# ECE59500 Project 1 Report

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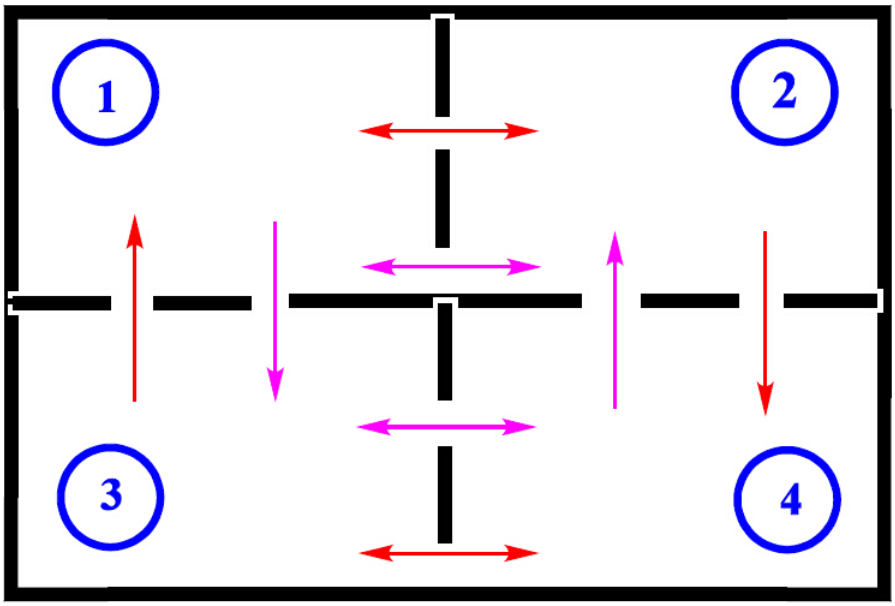


Figure 1. Layout of a house (Red for cat, Purple for mouse)

## 1 Build a Petri net model for movement of the cat

The Petri net model for movement of the cat was designed as following.

