IEEE INFOCOM

Looking for the Maximum Independent Set: A New Perspective on the Stable Path Problem

Yichao Cheng[1], Ning Luo[1], Jingxuan Zhang[1, 2],
Timos Antonopoulos[1], Ruzica Piskac[1], Qiao Xiang[3, 1],

[1]Yale University, [2]Tongji University, [3]Xiamen University

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Yale University

同濟大學 TONGJI UNIVERSITY

厦門大學 XIAMEN UNIVERSITY

What is the Stable Path Problem?

Powerful tool for BGP convergence analysis [1]

[1] Griffin, Timothy, et al. "The Stable Paths Problem and Interdomain Routing." *IEEE/ACM Transactions On Networking* 10, no. 2 (2002): 232-243.

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- An undirected graph G=(V, E)
  - n+1 nodes, node 0 represents the destination; a link represents neighborship
- Each node i
  - runs SPVP, an abstract version of BGP
  - has a set of permitted paths
  - has a ranking function to rank permitted paths

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  - belongs to its permitted paths, and
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- **Stable path problem (SPP)**: does a stable path assignment exists?

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Example: Good Gadget

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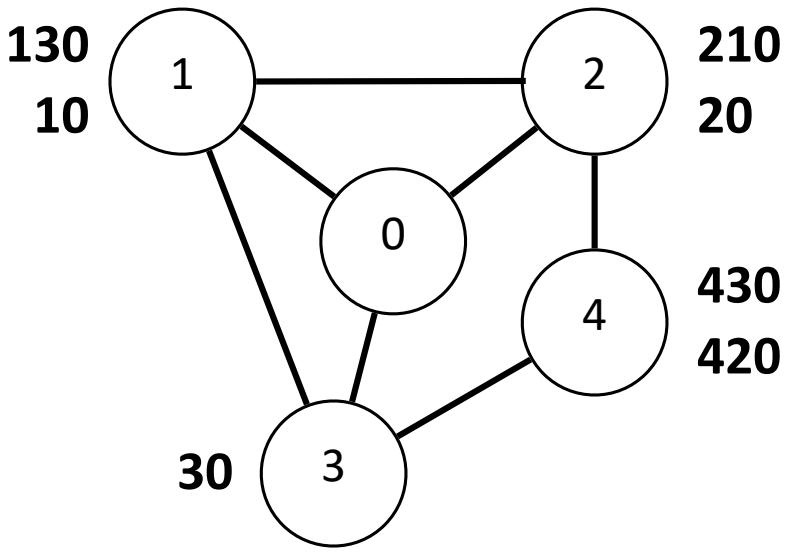

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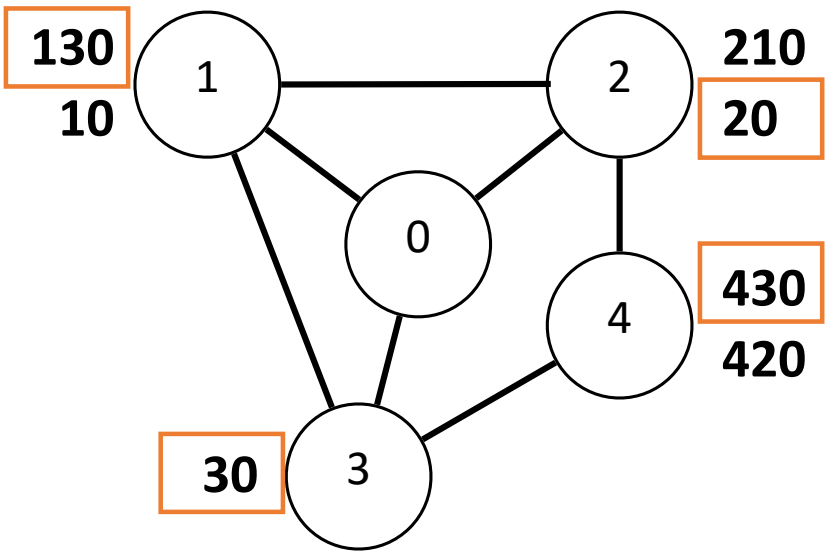

Good gadget

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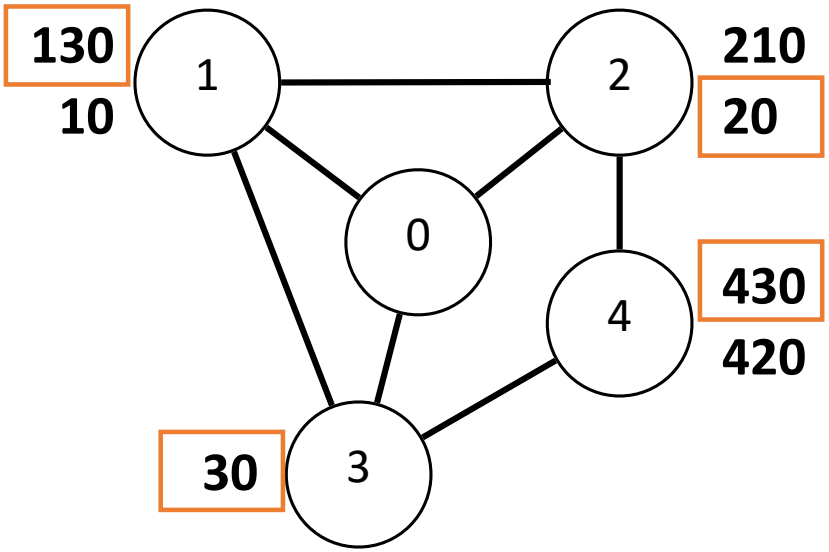
- A stable path assignment defines a tree rooted at node 0



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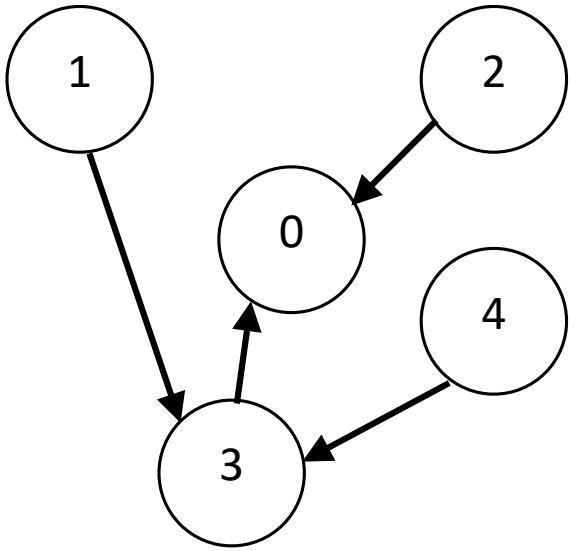

Stable path assignment of good gadget

Categories on SPP Instances

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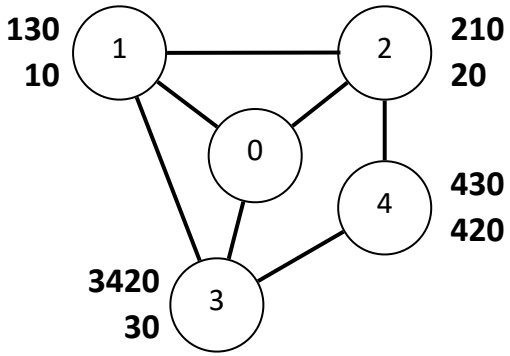

naughty gadget

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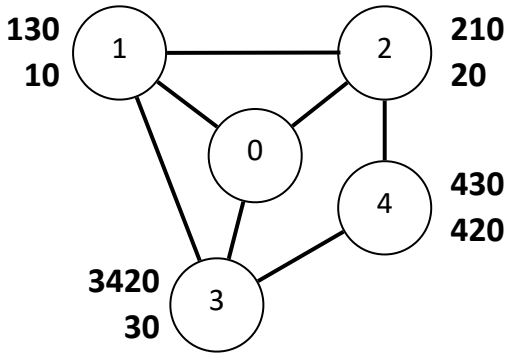
- **Solvable**: has at least one stable path assignment, e.g., disagree



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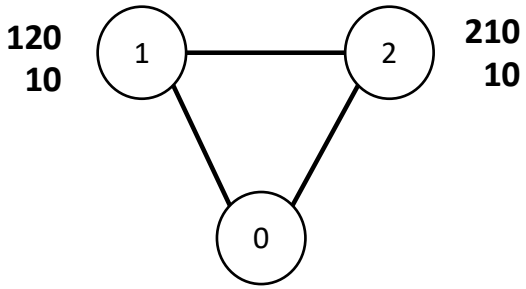

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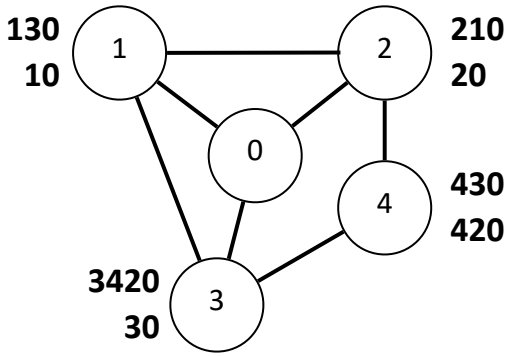
- **Unique**: has one and only one stable path assignment, e.g., good gadget and naughty gadget
- **Solvable**: has at least one stable path assignment, e.g., disagree
- **Unsolvable**: has no stable path assignment, e.g., bad gadget



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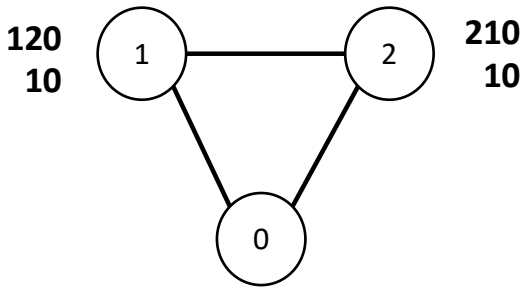

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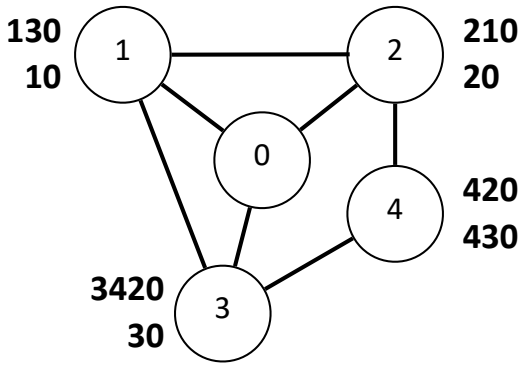

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Related Work on SPP

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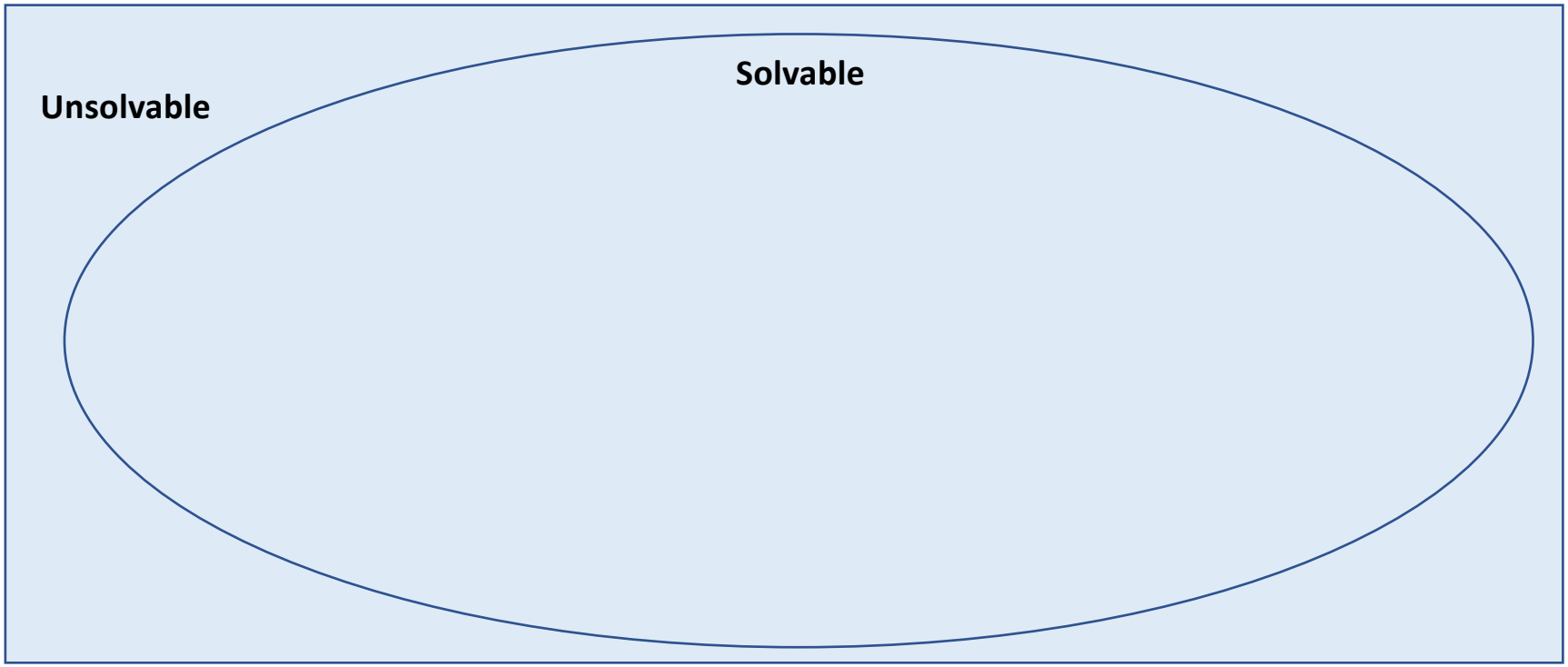

