12.1 Recursion: Introduction

An ***algorithm*** is a sequence of steps for solving a problem. For example, an algorithm for making lemonade is:

Figure 12.1.1: Algorithms are like recipes.

Make lemonade:

- Add sugar to pitcher
- Add lemon juice
- Add water
- Stir

Some problems can be solved using a recursive algorithm. A ***recursive algorithm*** is an algorithm that breaks the problem into smaller subproblems and applies the same algorithm to solve the smaller subproblems.

Figure 12.1.2: Mowing the lawn can be broken down into a recursive process.

- Mow the lawn
  - Mow the frontyard
    - Mow the left front
    - Mow the right front
  - Mow the backyard
    - Mow the left back
    - Mow the right back

The mowing algorithm consists of applying the mowing algorithm on smaller pieces of the yard and thus is a recursive algorithm.

At some point, a recursive algorithm must describe how to actually do something, known as the ***base case***. The mowing algorithm could thus be written as:

- *Mow* the lawn

- *If lawn is less than 100 square meters*
  - Push the lawnmower left-to-right in adjacent rows
- *Else*
  - Mow one half of the lawn
  - Mow the other half of the lawn

PARTICIPATION ACTIVITY 12.1.1: Recursion.

Which are recursive definitions/algorithms?

1) Helping N people:

   If N is 1, help that person.
   Else, help the first N/2 people, then help the second N/2 people.

   - True
   - False

2) Driving to the store:

   Go 1 mile.
   Turn left on Main Street.
   Go 1/2 mile.

   - True
   - False

3) Sorting envelopes by zipcode:

   If N is 1, done.
   Else, find the middle zipcode. Put all zipcodes less than the middle zipcode on the left, all greater ones on the right. Then sort the left, then sort the right.

   - True
   - False

12.2 Recursive functions

A function may call other functions, including calling itself. A function that calls itself is a ***recursive function***.

PARTICIPATION ACTIVITY 12.2.1: A recursive function example.

Animation captions:

1. The first call to CountDown( ) function comes from main. Each call to CountDown( ) effectively creates a new "copy" of the executing function, as shown on the right.
2. Then, the CountDown( ) function calls itself. CountDown(1) similarly creates a new "copy" of the executing function.
3. CountDown( ) function calls itself once more.
4. That last instance does not call CountDown( ) again, but instead returns. As each instance returns, that copy is deleted.

Each call to CountDown() effectively creates a new "copy" of the executing function, as shown on the right. Returning deletes that copy.

The example is for demonstrating recursion; counting down is otherwise better implemented with a loop.

Recursion may be direct, such as f() itself calling f(), or indirect, such as f() calling g() and g() calling f().

PARTICIPATION ACTIVITY 12.2.2: Thinking about recursion.

Refer to the above CountDown example for the following.

1) How many times is CountDown() called if main() calls CountDown(5)?

Check Show answer

2) How many times is CountDown() called if main() calls CountDown(0)?

Check Show answer

3) Is there a difference in how we define the parameters of a recursive versus

non-recursive function? Answer yes or no.

Check Show answer

CHALLENGE ACTIVITY 12.2.1: Calling a recursive function.

Write a statement that calls the recursive function BackwardsAlphabet() with parameter startingLetter.

```
#include <iostream>
using namespace std;

void BackwardsAlphabet(char currLetter){
   if (currLetter == 'a') {
      cout << currLetter << endl;
   }
   else{
      cout << currLetter << " ";
      BackwardsAlphabet(currLetter - 1);
   }
}

int main() {
   char startingLetter;

   cin >> startingLetter;
```

Run

12.3 Recursive algorithm: Search

Recursive search (general)

Consider a guessing game program where a friend thinks of a number from 0 to 100 and you try to guess the number, with the friend telling you to guess higher or lower until you guess correctly. What algorithm would you use to minimize the number of guesses?

A first try might implement an algorithm that simply guesses in increments of 1:

- Is it 0? Higher
- Is it 1? Higher
- Is it 2? Higher

This algorithm requires too many guesses (50 on average). A second try might implement an algorithm that guesses by 10s and then by 1s:

- Is it 10? Higher
- Is it 20? Higher
- Is it 30? Lower
- Is it 21? Higher
- Is it 22? Higher
- Is it 23? Higher

This algorithm does better but still requires about 10 guesses on average: 5 to find the correct tens digit and 5 to guess the correct ones digit. An even better algorithm uses a binary search. A ***binary search*** algorithm begins at the midpoint of the range and halves the range after each guess. For example:

- Is it 50 (the middle of 0-100)? Lower
- Is it 25 (the middle of 0-50)? Higher
- Is it 38 (the middle of 26-50)? Lower
- Is it 32 (the middle of 26-38)?

After each guess, the binary search algorithm is applied again, but on a smaller range, i.e., the algorithm is recursive.

PARTICIPATION ACTIVITY 12.3.1: Binary search: A well-known recursive algorithm.

Animation content:

undefined

Animation captions:

1. A friend thinks of a number from 0 to 100 and you try to guess the number, with the friend telling you to guess higher or lower until you guess correctly.
2. Using a binary search algorithm, you begin at the midpoint of the lower range. (highVal + lowVal) / 2 = (100 + 0) / 2, or 50.
3. The number is lower. The algorithm divides the range in half, then chooses the midpoint of that range.
4. After each guess, the binary search algorithm is applied, halving the range and guessing the midpoint or the corresponding range.
5. A recursive function is a natural match for the recursive binary search algorithm. A function GuessNumber(lowVal, highVal) has parameters that indicate the low and high sides of the

guessing range.

Recursive search function

A recursive function is a natural match for the recursive binary search algorithm. A function GuessNumber(lowVal, highVal) has parameters that indicate the low and high sides of the guessing range. The function guesses at the midpoint of the range. If the user says lower, the function calls GuessNumber(lowVal, midVal). If the user says higher, the function calls GuessNumber(midVal + 1, highVal)

The recursive function has an if-else statement. The if branch ends the recursion, known as the ***base case***. The else branch has recursive calls. Such an if-else pattern is common in recursive functions.

Figure 12.3.1: A recursive function carrying out a binary search algorithm.

```
#include <iostream>
using namespace std;

void GuessNumber(int lowVal, int highVal) {
   int midVal;            // Midpoint of low and high value
   char userAnswer;       // User response

   midVal = (highVal + lowVal) / 2;

   // Prompt user for input
   cout << "Is it " << midVal << "? (l/h/y): ";
   cin >> userAnswer;

   if( (userAnswer != 'l') && (userAnswer != 'h') ) { // Base case: found number
      cout << "Thank you!" << endl;
   }
   else {                                            // Recursive case: split into
lower OR upper half
      if (userAnswer == 'l') {                       // Guess in lower half
         GuessNumber(lowVal, midVal);                // Recursive call
      }
      else {                                         // Guess in upper half
         GuessNumber(midVal + 1, highVal);           // Recursive call
      }
   }
}

int main() {
   // Print game objective, user input commands
   cout << "Choose a number from 0 to 100." << endl;
   cout << "Answer with:" << endl;
   cout << "   l (your num is lower)" << endl;
   cout << "   h (your num is higher)" << endl;
   cout << "   any other key (guess is right)." << endl;

   // Call recursive function to guess number
   GuessNumber(0, 100);

   return 0;
}
```

```
Choose a number from 0 to 100.
Answer with:
   l (your num is lower)
   h (your num is higher)
   any other key (guess is right).
Is it 50? (l/h/y): l
Is it 25? (l/h/y): h
Is it 38? (l/h/y): l
Is it 32? (l/h/y): y
Thank you!
```


Calculating the middle value

Because midVal has already been checked, it need not be part of the new window, so midVal + 1 rather than midVal is used for the window's new low side, or midVal - 1 for the window's new high side. But the midVal - 1 can have the drawback of a non-intuitive base case (i.e., midVal < lowVal, because if the current window is say 4..5, midVal is 4, so the new window would be 4..4-1, or 4..3). rangeSize == 1 is likely more intuitive, and thus the algorithm uses midVal rather than midVal - 1. However, the algorithm uses midVal + 1 when searching higher, due to integer rounding. In particular, for window 99..100, midVal is 99 ((99 + 100) / 2 = 99.5, rounded to 99 due to truncation of the fraction in integer division). So the next window would again be 99..100, and the algorithm would repeat with this window forever. midVal + 1 prevents the problem, and doesn't miss any numbers because midVal was checked and thus need not be part of the window.

