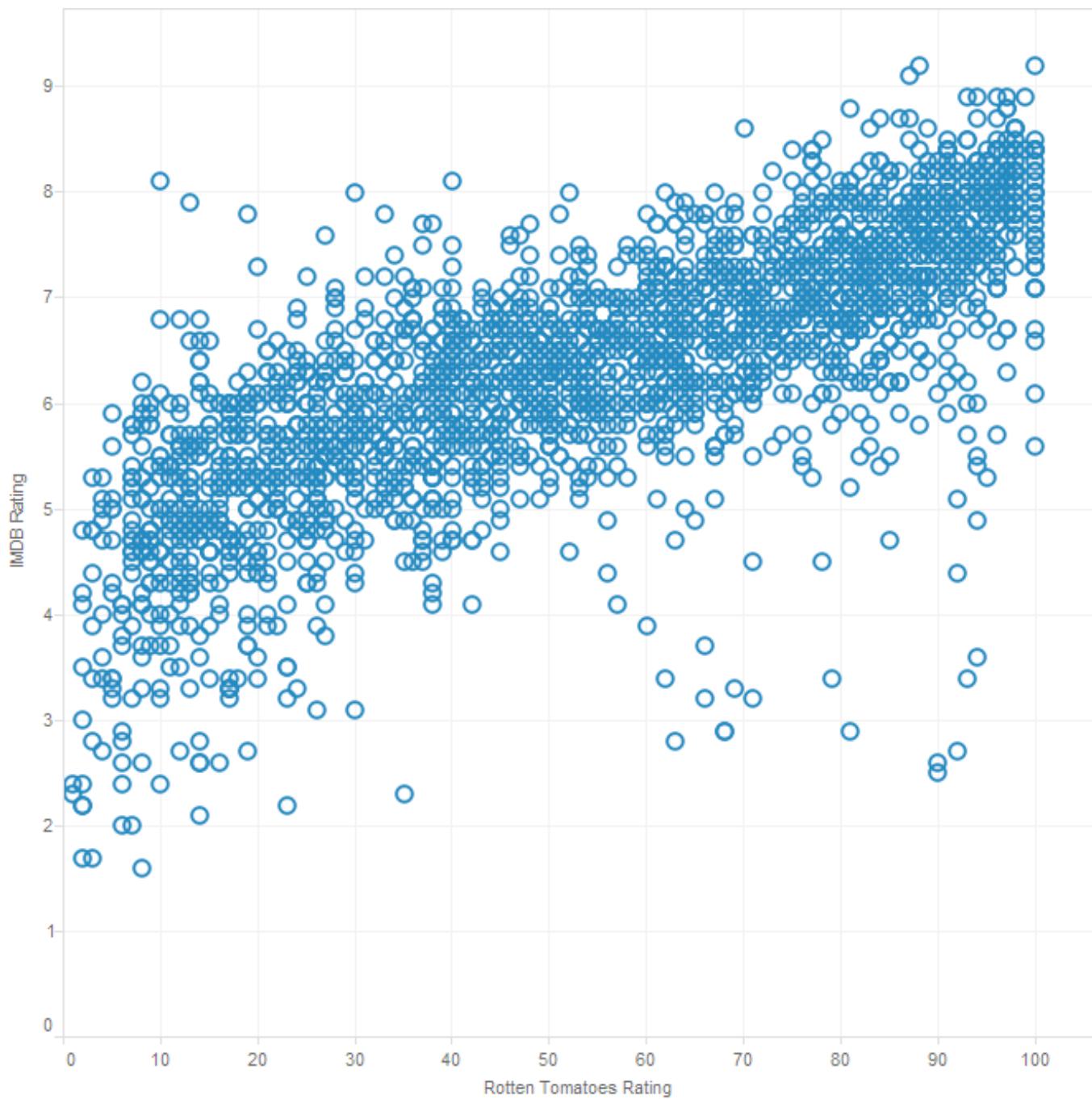
# Motion Pictures Data

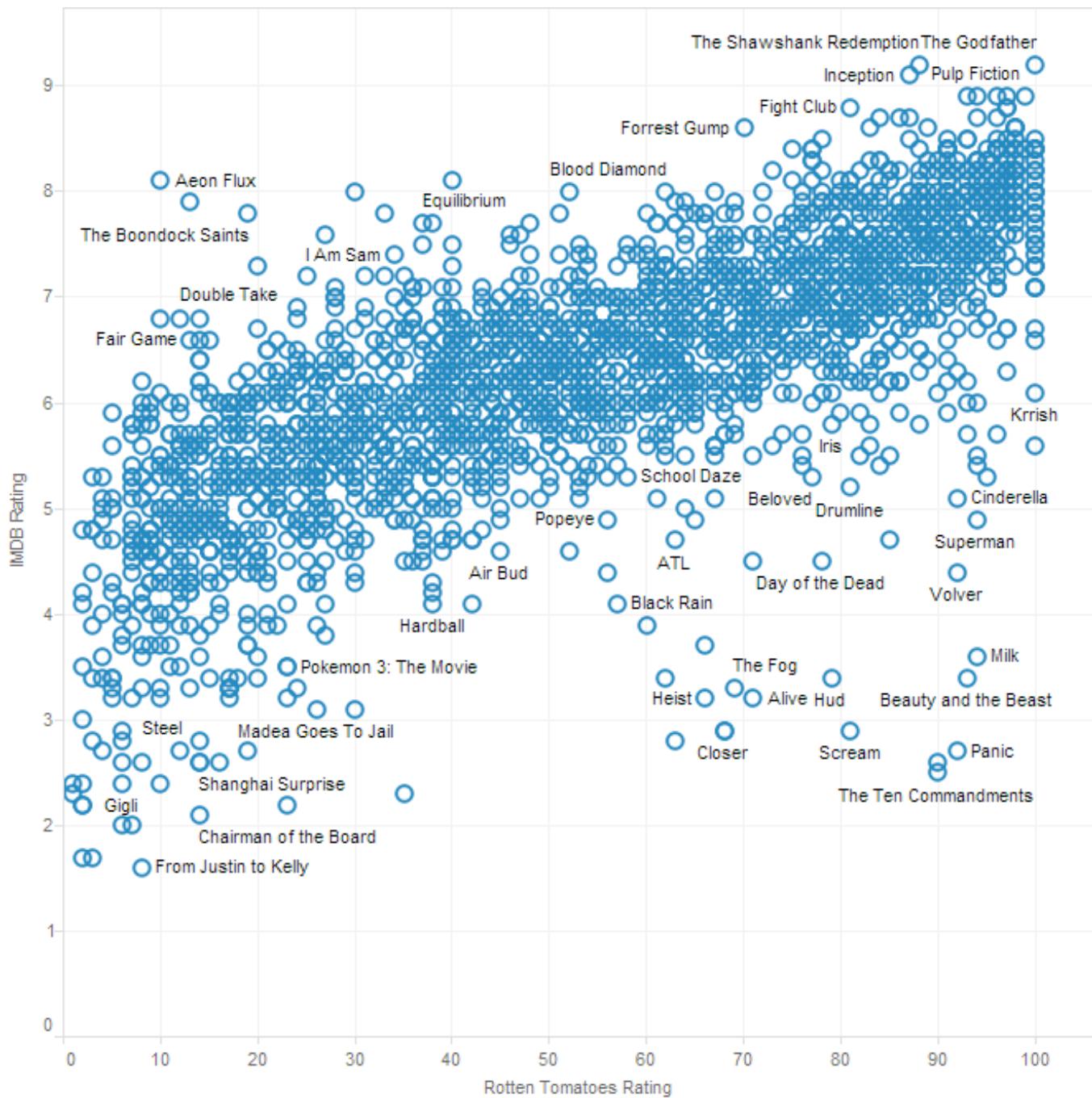
Title	String (N)
IMDB Rating	Number (Q)
Rotten Tomatoes Rating	Number (Q)
MPAA Rating	String (O)
Release Date	Date (T)

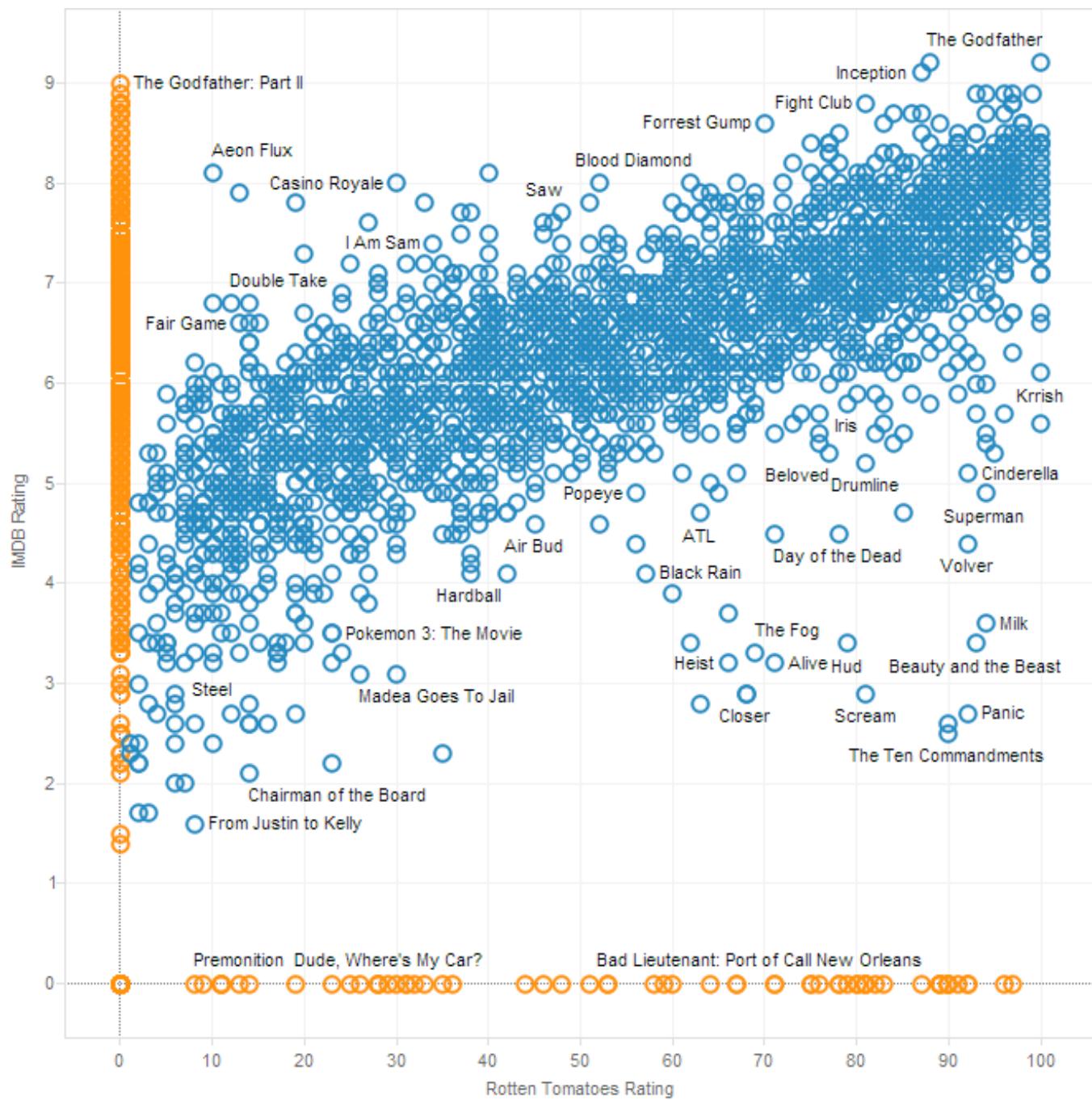


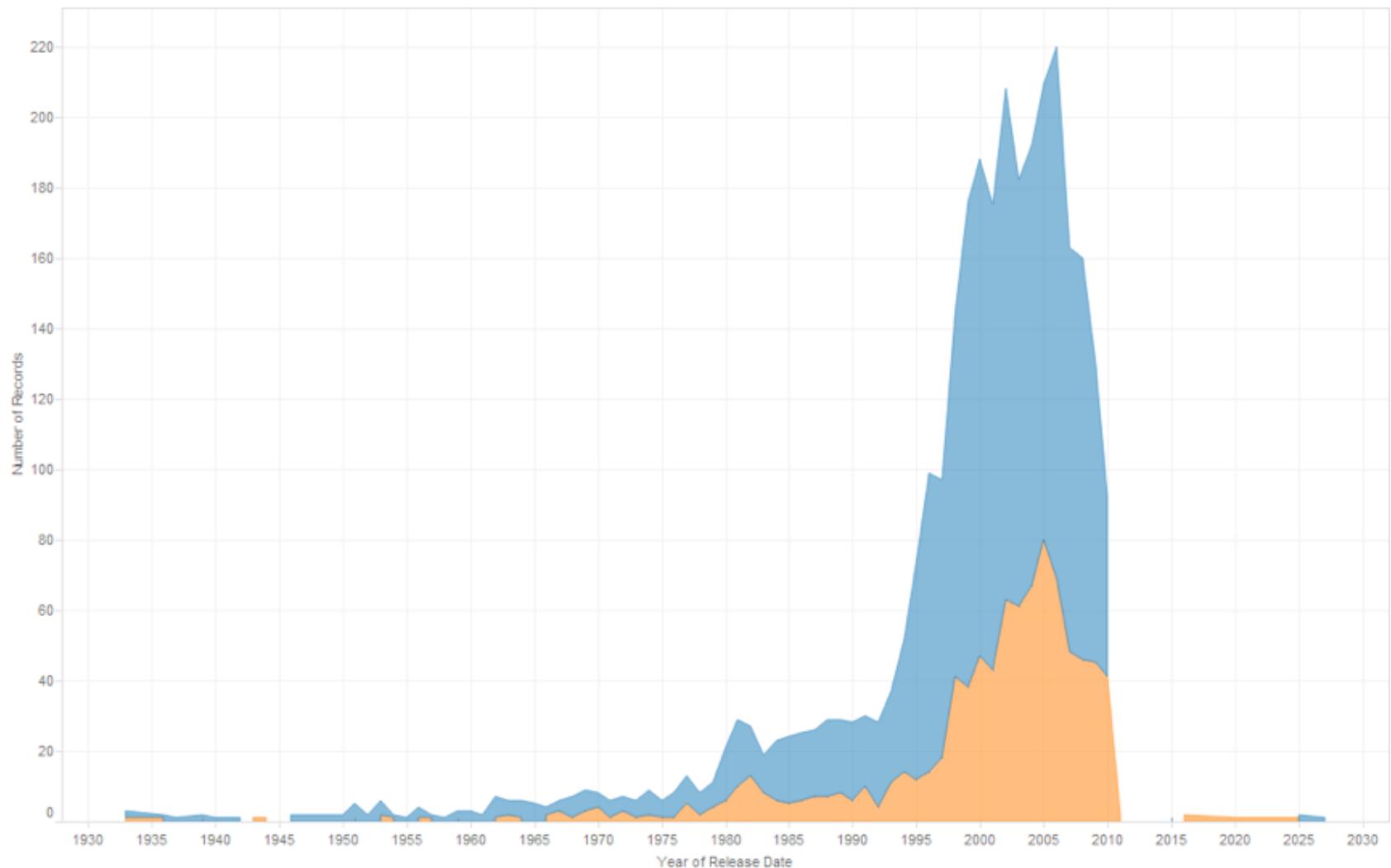
Rotten Tomatoes Rating (bin)











# Lesson: Exercise Skepticism

Check **data quality** and your **assumptions**.

Start with **univariate summaries**, then start to consider **relationships among variables**.

**Avoid premature fixation!**

# Analysis Example: Antibiotic Effectiveness

# Data Set: Antibiotic Effectiveness

Genus of Bacteria	String (N)
Species of Bacteria	String (N)
Antibiotic Applied	String (N)
Gram-Staining?	Pos / Neg (N)
Min. Inhibitory Concent. (g)	Number (Q)

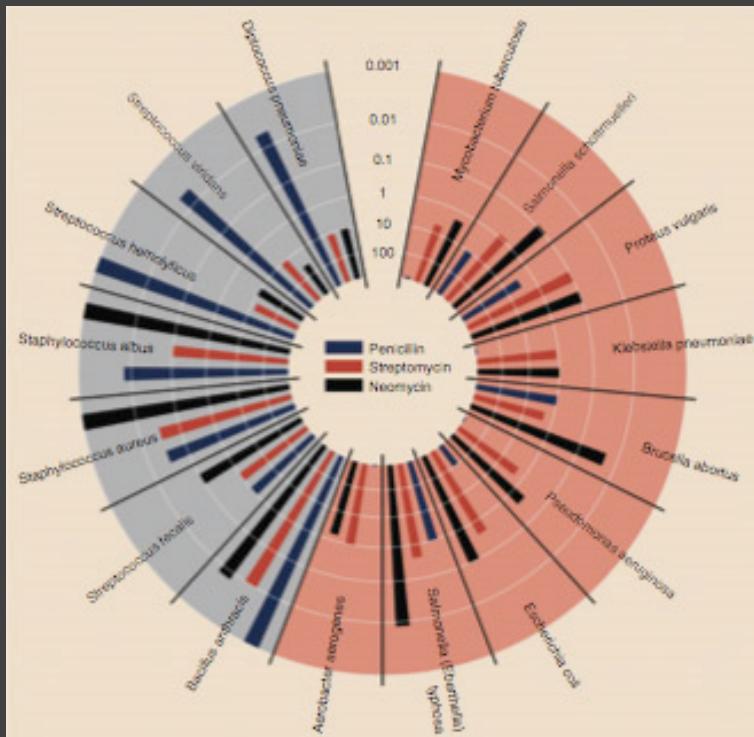
Collected prior to 1951.