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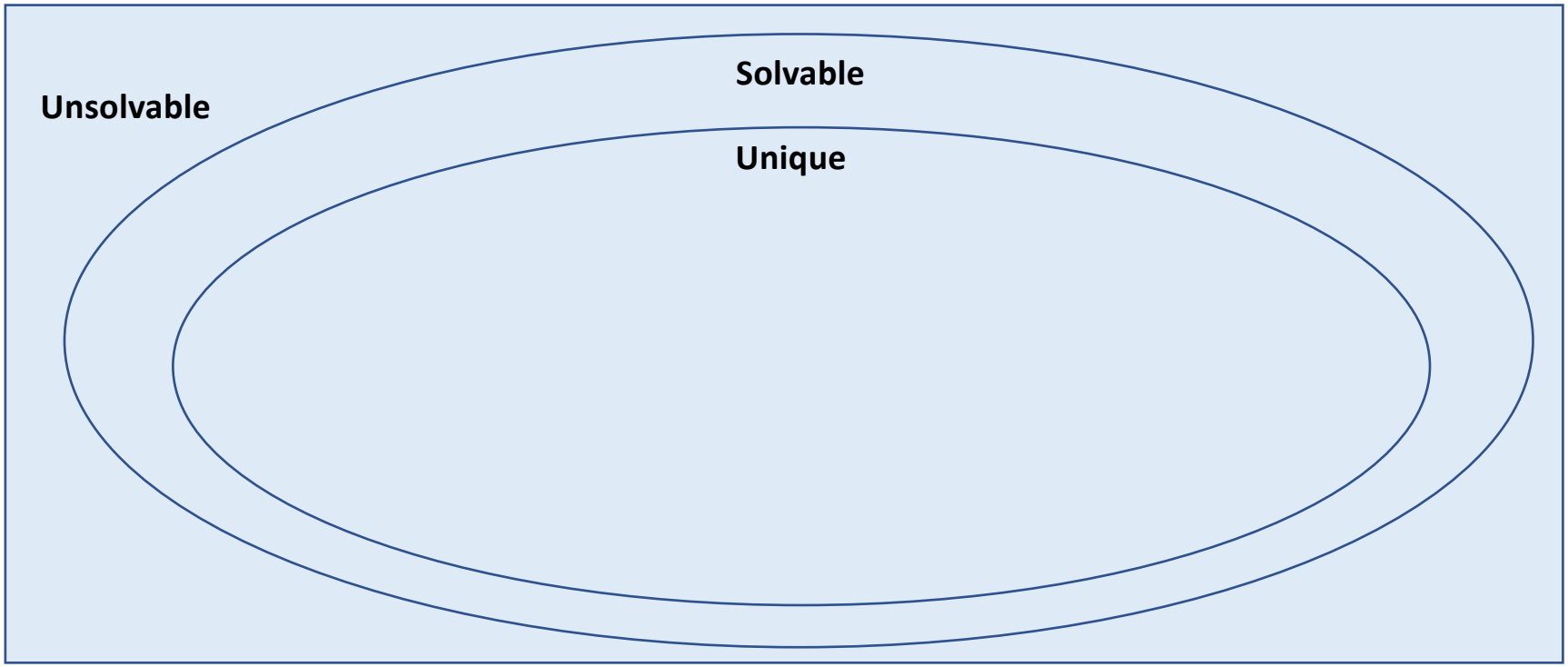

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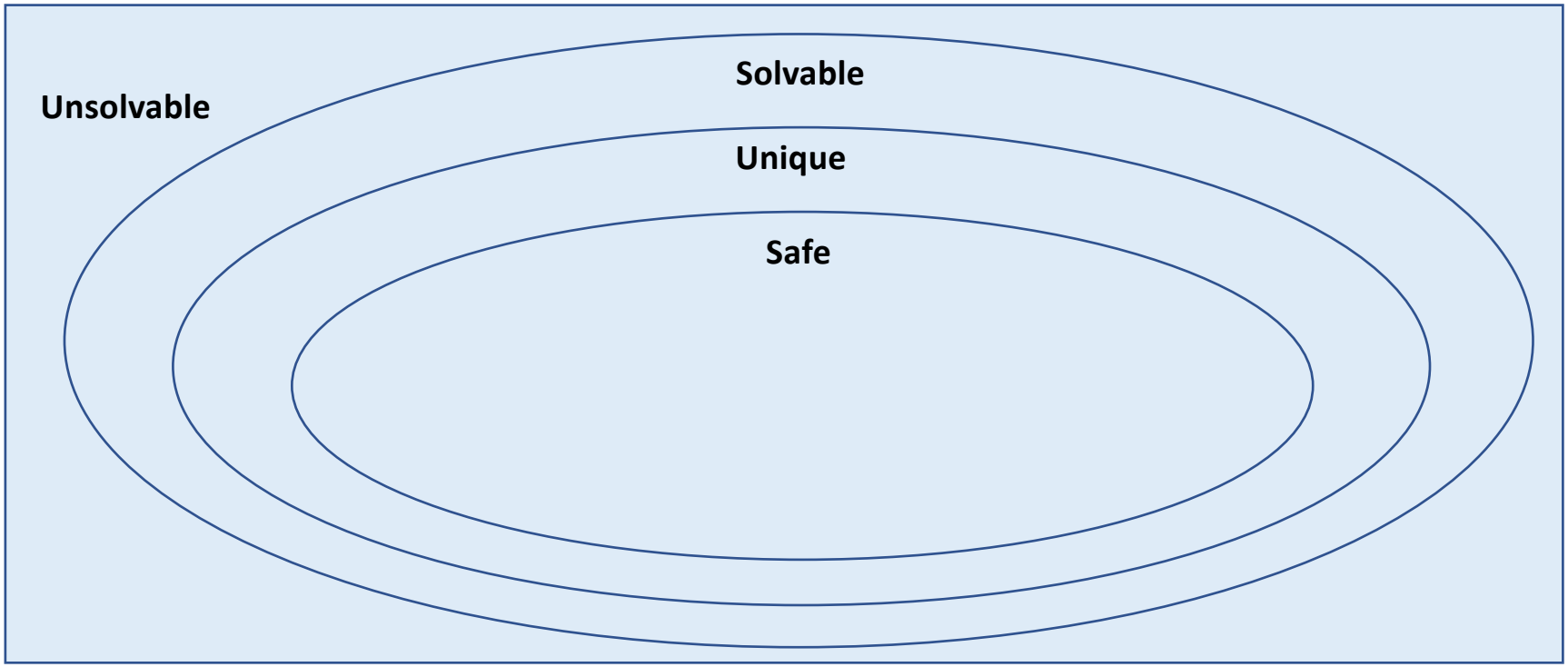

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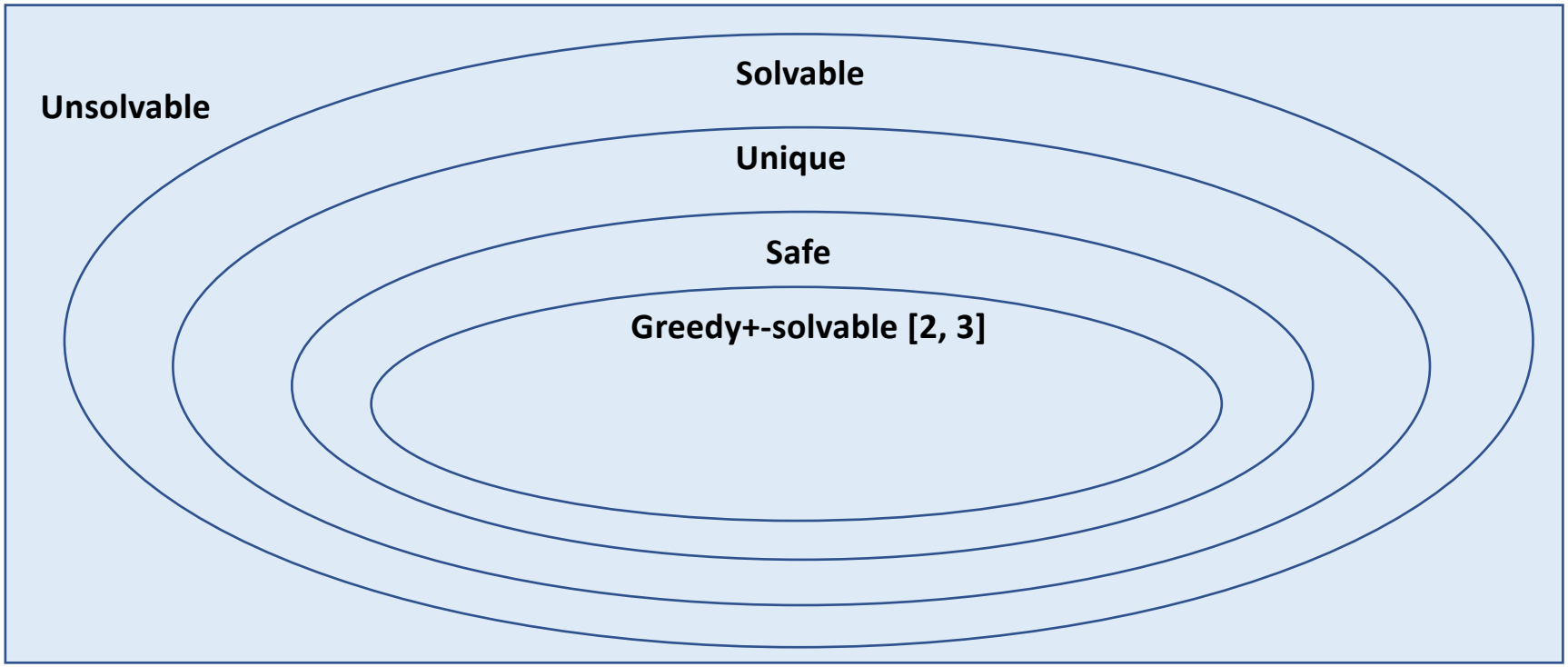

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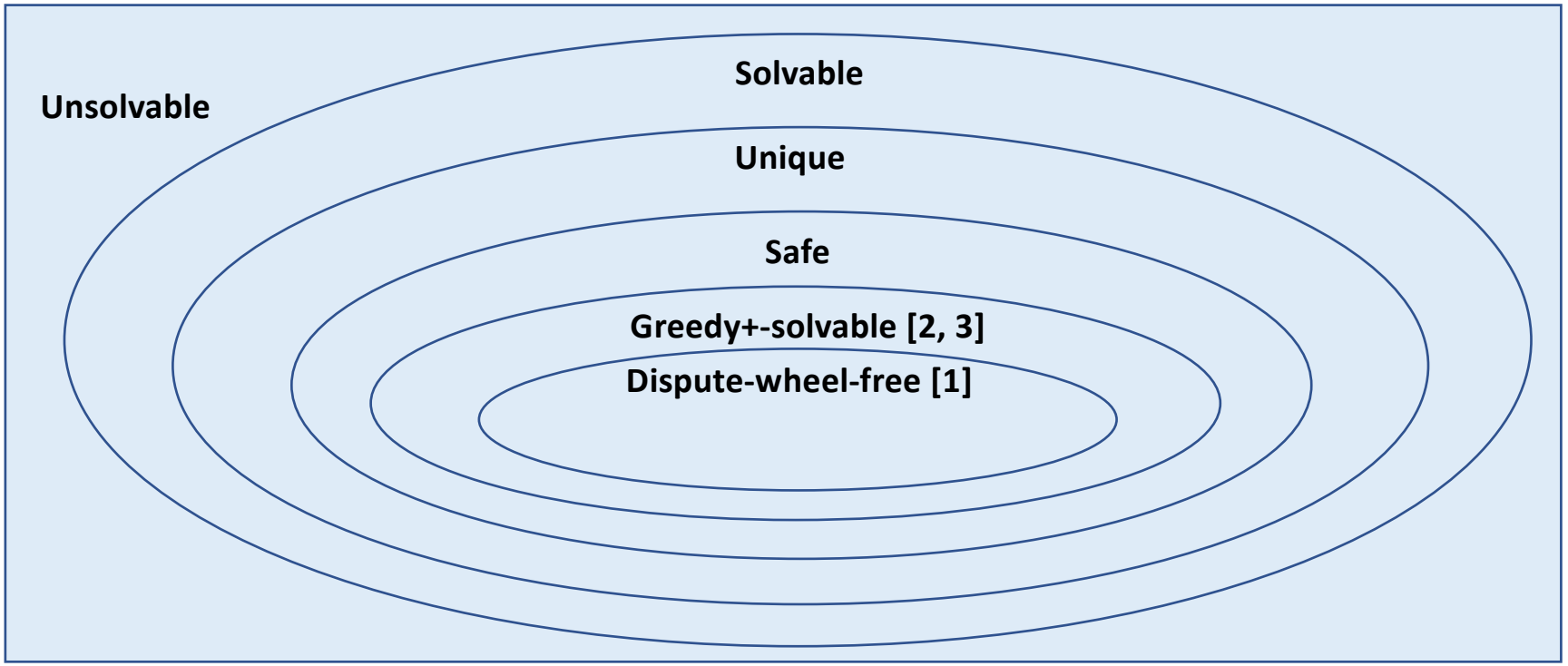