PARTICIPATION ACTIVITY 12.3.2: Binary search tree tool.

The following program guesses the hidden number known by the user. Assume the hidden number is 63.

```
#include <iostream>
using namespace std;

void Find(int low, int high) {
   int mid; // Midpoint of low..high
   char answer;

   mid = (high + low) / 2;

   cout << "Is it " << mid << "? (l/h/y): ";
   cin >> answer;

   if((answer != 'l') &&
      (answer != 'h')) { // Base case:
      cout << "Thank you!" << endl;  // Found number!
   }
   else { // Recursive case: Guess in
          // lower or upper half of range
      if (answer == 'l') { // Guess in lower half
         Find(low, mid); // Recursive call
      }
      else {               // Guess in upper half
         Find(mid + 1, high);     // Recursive call
      }
   }

   return;
}

int main() {
   cout << "Choose a number from 0 to 100." << endl;
   cout << "Answer with:" << endl;
   cout << "   l (your num is lower)" << endl;
   cout << "   h (your num is higher)" << endl;
   cout << "   any other key (guess is right)." << endl;

   Find(0, 100);

   return 0;
}
```


→ main()

```
int main() {
   cout << "Choose a number from 0 to 100." << endl;
   cout << "Answer with:" << endl;
   cout << "   l (your num is lower)" << endl;
   cout << "   h (your num is higher)" << endl;
   cout << "   any other key (guess is right)." << endl;

   Find(0, 100);

   return 0;
}
```

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Recursively searching a sorted list

Search is commonly performed to quickly find an item in a sorted list stored in an array or vector. Consider a list of attendees at a conference, whose names have been stored in alphabetical order in an array or vector. The following quickly determines whether a particular person is in attendance.

FindMatch() restricts its search to elements within the range lowVal to highVal. main() initially passes a range of the entire list: 0 to (list size - 1). FindMatch() compares to the middle element, returning that element's position if matching. If not matching, FindMatch() checks if the window's size is just one element, returning -1 in that case to indicate the item was not found. If neither of those two base cases are satisfied, then FindMatch() recursively searches either the lower or upper half of the range as appropriate.

Figure 12.3.2: Recursively searching a sorted list.

```
#include <iostream>
#include <string>
#include <vector>

using namespace std;

/* Finds index of string in vector of strings, else -1.
   Searches only with index range low to high
   Note: Upper/lower case characters matter
*/

int FindMatch(vector<string> stringsList, string itemMatch, int lowVal, int highVal) {
   int midVal;        // Midpoint of low and high values
   int itemPos;       // Position where item found, -1 if not found
   int rangeSize;     // Remaining range of values to search for match

   rangeSize = (highVal - lowVal) + 1;
   midVal = (highVal + lowVal) / 2;

   if (itemMatch == stringsList.at(midVal)) {   // Base case 1: item found at midVal
position
      itemPos = midVal;
   }
   else if (rangeSize == 1) {                   // Base case 2: match not found
      itemPos = -1;
   }
   else {                                       // Recursive case: search lower or upper
half
      if (itemMatch < stringsList.at(midVal)) { // Search lower half, recursive call
         itemPos = FindMatch(stringsList, itemMatch, lowVal, midVal);
      }
      else {                                    // Search upper half, recursive call
         itemPos = FindMatch(stringsList, itemMatch, midVal + 1, highVal);
      }
   }

   return itemPos;
}

int main() {
   vector<string> attendeesList(0); // List of attendees
   string attendeeName;             // Name of attendee to match
   int matchPos;                    // Matched position in attendee list

   // Omitting part of program that adds attendees
   // Instead, we insert some sample attendees in sorted order
   attendeesList.push_back("Adams, Mary");
   attendeesList.push_back("Carver, Michael");
   attendeesList.push_back("Domer, Hugo");
   attendeesList.push_back("Fredericks, Carlos");
   attendeesList.push_back("Li, Jie");

   // Prompt user to enter a name to find
   cout << "Enter person's name: Last, First: ";
   getline(cin, attendeeName); // Use getline to allow space in name

   // Call function to match name, output results
   matchPos = FindMatch(attendeesList, attendeeName, 0, attendeesList.size() - 1);
   if (matchPos >= 0) {
      cout << "Found at position " << matchPos << "." << endl;
   }
   else {
      cout << "Not found. " << endl;
   }

   return 0;
}
```

```
Enter person's name: Last, First: Meeks, Stan
Not found.

...

Enter person's name: Last, First: Adams, Mary
Found at position 0.

...

Enter person's name: Last, First: Li, Jie
Found at position 4.
```


PARTICIPATION ACTIVITY 12.3.3: Recursive search algorithm.

Consider the above FindMatch() function for finding an item in a sorted list.

1) If a sorted list has elements 0 to 50 and the item being searched for is at element 6, how many times will FindMatch() be called?

Check Show answer

2) If an alphabetically ascending list has elements 0 to 50, and the item at element 0 is "Bananas", how many calls to FindMatch() will be made during the failed search for "Apples"?

Check Show answer

PARTICIPATION ACTIVITY 12.3.4: Recursive calls.

A list has 5 elements numbered 0 to 4, with these letter values: 0: A, 1: B, 2: D, 3: E, 4: F.

1) To search for item C, the first call is FindMatch(0, 4). What is the second call to FindMatch()?

- FindMatch(0, 0)
- FindMatch(0, 2)
- FindMatch(3, 4)

2) In searching for item C, FindMatch(0, 2) is called. What happens next?

- Base case 1: item found at midVal.
- Base case 2: rangeSize == 1, so no match.
- Recursive call: FindMatch(2, 2)

CHALLENGE ACTIVITY 12.3.1: Enter the output of binary search.

Exploring further:

- Binary search from GeeksforGeeks.org

12.4 Adding output statements for debugging

Recursive functions can be particularly challenging to debug. Adding output statements can be helpful. Furthermore, an additional trick is to indent the print statements to show the current depth of recursion. The following program adds a parameter indent to a FindMatch() function that searches a sorted list for an item. All of FindMatch()'s print statements start with `cout << indentAmt << ...`. Indent is typically some number of spaces. main() sets indent to three spaces. Each recursive call *adds* three more spaces. Note how the output now clearly shows the recursion depth.

Figure 12.4.1: Output statements can help debug recursive functions, especially if indented based on recursion depth.

```
#include <iostream>
#include <string>
#include <vector>

using namespace std;
```

```
/* Finds index of string in vector of strings, else -1.
   Searches only with index range low to high
   Note: Upper/lower case characters matter
*/

int FindMatch(vector<string> stringsList, string itemMatch,
              int lowVal, int highVal, string indentAmt) { // indentAmt used for print
debug

   int midVal;         // Midpoint of low and high values
   int itemPos;        // Position where item found, -1 if not found
   int rangeSize;      // Remaining range of values to search for match

   cout << indentAmt << "Find() range " << lowVal << " " << highVal << endl;

   rangeSize = (highVal - lowVal) + 1;
   midVal = (highVal + lowVal) / 2;

   if (itemMatch == stringsList.at(midVal)) {   // Base case 1: item found at midVal
position
      cout << indentAmt << "Found person." << endl;
      itemPos = midVal;
   }
   else if (rangeSize == 1) {                   // Base case 2: match not found
      cout << indentAmt << "Person not found." << endl;
      itemPos = -1;
   }
   else {                                       // Recursive case: Search lower or upper
half
      if (itemMatch < stringsList.at(midVal)) { // Search lower half, recursive call
         cout << indentAmt << "Searching lower half." << endl;
         itemPos = FindMatch(stringsList, itemMatch, lowVal, midVal, indentAmt + "   ");
      }
      else {                                    // Search upper half, recursive call
         cout << indentAmt << "Searching upper half." << endl;
         itemPos = FindMatch(stringsList, itemMatch, midVal + 1, highVal, indentAmt + "
");
      }
   }

   cout << indentAmt << "Returning pos = " << itemPos << "." << endl;
   return itemPos;
}

int main() {
   vector<string> attendeesList(0); // List of attendees
   string attendeeName;             // Name of attendee to match
   int matchPos;                    // Matched position in attendee list

   // Omitting part of program that adds attendees
   // Instead, we insert some sample attendees in sorted order
   attendeesList.push_back("Adams, Mary");
   attendeesList.push_back("Carver, Michael");
   attendeesList.push_back("Domer, Hugo");
   attendeesList.push_back("Fredericks, Carlos");
   attendeesList.push_back("Li, Jie");

   // Prompt user to enter a name to find
   cout << "Enter person's name: Last, First: ";
   getline(cin, attendeeName); // Use getline to allow space in name

   // Call function to match name, output results
   matchPos = FindMatch(attendeesList, attendeeName, 0,
                        attendeesList.size() - 1, "   ");
   if (matchPos >= 0) {
      cout << "Found at position " << matchPos << "." << endl;
   }
   else {
      cout << "Not found. " << endl;
```

```
   }

   return 0;
}
```

```
Enter person's name: Last, First: Meeks, Stan
   Find() range 0 4
   Searching upper half.
      Find() range 3 4
      Searching upper half.
         Find() range 4 4
         Person not found.
         Returning pos = -1.
      Returning pos = -1.
   Returning pos = -1.
Not found.

...

Enter person's name: Last, First: Adams, Mary
   Find() range 0 4
   Searching lower half.
      Find() range 0 2
      Searching lower half.
         Find() range 0 1
         Found person.
         Returning pos = 0.
      Returning pos = 0.
   Returning pos = 0.
Found at position 0.
```


