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Homework 3

2 Programming

We are interested in solving the following low rank matrix problem. Given a sparse observation pattern $G \in \{0,1\}^{n \times n}$ and a data matrix $A \in \mathbb{R}^n$ our goal is to recover a low rank matrix pair, $B \in \mathbb{R}^{n \times r}, C \in \mathbb{R}^{r \times n}$ by minimizing

$$\min_{B,C} f(B,C) = \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(a_{ij} - e_i^T B(Ce_j))^2 + \frac{\mu}{2}(\|B\|_F^2 + \|C\|_F^2)$$

Note that this is the same as:

$$\min_{B,C} \Big\langle G, (A - BC) \circ (A - BC) \Big\rangle_F + \frac{\mu}{2}\Big\langle B, B \Big\rangle_F + \frac{\mu}{2}\Big\langle C, C \Big\rangle_F$$

Where $\circ$ is the Hadamard product, and $\langle \cdot, \cdot \rangle_F$ is the Frobenius inner product.

Deriving the gradient is an exercise in tensor calculus:

$$g(f) = \Big[\nabla_B f, \nabla_C f \Big]$$

and $\nabla_B f \in \mathbb{R}^{n \times r}$ likewise $\nabla_C f \in \mathbb{R}^{r \times n}$

$$\frac{\partial f}{\partial b_{lm}} = \frac{\partial}{\partial b_{lm}} \left(\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(a_{ij} - \sum_{k=1}^{r} b_{ik}c_{kj})(a_{ij} - \sum_{t=1}^{r} b_{it}c_{tj}) + \frac{\mu}{2}\sum_{i=1}^{n}\sum_{k=1}^{n}(b_{ik}b_{ik} + c_{ik}c_{ik}) \right)$$

I will first focus on the cost or least squares portion:

$$\frac{\partial}{\partial b_{lm}} \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(a_{ij} - \sum_{k=1}^{r} b_{ik}c_{kj})(a_{ij} - \sum_{t=1}^{r} b_{it}c_{tj})$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(-\sum_{k=1}^{r} \delta_{il}\delta_{km}c_{kj})(a_{ij} - \sum_{t=1}^{r} b_{it}c_{tj}) + \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(a_{ij} - \sum_{k=1}^{r} b_{ik}c_{kj})(-\sum_{t=1}^{r} \delta_{il}\delta_{tm}c_{tj})$$

$$= \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(-\delta_{il}c_{mj})(a_{ij} - \sum_{t=1}^{r} b_{it}c_{tj}) + \sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}(a_{ij} - \sum_{k=1}^{r} b_{ik}c_{kj})(-\delta_{il}c_{mj})$$

$$= -2\sum_{i=1}^{n}\sum_{j=1}^{n} \delta_{il}g_{ij}(a_{ij} - \sum_{k=1}^{r} b_{ik}c_{kj})c_{mj}$$

$$= -2\sum_{i=1}^{n}\sum_{j=1}^{n} \delta_{il}g_{ij}(A - BC)_{ij}c_{mj}$$

$$= -2\sum_{j=1}^{n} g_{lj}(A - BC)_{lj}c_{mj}$$

$$= -2 \sum_{j=1}^{n} \Big(G \circ (A - BC) \Big)_{lj} c_{mj}$$

$$= -2 \Bigg(\Big(G \circ (A - BC) \Big) C^T \Bigg)_{lm}$$

A derivative with respect to the regulation yields

$$\frac{\partial}{\partial b_{lm}} (b_{ik} b_{ik}) = 2 b_{ik} \delta_{il} \delta_{km} = 2 b_{lm}$$

The whole gradient can be represented in matrix form as:

$$\nabla_B f = -2 \Big(G \circ (A - BC) \Big) C^T + \mu B \in \mathbb{R}^{n \times r}$$

Similarly for the gradient with respect to C, I derived it as:

$$\nabla_C f = -2 B^T \Big(G \circ (A - BC) \Big) + \mu C$$

The Hessian action is needed at the very least to implement a trust region algorithm. The Hessian is a fourth order tensor, its action on a second order matrix results in a second order matrix (as is expected). Heuristically I will decompose the Hessian into a block structure as follows:

$$H = \begin{bmatrix} H_{BB} & H_{BC} \\ H_{CB} & H_{CC} \end{bmatrix}$$

each block has $n^2 r^2$ elements but the shapes vary as B and C have different shapes. Careful attention to indices is warranted.