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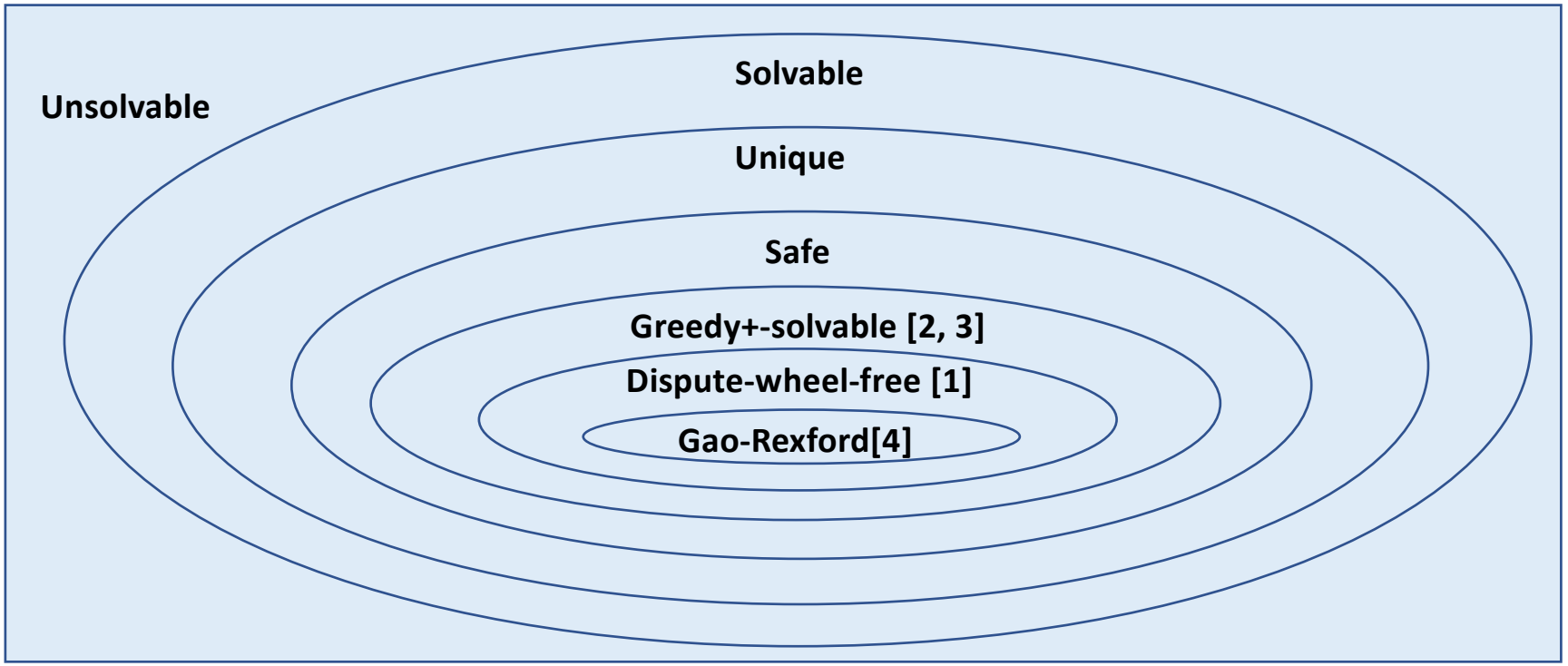

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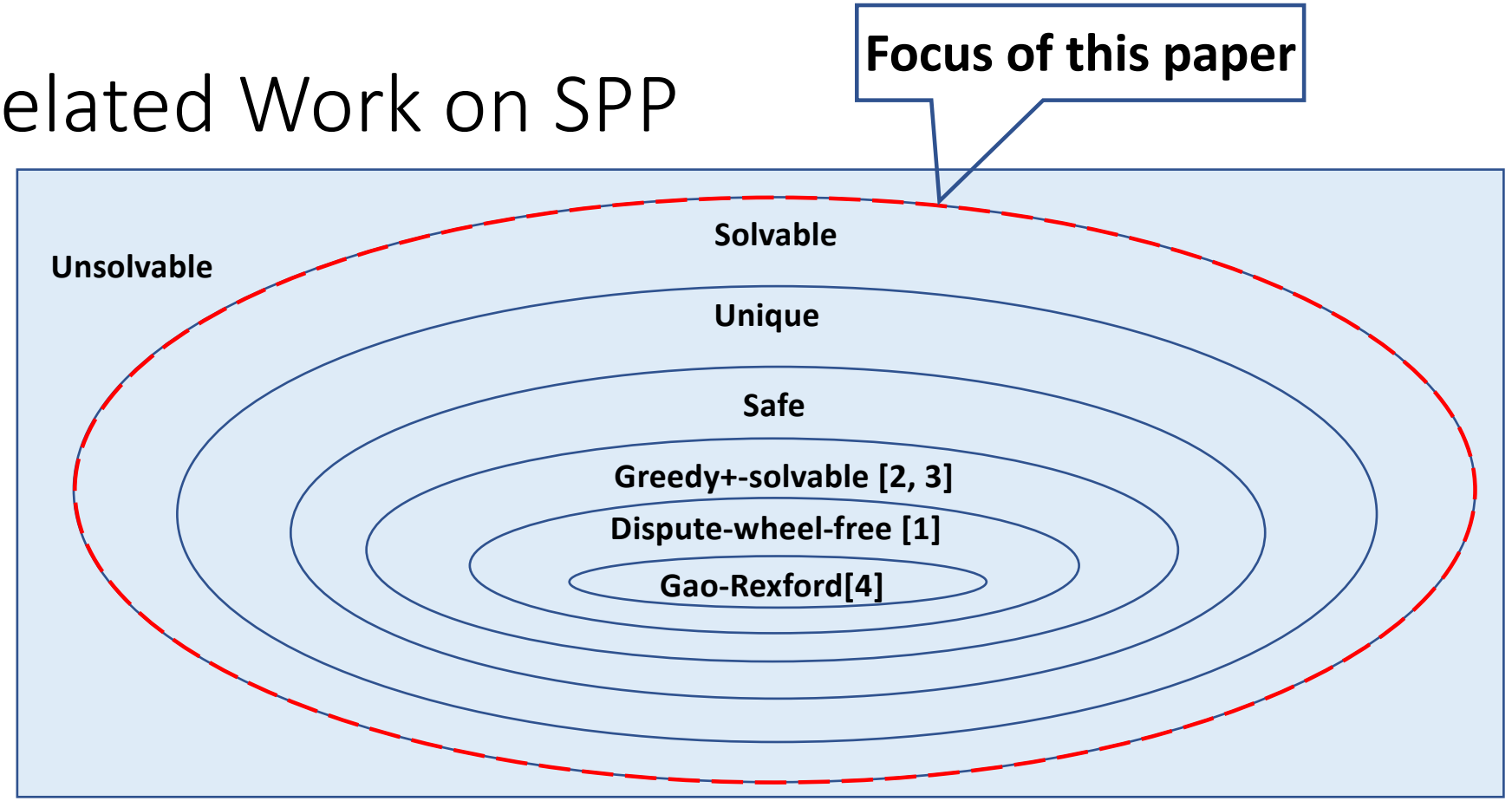

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Why Focus on SPP Solvability?

- Enable new interdomain routing use cases with more flexible policies
  - E.g., collaborative interdomain routing [4], software-defined-interdomain routing [5]
- Better understand the gap between solvable SPP and safe SPP
- Help improve the efficiency of network configuration verification

[4] Pouryousef, Shahrooz, et al. "Towards Logically Centralized Interdomain Routing." In *17th USENIX Symposium on Networked Systems Design and Implementation (NSDI'20)*, pp. 739-757. 2020.
[5] Xiang, Qiao, et al. "Toward Optimal Software-Defined Interdomain Routing." In *IEEE INFOCOM 2020-IEEE Conference on Computer Communications*, pp. 1529-1538. IEEE, 2020.

Outline

- Background on SPP
- Our Contribution

Contribution: New SPP Solvability Results and Interdomain Use Cases

1. **Theorem**: A novel solvability digraph for reasoning SPP solvability
2. **Algorithm**: A polynomial-time heuristic solves more SPP than state-of-the-art
3. **Use case**: collaborative interdomain routing
4. **Use case**: privacy-preserving collaborative interdomain routing

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- Solvability Digraph

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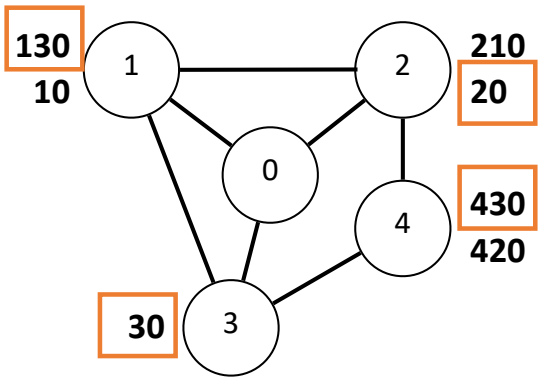

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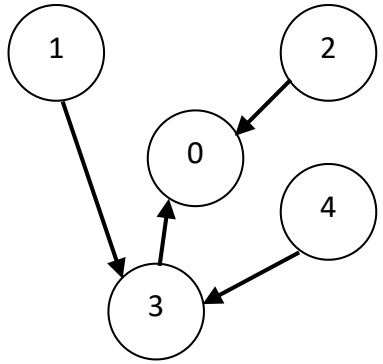

Stable path assignment for good gadget

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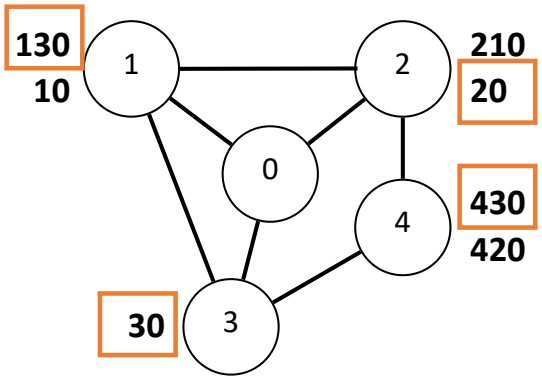

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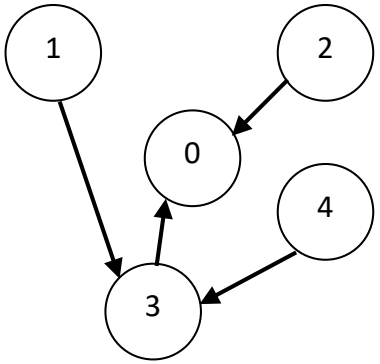

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In a stable path assignment:

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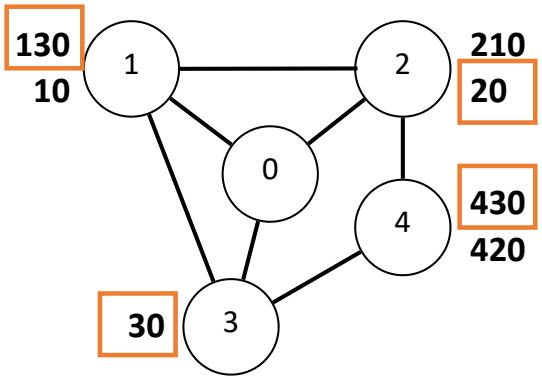

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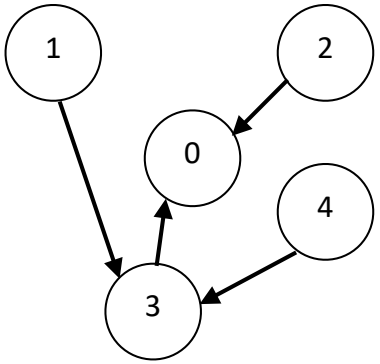

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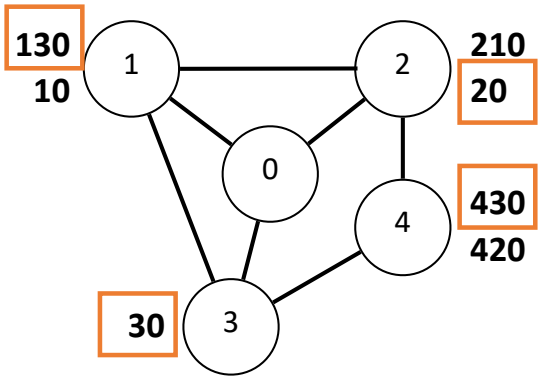

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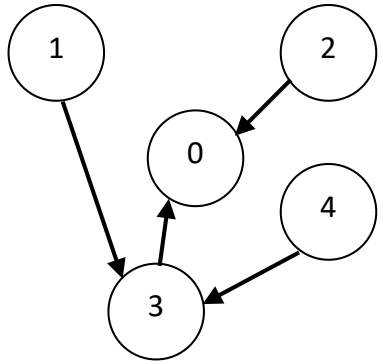

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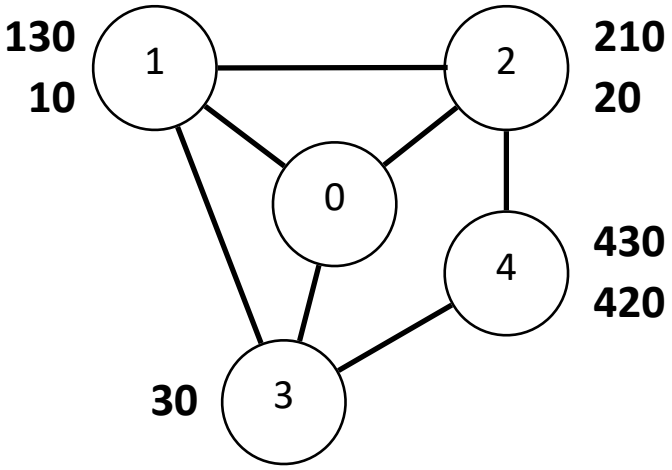
Solvability Digraph

Given an SPP instance, the corresponding solvability digraph is defined as:



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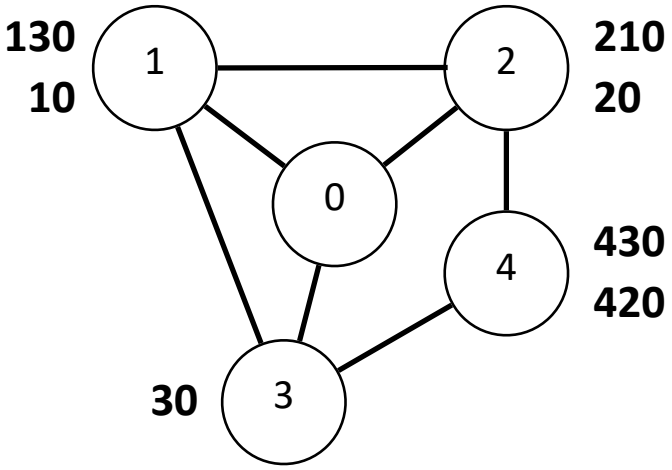

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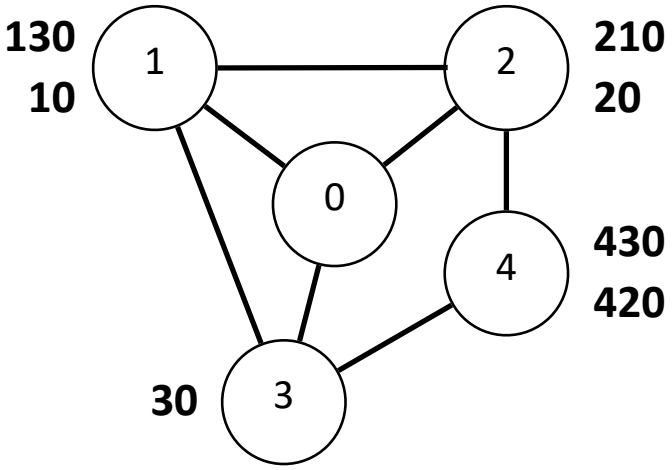

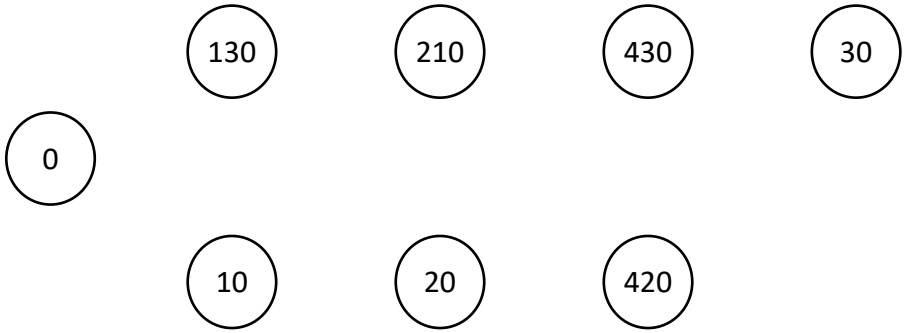

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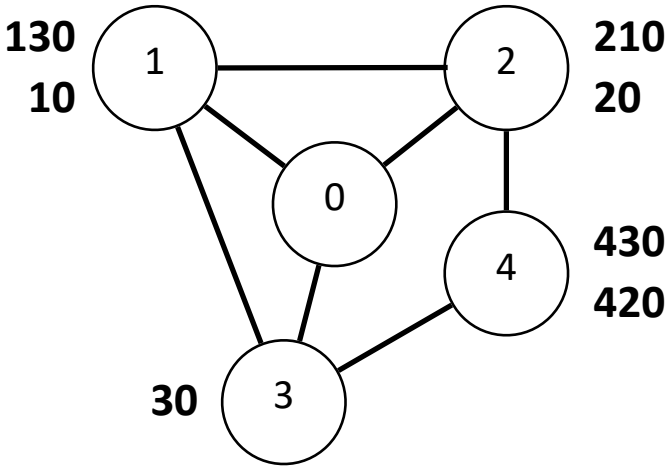

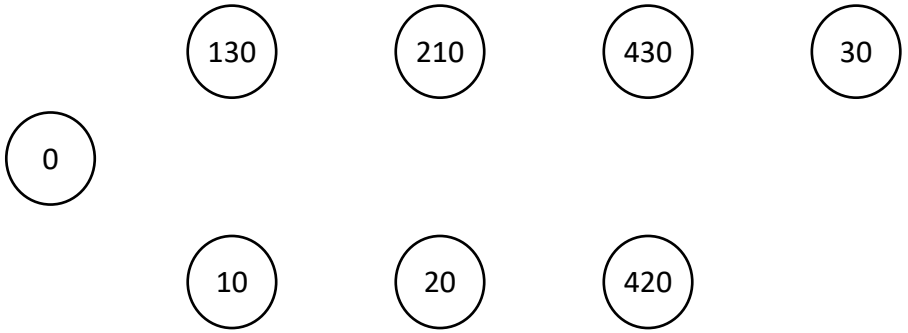

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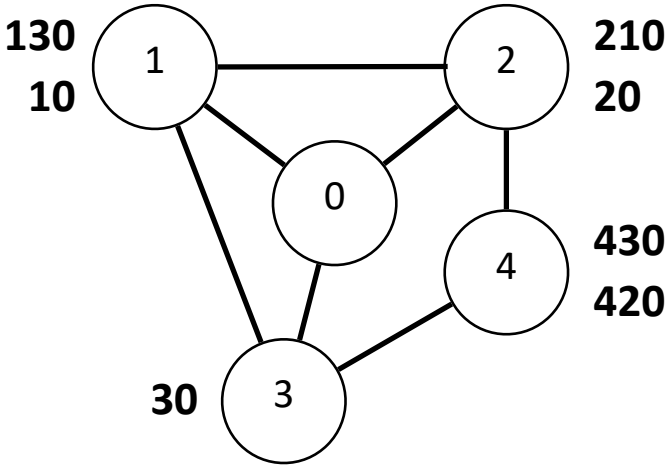
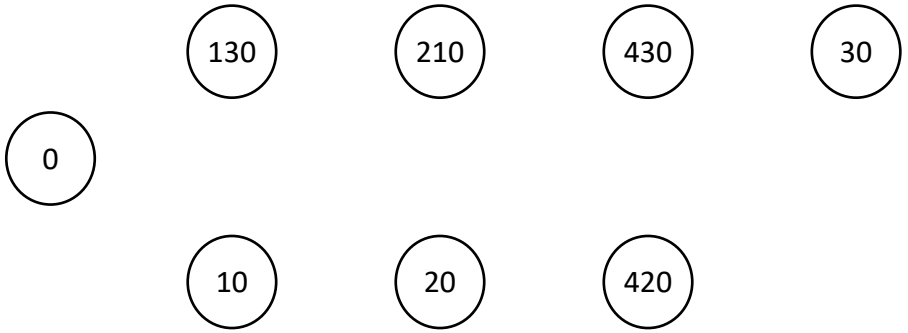
Type-1 edge (p1, p2): p1 and p2 from the same node's permitted paths, and p2 is more preferred





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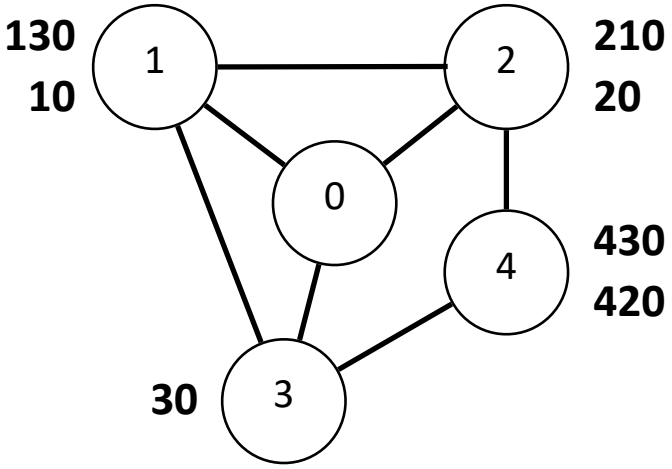

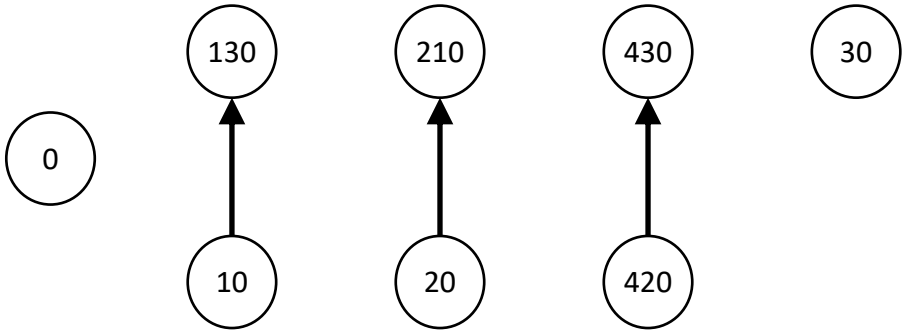

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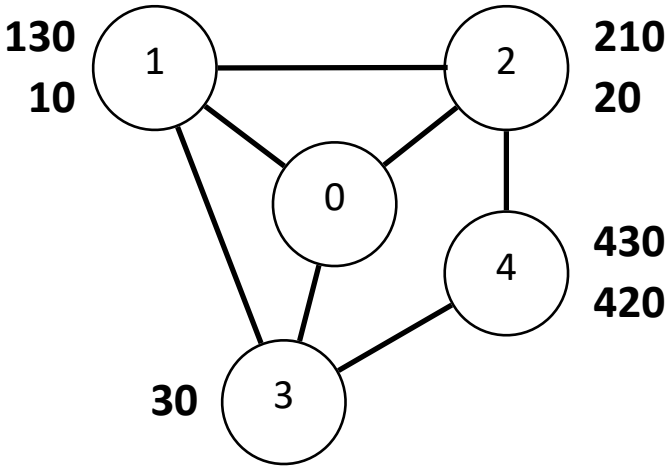

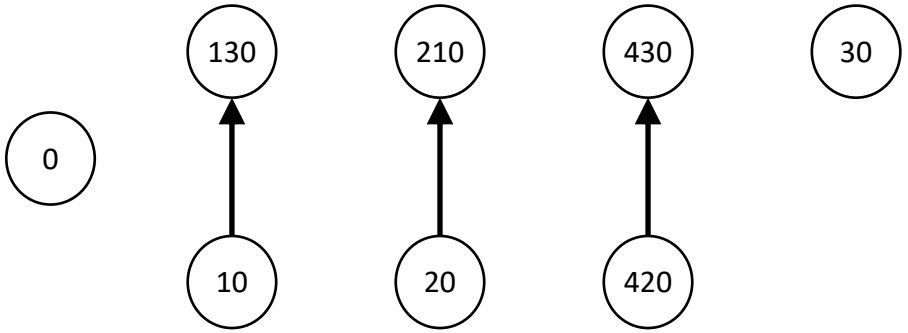

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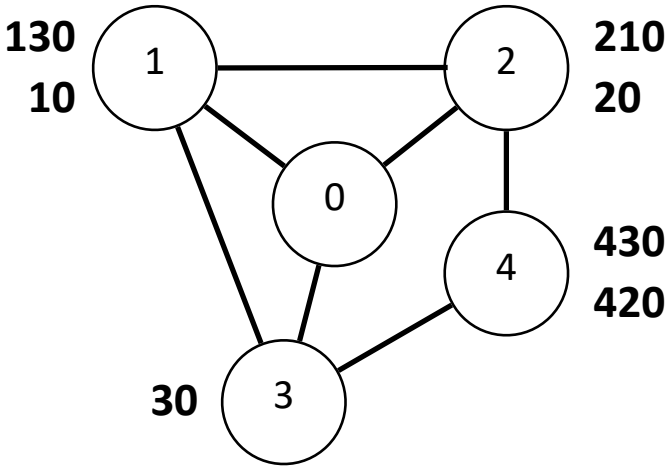

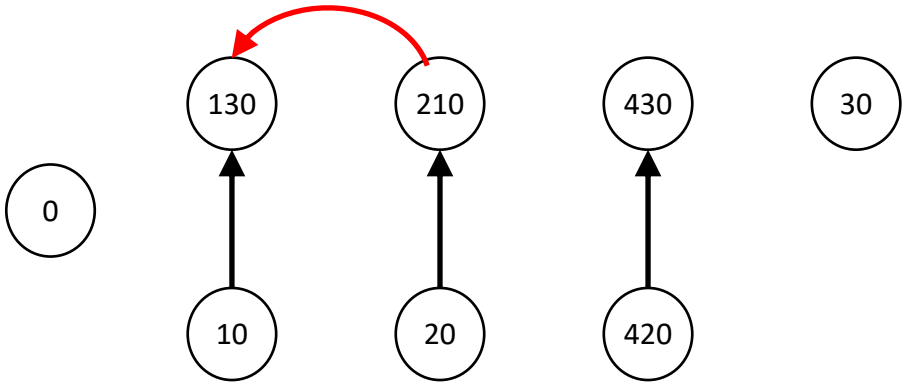

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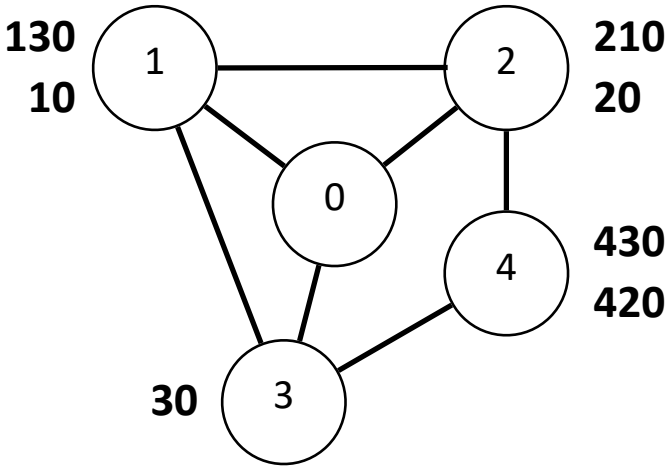