Some programmers like to leave the output statements in the code, commenting them out with "//" when not in use. The statements actually serve as a form of comment as well.

More advanced techniques for handling debug output exist too, such as ***conditional compilation*** (beyond this section's scope).

PARTICIPATION ACTIVITY 12.4.1: Recursive debug statements.

Refer to the above code using indented output statements.

1) The above debug approach requires an extra parameter be passed to indicate the amount of indentation.
   - True
   - False

2) Each recursive call should add a few spaces to the indent parameter.
   - True
   - False

3) The function should remove a few spaces from the indent parameter before returning.

- ○ True
- ○ False

zyDE 12.4.1: Output statements in a recursive function.

- Run the recursive program, and observe the output statements for debugging, and the person is correctly not found.
- Introduce an error by changing `itemPos = -1` to `itemPos = 0` in the range siz base case.
- Run the program, notice how the indented print statements help isolate the error c person incorrectly being found.

Load default template... **Run**

```
#include <iostream>
#include <string>
#include <vector>

using namespace std;

/* Finds index of string in vector of s
   Searches only with index range low t
   Note: Upper/lower case characters mc
*/

int FindMatch(vector<string> stringsLis
              int lowVal, int highVal,

   int midVal;        // Midpoint of lc
```


12.5 Creating a recursive function

Creating a recursive function can be accomplished in two steps.

- **Write the base case** -- Every recursive function must have a case that returns a value without performing a recursive call. That case is called the ***base case***. A programmer may write that part

of the function first, and then test. There may be multiple base cases.

- **Write the recursive case** -- The programmer then adds the recursive case to the function.

The following illustrates a simple function that computes the factorial of N (i.e. N!). The base case is N = 1 or 1! which evaluates to 1. The base case is written as `if (N <= 1) { fact = 1; }`. The recursive case is used for N > 1, and written as `else { fact = N * NFact( N - 1 ); }`.

PARTICIPATION ACTIVITY 12.5.1: Writing a recursive function for factorial: First write the base case, then add the recursive case.

Animation captions:

1. The base case, which returns a value without performing a recursive call, is written and tested first. If N is less than or equal to 1, then the NFact() function returns 1.
2. Next the recursive case, which calls itself, is written and tested. If N is greater than 1, then the NFact() function returns N * NFact(N - 1).

A common error is to not cover all possible base cases in a recursive function. Another common error is to write a recursive function that doesn't always reach a base case. Both errors may lead to infinite recursion, causing the program to fail.

Typically, programmers will use two functions for recursion. An "outer" function is intended to be called from other parts of the program, like the function `int CalcFactorial(int inVal)`. An "inner" function is intended only to be called from that outer function, for example a function `int CalcFactorialHelper(int inVal)`. The outer function may check for a valid input value, e.g., ensuring inVal is not negative, and then calling the inner function. Commonly, the inner function has parameters that are mainly of use as part of the recursion, and need not be part of the outer function, thus keeping the outer function more intuitive.

PARTICIPATION ACTIVITY 12.5.2: Creating recursion.

1) Recursive functions can be accomplished in one step, namely repeated calls to itself.
   - True
   - False

2) A recursive function with parameter N counts up from any negative number to 0. An appropriate base case would be `N == 0`.
   - True
   -

False

3) A recursive function can have two base cases, such as `N == 0` returning 0, and `N == 1` returning 1.

- True
- False

Before writing a recursive function, a programmer should determine:

1. Does the problem naturally have a recursive solution?
2. Is a recursive solution better than a non-recursive solution?

For example, computing N! (N factorial) does have a natural recursive solution, but a recursive solution is not better than a non-recursive solution. The figure below illustrates how the factorial computation can be implemented as a loop. Conversely, binary search has a natural recursive solution, and that solution may be easier to understand than a non-recursive solution.

Figure 12.5.1: Non-recursive solution to compute N!

```
for (i = inputNum; i > 1; --i) {
   facResult = facResult * i;
}
```

PARTICIPATION ACTIVITY 12.5.3: When recursion is appropriate.

1) N factorial (N!) is commonly implemented as a recursive function due to being easier to understand and executing faster than a loop implementation.

- True
- False

zyDE 12.5.1: Output statements in a recursive function.

Implement a recursive function to determine if a number is prime. Skeletal code is provi
the IsPrime function.

Load default template... Run

```
#include <iostream>
using namespace std;

// Returns false if value is not prime,
bool IsPrime(int testVal, int divVal)
{
   // Base case 1: 0 and 1 are not prim

   // Base case 2: testVal only divisib

   // Recursive Case
      // Check if testVal can be evenly
      // Hint: use the % operator

      // If not, recursive call to isPr
   return 0;
```


CHALLENGE ACTIVITY 12.5.1: Recursive function: Writing the base case.

Write code to complete DoublePennies()'s base case. Sample output for below program with inputs 1 and 10:

```
Number of pennies after 10 days: 1024
```

Note: If the submitted code has an infinite loop, the system will stop running the code after a few seconds, and report "Program end never reached." The system doesn't print the test case that caused the reported message.

```
#include <iostream>
using namespace std;

// Returns number of pennies if pennies are doubled numDays times
long long DoublePennies(long long numPennies, int numDays){
   long long totalPennies;

   /* Your solution goes here  */

   else {
      totalPennies = DoublePennies((numPennies * 2), numDays - 1);
   }

   return totalPennies;
}

// Program computes pennies if you have 1 penny today,
```

Run

CHALLENGE ACTIVITY 12.5.2: Recursive function: Writing the recursive case.

Write code to complete PrintFactorial()'s recursive case. Sample output if input is 5:

```
5! = 5 * 4 * 3 * 2 * 1 = 120
```



```
#include <iostream>
using namespace std;

void PrintFactorial(int factCounter, int factValue){
   int nextCounter;
   int nextValue;

   if (factCounter == 0) {            // Base case: 0! = 1
      cout << "1" << endl;
   }
   else if (factCounter == 1) {       // Base case: Print 1 and result
      cout << factCounter << " = " << factValue << endl;
   }
   else {                             // Recursive case
      cout << factCounter << " * ";
      nextCounter = factCounter - 1;
      nextValue = nextCounter * factValue;
```

Run

12.6 Recursive math functions

Fibonacci sequence

Recursive functions can solve certain math problems, such as computing the Fibonacci sequence. The ***Fibonacci sequence*** is 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc.; starting with 0, 1, the pattern is to compute the next number by adding the previous two numbers.

Below is a program that outputs the Fibonacci sequence values step-by-step, for a user-entered number of steps. The base case is that the program has output the requested number of steps. The

recursive case is that the program needs to compute the number in the Fibonacci sequence.