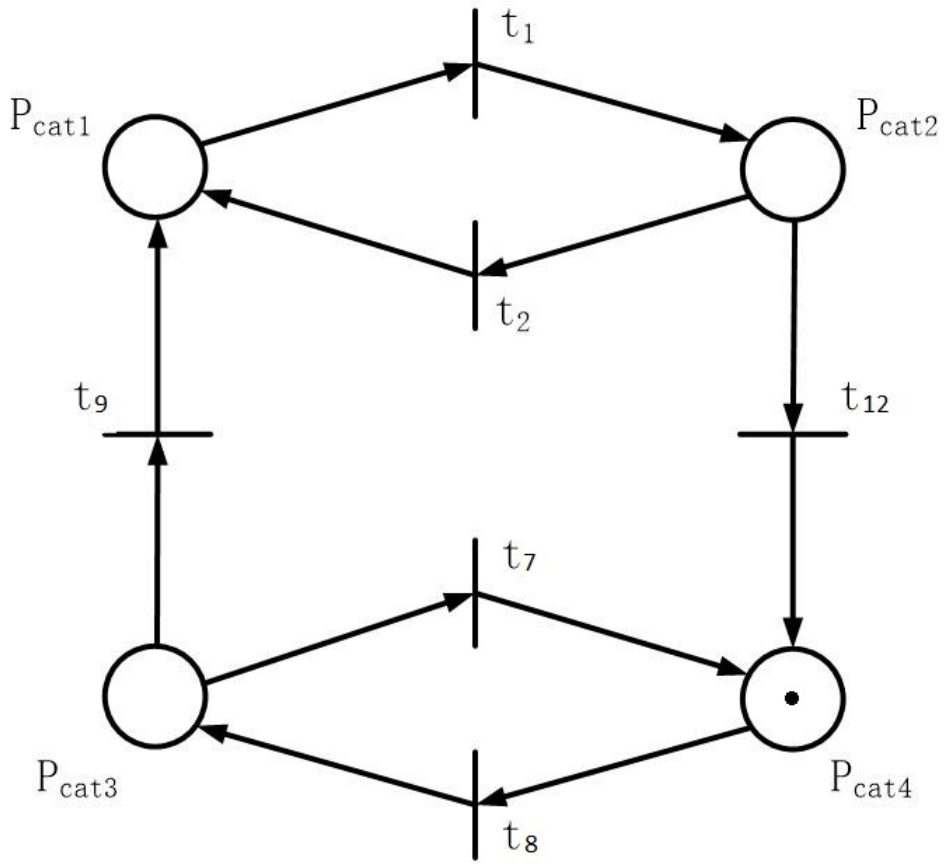


Figure 2. Petri Net for movement of the cat

As Figure 2 showing, each of the four places represent whether the cat is in that rooms or not. And each transtion represent one possible movement of cat. For example, assume there is a token in a place $P_{cat2}$, it means the cat is in room 2, $t_2$ and $t_{12}$ it enabled, it means from room 2 the cat can go to room 1 or room 4.
Figure 2 showes the initial situation, the cat is in room 4.

# 2 Build a Petri net model for the movement of the mouse

The Petri net model for movement of the mouse is designed as following. It was designed under the same concept of Petri Net of cat's movement.

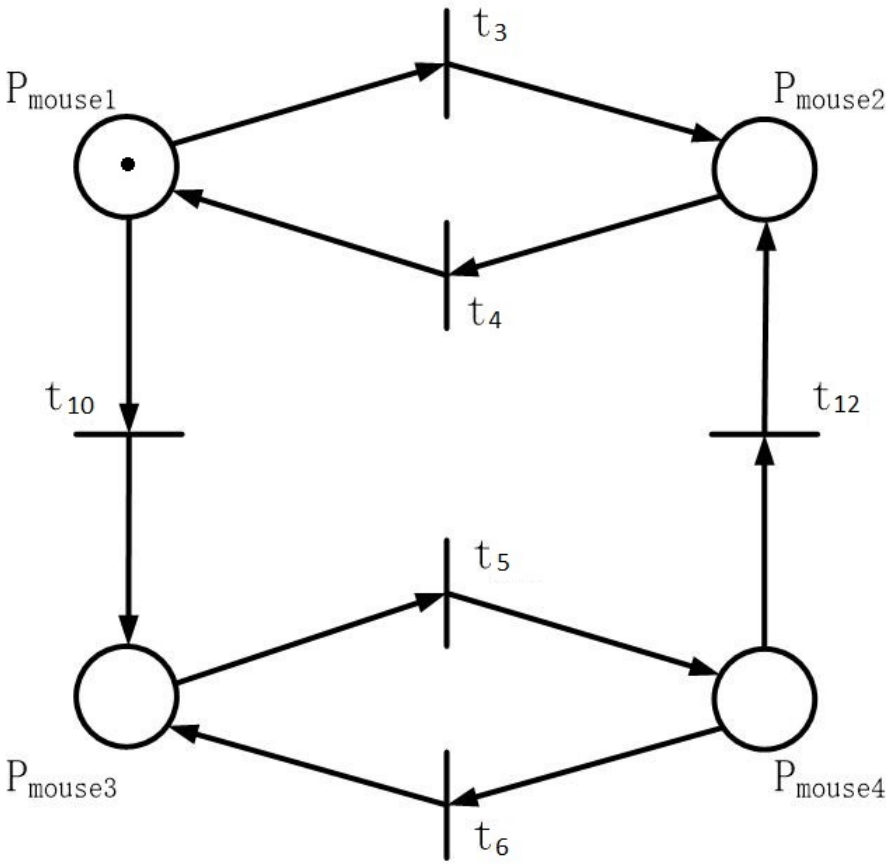


Figure 2. Petri Net for movement of the mouse

# 3 Design a Petri net controller to guarantee that the cat and mouse can never be in the same room

To design a controller, incident matrix $B$ and initial state $M0$ is needed. The incident matrix B is showen as following.