# What questions might we ask?

Table 1: Burtin's data.

Bacteria	Antibiotic			Gram Staining
	Penicillin	Streptomycin	Neomycin	
<i>Aerobacter aerogenes</i>	870	1	1.6	negative
<i>Brucella abortus</i>	1	2	0.02	negative
<i>Brucella anthracis</i>	0.001	0.01	0.007	positive
<i>Diplococcus pneumoniae</i>	0.005	11	10	positive
<i>Escherichia coli</i>	100	0.4	0.1	negative
<i>Klebsiella pneumoniae</i>	850	1.2	1	negative
<i>Mycobacterium tuberculosis</i>	800	5	2	negative
<i>Proteus vulgaris</i>	3	0.1	0.1	negative
<i>Pseudomonas aeruginosa</i>	850	2	0.4	negative
<i>Salmonella (Eberthella) typhosa</i>	1	0.4	0.008	negative
<i>Salmonella schottmuelleri</i>	10	0.8	0.09	negative
<i>Staphylococcus albus</i>	0.007	0.1	0.001	positive
<i>Staphylococcus aureus</i>	0.03	0.03	0.001	positive
<i>Streptococcus fecalis</i>	1	1	0.1	positive
<i>Streptococcus hemolyticus</i>	0.001	14	10	positive
<i>Streptococcus viridans</i>	0.005	10	40	positive

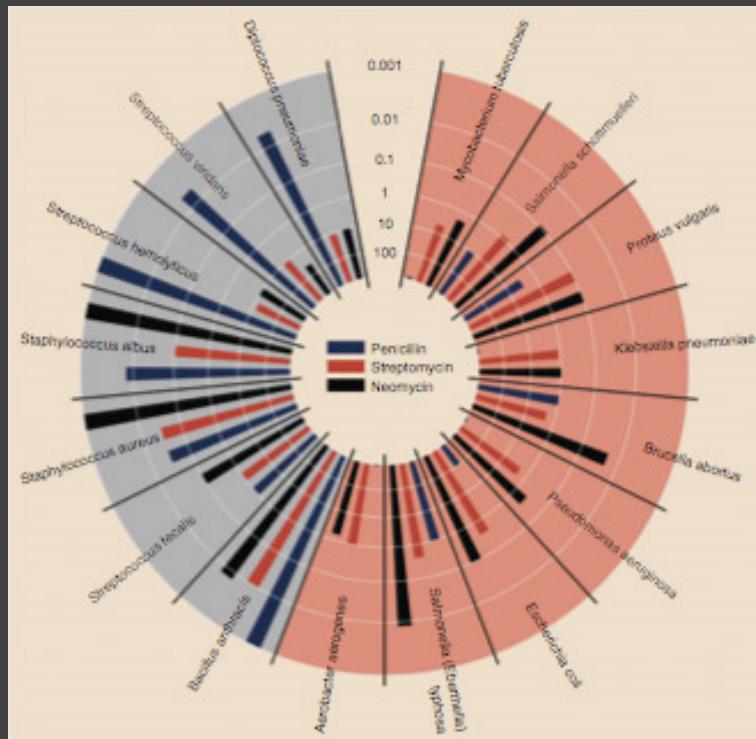
# How do the drugs compare?



Bacteria	Penicillin	Antibiotic Streptomycin	Neomycin	Gram stain
<i>Aerobacter aerogenes</i>	870	1	1.6	-
<i>Brucella abortus</i>	1	2	0.02	-
<i>Bacillus anthracis</i>	0.001	0.01	0.007	+
<i>Diplococcus pneumoniae</i>	0.005	11	10	+
<i>Escherichia coli</i>	100	0.4	0.1	-
<i>Klebsiella pneumoniae</i>	850	1.2	1	-
<i>Mycobacterium tuberculosis</i>	800	5	2	-
<i>Proteus vulgaris</i>	3	0.1	0.1	-
<i>Pseudomonas aeruginosa</i>	850	2	0.4	-
<i>Salmonella (Eberthella) typhosa</i>	1	0.4	0.008	-
<i>Salmonella schottmuelleri</i>	10	0.8	0.09	-
<i>Staphylococcus albus</i>	0.007	0.1	0.001	+
<i>Staphylococcus aureus</i>	0.03	0.03	0.001	+
<i>Streptococcus faecalis</i>	1	1	0.1	+
<i>Streptococcus hemolyticus</i>	0.001	14	10	+
<i>Streptococcus viridans</i>	0.005	10	40	+

Original graphic by Will Burtin, 1951

# How do the drugs compare?



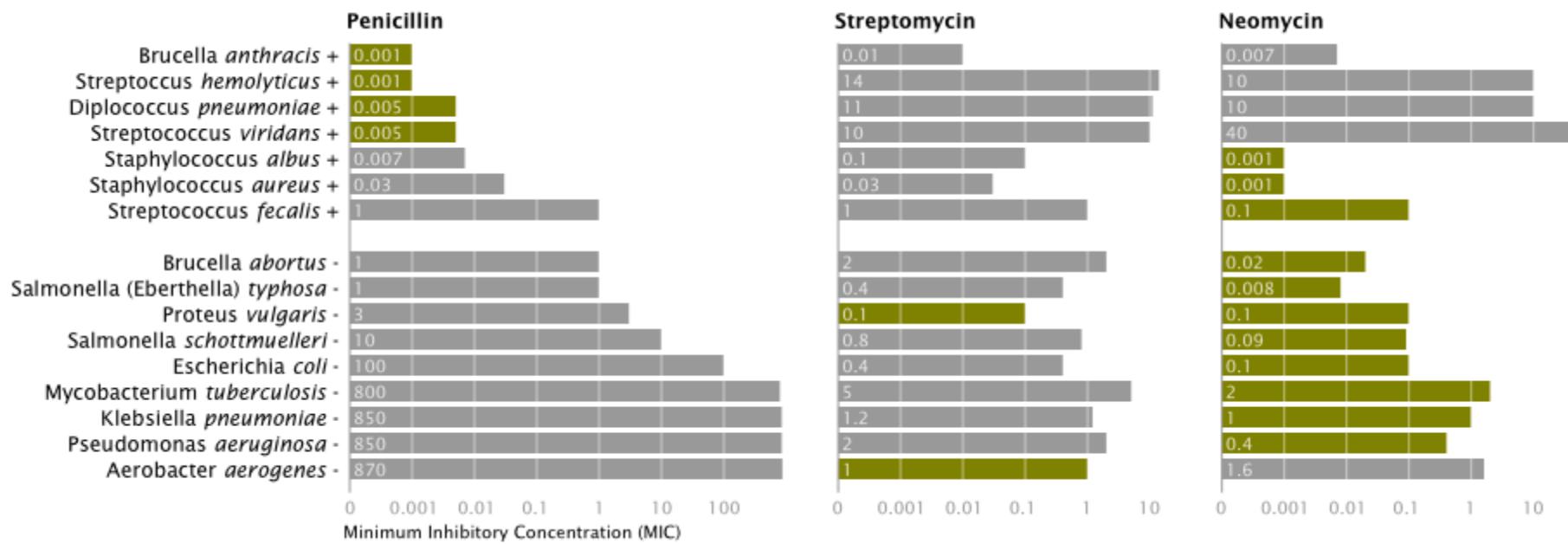
Bacteria	Penicillin	Antibiotic Streptomycin	Neomycin	Gram stain
<i>Aerobacter aerogenes</i>	870	1	1.6	-
<i>Brucella abortus</i>	1	2	0.02	-
<i>Bacillus anthracis</i>	0.001	0.01	0.007	+
<i>Diplococcus pneumoniae</i>	0.005	11	10	+
<i>Escherichia coli</i>	100	0.4	0.1	-
<i>Klebsiella pneumoniae</i>	850	1.2	1	-
<i>Mycobacterium tuberculosis</i>	800	5	2	-
<i>Proteus vulgaris</i>	3	0.1	0.1	-
<i>Pseudomonas aeruginosa</i>	850	2	0.4	-
<i>Salmonella (Eberthella) typhosa</i>	1	0.4	0.008	-
<i>Salmonella schottmuelleri</i>	10	0.8	0.09	-
<i>Staphylococcus albus</i>	0.007	0.1	0.001	+
<i>Staphylococcus aureus</i>	0.03	0.03	0.001	+
<i>Streptococcus fecalis</i>	1	1	0.1	+
<i>Streptococcus hemolyticus</i>	0.001	14	10	+
<i>Streptococcus viridans</i>	0.005	10	40	+

Radius:  $1 / \log(\text{MIC})$

Bar Color: Antibiotic

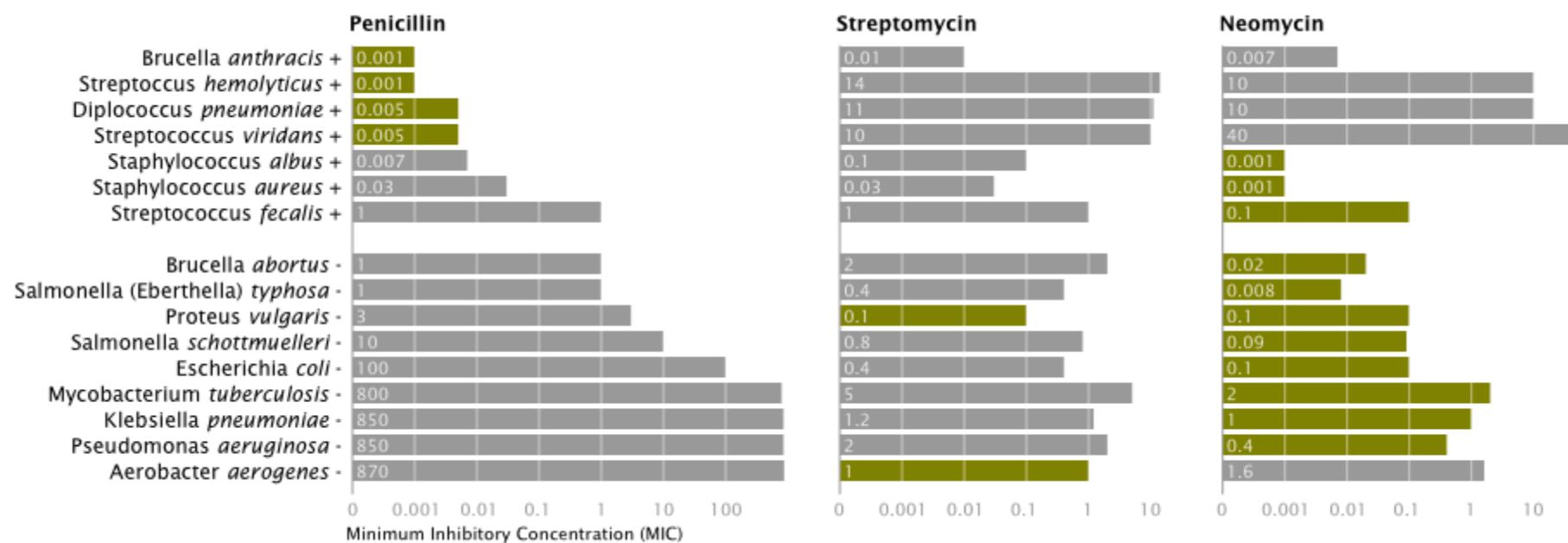
Background Color: Gram Staining

# How do the drugs compare?



Mike Bostock  
Stanford CS448B, Winter 2009

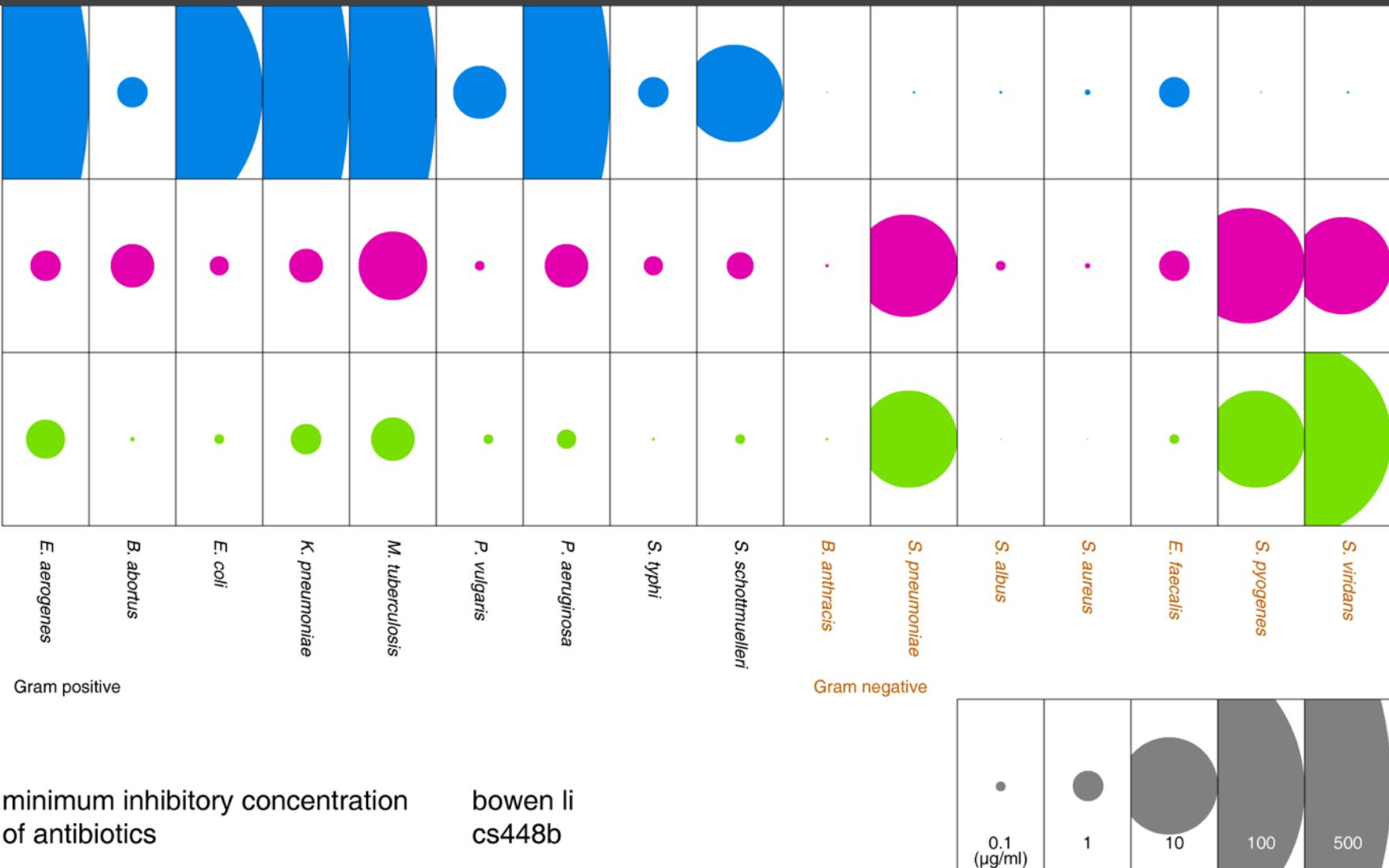
# How do the drugs compare?