Figure 12.6.1: Fibonacci sequence step-by-step.

```
#include <iostream>
using namespace std;

/* Output the Fibonacci sequence step-by-step.
   Fibonacci sequence starts as:
   0 1 1 2 3 5 8 13 21 ... in which the first
   two numbers are 0 and 1 and each additional
   number is the sum of the previous two numbers
*/

void ComputeFibonacci(int fibNum1, int fibNum2, int
runCnt) {

   cout << fibNum1 << " + " << fibNum2 << " = "
        << fibNum1 + fibNum2 << endl;

   if (runCnt <= 1) { // Base case: Ran for user
specified
                      // number of steps, do nothing
   }
   else {             // Recursive case: compute next
value
      ComputeFibonacci(fibNum2, fibNum1 + fibNum2, runCnt
- 1);
   }
}

int main() {
   int runFor;       // User specified number of values
computed

   // Output program description
   cout << "This program outputs the" << endl
   << "Fibonacci sequence step-by-step," << endl
   << "starting after the first 0 and 1." << endl <<
endl;

   // Prompt user for number of values to compute
   cout << "How many steps would you like? ";
   cin >> runFor;

   // Output first two Fibonacci values, call recursive
function
   cout << "0" << endl << "1" << endl;
   ComputeFibonacci(0, 1, runFor);

   return 0;
}
```

```
This program outputs the
Fibonacci sequence step-by-
step,
starting after the first 0
and 1.

How many steps would you
like? 10
0
1
0 + 1 = 1
1 + 1 = 2
1 + 2 = 3
2 + 3 = 5
3 + 5 = 8
5 + 8 = 13
8 + 13 = 21
13 + 21 = 34
21 + 34 = 55
34 + 55 = 89
```


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zyDE 12.6.1: Recursive Fibonacci.

Complete ComputeFibonacci() to return F_N, where F_0 is 0, F_1 is 1, F_2 is 1, F_3 is 2, F_4 is 3, continuing: F_N is F_{N-1} + F_{N-2}. Hint: Base cases are N == 0 and N == 1.

Load default templ

```
1
2  #include <iostream>
3  using namespace std;
4
5  int ComputeFibonacci(int N) {
6
7     cout << "FIXME: Complete this function." << endl;
8     cout << "Currently just returns 0." << endl;
9
10    return 0;
11 }
12
13 int main() {
14    int N;       // F_N, starts at 0
15
16    N = 4;
17
18    cout << "F_" << N << " is "
```


Run

Greatest common divisor (GCD)

Recursion can solve the greatest common divisor problem. The ***greatest common divisor*** (GCD) is the largest number that divides evenly into two numbers, e.g. GCD(12, 8) = 4. One GCD algorithm (described by Euclid around 300 BC) subtracts the smaller number from the larger number until both numbers are equal. Ex:

- GCD(12, 8): Subtract 8 from 12, yielding 4.
- GCD(4, 8): Subtract 4 from 8, yielding 4.
- GCD(4, 4): Numbers are equal, return 4

The following recursively computes the GCD of two numbers. The base case is that the two numbers are equal, so that number is returned. The recursive case subtracts the smaller number from the larger number and then calls GCD with the new pair of numbers.

Figure 12.6.2: Calculate greatest common divisor of two numbers.

```
#include <iostream>
using namespace std;

/* Determine the greatest common divisor
   of two numbers, e.g. GCD(8, 12) = 4
*/
int GCDCalculator(int inNum1, int inNum2) {
   int gcdVal;      // Holds GCD results

   if(inNum1 == inNum2) {     // Base case: Numbers are
equal
      gcdVal = inNum1;        // Return value
   }
   else {                     // Recursive case: subtract
smaller from larger
      if (inNum1 > inNum2) { // Call function with new
values
         gcdVal = GCDCalculator(inNum1 - inNum2, inNum2);
      }
      else {
         gcdVal= GCDCalculator(inNum1, inNum2 - inNum1);
      }
   }

   return gcdVal;
}

int main() {
   int gcdInput1;      // First input to GCD calc
   int gcdInput2;      // Second input to GCD calc
   int gcdOutput;      // Result of GCD

   // Print program function
   cout << "Program outputs the greatest \n"
        << "common divisor of two numbers." << endl;

   // Prompt user for input
   cout << "Enter first number: ";
   cin >> gcdInput1;

   cout << "Enter second number: ";
   cin >> gcdInput2;

   // Check user values are > 1, call recursive GCD
function
   if ((gcdInput1 < 1) || (gcdInput2 < 1)) {
      cout << "Note: Neither value can be below 1." <<
endl;
   }
   else {
      gcdOutput = GCDCalculator(gcdInput1, gcdInput2);
      cout << "Greatest common divisor = " << gcdOutput <<
endl;
   }

   return 0;
}
```

```
Program outputs the
greatest
common divisor of two
numbers.
Enter first number: 12
Enter second number: 8
Greatest common divisor = 4

...

Program outputs the
greatest
common divisor of two
numbers.
Enter first number: 456
Enter second number: 784
Greatest common divisor = 8

...

Program outputs the
greatest
common divisor of two
numbers.
Enter first number: 0
Enter second number: 10
Note: Neither value can be
below 1.
```


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PARTICIPATION ACTIVITY 12.6.1: Recursive GCD example.

1) How many calls are made to GCDCalculator() function for input values 12 and 8?

- 1
- 2
- 3

2) What is the base case for the GCD algorithm?

- When both inputs to the function are equal.
- When both inputs are greater than 1.
- When inNum1 > inNum2.

Exploring further:

- Fibonacci number from Wolfram.
- Greatest Common Divisor from Wolfram.

CHALLENGE ACTIVITY 12.6.1: Writing a recursive math function.

Write code to complete RaiseToPower(). Sample output if userBase is 4 and userExponent is 2 is shown below. Note: This example is for practicing recursion; a non-recursive function, or using the built-in function pow(), would be more common.

```
4^2 = 16
```



```
#include <iostream>
using namespace std;

int RaiseToPower(int baseVal, int exponentVal){
   int resultVal;

   if (exponentVal == 0) {
      resultVal = 1;
   }
   else {
```

```
11        resultVal = baseVal * /* Your solution goes here  */;
12     }
13
14     return resultVal;
15  }
16
17  int main() {
```

Run

12.7 Recursive exploration of all possibilities

Recursion is a powerful technique for exploring all possibilities, such as all possible reorderings of a word's letters, all possible subsets of items, all possible paths between cities, etc. This section provides several examples.

Word scramble

Consider printing all possible combinations (or "scramblings") of a word's letters. The letters of abc can be scrambled in 6 ways: abc, acb, bac, bca, cab, cba. Those possibilities can be listed by making three choices: Choose the first letter (a, b, or c), then choose the second letter, then choose the third letter. The choices can be depicted as a tree. Each level represents a choice. Each node in the tree shows the unchosen letters on the left, and the chosen letters on the right.

PARTICIPATION ACTIVITY 12.7.1: Exploring all possibilities viewed as a tree of choices.

Animation captions:

1. Consider printing all possible combinations of a word's letters. Those possibilities can be listed by choosing the first letter, then the second letter, then the third letter.
2. The choices can be depicted as a tree. Each level represents a choice.
3. A recursive exploration function is a natural match to print all possible combinations of a string's letters. Each call to the function chooses from the set of unchosen letters, continuing until no unchosen letters remain.

The tree guides creation of a recursive exploration function to print all possible combinations of a string's letters. The function takes two parameters: unchosen letters, and already chosen letters. The base case is no unchosen letters, causing printing of the chosen letters. The recursive case calls the function once for each letter in the unchosen letters. The above animation depicts how the recursive algorithm traverses the tree. The tree's leaves (the bottom nodes) are the base cases.

The following program prints all possible ordering of the letters of a user-entered word.