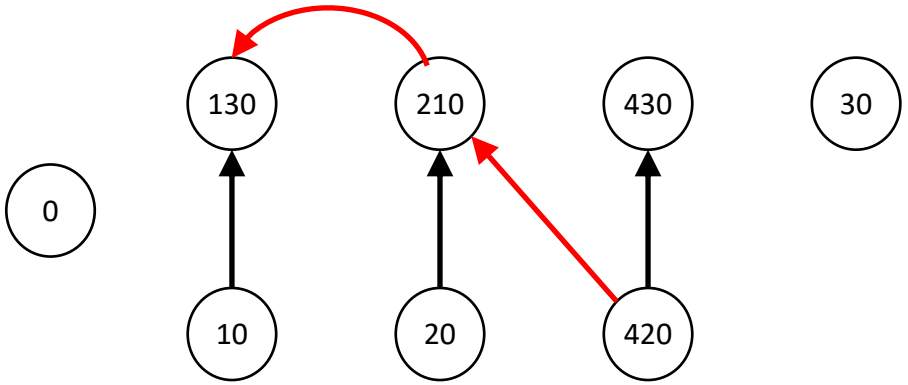

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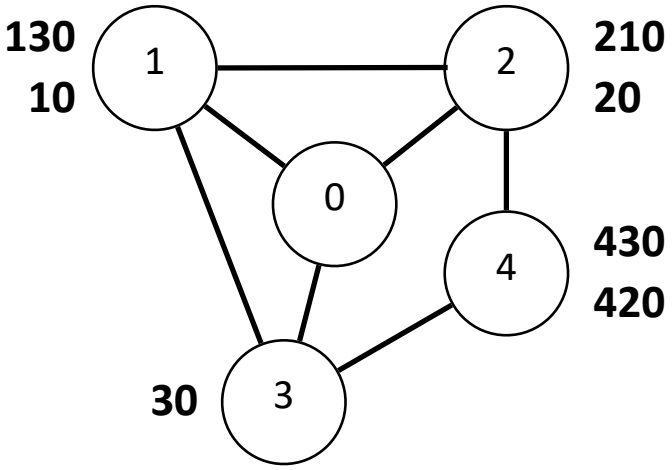

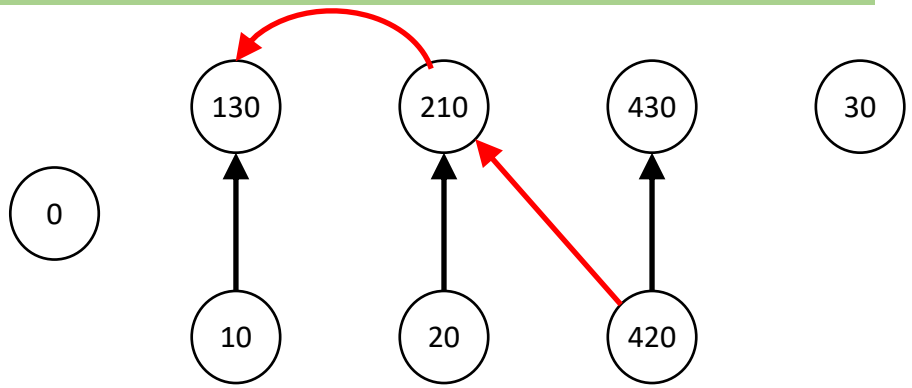

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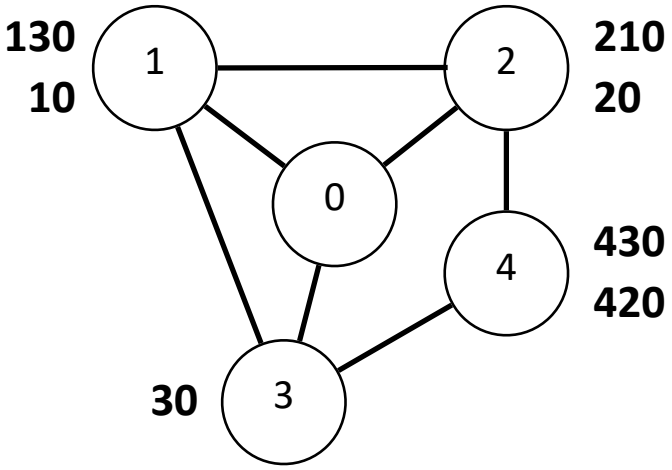

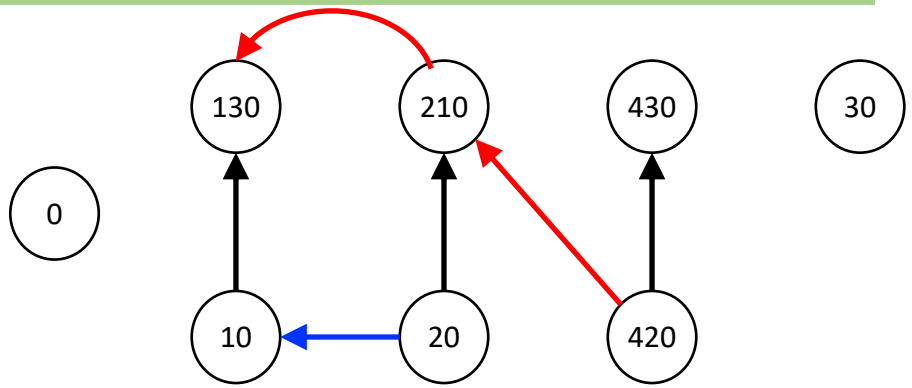

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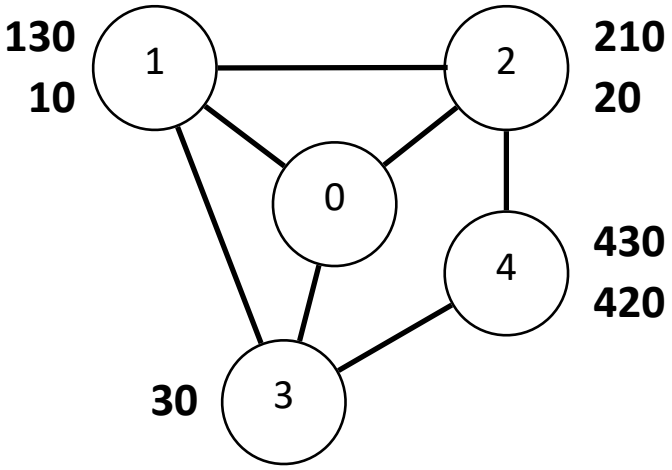

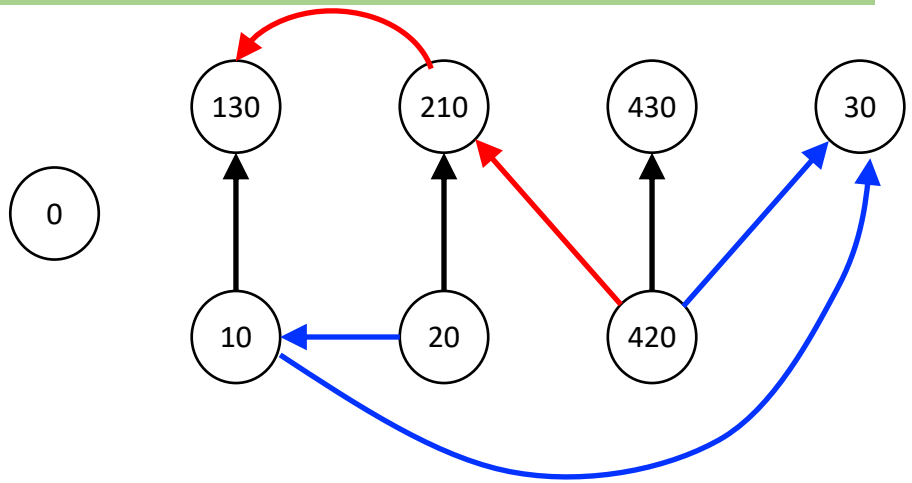

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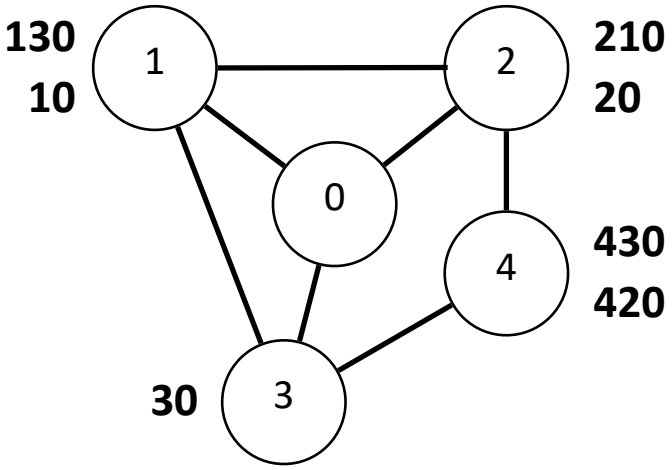

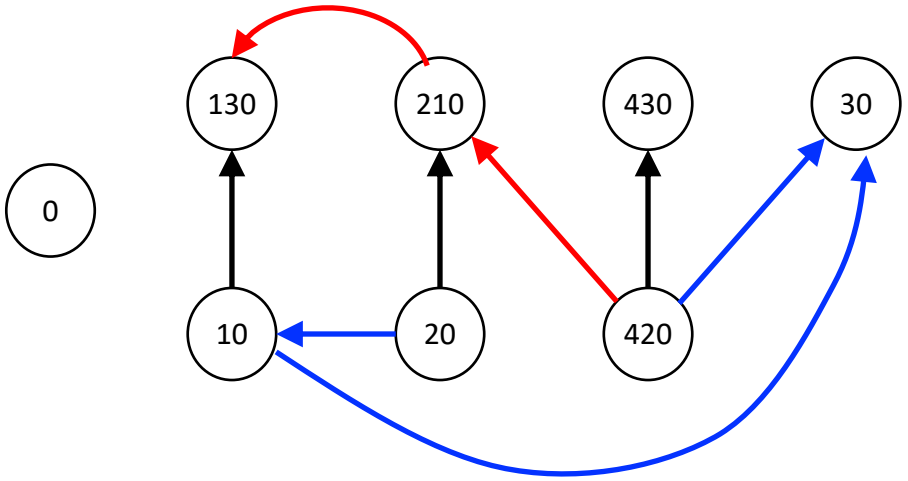

Solvability Graph - More Examples

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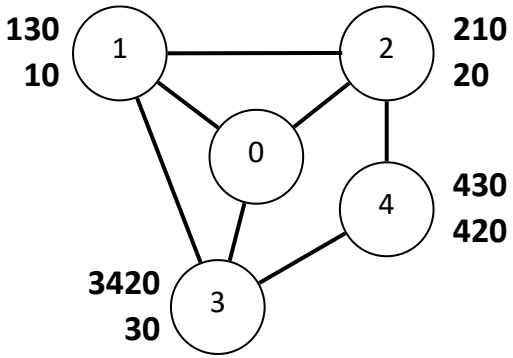
Naughty Gadget



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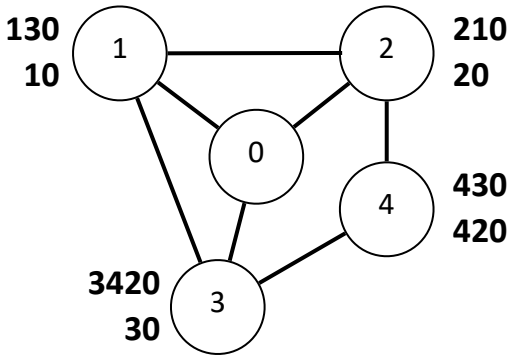

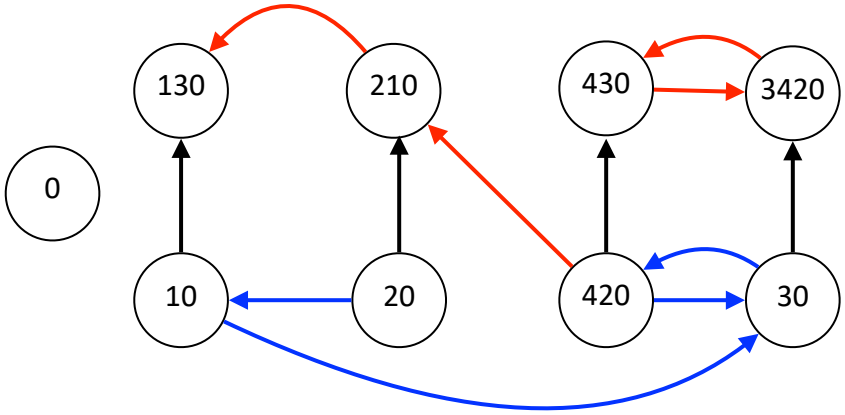

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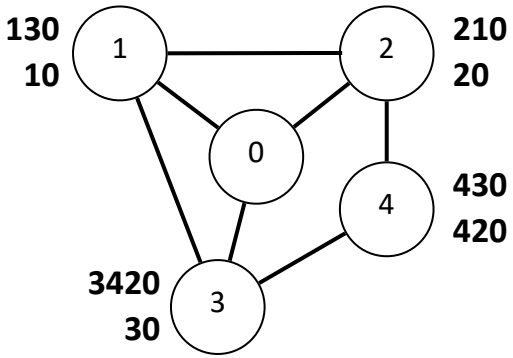