X-axis: Antibiotic |  $\log(\text{MIC})$

Y-axis: Gram-Staining | Species

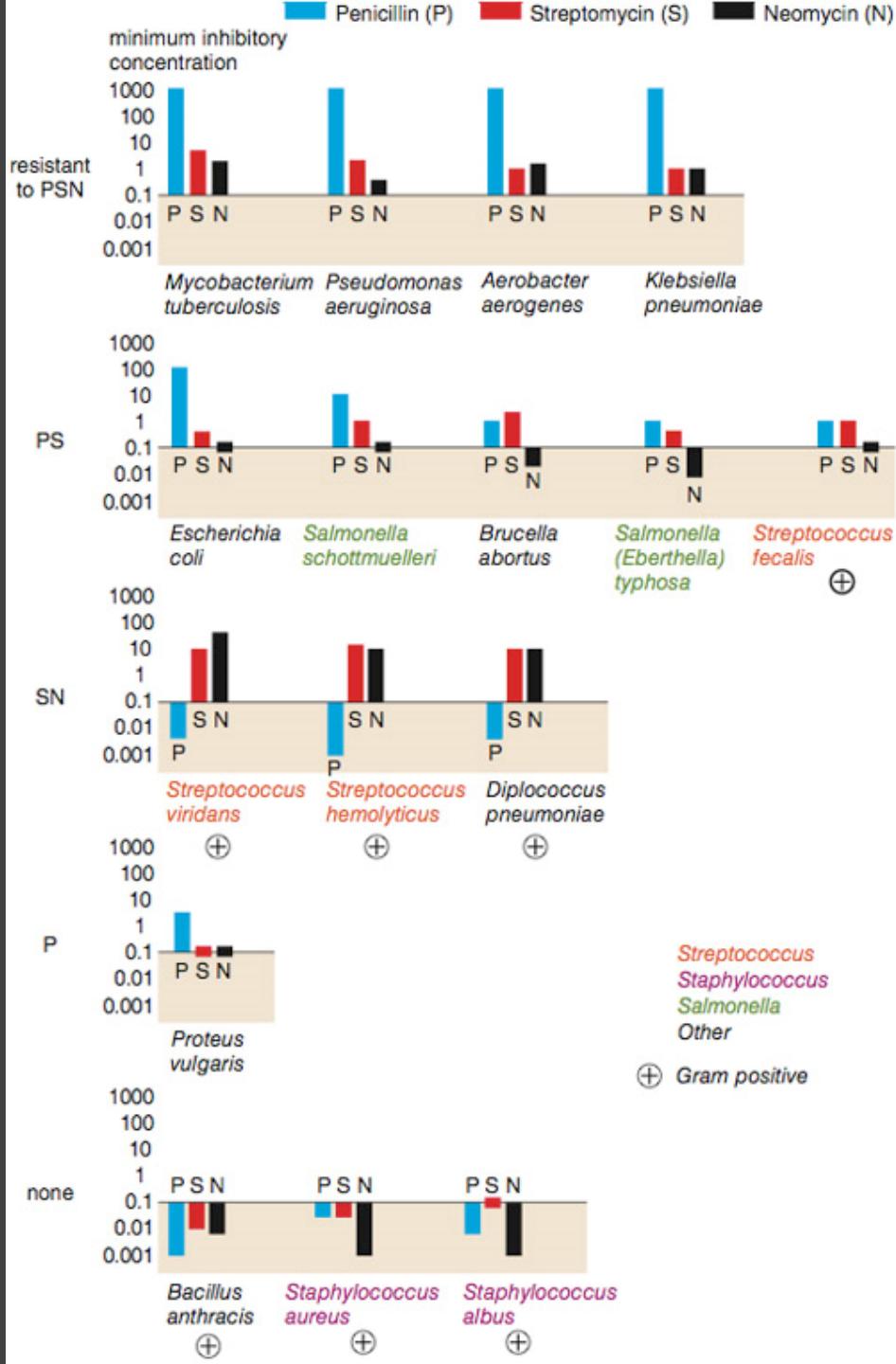
Color: Most-Effective?



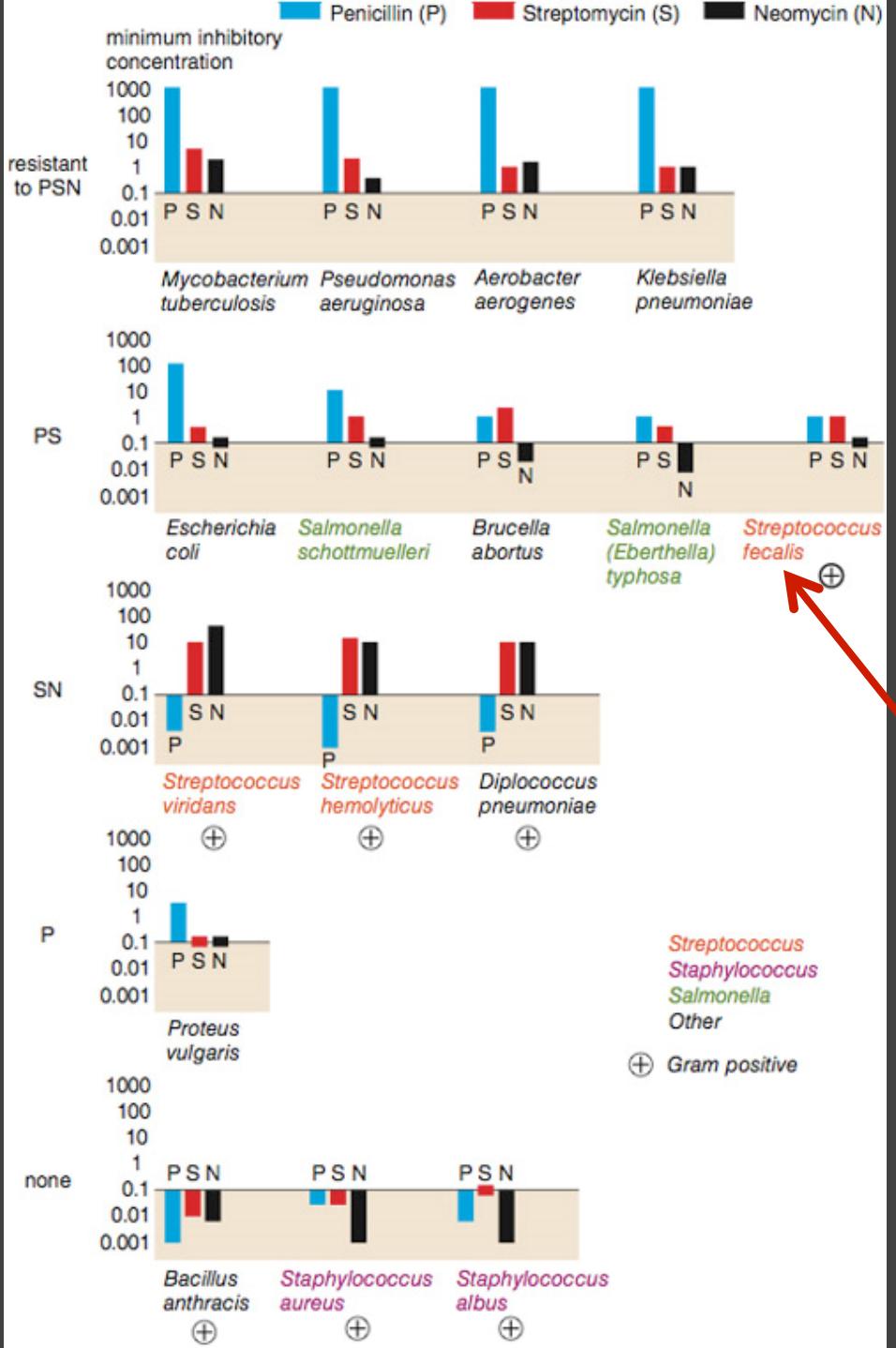
Bowen Li  
Stanford CS448B, Fall 2009

Do the bacteria  
group by antibiotic  
resistance?

# Do the bacteria group by antibiotic resistance?



Wainer & Lysen  
American Scientist, 2009

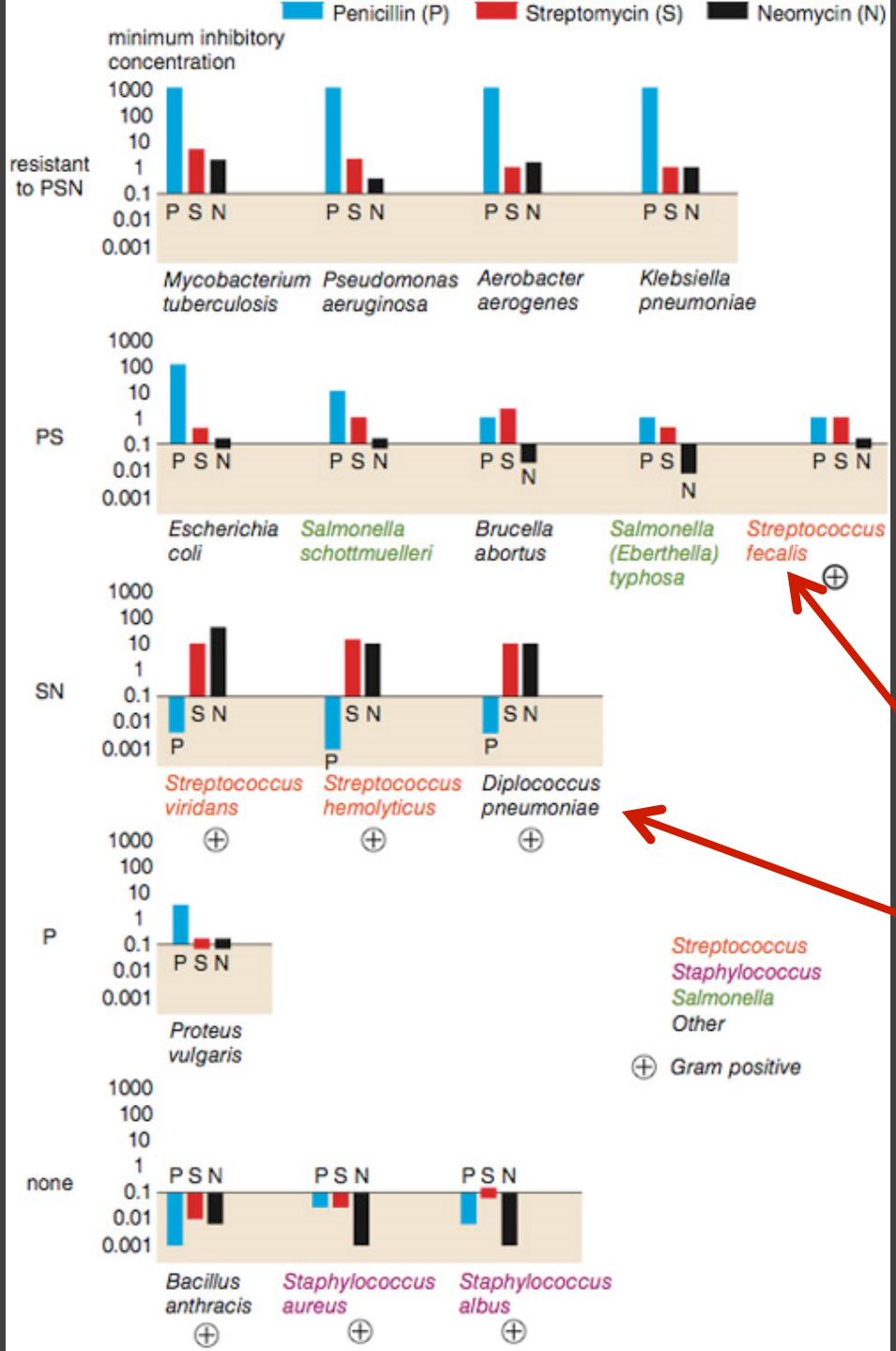


# Do the bacteria group by antibiotic resistance?

Not a streptococcus!  
(realized ~30 yrs later)

Wainer & Lysen  
American Scientist, 2009

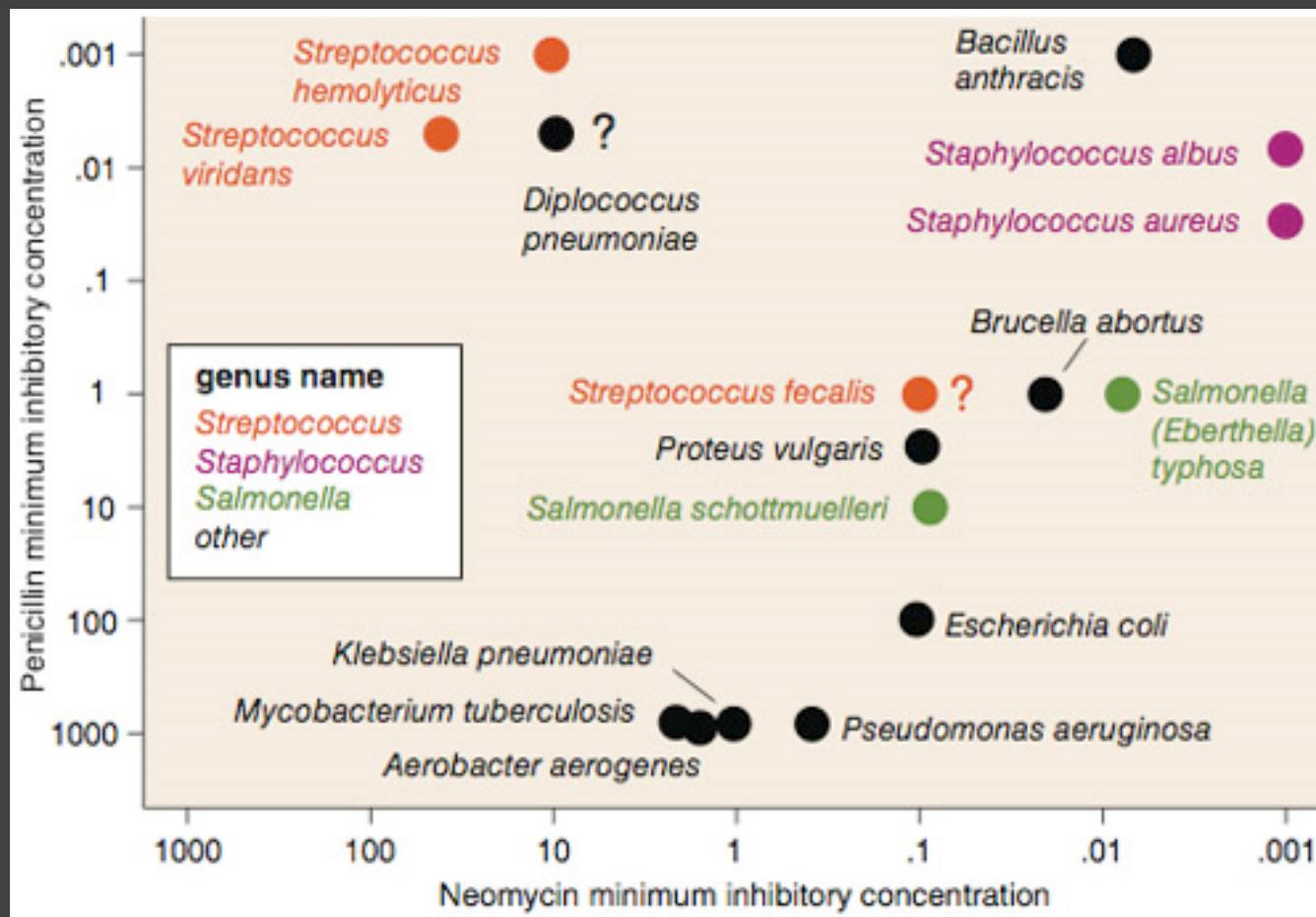
# Do the bacteria group by antibiotic resistance?



Not a streptococcus!  
(realized ~30 yrs later)

Really a streptococcus!  
(realized ~20 yrs later)

**Do the bacteria group by resistance?  
Do different drugs correlate?**



Do the bacteria group by resistance?  
Do different drugs correlate?

Wainer & Lysen  
*American Scientist, 2009*

# Lesson: Iterative Exploration

## Exploratory Process

- 1 Construct graphics to address questions
- 2 Inspect “answer” and assess new questions
- 3 Repeat...

**Transform data** appropriately (e.g., invert, log)

**Show data variation, not design variation** [Tufte]

# Common Data Transformations

Normalize

$$y_i / \sum_i y_i$$

Log

$$\log y$$

Power

$$y^{1/k}$$

Box-Cox Transform

$$(y^\lambda - 1) / \lambda \quad \text{if } \lambda \neq 0$$

$$\log y \quad \text{if } \lambda = 0$$

Binning

e.g., histograms

Grouping

e.g., merge categories

Often performed to aid comparison (% or scale difference) or better approx. normal distribution

# Analysis Example: MTurk Participation

# Data Set: Turker Participation

Turker ID

String (N)

Avg. Completion Rate

Number [0,1] (Q)

Collected in 2009 by Heer & Bostock.

What questions might we ask of the data?

What charts might provide insight?