Figure 12.7.1: Scramble a word's letters in every possible way.

```
#include <iostream>
#include <string>
using namespace std;

/* Output every possible combination of a word.
   Each recursive call moves a letter from
   remainLetters to scramLetters.
*/
void ScrambleLetters(string remainLetters,  // Remaining
letters
                     string scramLetters) { // Scrambled
letters
   string tmpString; // Temp word combination
   unsigned int i;   // Loop index

   if (remainLetters.size() == 0) { // Base case: All
letters used
      cout << scramLetters << endl;
   }
   else {                           // Recursive case:
move a letter from
                                    // remaining to
scrambled letters
      for (i = 0; i < remainLetters.size(); ++i) {
         // Move letter to scrambled letters
         tmpString = remainLetters.substr(i, 1);
         remainLetters.erase(i, 1);
         scramLetters = scramLetters + tmpString;

         ScrambleLetters(remainLetters, scramLetters);

         // Put letter back in remaining letters
         remainLetters.insert(i, tmpString);
         scramLetters.erase(scramLetters.size() - 1, 1);
      }
   }
}

int main() {
   string wordScramble; // User defined word to scramble

   // Prompt user for input
   cout << "Enter a word to be scrambled: ";
   cin >> wordScramble;

   // Call recursive function
   ScrambleLetters(wordScramble, "");

   return 0;
}
```

```
Enter a word to be
scrambled: cat
cat
cta
act
atc
tca
tac
```


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PARTICIPATION ACTIVITY 12.7.2: Letter scramble.

1) What is the output of ScrambleLetters("xy", "")? Determine

your answer by manually tracing the code, not by running the program.

- yx xy
- xx yy xy yx
- xy yx

Shopping spree

Recursion can find all possible subsets of a set of items. Consider a shopping spree in which a person can select any 3-item subset from a larger set of items. The following program prints all possible 3-item subsets of a given larger set. The program also prints the total price of each subset.

ShoppingBagCombinations() has a parameter for the current bag contents, and a parameter for the remaining items from which to choose. The base case is that the current bag already has 3 items, which prints the items. The recursive case moves one of the remaining items to the bag, recursively calling the function, then moving the item back from the bag to the remaining items.

Figure 12.7.2: Shopping spree in which a user can fit 3 items in a shopping bag.

```
#include <iostream>
#include <string>
#include <vector>
using namespace std;

class Item {
public:
   string itemName;  // Name of item
   int priceDollars; // Price of item
};

const unsigned int MAX_ITEMS_IN_SHOPPING_BAG = 3; // Max num items

/* Output every combination of items that fit
   in a shopping bag. Each recursive call moves
   one item into the shopping bag.
*/
void ShoppingBagCombinations(vector<Item> currBag,          // Bag
contents
                             vector<Item> remainingItems) { //
Available items
   int bagValue;        // Cost of items in shopping bag
   Item tmpGroceryItem; // Grocery item to add to bag
   unsigned int i;      // Loop index

   if( currBag.size() == MAX_ITEMS_IN_SHOPPING_BAG ) { // Base case:
Shopping bag full
      bagValue = 0;
      for(i = 0; i < currBag.size(); ++i) {
         bagValue += currBag.at(i).priceDollars;
         cout << currBag.at(i).itemName << "  ";
      }
      cout << "= $" << bagValue << endl;
   }
```

```
   else {                                              // Recursive
case: move one
      for(i = 0; i < remainingItems.size(); ++i) {     // item to
bag
         // Move item into bag
         tmpGroceryItem = remainingItems.at(i);
         remainingItems.erase(remainingItems.begin() + i);
         currBag.push_back(tmpGroceryItem);

         ShoppingBagCombinations(currBag, remainingItems);

         // Take item out of bag
         remainingItems.insert(remainingItems.begin() +
i,tmpGroceryItem);
         currBag.pop_back();
      }
   }
}

int main() {
   vector<Item> possibleItems(0); // Possible shopping items
   vector<Item> shoppingBag(0);   // Current shopping bag
   Item tmpGroceryItem;           // Temp item

   // Populate grocery with different items
   tmpGroceryItem.itemName = "Milk";
   tmpGroceryItem.priceDollars = 2;
   possibleItems.push_back(tmpGroceryItem);

   tmpGroceryItem.itemName = "Belt";
   tmpGroceryItem.priceDollars = 24;
   possibleItems.push_back(tmpGroceryItem);

   tmpGroceryItem.itemName = "Toys";
   tmpGroceryItem.priceDollars = 19;
   possibleItems.push_back(tmpGroceryItem);

   tmpGroceryItem.itemName = "Cups";
   tmpGroceryItem.priceDollars = 12;
   possibleItems.push_back(tmpGroceryItem);

   // Try different combinations of three items
   ShoppingBagCombinations(shoppingBag, possibleItems);

   return 0;
}
```

```
Milk  Belt  Toys
= $45
Milk  Belt  Cups
= $38
Milk  Toys  Belt
= $45
Milk  Toys  Cups
= $33
Milk  Cups  Belt
= $38
Milk  Cups  Toys
= $33
Belt  Milk  Toys
= $45
Belt  Milk  Cups
= $38
Belt  Toys  Milk
= $45
Belt  Toys  Cups
= $55
Belt  Cups  Milk
= $38
Belt  Cups  Toys
= $55
Toys  Milk  Belt
= $45
Toys  Milk  Cups
= $33
Toys  Belt  Milk
= $45
Toys  Belt  Cups
= $55
Toys  Cups  Milk
= $33
Toys  Cups  Belt
= $55
Cups  Milk  Belt
= $38
Cups  Milk  Toys
= $33
Cups  Belt  Milk
= $38
Cups  Belt  Toys
= $55
Cups  Toys  Milk
= $33
Cups  Toys  Belt
= $55
```


PARTICIPATION ACTIVITY 12.7.3: All letter combinations.

1) When main() calls ShoppingBagCombinations(), how many items are in the remainingItems list?

- None
- 3
- 4

2) When main() calls ShoppingBagCombinations(), how many items are in currBag list?

- None
- 1
- 4

3) After main() calls ShoppingBagCombinations(), what happens first?

- The base case prints Milk, Belt, Toys.
- The function bags one item, makes recursive call.
- The function bags 3 items, makes recursive call.

4) Just before ShoppingBagCombinations() returns back to main(), how many items are in the remainingItems list?

- None
- 4

5) How many recursive calls occur before the first combination is printed?

- None
- 1
- 3

6) What happens if main() only put 2, rather than 4, items in the possibleItems list?

- Base case never executes; nothing printed.
- Infinite recursion occurs.

Traveling salesman

Recursion is useful for finding all possible paths. Suppose a salesman must travel to 3 cities: Boston, Chicago, and Los Angeles. The salesman wants to know all possible paths among those three cities, starting from any city. A recursive exploration of all travel paths can be used. The base case is that the salesman has traveled to all cities. The recursive case is to travel to a new city, explore possibilities, then return to the previous city.

Figure 12.7.3: Find distance of traveling to 3 cities.

```
#include <iostream>
#include <iomanip>
#include <vector>
using namespace std;

const unsigned int NUM_CITIES = 3;          // Number of
cities
int cityDistances[NUM_CITIES][NUM_CITIES]; // Distance
between cities
string cityNames[NUM_CITIES];               // City names

/* Output every possible travel path.
   Each recursive call moves to a new city.
*/
void TravelPaths(vector<int> currPath, vector<int>
needToVisit) {
   int totalDist;     // Total distance given current
path
   int tmpCity;       // Next city distance
   unsigned int i;    // Loop index

   if( currPath.size() == NUM_CITIES ) { // Base case:
Visited all cities
      totalDist = 0;                     // return total
path distance
      for (i = 0; i < currPath.size(); ++i) {
         cout << cityNames[currPath.at(i)] << "   ";

         if (i > 0) {
            totalDist += cityDistances[currPath.at(i -
1)][currPath.at(i)];
         }
      }

      cout << "= " << totalDist << endl;
   }
   else {                                // Recursive
case: pick next city
      for(i = 0; i < needToVisit.size(); ++i) {
         // Add city to travel path
         tmpCity = needToVisit.at(i);
         needToVisit.erase(needToVisit.begin() + i);
         currPath.push_back(tmpCity);

         TravelPaths(currPath, needToVisit);

         // Remove city from travel path
         needToVisit.insert(needToVisit.begin() + i,
tmpCity);
         currPath.pop_back();
      }
   }
```

```
Boston   Chicago   Los
Angeles   = 2971
Boston   Los Angeles
Chicago   = 4971
Chicago   Boston   Los
Angeles   = 3920
Chicago   Los Angeles
Boston   = 4971
Los Angeles   Boston
Chicago   = 3920
Los Angeles   Chicago
Boston   = 2971
```

```
}

int main() {
   vector<int> needToVisit(0); // Cities left to visit
   vector<int> currPath(0);    // Current path traveled

   // Initialize distances array
   cityDistances[0][0] = 0;
   cityDistances[0][1] = 960;  // Boston-Chicago
   cityDistances[0][2] = 2960; // Boston-Los Angeles
   cityDistances[1][0] = 960;  // Chicago-Boston
   cityDistances[1][1] = 0;
   cityDistances[1][2] = 2011; // Chicago-Los Angeles
   cityDistances[2][0] = 2960; // Los Angeles-Boston
   cityDistances[2][1] = 2011; // Los Angeles-Chicago
   cityDistances[2][2] = 0;

   cityNames[0] = "Boston";
   cityNames[1] = "Chicago";
   cityNames[2] = "Los Angeles";

   needToVisit.push_back(0); // Boston
   needToVisit.push_back(1); // Chicago
   needToVisit.push_back(2); // Los Angeles

   // Explore different paths
   TravelPaths(currPath, needToVisit);

   return 0;
}
```


PARTICIPATION ACTIVITY 12.7.4: Recursive exploration.