$$B = \begin{array}{c} \\ P_{cat1} \\ P_{cat2} \\ P_{cat3} \\ P_{cat4} \\ P_{mouse1} \\ P_{mouse2} \\ P_{mouse3} \\ P_{mouse4} \end{array} \begin{array}{c} \begin{array}{cccccccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & t_{10} & t_{11} & t_{12} \end{array} \\ \left[\begin{array}{cccccccccccc} -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \end{array}\right] \end{array}$$

Initially the cat is in Room 4 and the mouse is in Room 1. So for state $M_0$ there is a token in $P_{cat4}$, which is the 4th row. And there is a token in $P_{mouse1}$, which is the 5th row. $M_0$ is showen as following.

$$M_0 = \begin{array}{c} P_{cat1} \\ P_{cat2} \\ P_{cat3} \\ P_{cat4} \\ P_{mouse1} \\ P_{mouse2} \\ P_{mouse3} \\ P_{mouse4} \end{array} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

To guarantee that the cat and mouse can never be in the same room, there are four constraints.

- Cat and mouse cannot both in room 1.
  $M(P_{cat1}) + M(P_{mouse1}) \leq 1$
- Cat and mouse cannot both in room 2.
  $M(Pcat2) + M(Pmouse2) \leq 1$
- Cat and mouse cannot both in room 3.
  $M(Pcat3) + M(Pmouse3) \leq 1$
- Cat and mouse cannot both in room 4.
  $M(Pcat4) + M(Pmouse4) \leq 1$

$$\Rightarrow L = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}, b = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

$$B_c = -L * B$$

$$\Rightarrow B_c = \begin{array}{c} \\ P_{C1} \\ P_{C2} \\ P_{C3} \\ P_{C4} \end{array} \begin{array}{c} \begin{array}{cccccccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 & t_9 & t_{10} & t_{11} & t_{12} \end{array} \\ \begin{bmatrix} 1 & -1 & -1 & 1 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1 & -1 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & -1 & 1 & 0 & 0 & 1 & -1 \end{bmatrix} \end{array}$$

$$M_c = b - L * M$$

$$\Rightarrow M_c = \begin{array}{c} \\ P_{C1} \\ P_{C2} \\ P_{C3} \\ P_{C4} \end{array} \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}$$

The Peri net guarantees that the cat and mouse can never be in the same room based on this controller design is showen as following.