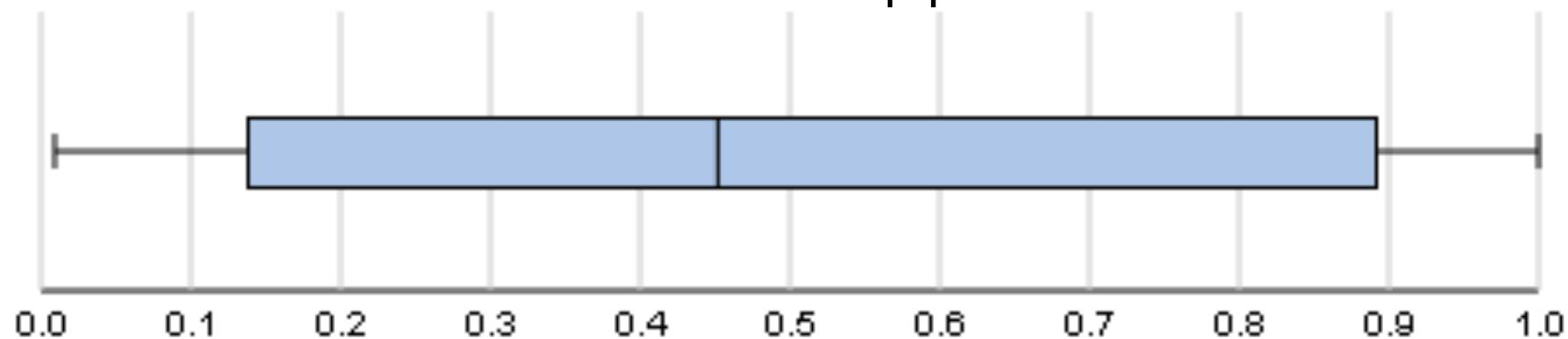
Min

Median

Max

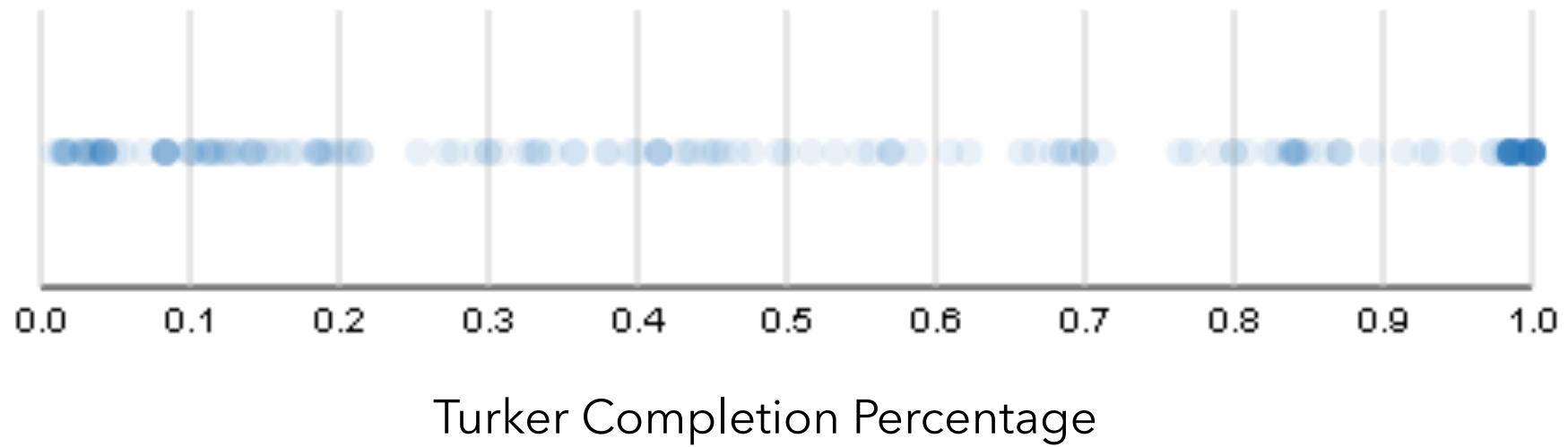
Lower Quartile

Upper Quartile

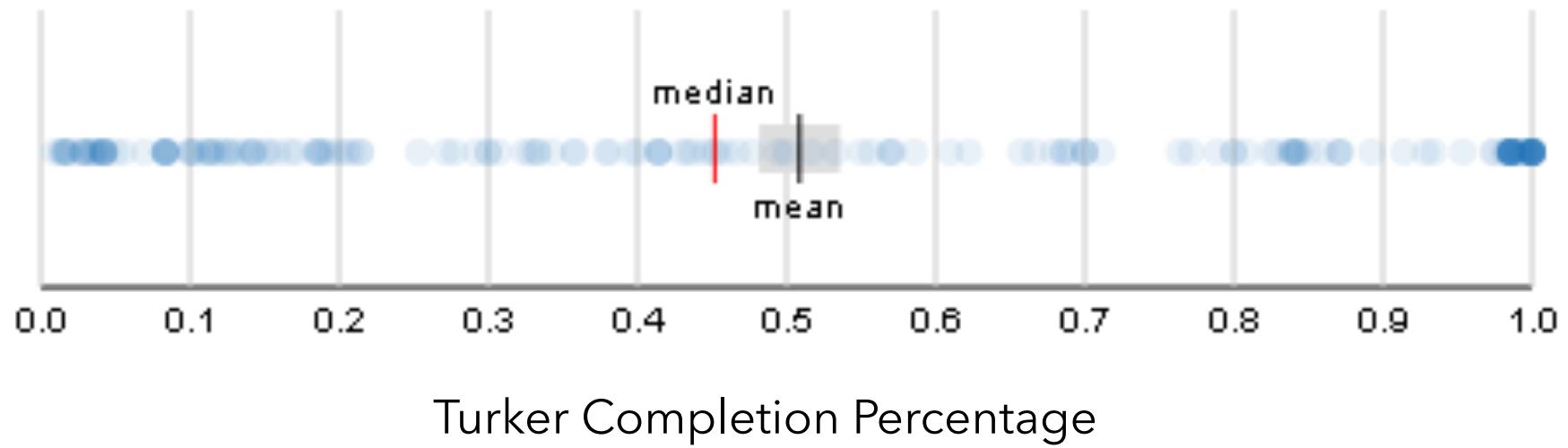


Turker Completion Percentage

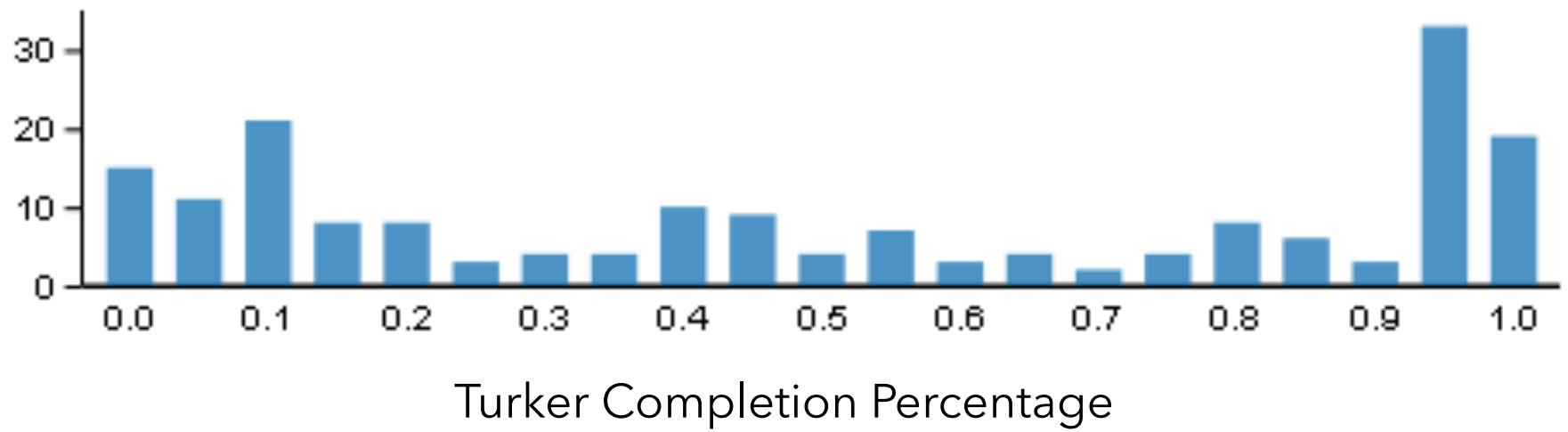
**Box Plot**



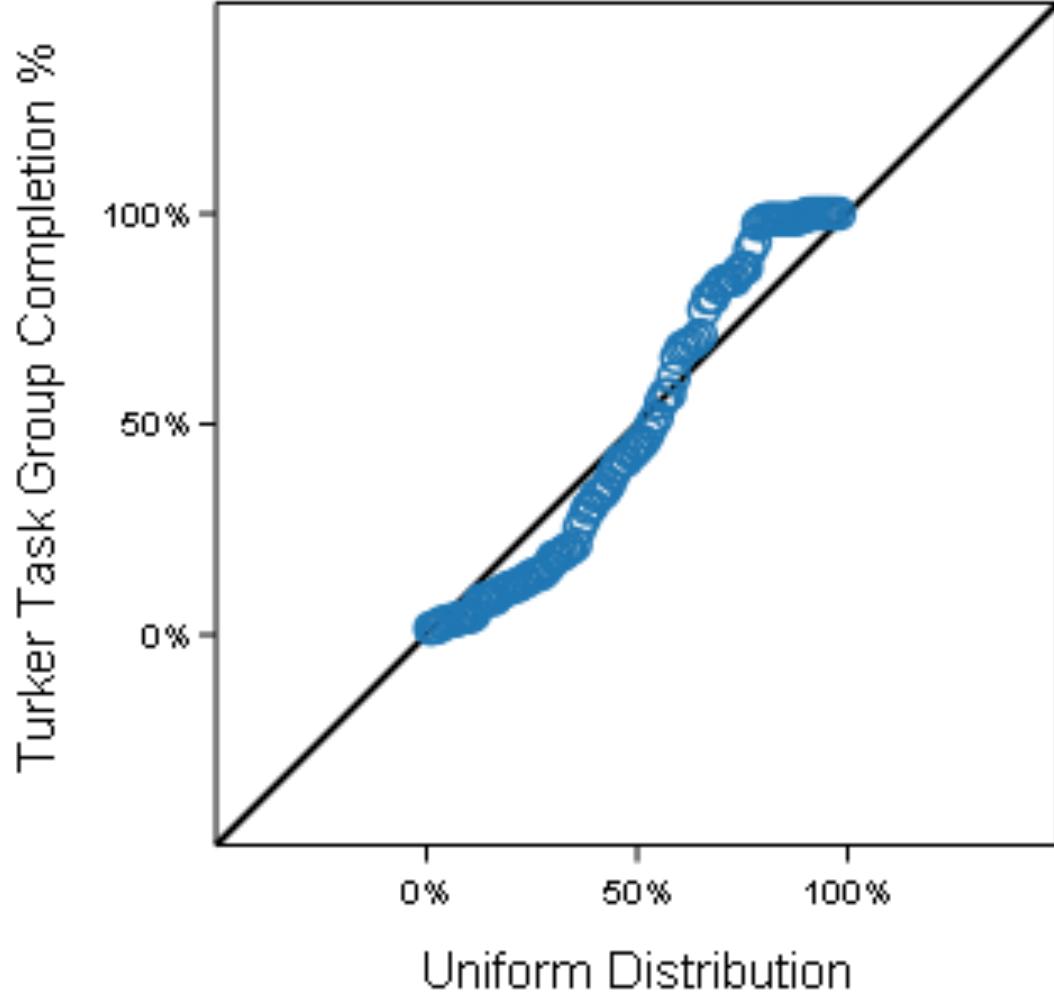
**Dot Plot (with transparency for overlap)**



## Dot Plot (with Reference Lines)



**Histogram (binned counts)**

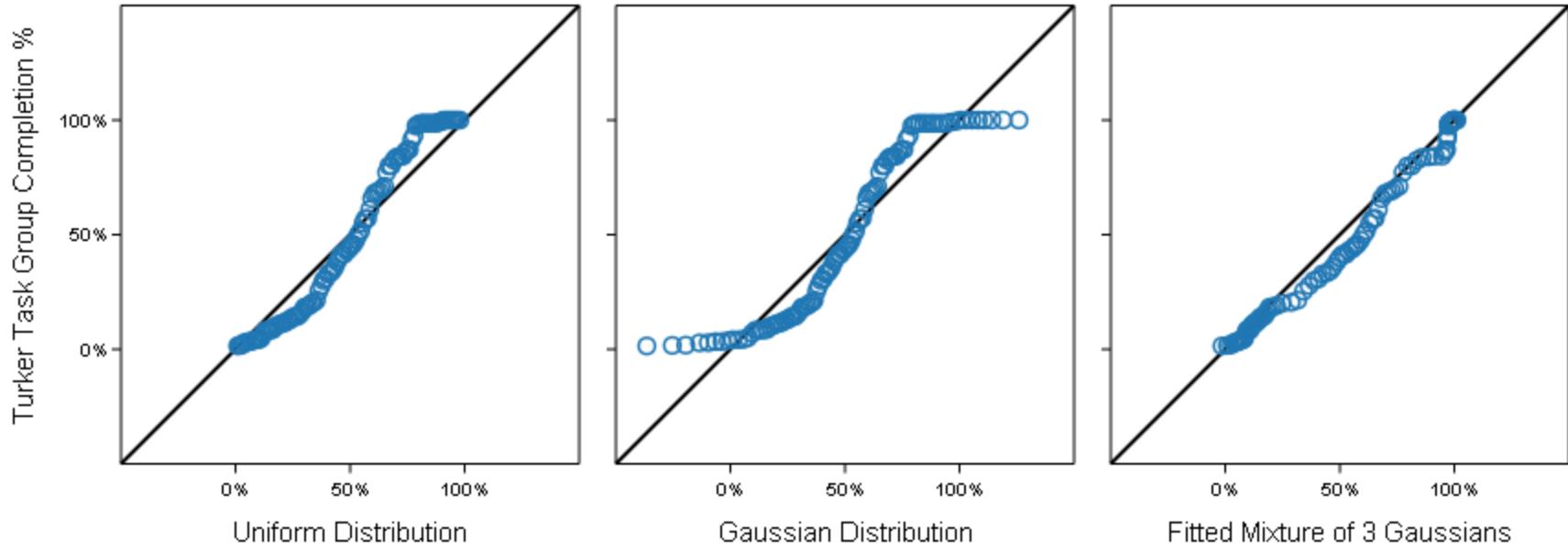


## Quantile-Quantile Plot

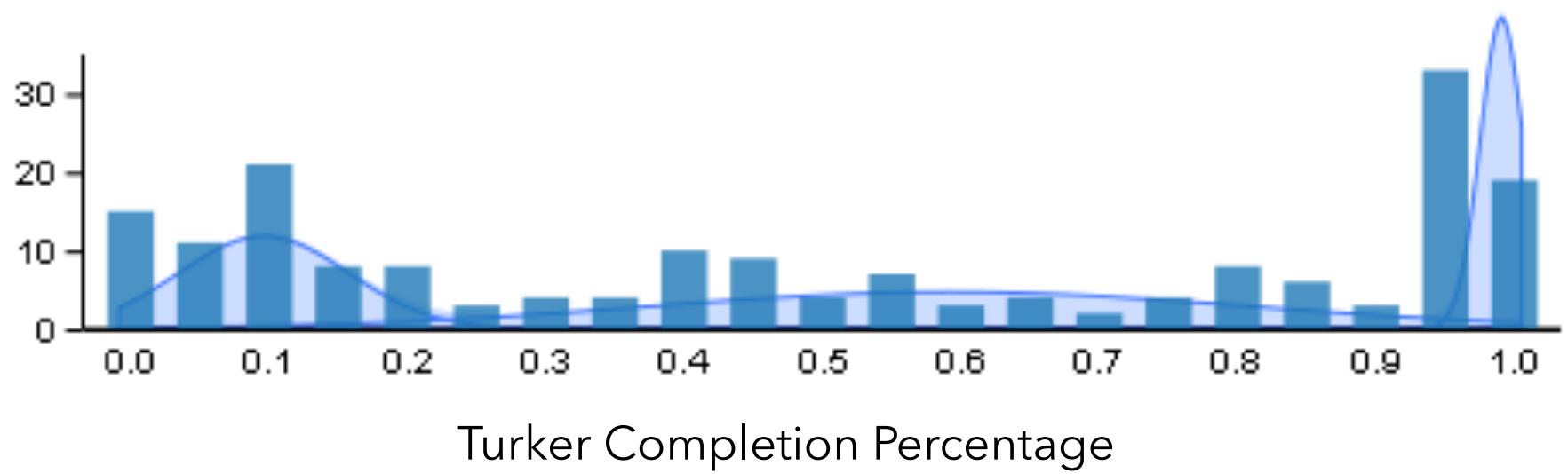
Used to compare two distributions; in this case, one actual and one theoretical.

Plots the quantiles (here, the percentile values) against each other.

Similar distributions lie along the diagonal. If linearly related, values will lie along a line, but with potentially varying slope and intercept.



# Quantile-Quantile Plots



**Histogram (+ Fitted Mixture of 3 Gaussians)**

# Lessons

Even for “simple” data, a variety of graphics might provide insight. Tailor the choice of graphic to the questions being asked, but be open to surprises.

Graphics can be used to guide and help assess the quality of statistical models.

Premature commitment to a model and lack of verification can lead an analysis astray.