1) You wish to generate all possible 3-letter subsets from the letters in an N-letter word (N>3). Which of the above recursive functions is the closest?

   - ShoppingBagCombinations
   - ScrambleLetters
   - main()

CHALLENGE ACTIVITY 12.7.1: Enter the output of recursive exploration.

Exploring further:

- Recursive Algorithms from khanacademy.org

12.8 Stack overflow

Recursion enables an elegant solution to some problems. But, for large problems, deep recursion can cause memory problems. Part of a program's memory is reserved to support function calls. Each function call places a new ***stack frame*** on the stack, for local parameters, local variables, and more function items. Upon return, the frame is deleted.

Deep recursion could fill the stack region and cause a ***stack overflow***, meaning a stack frame extends beyond the memory region allocated for stack, Stack overflow usually causes the program to crash and report an error like: segmentation fault, access violation, or bad access.

PARTICIPATION ACTIVITY 12.8.1: Recursion causing stack overflow.

Animation captions:

1. Deep recursion may cause stack overflow, causing a program to crash.

The animation showed a tiny stack region for easy illustration of stack overflow.

The size of parameters and local variables results in a larger stack frame. Large vectors, arrays, or strings declared as local variables, or passed by copy, can lead to faster stack overflow.

A programmer can estimate recursion depth and stack size to determine whether stack overflow might occur. Sometimes a non-recursive algorithm must be developed to avoid stack overflow.

PARTICIPATION ACTIVITY 12.8.2: Stack overflow.

1) A memory's stack region can store at most one stack frame.
   - ○ True
   - ○ False

2) The size of the stack is unlimited.
   - ○ True
   - ○ False

3) A stack overflow occurs when the stack frame for a function call extends past the end of the stack's memory.
   - ○

True

False

4) The following recursive function will result in a stack overflow.

```
int RecAdder(int inValue) {
   return RecAdder(inValue + 1);
}
```

True

False

12.9 Quicksort

Quicksort is a sorting algorithm that repeatedly partitions the input into low and high parts (each part unsorted), and then recursively sorts each of those parts. To partition the input, quicksort chooses a pivot to divide the data into low and high parts. The ***pivot*** can be any value within the array being sorted, commonly the value of the middle array element. Ex: For the list {4 34 10 25 1}, the middle element is located at index 2 (the middle of indices 0..4) and has a value of 10.

To partition the input, the quicksort algorithm divides the array into two parts, referred to as the low partition and the high partition. All values in the low partition are less than or equal to the pivot value. All values in the high partition are greater than or equal to the pivot value. The values in each partition are not necessarily sorted. Ex: Partitioning {4 34 10 25 1} with a pivot value of 10 results in a low partition of {4 10 1} and a high partition of {34 25}. Values equal to the pivot may appear in either or both of the partitions.

PARTICIPATION ACTIVITY 12.9.1: Quicksort partitions data into a low part with data less than/equal to a pivot value and a high part with data greater than/equal to a pivot value.

Animation captions:

1. The pivot value is the value of the middle element.
2. Index l begins at element i and is incremented until a value greater than the pivot is found.
3. Index h begins at element k, and is decremented until a value less than the pivot is found.
4. Elements at indices l and h are swapped, moving those elements to the correct partitions.
5. The partition process repeats until indices l and h reach or pass each other (l >= h), indicating all elements have been partitioned.
6. Once partitioned, the algorithm returns h indicating the highest index of the low partition. The partitions are not yet sorted.

The partitioning algorithm uses two index variables l and h (low and high), initialized to the left and right sides of the current elements being sorted. As long as the value at index l is less than the pivot value, the algorithm increments l, because the element should remain in the low partition. Likewise, as long as the value at index h is greater than the pivot value, the algorithm decrements h, because the element should remain in the high partition. Then, if l >= h, all elements have been partitioned, and the partitioning algorithm returns h, which is the index of the last element in the low partition. Otherwise, the elements at indices l and h are swapped to move those elements to the correct partitions. The algorithm then increments l, decrements h, and repeats.

PARTICIPATION ACTIVITY 12.9.2: Quicksort pivot location and value.

Determine the midpoint and pivot values.

1) numbers = {1 2 3 4 5}, i = 0, k = 4

`midpoint =` [ ]

Check Show answer

2) numbers = {1 2 3 4 5}, i = 0, k = 4

`pivot =` [ ]

Check Show answer

3) numbers = {200 11 38 9}, i = 0, k = 3

`midpoint =` [ ]

Check Show answer

4) numbers = {200 11 38 9}, i = 0, k = 3

`pivot =` [ ]

Check Show answer

5) numbers = {55 7 81 26 0 34 68 125}, i = 3, k = 7

`midpoint =` [ ]

Check Show answer

6) numbers = {55 7 81 26 0 34 68 125}, i = 3, k = 7

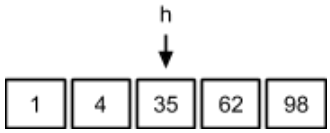
pivot =

Check Show answer

PARTICIPATION ACTIVITY 12.9.3: Low and high partitions.

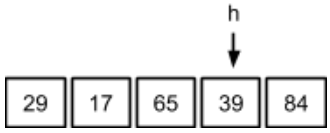
Determine if the low and high partitions are correct given h and pivot.

1) pivot = 35



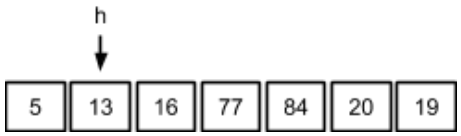
- True
- False

2) pivot = 65



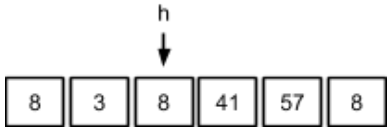
- True
- False

3) pivot = 5



- True
- False

4) pivot = 8



- True
- False

Once partitioned, each partition needs to be sorted. Quicksort is typically implemented as a recursive algorithm using calls to quicksort to sort the low and high partitions. This recursive sorting process continues until a partition has one or zero elements, and thus already sorted.

PARTICIPATION

ACTIVITY 12.9.4: Quicksort.

Animation captions:

1. List contains more than one element. Partition the list.
2. Recursively call quicksort on the low and high partitions.
3. Low partition contains more than one element. Partition the low partition and recursively call quicksort.
4. Low partition contains one element, so partition is already sorted. High partition contains one element, so partition is already sorted.
5. High partition contains more than one element. Partition the high partition and recursively call quicksort.
6. Low partition contains more than one element. Partition the low partition and recursively call quicksort.
7. Low partition contains one element, so partition is already sorted. High partition contains one element, so partition is already sorted.
8. High partition contains one element, so partition is already sorted.
9. All elements are sorted.

Below is the recursive quicksort algorithm, including its key component the partitioning function.