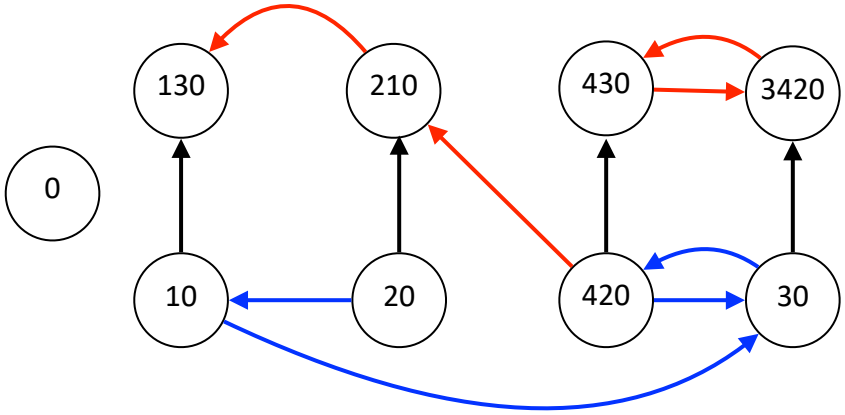

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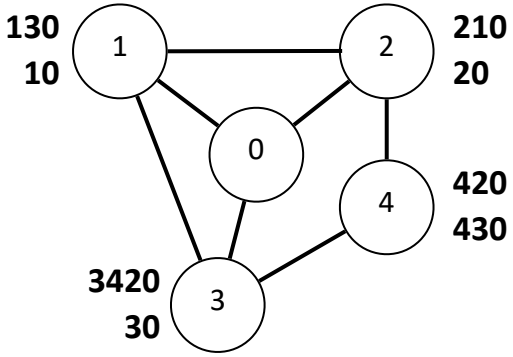

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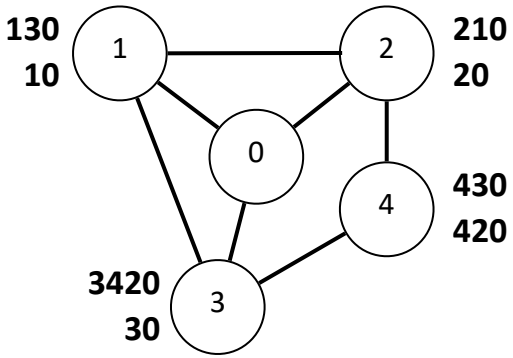

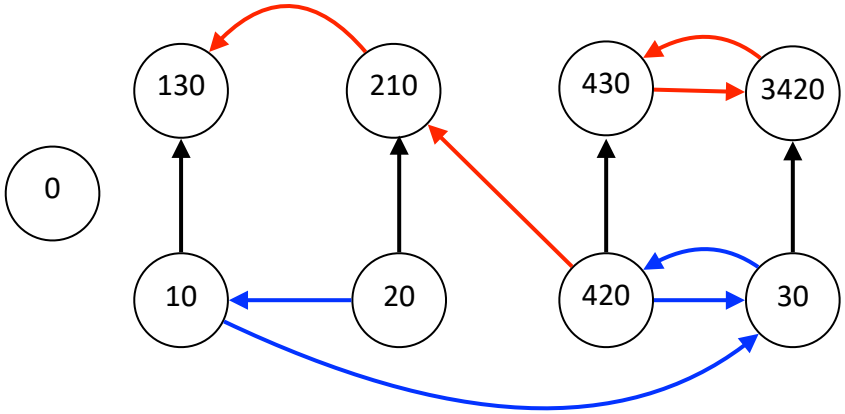

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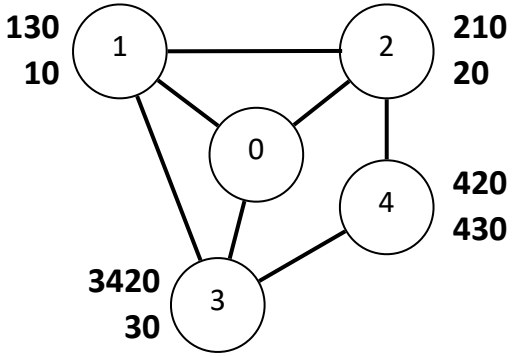

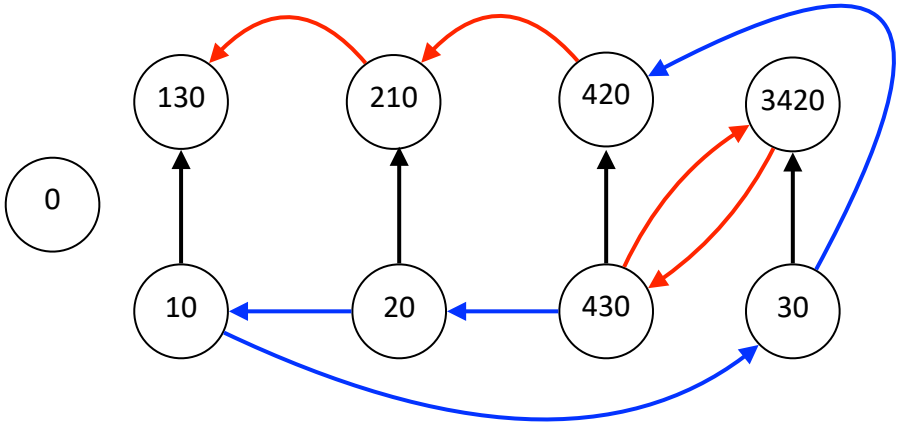

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Theorem 1: Given an SPP instance with n+1 nodes, it is solvable if and only if the corresponding solvability digraph has a maximum independent set of size n+1. (proof in the paper)

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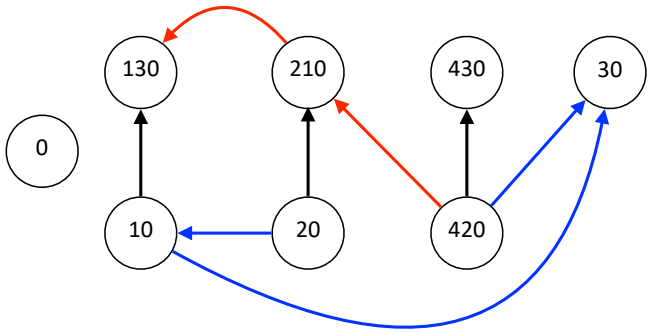

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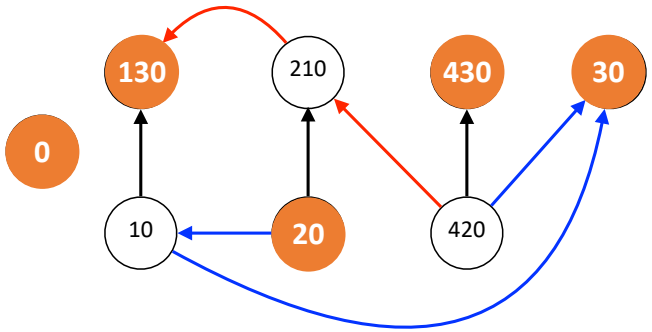

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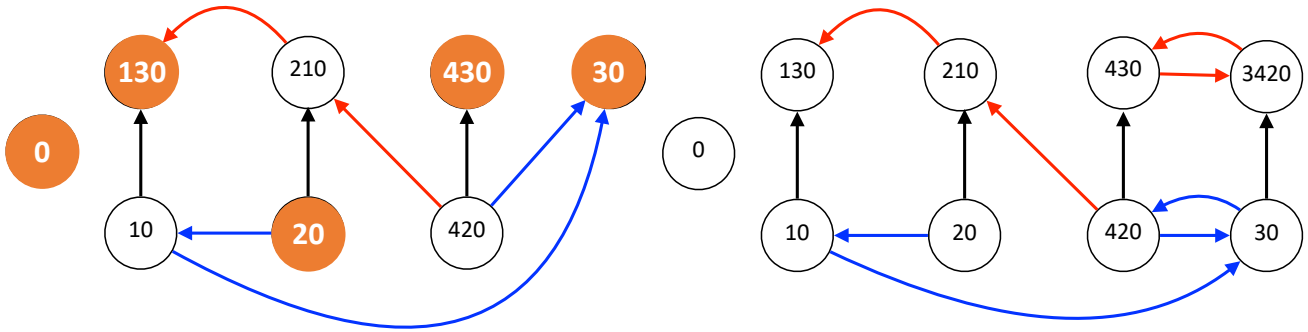

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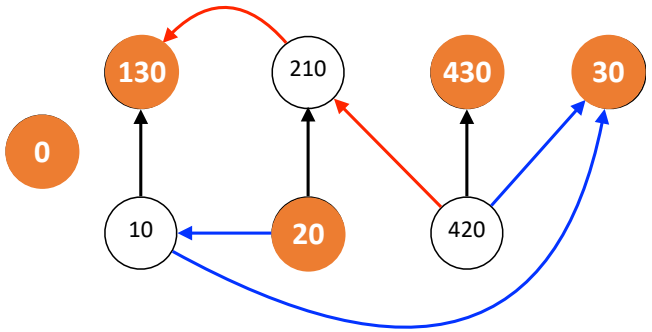

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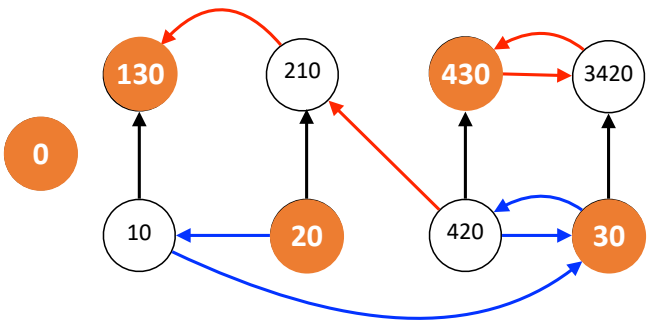

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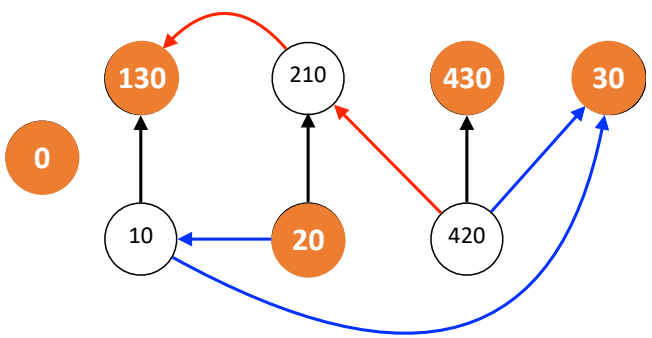

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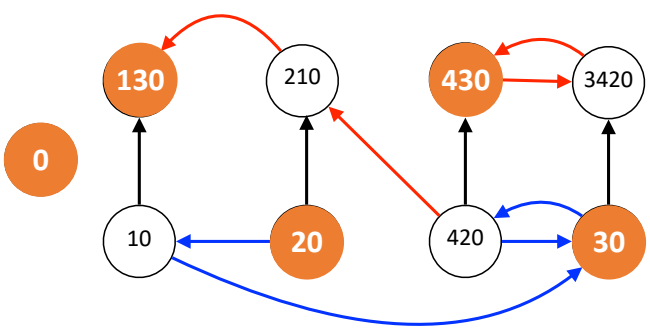

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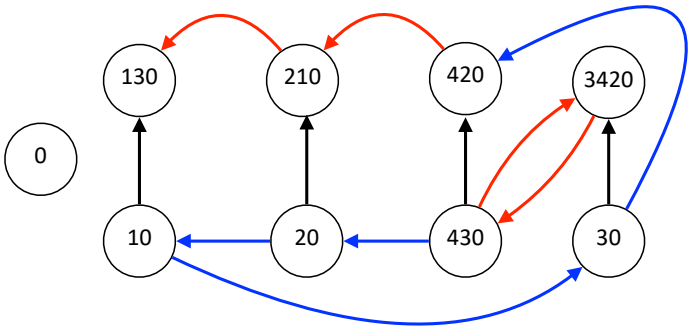

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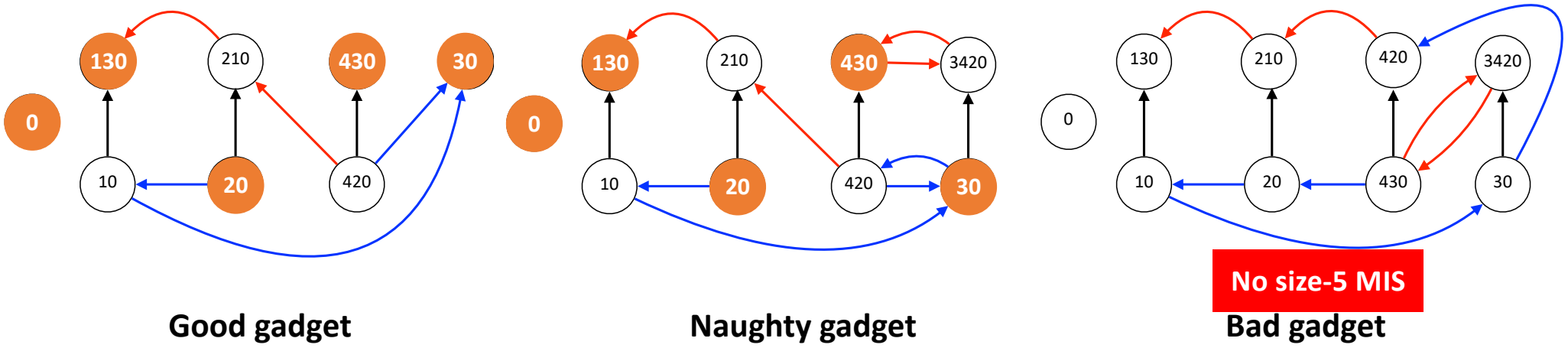

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Outline

- Background on SPP
- Our Contribution
- Solvability Digraph
- GreedyMIS: A Polynomial-Time Heuristic for SPP