# Administrivia

# A2: Exploratory Data Analysis

Use visualization software to form & answer questions

## First steps:

Step 1: Pick domain & data

Step 2: Pose questions

Step 3: Profile the data

Iterate as needed

## Create visualizations

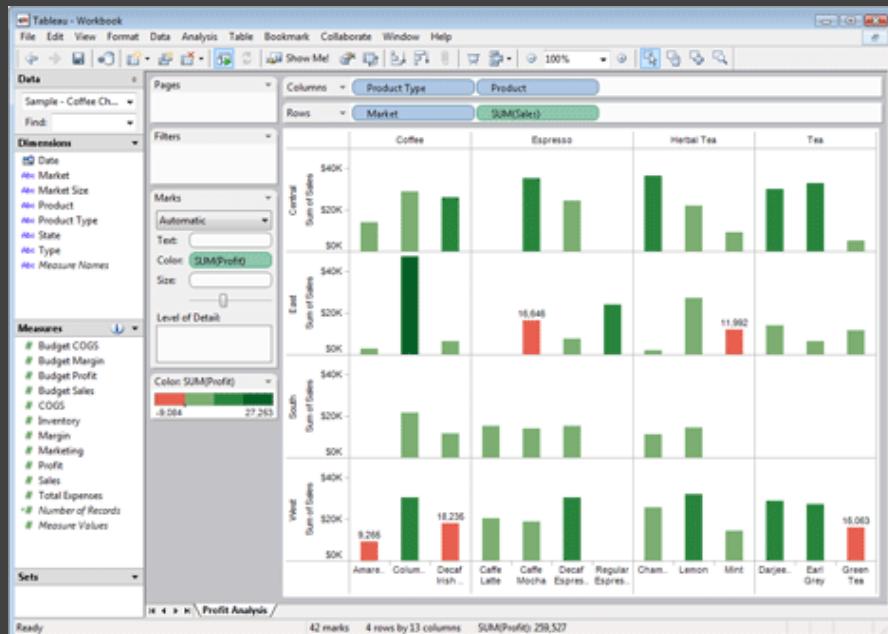
Interact with data

Refine your questions

## Make a notebook

Keep record of your analysis

Prepare a final graphic and caption



Due by 5:00pm

**Friday, April 15**

# Tutorials!

## Visualization Tools

Tue 4/12, 3:00-4:20pm PAA 114A

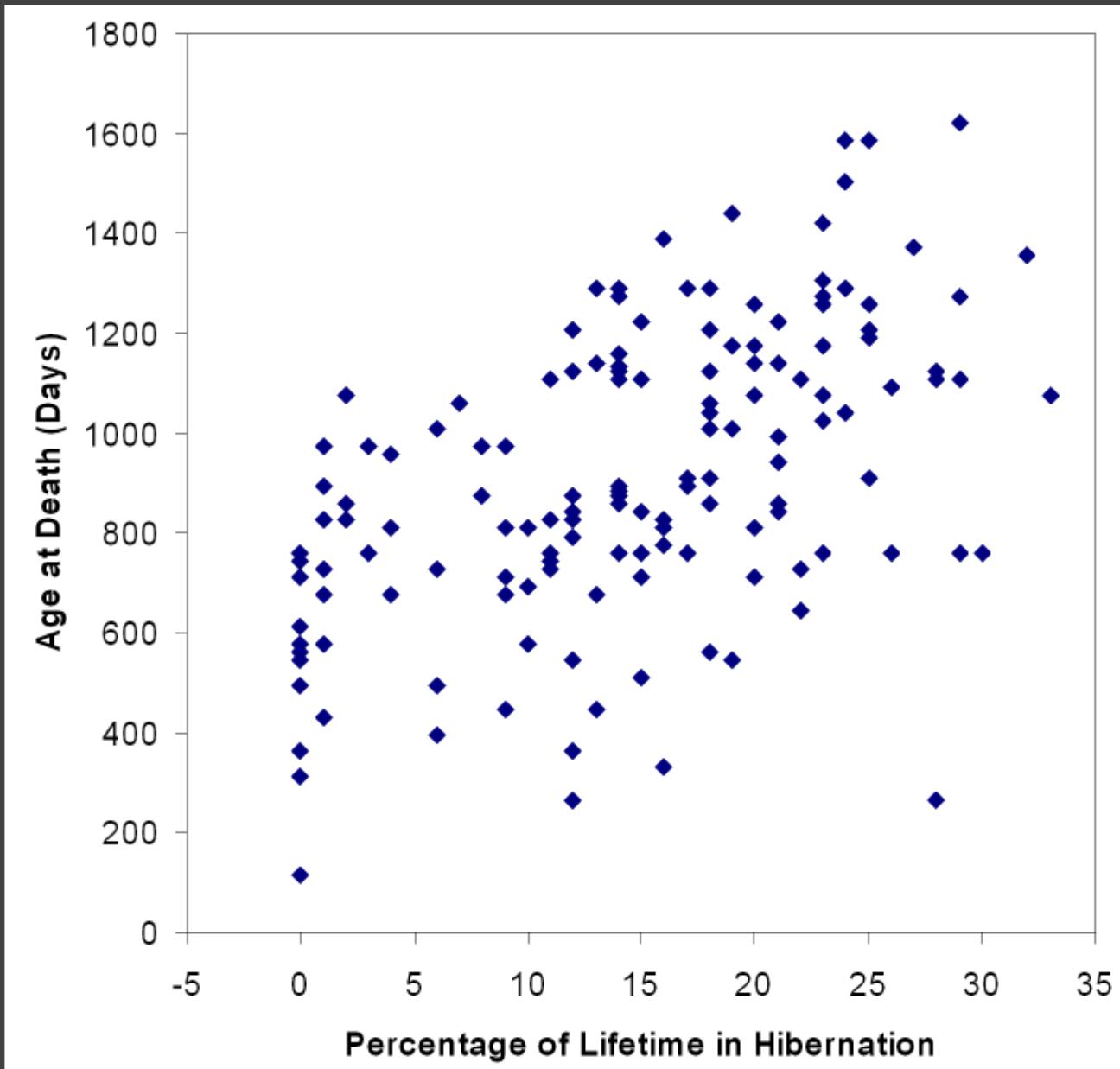
Introduction to Tableau, plus a few others.

## d3.js: Data-Driven Documents

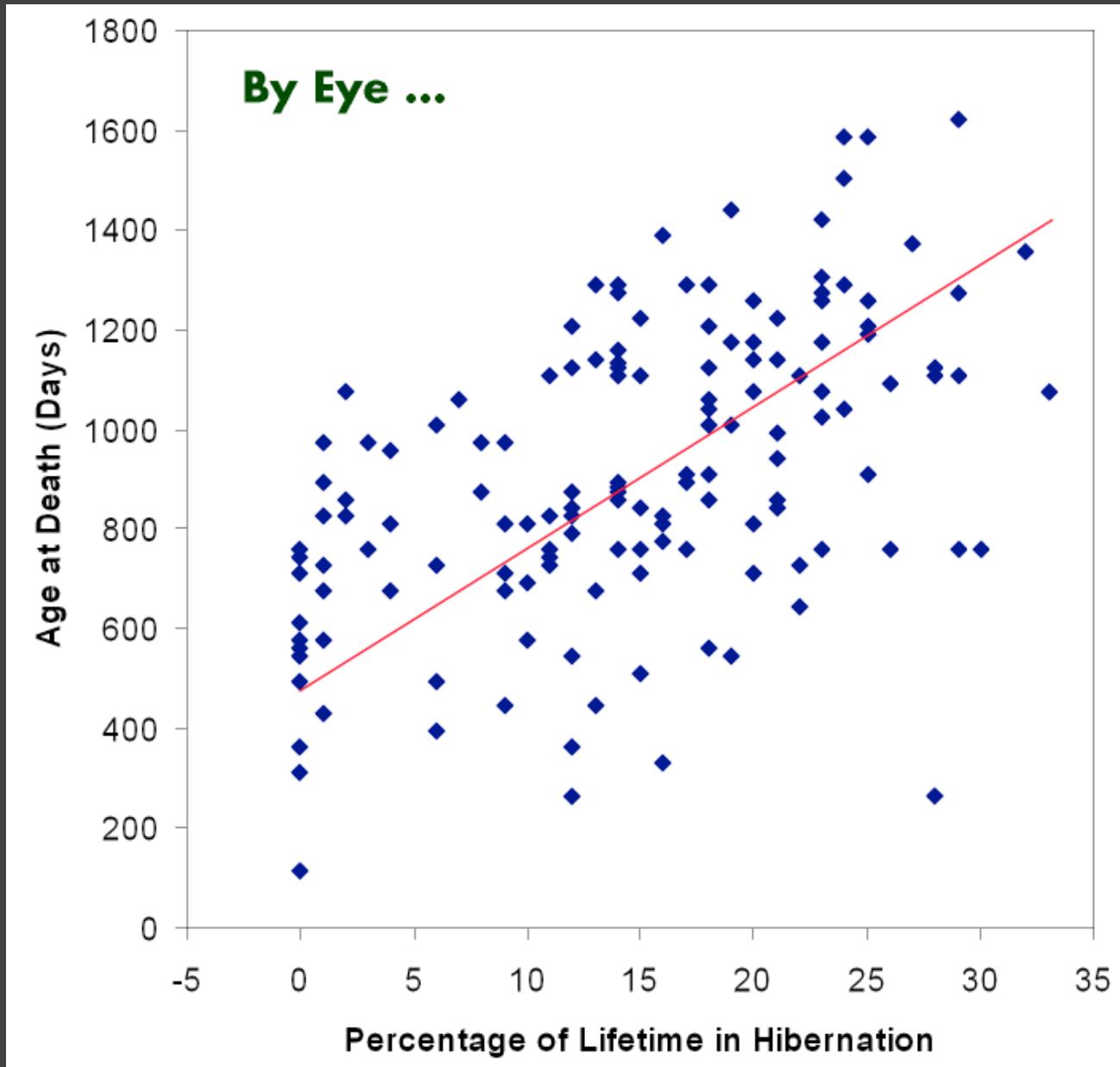
Tue 4/19, 3:00-4:20pm PAA 114A

Focus on D3, touches on HTML/CSS/JS

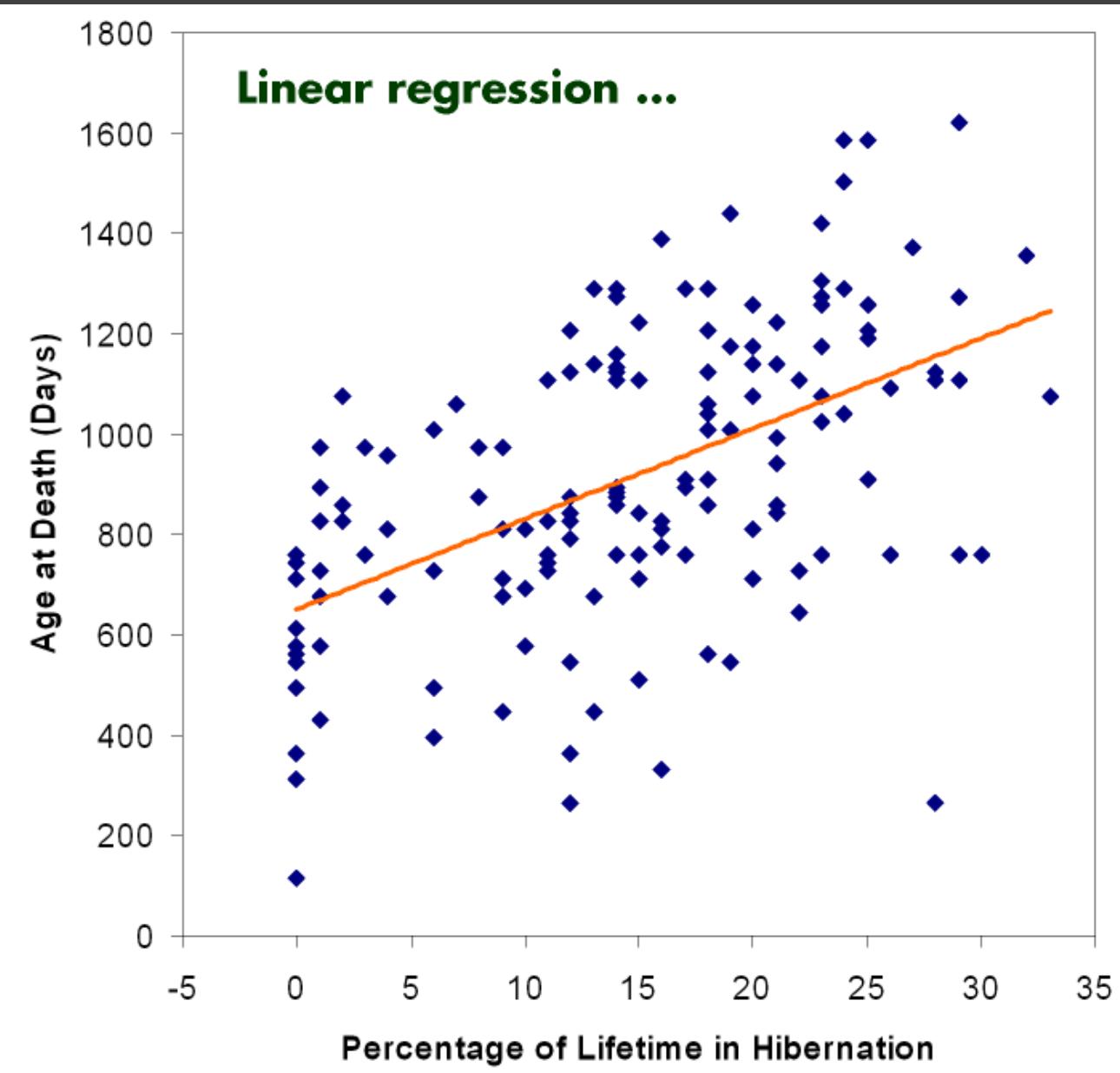
# Visualization + Statistics



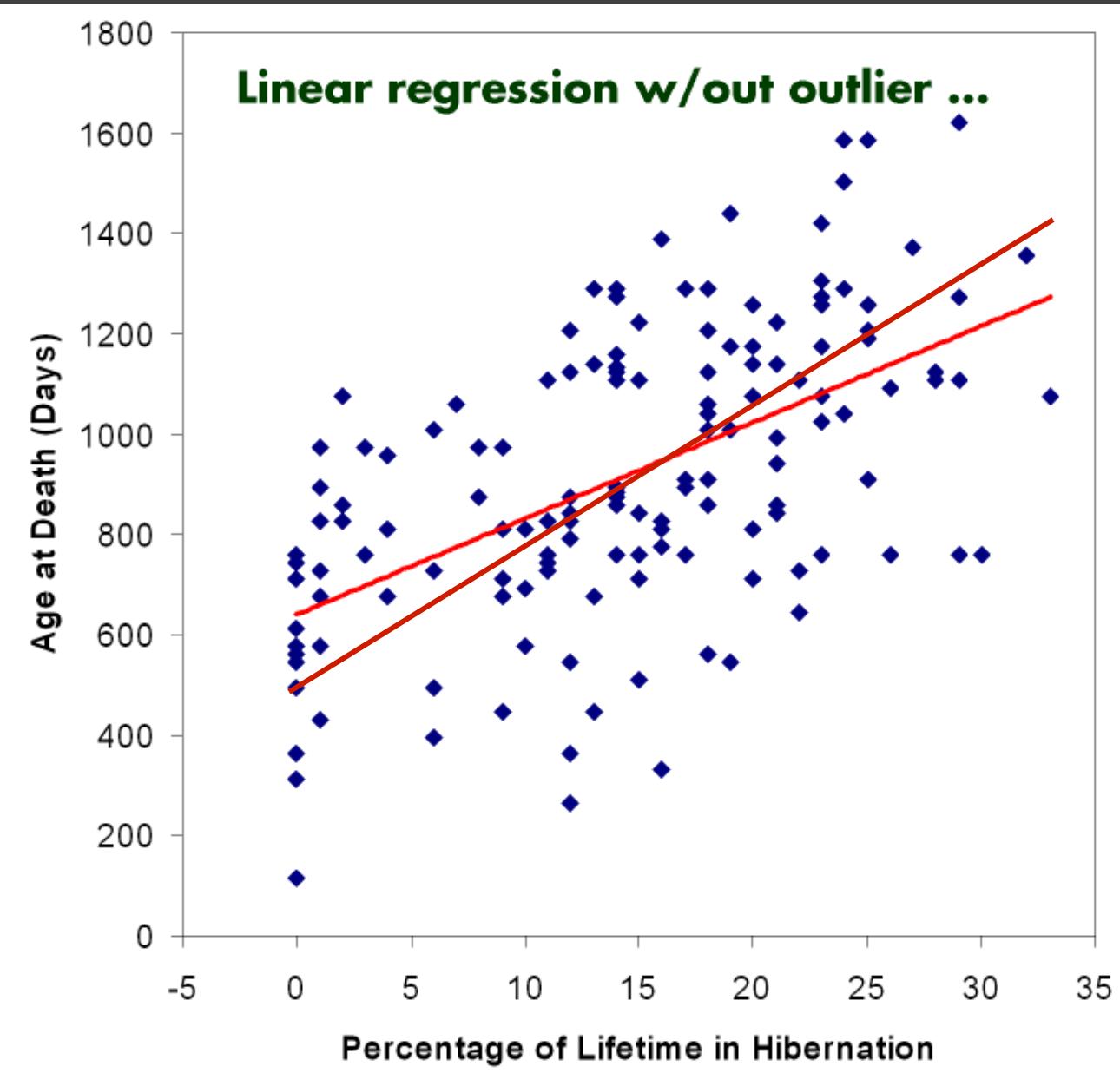
[The Elements of Graphing Data. Cleveland 94]



[The Elements of Graphing Data. Cleveland 94]