Figure 12.9.1: Quicksort algorithm.

```
#include <iostream>
using namespace std;


int Partition(int numbers[], int i, int k) {
   int l;
   int h;
   int midpoint;
   int pivot;
   int temp;
   bool done;

   /* Pick middle element as pivot */
   midpoint = i + (k - i) / 2;
   pivot = numbers[midpoint];

   done = false;
   l = i;
   h = k;

   while (!done) {

      /* Increment l while numbers[l] < pivot */
      while (numbers[l] < pivot) {
         ++l;
      }

      /* Decrement h while pivot < numbers[h] */
      while (pivot < numbers[h]) {
         --h;
```

```
      }

      /* If there are zero or one elements remaining,
       all numbers are partitioned. Return h */
      if (l >= h) {
         done = true;
      }
      else {
         /* Swap numbers[l] and numbers[h],
          update l and h */
         temp = numbers[l];
         numbers[l] = numbers[h];
         numbers[h] = temp;

         ++l;
         --h;
      }
   }

   return h;
}

void Quicksort(int numbers[], int i, int k) {
   int j;

   /* Base case: If there are 1 or zero elements to sort,
    partition is already sorted */
   if (i >= k) {
      return;
   }

   /* Partition the data within the array. Value j returned
    from partitioning is location of last element in low partition. */
   j = Partition(numbers, i, k);

   /* Recursively sort low partition (i to j) and
    high partition (j + 1 to k) */
   Quicksort(numbers, i, j);
   Quicksort(numbers, j + 1, k);
}

int main() {
   int numbers[] = { 10, 2, 78, 4, 45, 32, 7, 11 };
   const int NUMBERS_SIZE = 8;
   int i;

   cout << "UNSORTED: ";
   for(i = 0; i < NUMBERS_SIZE; ++i) {
      cout << numbers[i] << " ";
   }
   cout << endl;

   /* Initial call to quicksort */
   Quicksort(numbers, 0, NUMBERS_SIZE - 1);

   cout << "SORTED: ";
   for(i = 0; i < NUMBERS_SIZE; ++i) {
      cout << numbers[i] << " ";
   }
   cout << endl;

   return 0;
}
```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```
UNSORTED: 10 2 78 4 45 32 7 11
SORTED: 2 4 7 10 11 32 45 78
```

The following activity helps build intuition as to how partitioning a list into two unsorted parts, one part <= a pivot value and the other part >= a pivot value, and then recursively sorting each part, ultimately leads to a sorted list.

PARTICIPATION ACTIVITY 12.9.5: Quicksort tool.

Select all values in the current window that are less than the pivot for the left part, then press "Partition".

The quicksort algorithm's runtime is typically O(N log N). Quicksort has several partitioning levels , the first level dividing the input into 2 parts, the second into 4 parts, the third into 8 parts, etc. At each level, the algorithm does at most N comparisons moving the l and h indices. If the pivot yields two equal-sized parts, then there will be log N levels, requiring the N * log N comparisons.

PARTICIPATION ACTIVITY 12.9.6: Quicksort runtime.

Assume quicksort always chooses a pivot that divides the elements into two equal parts.

1) How many partitioning levels are required for a list of 8 elements?

Check Show answer

2) How many partitioning "levels" are required for a list of 1024 elements?

Check Show answer

3) How many total comparisons are required to sort a list of 1024 elements?

Check Show answer

For typical unsorted data, such equal partitioning occurs. However, partitioning may yield unequal sized part in some cases. If the pivot selected for partitioning is the smallest or largest element, one partition will have just 1 element, and the other partition will have all other elements. If this unequal partitioning happens at every level, there will be N - 1 levels, yielding $(N - 1) \cdot N)$, which is $O(N^2)$. So the worst case runtime for the quicksort algorithm is $O(N^2)$. Fortunately, this worst case runtime rarely occurs.

PARTICIPATION ACTIVITY 12.9.7: Worst case quicksort runtime.

Assume quicksort always chooses the smallest element as the pivot.

1) Given numbers = {7 4 2 25 19}, i = 0, and k = 4, what is contents of the low partition? Use curly braces in your answer.

Check Show answer

2) How many partitioning "levels" of are required for a list of 5 elements?

Check Show answer

3) How many partitioning "levels" are required for a list of 1024 elements?

Check Show answer

4) How many total comparisons are required to sort a list of 1024 elements?

Check Show answer

12.10 Merge sort

Merge sort is a sorting algorithm that divides a list into two halves, recursively sorts each half, and then merges the sorted halves to produce a sorted list. The recursive partitioning continues until a list of 1 element is reached, as list of 1 element is already sorted.

PARTICIPATION ACTIVITY | 12.10.1: Merge sort recursively divides the input into two halves, sorts each half, and merges the lists together.

Animation captions:

1. Merge sort recursively divides the list into two halves.
2. The list is divided until a list of 1 element is found.
3. A list of 1 element is already sorted.
4. At each level, the sorted lists are merged together while maintaining the sorted order.

The merge sort algorithm uses three index variables to keep track of the elements to sort for each recursive function call. The index variable i is the index of first element in the list, and the index variable k is the index of the last element. The index variable j is used to divide the list into two halves. Elements from i to j are in the left half, and elements from j + 1 to k are in the right half.

PARTICIPATION ACTIVITY | 12.10.2: Merge sort partitioning.

Determine the index j and the left and right partitions.

1) numbers = {1 2 3 4 5}, i = 0, k = 4

```
j =
```

Check Show answer

2) numbers = {1 2 3 4 5}, i = 0, k = 4

```
Left partition  = {  }
```

Check Show answer

3) numbers = {1 2 3 4 5}, i = 0, k = 4

```
Right partition  = {  }
```

Check Show answer

4) numbers = {34 78 14 23 8 35}, i = 3, k = 5

```
j =
```

Check Show answer

5) numbers = {34 78 14 23 8 35}, i = 3, k = 5


```
Left partition  = {
```

Check Show answer

6) numbers = {34 78 14 23 8 35}, i = 3, k = 5

```
Right partition  = {
```

Check Show answer

Merge sort merges the two sorted partitions into a single list by repeatedly selecting the smallest element from either the left or right partition and adding that element to a temporary merged list. Once fully merged, the elements in the temporary merged list are copied back to the original list.

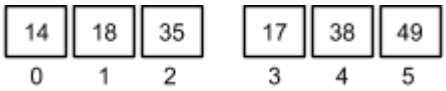
PARTICIPATION ACTIVITY 12.10.3: Merging partitions: Smallest element from left or right partition is added one at a time to a temporary merged list. Once merged, temporary list is copied back to the original list.

Animation captions:

1. Create a temporary list for merged numbers. Initialize mergePos, leftPos, and rightPos to the first element of each of the corresponding list.
2. Compare the element in the left and right partitions. Add the smallest value to the temporary list and update the relevant indices.
3. Continue to compare the elements in the left and right partitions until one of the partitions is empty.
4. If a partition is not empty, copy the remaining elements to the temporary list. The elements are already in sorted order.
5. Lastly, the elements in the temporary list are copied back to the original list.

PARTICIPATION ACTIVITY 12.10.4: Tracing merge operation.

Trace the merge operation by determining the next value added to mergedNumbers.



1) leftPos = 0, rightPos = 3

Check Show answer

2) leftPos = 1, rightPos = 3

Check Show answer

3) leftPos = 1, rightPos = 4

Check Show answer

4) leftPos = 2, rightPos = 4

Check Show answer

5) leftPos = 3, rightPos = 4

Check Show answer

6) leftPos = 3, rightPos = 5

Check Show answer

Figure 12.10.1: Merge sort algorithm.