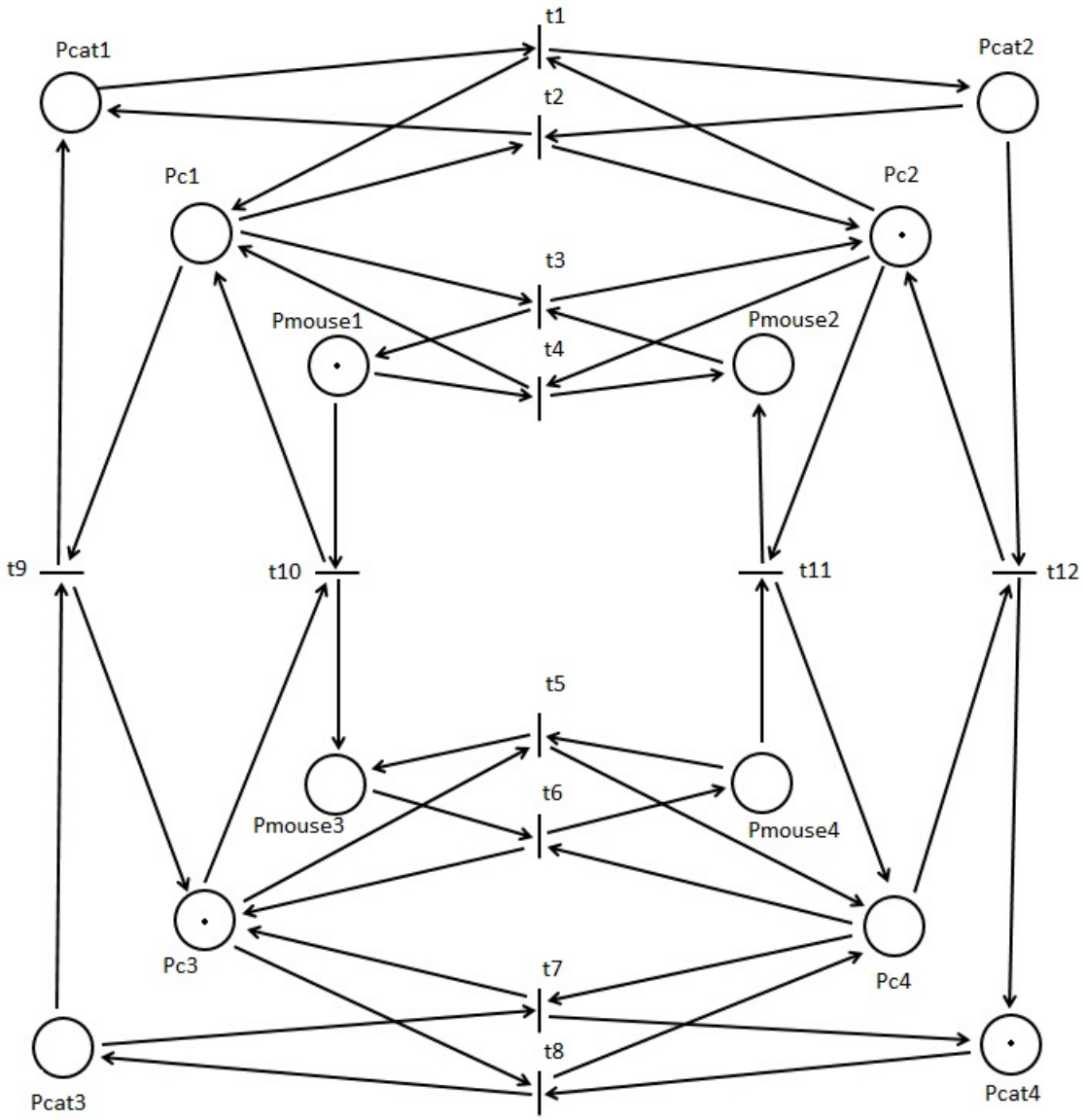


Figure 2. Petri Net for movement of the mouse and cat with a controller

# 4 Computer program for the Controlled Petri net to calculate all possible reachable states

The result from the program is showing as following.

$$states = \begin{array}{c} \\ P_{cat1} \\ P_{cat2} \\ P_{cat3} \\ P_{cat4} \\ P_{mouse1} \\ P_{mouse2} \\ P_{mouse3} \\ P_{mouse4} \\ P_{C1} \\ P_{C2} \\ P_{C3} \\ P_{C4} \end{array} \begin{array}{c} \begin{array}{cccccc} M_0 & M_1 & M_2 & M_3 & M_4 & M_5 \end{array} \\ \left[\begin{array}{cccccc} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \end{array}$$

The states connections are showing as following.

- $M_0 \xrightarrow{t_4} M_1$
- $M_0 \xrightarrow{t_8} M_2$
- $M_0 \xrightarrow{t_{10}} M_3$
- $M_1 \xrightarrow{t_3} M_0$
- $M_1 \xrightarrow{t_8} M_4$
- $M_2 \xrightarrow{t_4} M_4$
- $M_2 \xrightarrow{t_7} M_0$
- $M_3 \xrightarrow{0} 0$
- $M_4 \xrightarrow{t_3} M_2$

- $M_4 \xrightarrow{t_7} M_1$
- $M_4 \xrightarrow{t_9} M_5$
- $M_5 \xrightarrow{0} 0$

Computer program was writen with MATLAB. The files are compressed in a zip file and attached with the submission. To run the program, open the script *Project1.m* and run.