GreedyMIS: Basic Idea

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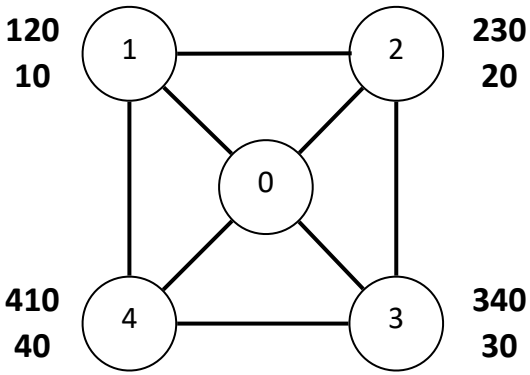
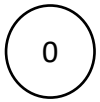
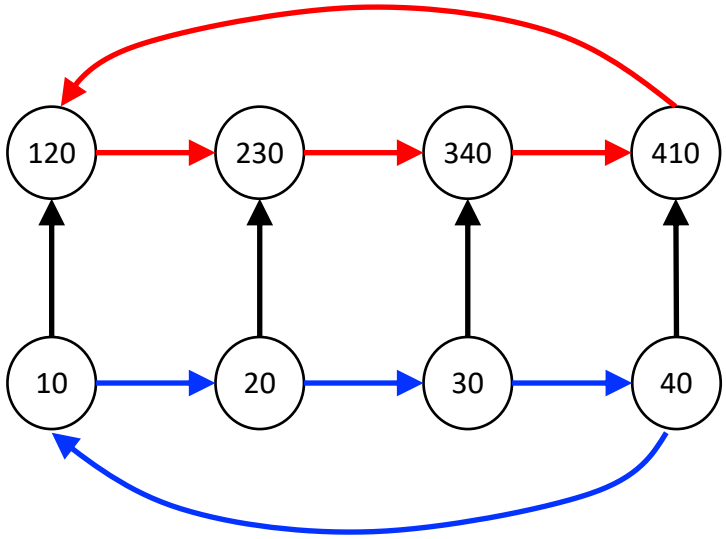
- Terminates when
  - (**solvable**) one of size n+1 is found, or
  - (**non-solvable**) no more nodes can be added into the set

GreedyMIS: Example







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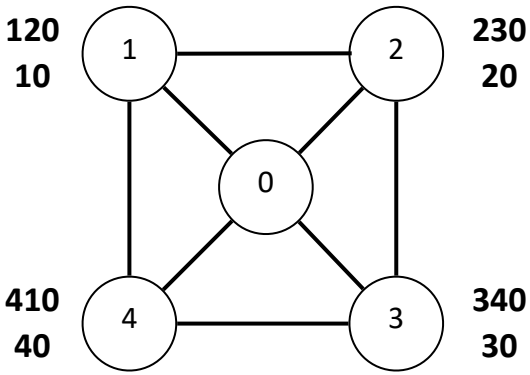

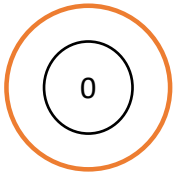

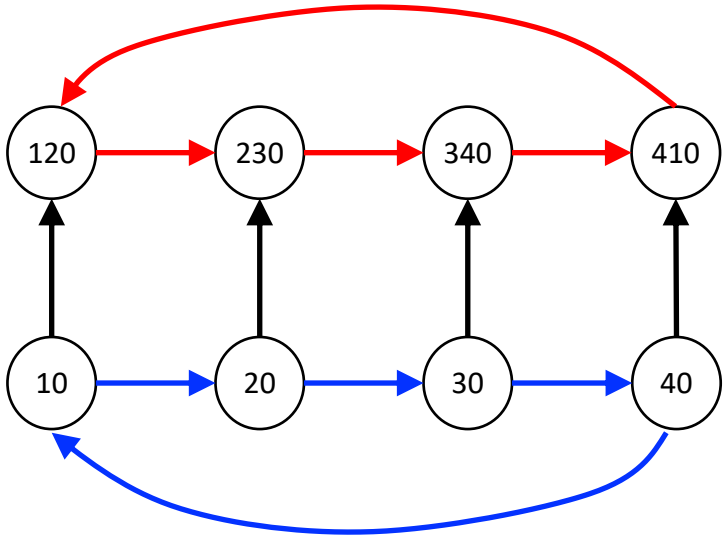

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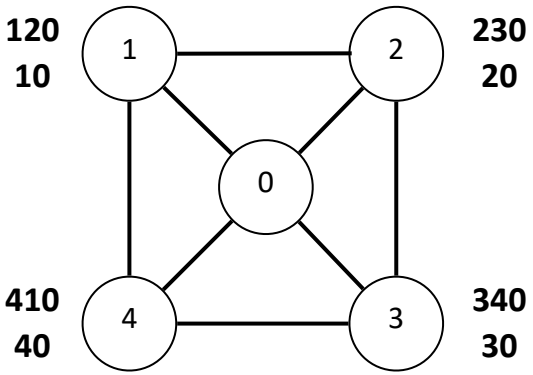

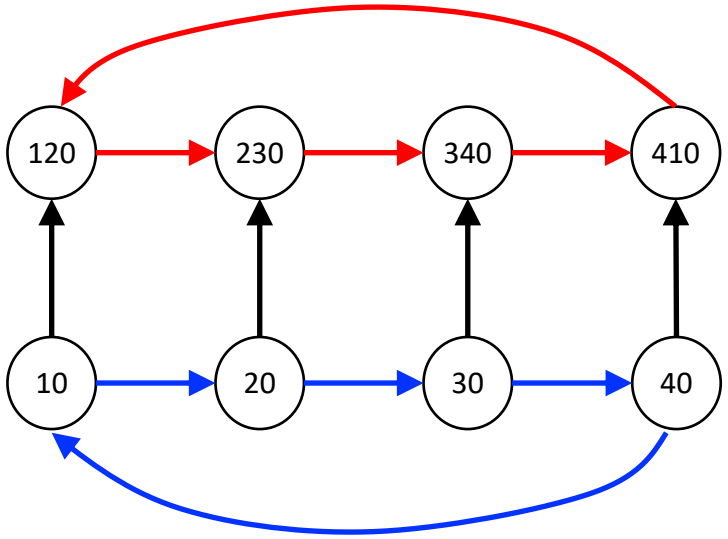

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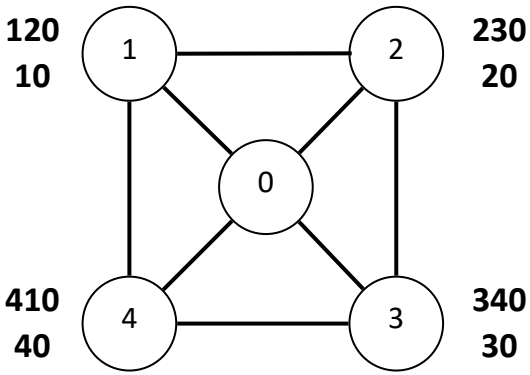

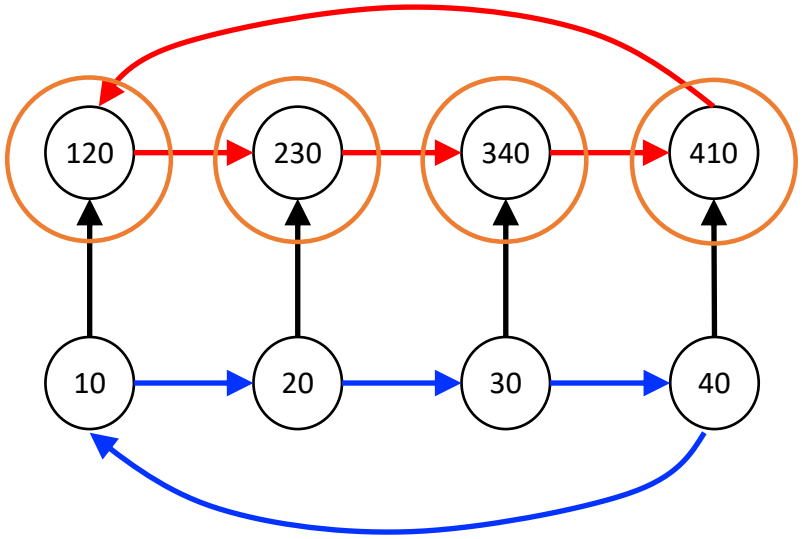

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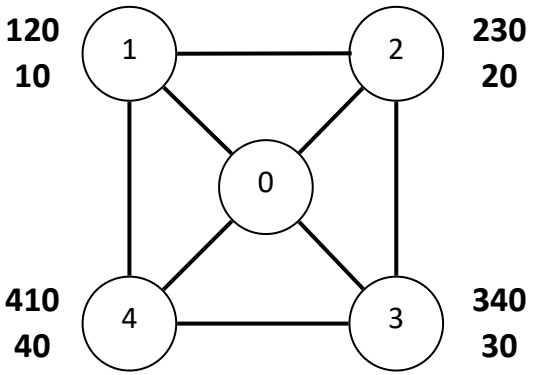

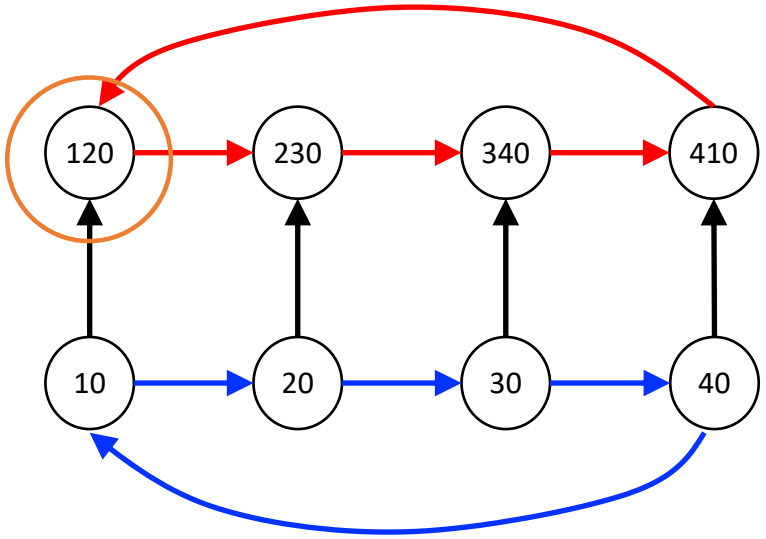

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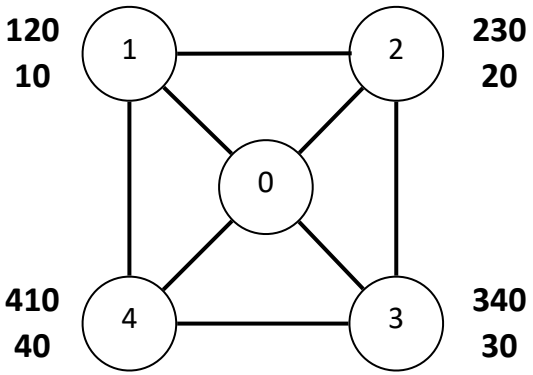

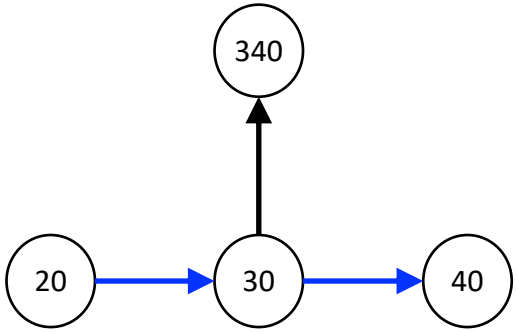

- Iteration 1: 0 is added (zero out edges)
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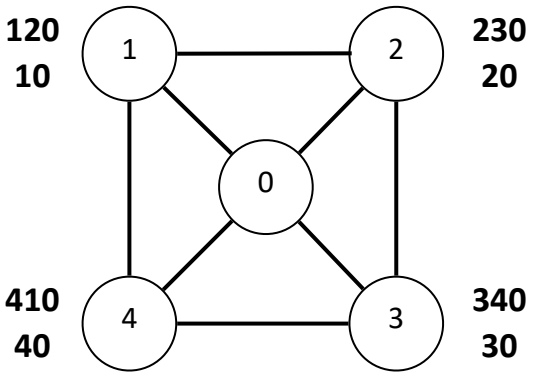
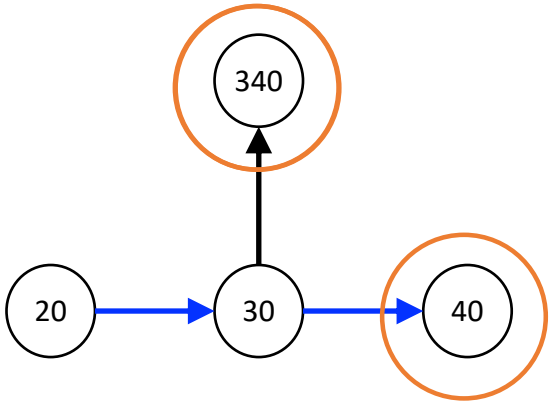
GreedyMIS: Example





- Iteration 1: 0 is added (zero out edges)
- Iteration 2: {120, 230, 340, 410} all have out degree of 1, randomly add 120 (one with lowest out degree)
- Iteration 3: 340, 40 are added (zero out edges)

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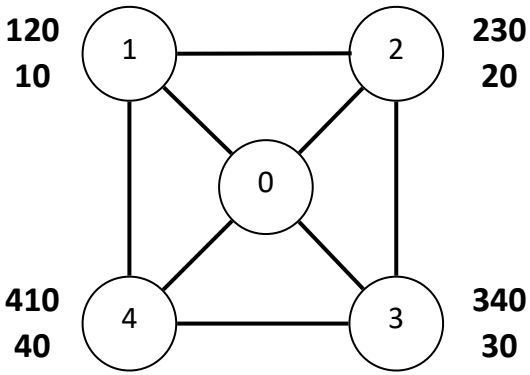