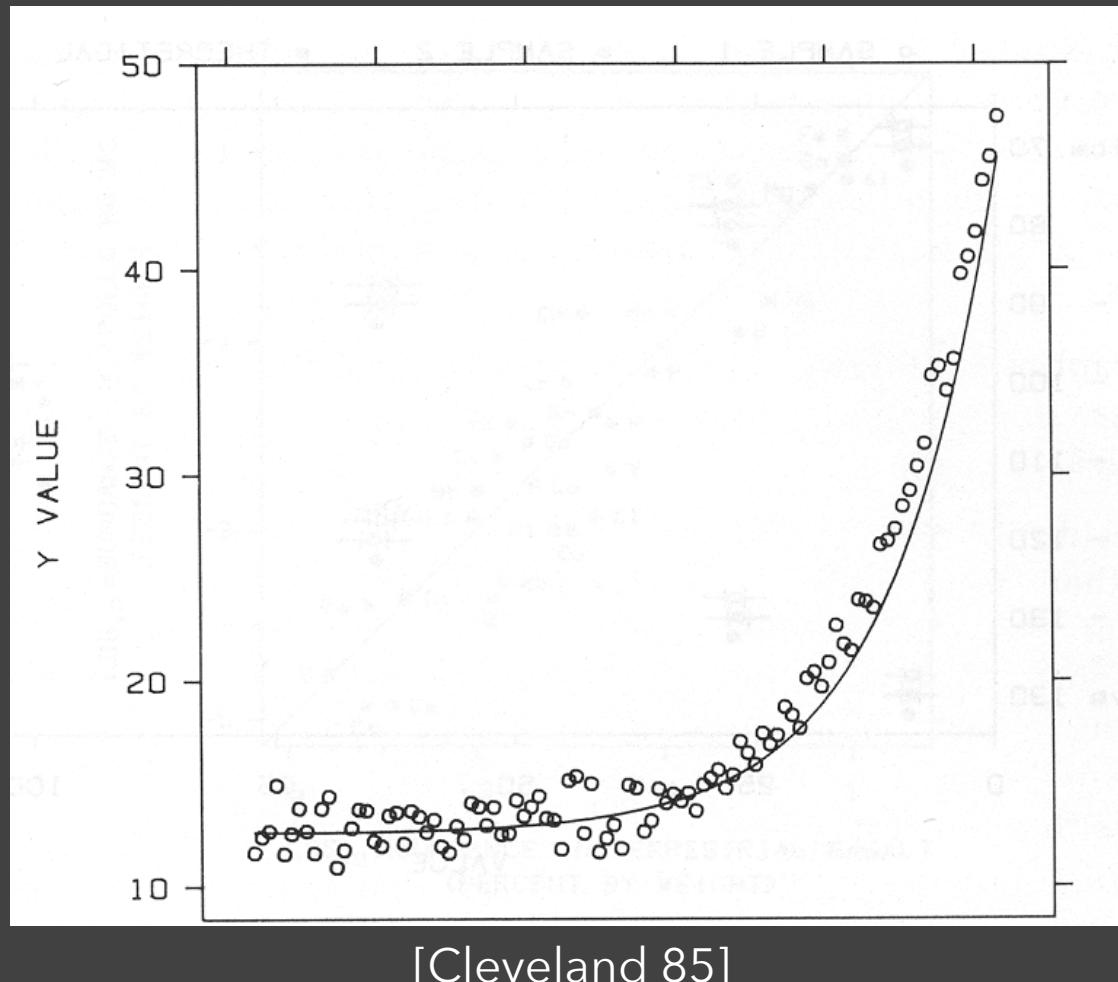
[The Elements of Graphing Data. Cleveland 94]



[The Elements of Graphing Data. Cleveland 94]

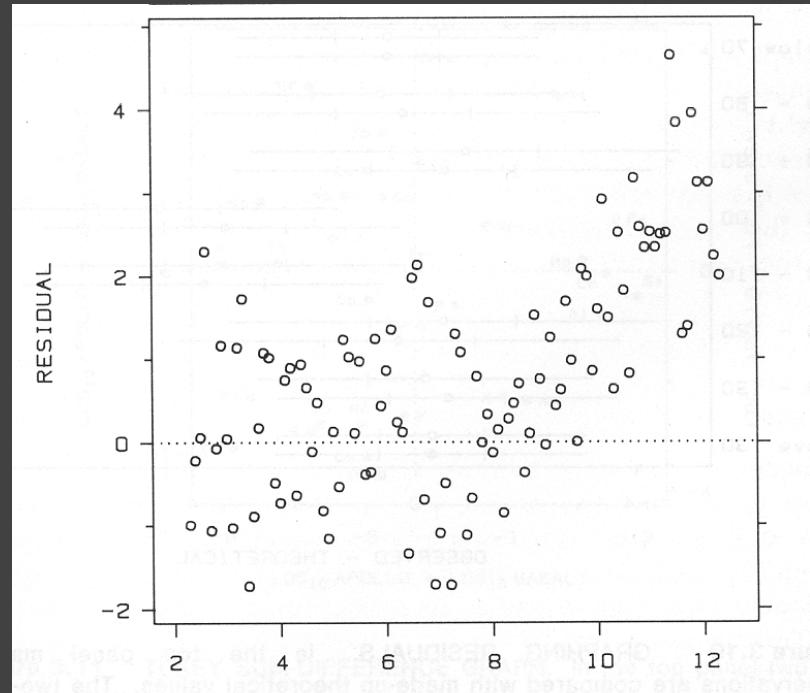
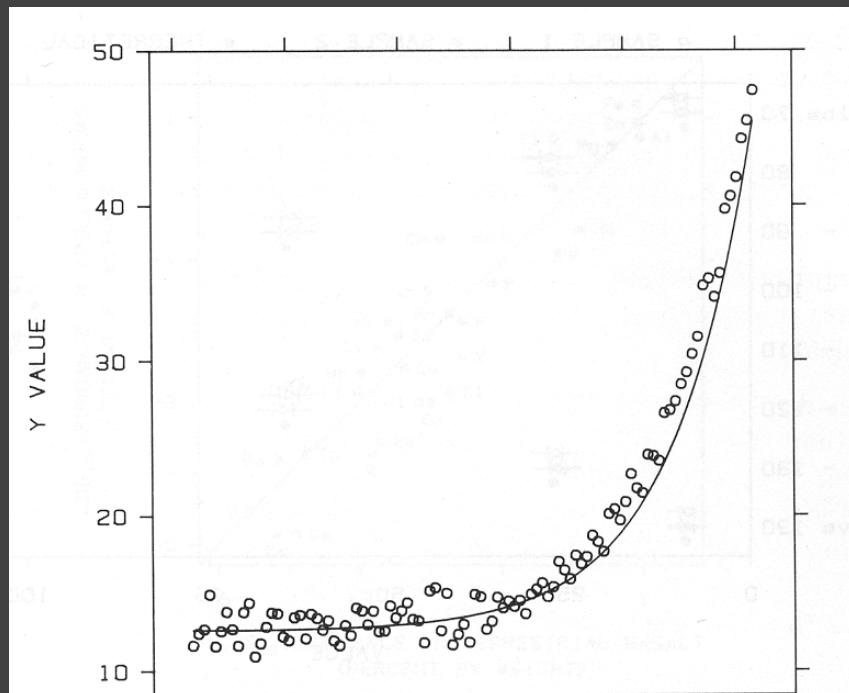
# Transforming Data

How well does the curve fit the data?



# Plot the Residuals

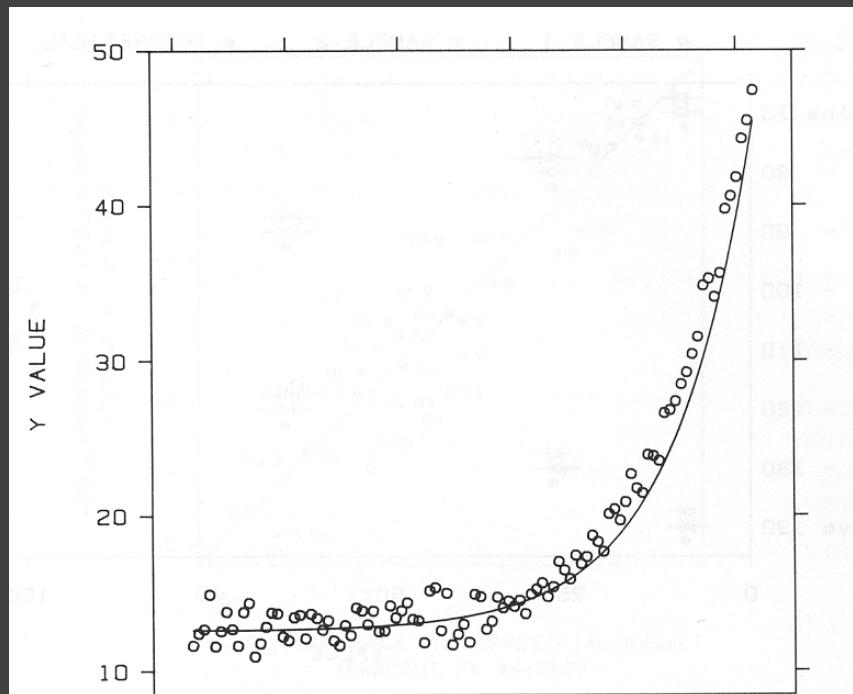
Plot vertical distance from best fit curve  
Residual graph shows accuracy of fit



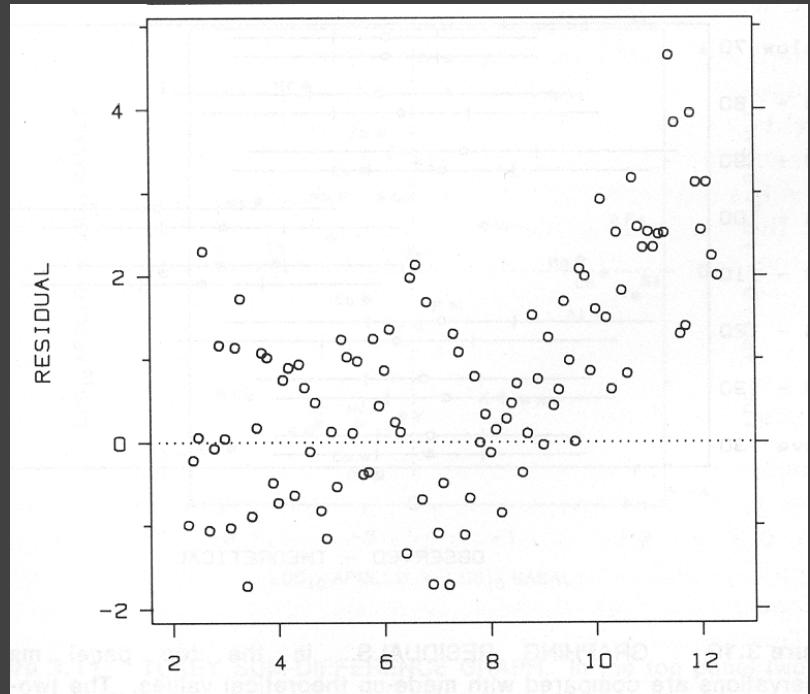
[Cleveland 85]

# Multiple Plotting Options

Plot model in data space



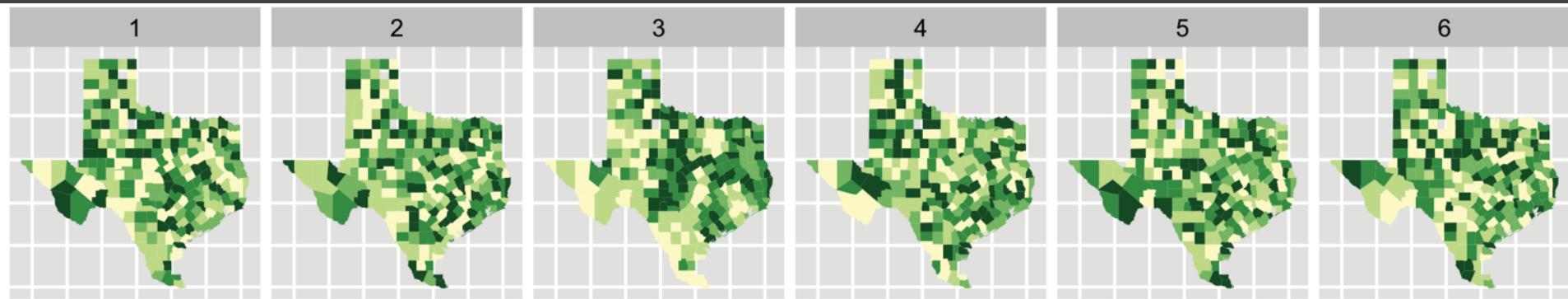
Plot data in model space



[Cleveland 85]

# Graphical Inference

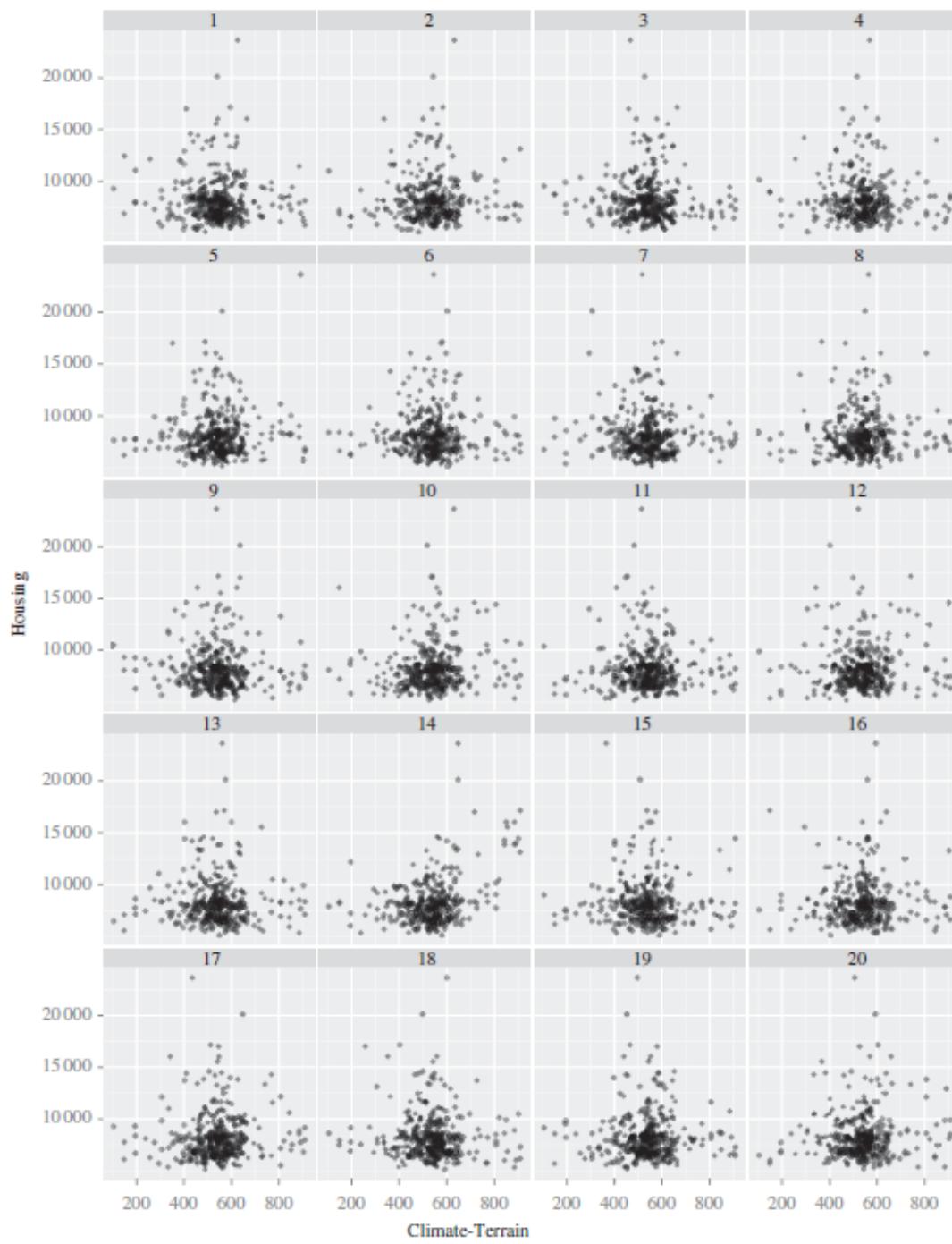
[Buja, Cook, Hofmann, Wickham, et al.]