```
#include <iostream>
using namespace std;

void Merge(int numbers[], int i, int j, int k) {
   int mergedSize;                              // Size of merged partition
```

```
   int mergePos;                           // Position to insert merged number
   int leftPos;                            // Position of elements in left partition
   int rightPos;                           // Position of elements in right partition
   int* mergedNumbers = nullptr;

   mergePos = 0;
   mergedSize = k - i + 1;
   leftPos = i;                            // Initialize left partition position
   rightPos = j + 1;                       // Initialize right partition position
   mergedNumbers = new int[mergedSize];    // Dynamically allocates temporary array
                                           // for merged numbers

   // Add smallest element from left or right partition to merged numbers
   while (leftPos <= j && rightPos <= k) {
      if (numbers[leftPos] < numbers[rightPos]) {
         mergedNumbers[mergePos] = numbers[leftPos];
         ++leftPos;
      }
      else {
         mergedNumbers[mergePos] = numbers[rightPos];
         ++rightPos;

      }
      ++mergePos;
   }

   // If left partition is not empty, add remaining elements to merged numbers
   while (leftPos <= j) {
      mergedNumbers[mergePos] = numbers[leftPos];
      ++leftPos;
      ++mergePos;
   }

   // If right partition is not empty, add remaining elements to merged numbers
   while (rightPos <= k) {
      mergedNumbers[mergePos] = numbers[rightPos];
      ++rightPos;
      ++mergePos;
   }

   // Copy merge number back to numbers
   for (mergePos = 0; mergePos < mergedSize; ++mergePos) {
      numbers[i + mergePos] = mergedNumbers[mergePos];
   }
   delete[] mergedNumbers;
}

void MergeSort(int numbers[], int i, int k) {
   int j;

   if (i < k) {
      j = (i + k) / 2;  // Find the midpoint in the partition

      // Recursively sort left and right partitions
      MergeSort(numbers, i, j);
      MergeSort(numbers, j + 1, k);

      // Merge left and right partition in sorted order
      Merge(numbers, i, j, k);
   }
}

int main() {
   int numbers[] = { 10, 2, 78, 4, 45, 32, 7, 11 };
   const int NUMBERS_SIZE = 8;
   int i;

   cout << "UNSORTED: ";
   for(i = 0; i < NUMBERS_SIZE; ++i) {
      cout << numbers[i] << " ";
```

```
   }
   cout << endl;

   MergeSort(numbers, 0, NUMBERS_SIZE - 1);

   cout << "SORTED: ";
   for(i = 0; i < NUMBERS_SIZE; ++i) {
      cout << numbers[i] << " ";
   }
   cout << endl;

   return 0;
}
```



```
UNSORTED: 10 2 78 4 45 32 7 11
SORTED: 2 4 7 10 11 32 45 78
```

The merge sort algorithm's runtime is O(N log N). Merge sort divides the input in half until a list of 1 element is reached, which requires log N partitioning levels. At each level, the algorithm does about N comparisons selecting and copying elements from the left and right partitions, yielding N * log N comparisons.

Merge sort requires O(N) additional memory elements for the temporary array of merged elements. For the final merge operation, the temporary list has the same number of elements as the input. Some sorting algorithms sort the list elements in place and require no additional memory, but are more complex to write and understand.

To allocate the temporary array, the `Merge()` function dynamically allocates the array. mergedNumbers is a pointer variable that points to the dynamically allocated array, and `new int[mergedSize]` allocates the array with mergedSize elements. Alternatively, instead of allocating the array within the `Merge()` function, a temporary array with the same size as the array being sorted can be passed as an argument.

PARTICIPATION ACTIVITY 12.10.5: Merge sort runtime and memory complexity.

1) How many recursive partitioning levels are required for a list of 8 elements?

Check Show answer

2) How many recursive partitioning levels are required for a list of 2048 elements?

Check Show answer

3) How many elements will the temporary merge list have for merging two partitions with 250 elements each?

Check Show answer

12.11 C++ example: Recursively output permutations

zyDE 12.11.1: Recursively output permutations.

The below program prints all permutations of an input string of letters, one permutation line. Ex: The six permutations of "cab" are:

```
cab
cba
acb
abc
bca
bac
```

Below, the PermuteString function works recursively by starting with the first character a permuting the remainder of the string. The function then moves to the second characte permutes the string consisting of the first character and the third through the end of the and so on.

1. Run the program and input the string "cab" (without quotes) to see that the above is produced.
2. Modify the program to print the permutations in the opposite order, and also to ou permutation count on each line.
3. Run the program again and input the string cab. Check that the output is reversed
4. Run the program again with an input string of abcdef. Why did the program take lo to produce the results?

Load default templ

```
1 #include <iostream>
```

```
#include <string>
using namespace std;

// FIXME: Use a static variable to count permutations. Why should the variable

void PermuteString(string head, string tail) {
   char current;
   string newPermute;
   int len;
   int i;

   current = '?';

   len = tail.size();
   if (len <= 1) {
      // FIXME: Output the permutation count on each line too
```


cab

Run

zyDE 12.11.2: Recursively output permutations (solution).

Below is the solution to the above problem.

Load default templ

```
#include <iostream>
#include <string>
using namespace std;

static int permutationCount = 0;

void PermuteString(string head, string tail) {
   char current;
   string newPermute;
   int len;
   int i;

   current = '?';

   len = tail.size();
   if (len <= 1) {
      ++permutationCount;
```

```
cab
abcdef
```

Run

12.12 LAB: Descending selection sort with output during execution

Write a void function SelectionSortDescendTrace() that takes an integer array, and sorts the array into descending order. The function should use nested loops and output the array after each iteration of the outer loop, thus outputting the array N-1 times (where N is the size). Complete main() to read in a list of up to 10 positive integers (ending in -1) and then call the SelectionSortDescendTrace() function.

If the input is:

```
20 10 30 40 -1
```

then the output is:

```
40 10 30 20
40 30 10 20
40 30 20 10
```

LAB ACTIVITY | 12.12.1: LAB: Descending selection sort with output during execution | 0 / 10

main.cpp — Load default template...


```
1 Loading latest submission...
```

Develop mode | Submit mode

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

If your code requires input values, provide them here.

Run program

Input (from above) → **main.cpp** (Your program) → Outp

Program output displayed here

Signature of your work What is this?

```
History of your effort will appear here once you begin working
on this zyLab.
```

12.13 LAB: Sorting user IDs

Given a main() that reads user IDs (until -1), complete the Quicksort() and Partition() functions to sort the IDs in ascending order using the Quicksort algorithm, and output the sorted IDs one per line.

Ex. If the input is:


```
kaylasimms
julia
myron1994
kaylajones
-1
```

the output is:

```
julia
kaylajones
kaylasimms
myron1994
```

LAB ACTIVITY | 12.13.1: LAB: Sorting user IDs

0 / 10

main.cpp Load default template...

```
1 Loading latest submission...
```

Develop mode | **Submit mode**

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

If your code requires input values, provide them here.

Run program

Input (from above) → **main.cpp** (Your program) → Outp

Program output displayed here

Signature of your work What is this?

```
History of your effort will appear here once you begin working
on this zyLab.
```


12.14 LAB: All permutations of names

Write a program that lists all ways people can line up for a photo (all permutations of a list of strings). The program will read a list of one word names (until -1), and use a recursive method to create and output all possible orderings of those names, one ordering per line.

When the input is:

```
Julia Lucas Mia -1
```

then the output is (must match the below ordering):

```
Julia Lucas Mia
Julia Mia Lucas
Lucas Julia Mia
Lucas Mia Julia
Mia Julia Lucas
Mia Lucas Julia
```

LAB ACTIVITY 12.14.1: LAB: All permutations of names 0 / 10

main.cpp Load default template...

```
1 Loading latest submission...
```

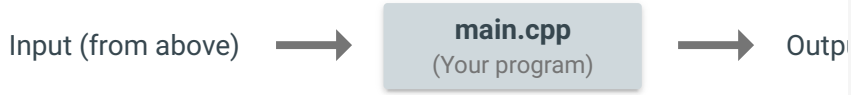
Develop mode | Submit mode

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

If your code requires input values, provide them here.

Run program



Program output displayed here

Signature of your work What is this?

```
History of your effort will appear here once you begin working
on this zyLab.
```

12.15 LAB: Number pattern

Write a recursive function called PrintNumPattern() to output the following number pattern.

Given a positive integer as input (Ex: 12), subtract another positive integer (Ex: 3) continually until 0 or a negative value is reached, and then continually add the second integer until the first integer is again reached.

Ex. If the input is:

```
12
3
```


the output is:

```
12 9 6 3 0 3 6 9 12
```

LAB 12.15.1: LAB: Number pattern

12.15.1: LAB: Number pattern

ACTIVITY 0 / 10

main.cpp Load default template...

```
1 Loading latest submission...
```


Develop mode | Submit mode

Run your program as often as you'd like, before submitting for grading. Below, type any needed input values in the first box, then click **Run program** and observe the program's output in the second box.

Enter program input (optional)

If your code requires input values, provide them here.

Run program

Input (from above) → main.cpp (Your program) → Outp

Program output displayed here

Signature of your work What is this?

```
History of your effort will appear here once you begin working
on this zyLab.
```