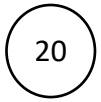

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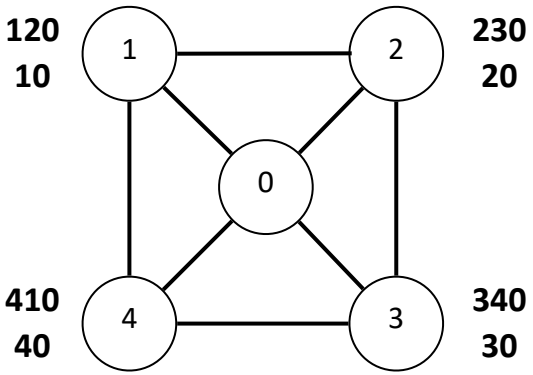

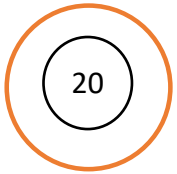

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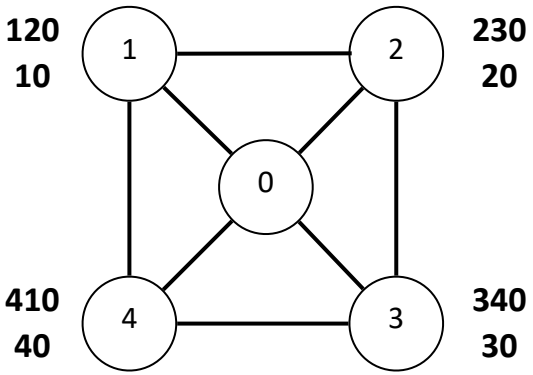
- Iteration 4: 20 is added (zero out edges)

GreedyMIS: Example



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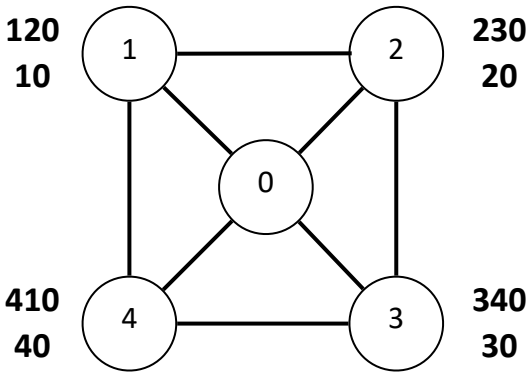

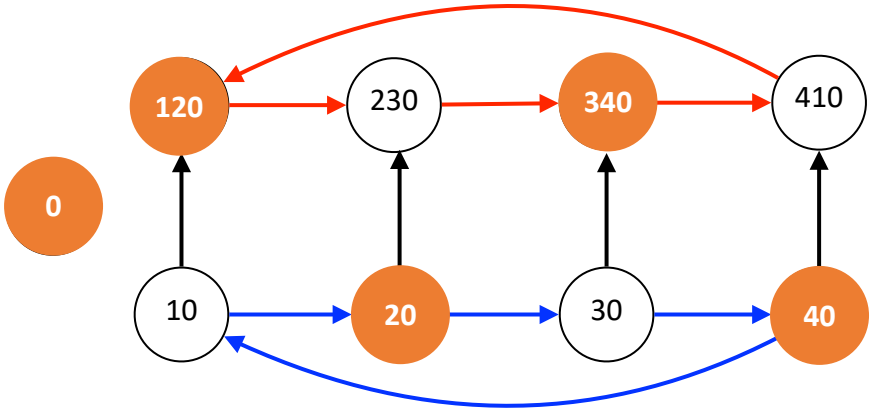

- Iteration 1: 0 is added (zero out edges)
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- Iteration 3: 340, 40 are added (zero out edges)
- Iteration 4: 20 is added (zero out edges)
- **Final result**: {0, 120, 340, 40, 20}

GreedyMIS: Analysis

Theorem 2: Any SPP instance that is solvable by Greedy+ [3], , the best known polynomial-time heuristic algorithm for SPP, is also solvable by GreedyMIS. (proof in the paper)

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Theorem 3: GreedyMIS solves strictly more SPP instances than Greedy+.

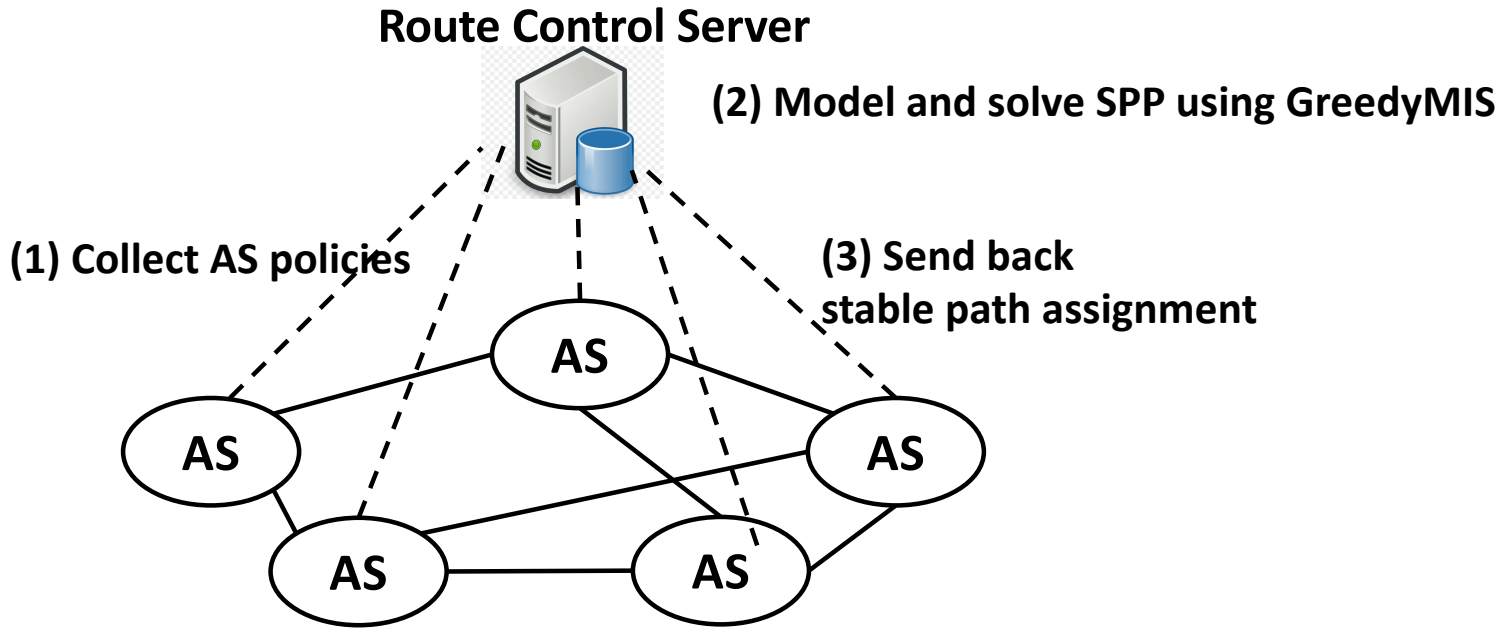
- A direct result from Theorem 2 and the example in the last slide.

Outline

- Background on SPP
- Our Contribution
- Solvability Digraph
- GreedyMIS: A Polynomial-Time Heuristic for SPP
- Use Cases

Use Case 1: Collaborative Interdomain Routing

- **Advantage**: allow more flexible routing policies, i.e., doesn't haven't to be a "safe" SPP
- **Scenario**: collaborative science networks such as Large Hadron Collider



Use Case 2: Privacy-Preserving, Collaborative Interdomain Routing

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- **Protocol**: an privacy-preserving version of GreedyMIS that employs linear secret sharing

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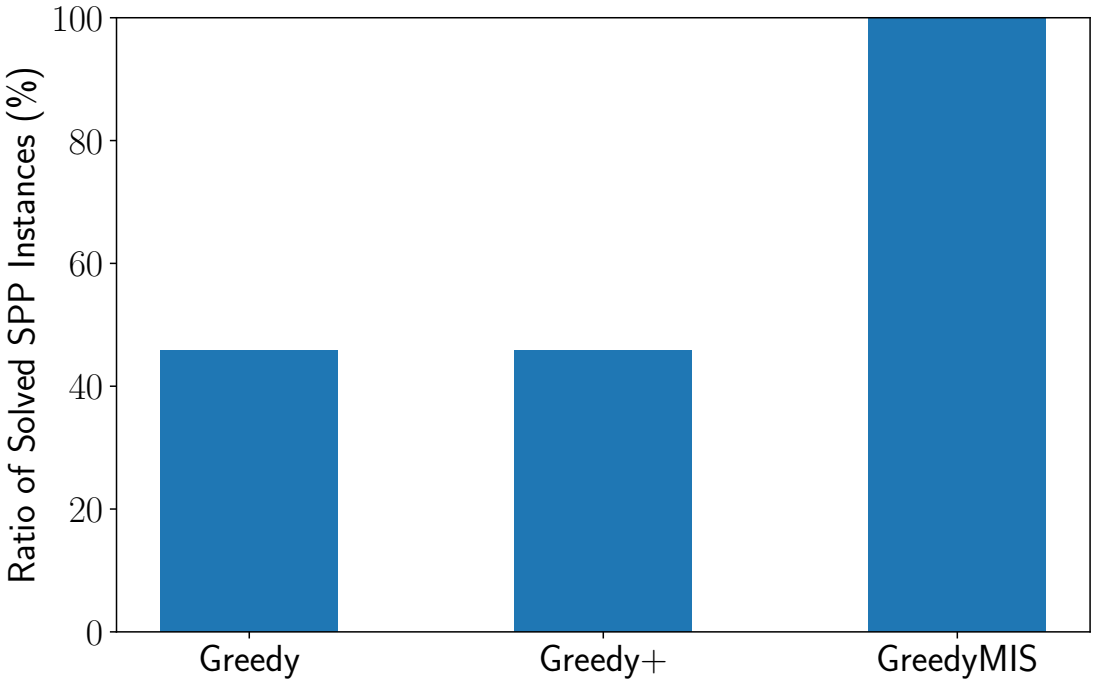
- Performance Evaluation

Performance Evaluation: Settings

- **Topology**: real AS-level topologies from CAIDA Internet topology dataset
- **AS export policies**: (1) C/P relationship, (2) blacklist ASes, (3) forbidden segments.
- **SPP instances**: 11-50 ASes, 8-12k permitted paths
- **Comparison**: GreedyMIS vs. Greedy [1] and Greedy+ [3]

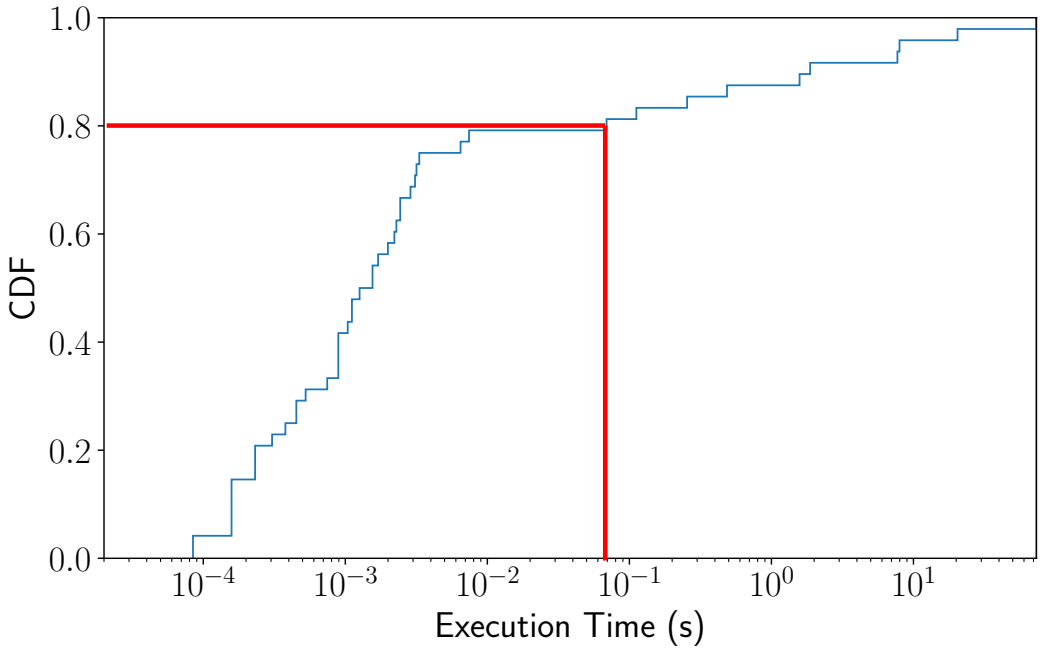
Results: Solvability of GreedyMIS

- GreedyMIS solves all SPP instances in the experiment, while the other two algorithms solves less than 50%.



Results: Efficiency of GreedyMIS

- In 80% of the experiments, GreedyMIS solves SPP in no more than 0.01s



Conclusion and Future Work

- Propose the solvability digraph data structure to reason about SPP solvability
- Design GreedyMIS, a polynomial-time heuristic that solves strictly more SPP instances than known state-of-the-art
- Tailor GreedyMIS to important interdomain routing use cases
- Demonstrate the efficiency and potentials of GreedyMIS via evaluation on real topologies

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Future Work

- Generic exact/approximated MIS algorithm for solving SPP
- Apply the SPP-MIS theorem to network configuration verification

Backup Slides