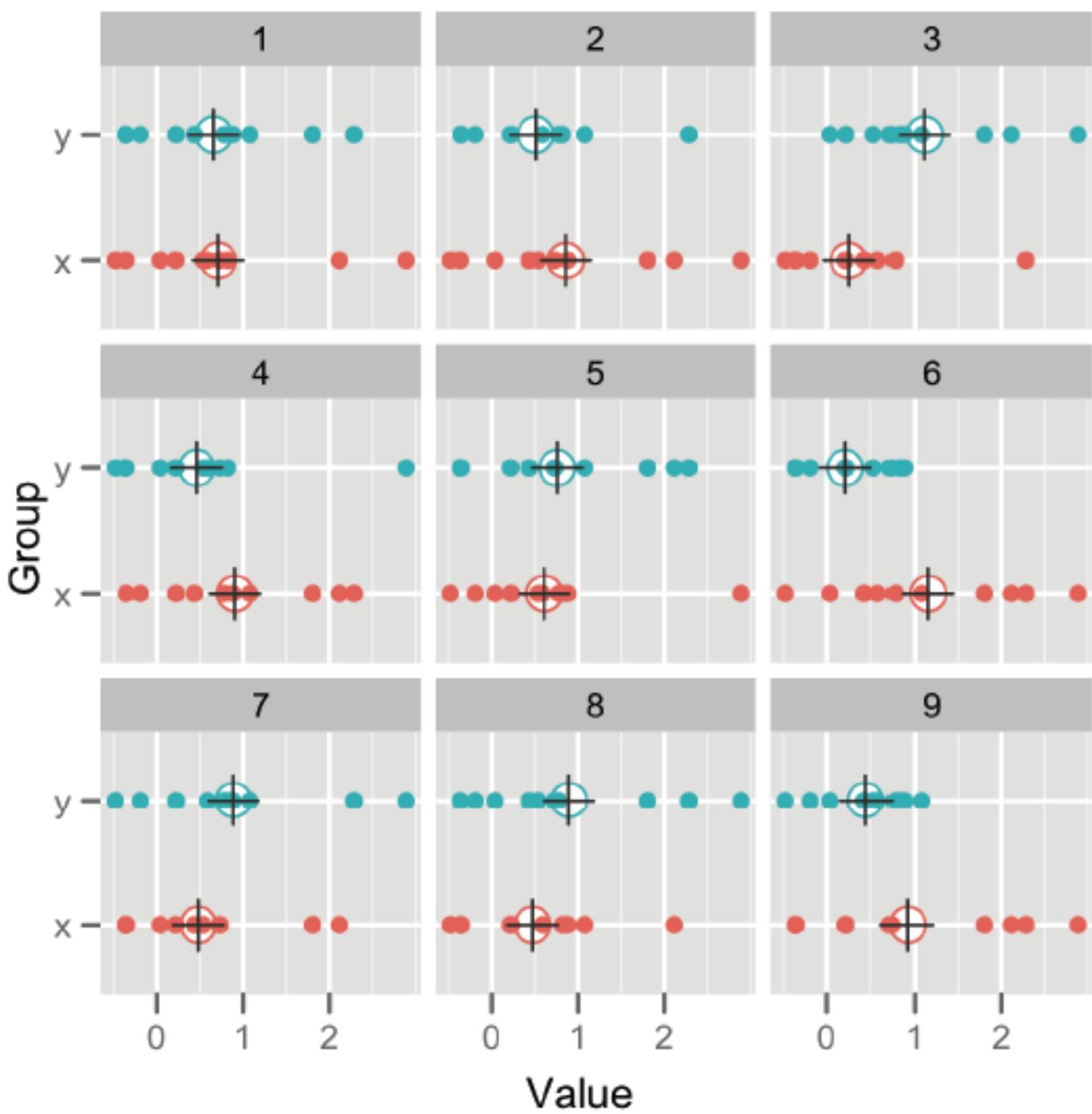


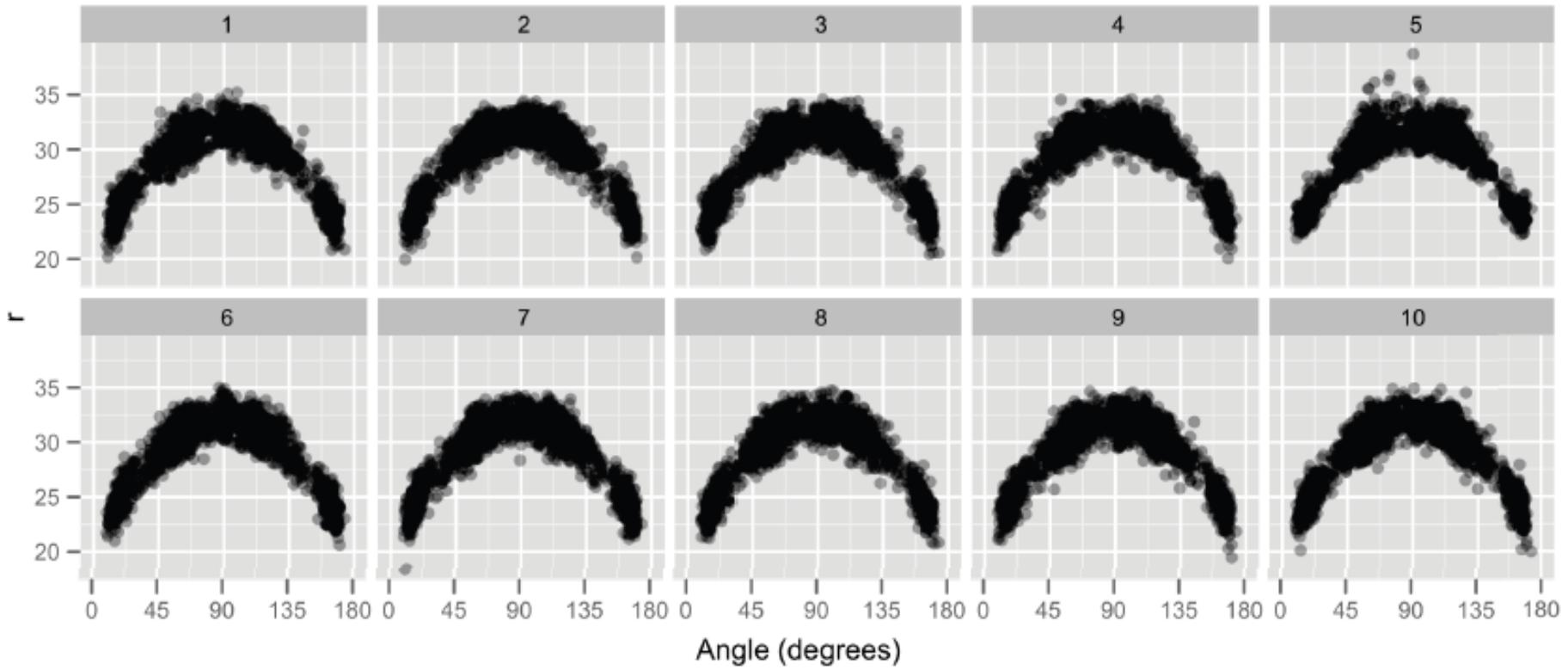
## Choropleth maps of cancer deaths in Texas.

One plot shows a real data set. The others are simulated under the null hypothesis of spatial independence.

Can you spot the real data? If so, you have some evidence of spatial dependence in the data.

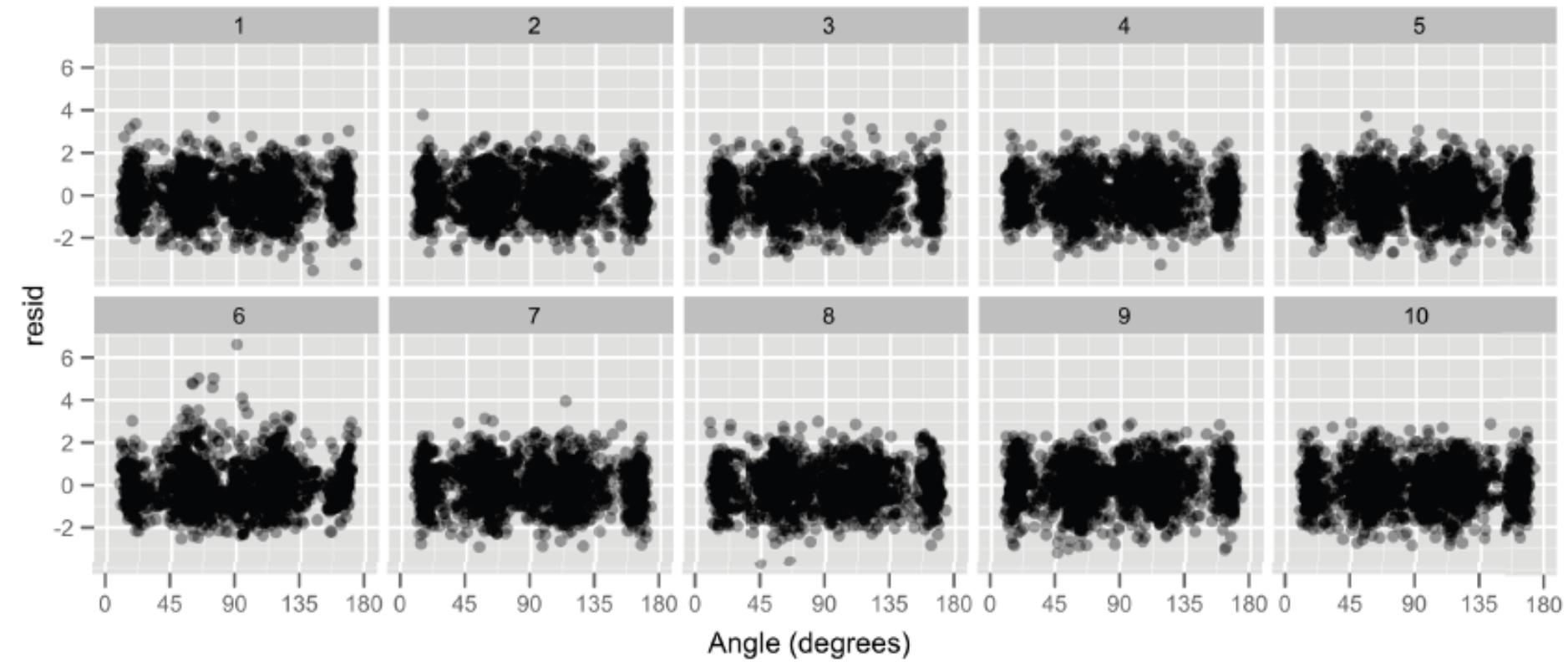






**Distance vs. angle for 3 point shots by the LA Lakers.**

One plot is the real data. The others are generated according to a null hypothesis of quadratic relationship.



**Distance vs. angle for 3 point shots by the LA Lakers.**

One plot is the real data. The others are generated according to a null hypothesis of quadratic relationship.

# Summary

Exploratory analysis may combine graphical methods, data transformations, and statistics.

Use questions to uncover more questions.

Formal methods may be used to confirm, sometimes on held-out or new data.

Visualization can further aid assessment of fitted statistical models.

# Data Quality

# A Detective Story

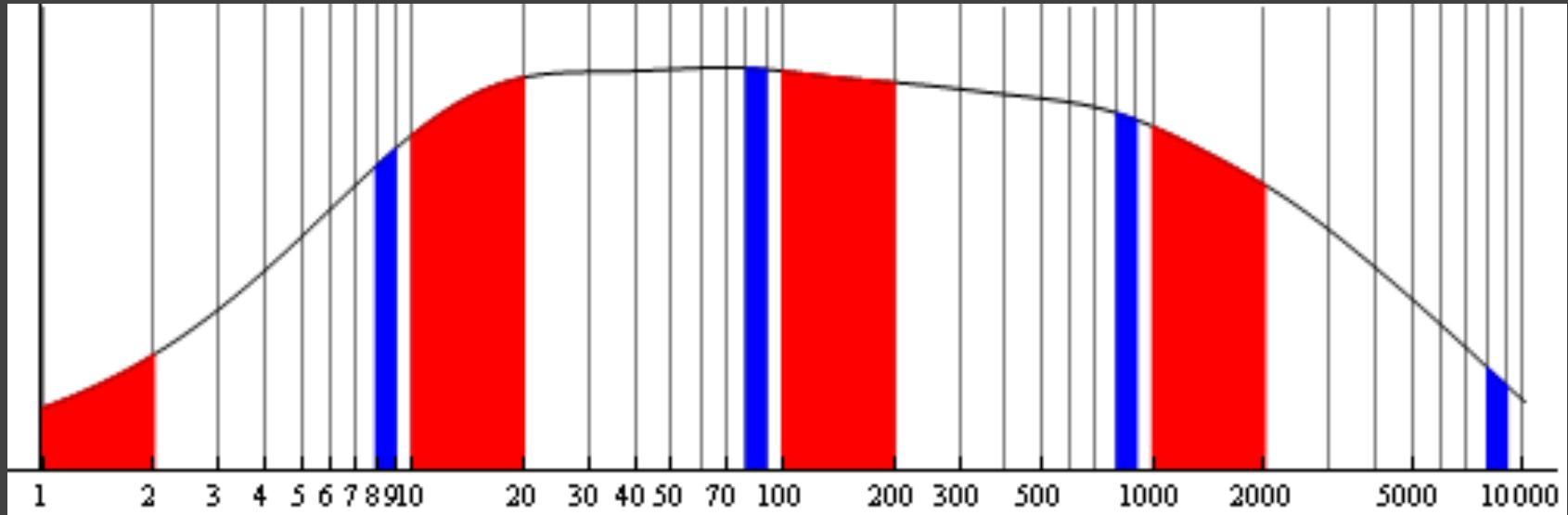
You have accounting records for two firms  
that are in dispute. One is lying. *How to tell?*

<i>Firm A</i>		<i>Firm B</i>	<b>LIARS!</b>
283.08	25.23	283.08	75.23
153.86	385.62	353.86	185.25
1448.97	12371.32	5322.79	9971.42
18595.91	1280.76	8795.64	4802.43
21.33	257.64	61.33	57.64
Amt. Paid: \$34823.72		Amt. Rec'd: \$29908.67	

# Benford's Law

(Benford 1938, Newcomb 1881)

The *logarithms* of the values (not the values themselves) are uniformly randomly distributed.



Hence the leading digit **1** has a ~30% likelihood.  
Larger digits are increasingly less likely.

# Benford's Law

(Benford 1938, Newcomb 1881)

The *logarithms* of the values (not the values themselves) are uniformly randomly distributed.

Holds for many (but certainly not all) real-life data sets: Addresses, Bank accounts, Building heights, ...

Data must span multiple orders of magnitude.

Evidence that records do not follow Benford's Law is admissible in a court of law!

# Model-Driven Data Validation

Deviations from the model may represent errors

## Find Statistical Outliers

# std dev, Mahalanobis dist, nearest-neighbor,  
non-parametric methods, time-series models

Robust statistics to combat noise, masking

## Data Entry Errors

Product codes: PZV, PZV, PZR, PZC, PZV

Which of the above is most likely in error?

**Opportunity:** combine with visualization methods

# Confirmatory Analysis

# Incorporating Models

**Hypothesis testing:** What is the probability that the pattern might have arisen by chance?

**Prediction:** How well do one (or more) data variables predict values of interest?

**Summarization:** With what parameters does data fit a given function? What is the goodness of fit?

**Scientific theory:** Which model explains reality?

# Example: Heights by Gender

Gender	Male / Female
Height (in)	Number

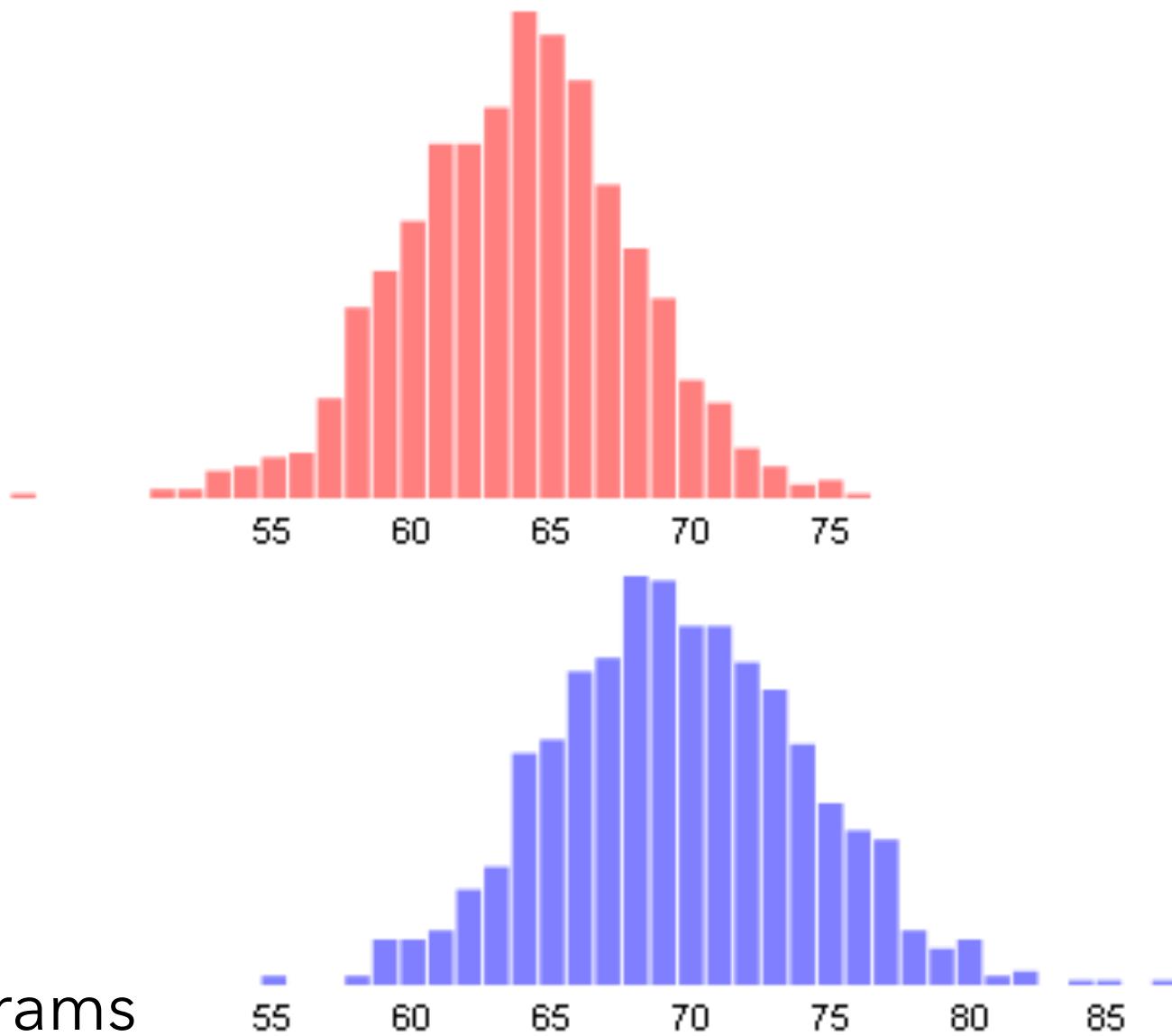
$$\mu_m = 69.4 \quad \sigma_m = 4.69 \quad N_m = 1000$$

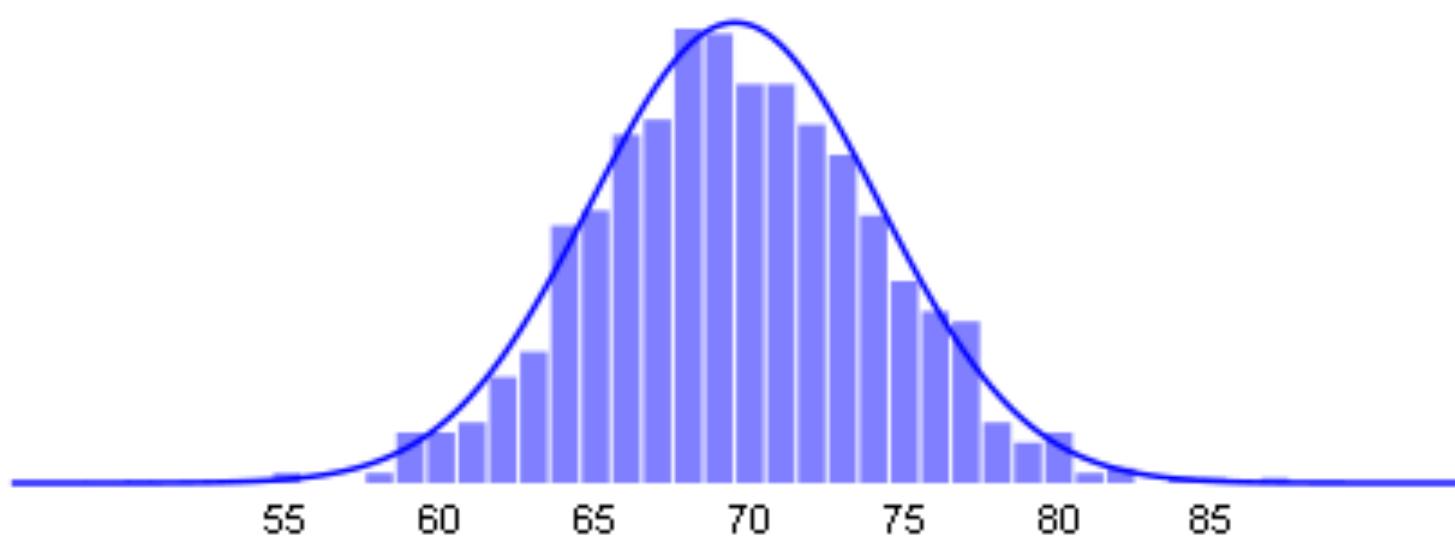
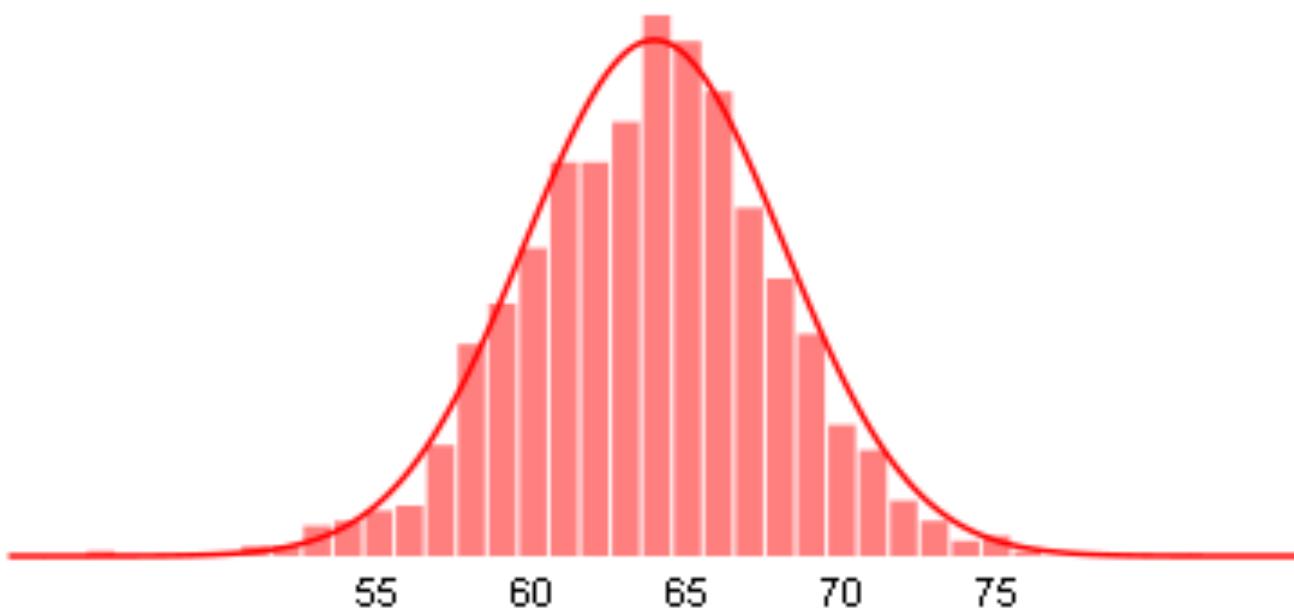
$$\mu_f = 63.8 \quad \sigma_f = 4.18 \quad N_f = 1000$$

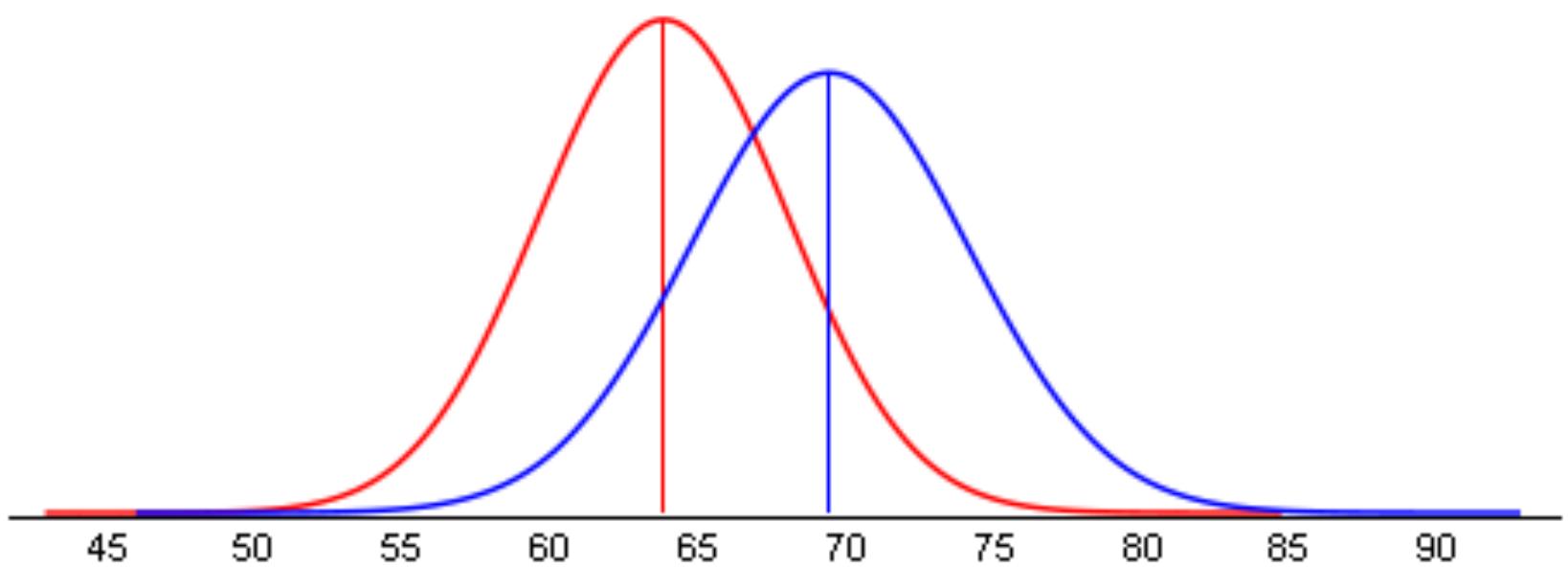
Is the difference in heights significant?

In other words: assuming no true difference, what is the probability that our data is due to chance?

# Histograms







# Formulating a Hypothesis

Null Hypothesis ( $H_0$ ):  $\mu_m = \mu_f$  (population)

Alternate Hypothesis ( $H_a$ ):  $\mu_m \neq \mu_f$  (population)

A **statistical hypothesis test** assesses the likelihood of the null hypothesis.

What is the probability of sampling the observed data assuming the population means are equal?

This is called the **p-value**.

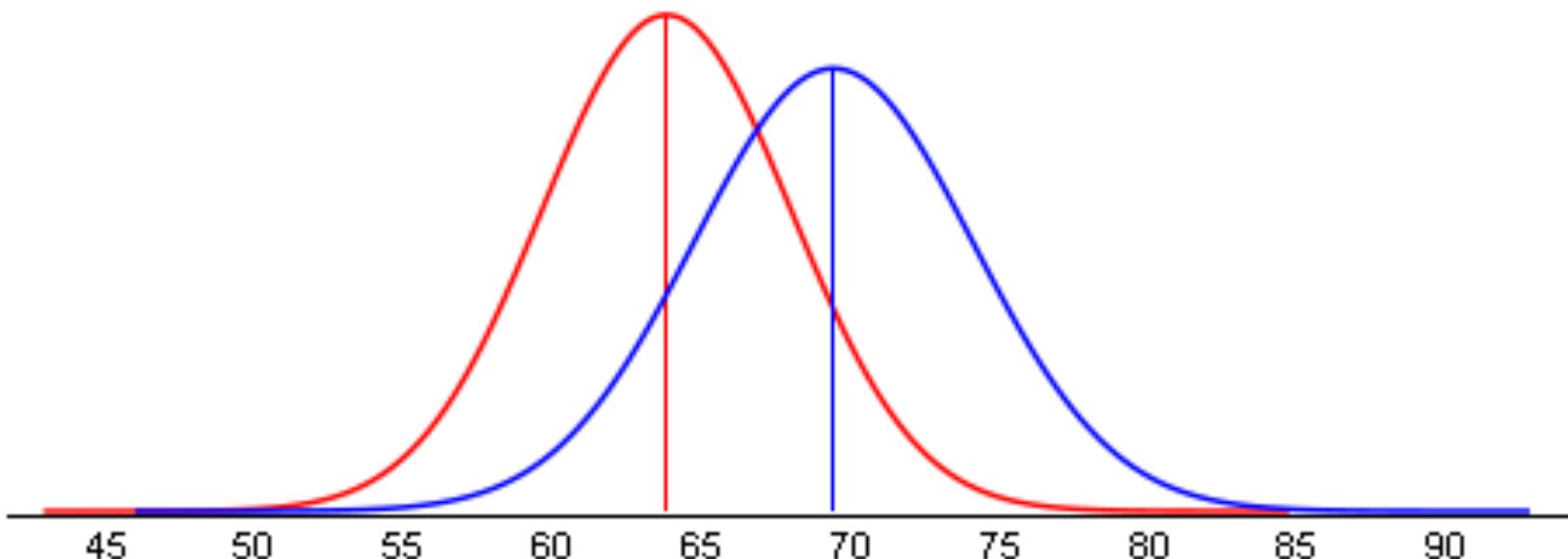
# Testing Procedure

Compute a **test statistic**. This is a number that in essence summarizes the difference.

# Calculate the Test Statistic

$$Z = \frac{\mu_m - \mu_f}{\sqrt{\sigma_m^2 / N_m + \sigma_f^2 / N_f}}$$

$$\mu_m - \mu_f = 5.6$$



# Testing Procedure

Compute a **test statistic**. This is a number that in essence summarizes the difference.

The possible values of this statistic come from a **known probability distribution**.

According to this distribution, determine the probability of seeing a value meeting or exceeding the test statistic. This is the **p-value**.

# Lookup Probability of Test Statistic

Normal Distribution

$$\mu = 0, \sigma = 1$$

$$Z \sim N(0, 1)$$

$$Z = .2$$

$$Z > +1.96$$

95% of Probability Mass

-1.96

$p > 0.05$

+1.96

$p < 0.05$

Diagram illustrating the standard normal distribution curve. The horizontal axis represents the test statistic Z. The area under the curve between -1.96 and +1.96 is shaded gray and labeled "95% of Probability Mass". The area to the right of +1.96 is shaded gray and labeled "p < 0.05". The area to the left of -1.96 is shaded gray and labeled "p > 0.05". A vertical line at Z = .2 is labeled "Z = .2".

# Statistical Significance

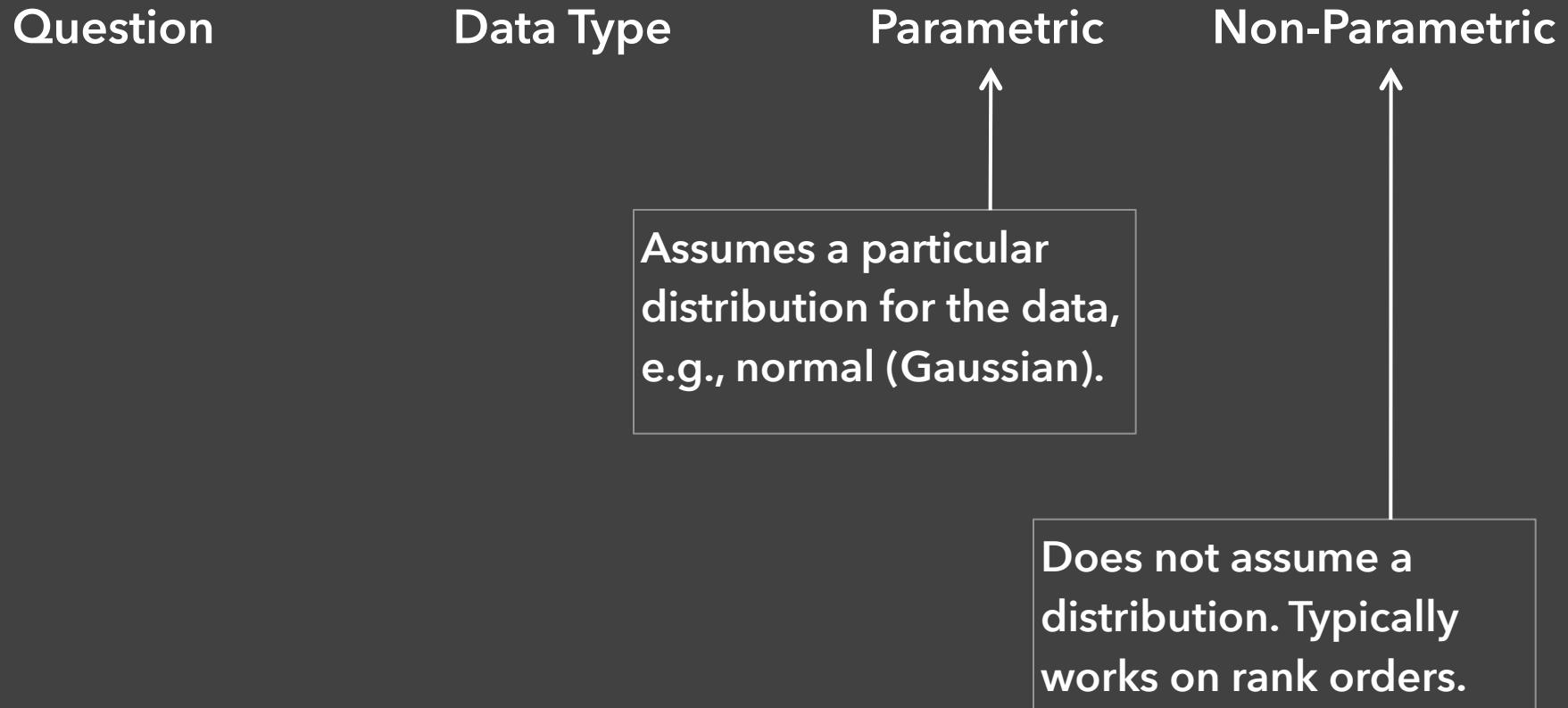
The threshold at which we consider it safe (or reasonable?) to *reject the null hypothesis*.

If  $p < 0.05$ , we typically say that the observed effect or difference is **statistically significant**.

This means that there is a less than 5% chance that the observed data is due to chance.

Note that the choice of 0.05 is a somewhat arbitrary threshold (chosen by R. A. Fisher)

# Common Statistical Methods



# Common Statistical Methods

Question	Data Type	Parametric	Non-Parametric
Do data distributions have different “centers”? (aka “location” tests)	2 uni. dists > 2 uni. dists > 2 multi. dists	t-Test ANOVA MANOVA	Mann-Whitney U Kruskal-Wallis Median Test
Are observed counts significantly different?	Counts in categories		$\chi^2$ (chi-squared)
Are two vars related?	2 variables	Pearson coeff.	Rank correl.
Do 1 (or more) variables predict another?	Continuous Binary	Linear regression Logistic regression	