$$(H_{BB})_{lmpq} = \frac{\partial}{\partial b_{pq}}(\nabla_B f)_{lm} = 2\sum_{j=1}^{n} g_{lj}\delta_{lp}c_{qj}c_{mj} + \mu\delta_{lp}\delta_{kq}$$

$$(H_{BC})_{lmpq} = \frac{\partial}{\partial c_{qp}}(\nabla_B f)_{lm} = 2g_{lp}b_{lq}c_{mp} - 2(G \circ (A - BC))_{lp}\delta_{mq}$$

$$(H_{CB})_{lmpq} = \frac{\partial}{\partial b_{pq}}(\nabla_C f)_{lm} = 2g_{pl}b_{pm}c_{ql} - 2(G \circ (A - BC))_{pl}\delta_{mq}$$

$$(H_{CC})_{lmpq} = \frac{\partial}{\partial c_{qp}}(\nabla_C f)_{lm} = 2\sum_{i=1}^{n} b_{im}g_{il}b_{iq}\delta_{lp} + \mu\delta_{mq}\delta_{lp}$$

In order to implement a trust region algorithm we only need the Hessian action (we can also do Newton with just the action if we use a Krylov solver).

The Hessian Action can be defined as follows:

$$\begin{bmatrix} H_{BB} & H_{BC} \\ H_{CB} & H_{CC} \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} H_{BB}X + H_{BC}Y \\ H_{CB}X + H_{CC}Y \end{bmatrix}$$

Each block multiplication is as follows:

$$H_{BB}X = [G \circ (XC)]C^T + \mu X$$

$$H_{BC}Y = 2C[G \circ (BY)]^T - 2Y[G \circ (A - BC)]^T$$

$$H_{CB}X = 2[G \circ (XC)]^T B - 2[G \circ (A - BC)]^T X$$

$$H_{CC}Y = B^T[G \circ (BY)] + \mu Y$$

I will now derive a class for this problem that will encapsulate all the important information.

In [1]:
```
import numpy as np
import numpy.linalg as la

class low_rank_matrix_handler:
    def __init__(self,A,B,C,G):
        self.A = A
        self.B = B
        self.C = C
        self.G = G
        self.mu = 1.0
        self.n,self.r = self.B.shape
```

```
    def control(self):
        return np.array([self.B,self.C])

    def objective(self,B=None,C=None):
        if B is None:
            B = self.B
        if C is None:
            C = self.C
        temp = self.A - np.matmul(B,C)
        temp = np.multiply(temp,temp)
        cost = np.sum(np.multiply(self.G,temp))
        reg_B = 0.5*self.mu*np.sum(np.multiply(B,B))
        reg_C = 0.5*self.mu*np.sum(np.multiply(C,C))
        return cost + reg_B + reg_C

    def gradient(self):
        # Initialize to the regularization portion
        reg_B = self.mu*self.B
        reg_C = self.mu*self.C
        # Add the least squares (cost) portion
        A_BC = self.A - np.matmul(self.B,self.C)
        G_had_A_BC = np.multiply(self.G,A_BC)
        cost_B = -2*np.matmul(G_had_A_BC,self.C.T)
        cost_C = -2*np.matmul(self.B.T,G_had_A_BC)
        grad_B = cost_B + reg_B
        grad_C = cost_C + reg_C
        return grad_B, grad_C

    def Hessian_action(self, X = None, Y=None):
        if X is None:
            X = np.zeros_like(self.B)
        if Y is None:
            Y = np.zeros_like(self.C)
        shape_B = self.B.shape
        shape_C = self.C.shape
        if X.shape != shape_B:
            raise ValueError(X)
        if Y.shape != shape_C:
            raise ValueError(Y)
        XC = np.matmul(X,self.C)
        G_had_XC = np.multiply(self.G,XC)
        H_BB_X = np.matmul(G_had_XC,self.C.T) + self.mu*X
        A_BC = self.A - np.matmul(self.B,self.C)
        BY = np.matmul(self.B,Y)
        G_had_BY = np.multiply(self.G,BY)
        G_had_A_BC = np.multiply(self.G,A_BC)
        H_BC_Y = 2*np.matmul(self.C,G_had_BY.T) - 2*np.matmul(Y,G_had_A_
BC.T)
        H_CB_X = 2*np.matmul(G_had_XC.T,self.B) -
2*np.matmul(G_had_A_BC.T,X)
        H_CC_Y = np.matmul(self.B.T,G_had_BY) + self.mu*Y
        return [H_BB_X+H_BC_Y.T,H_CB_X+ H_CC_Y.T]

    def Hessian_inner(self, X = None,Y=None):
        if X is None:
            X = np.zeros_like(self.B)
        if Y is None:
```

```
            Y = np.zeros_like(self.C)
        H_XY = self.Hessian_action(X,Y)
        H_X = H_XY[0]
        H_Y = H_XY[1]
        X_H_X = np.multiply(X,H_X)
        Y_H_Y = np.multiply(Y.T,H_Y)
        return np.sum(X_H_X) + np.sum(Y_H_Y)

    def update_control(self, P_B= None,P_C = None,alpha = 1.):
        if P_B == None:
            P_B = np.zeros_like(self.B)
        if P_C == None:
            P_C = np.zeros_like(self.C)
        self.B += alpha*P_B
        self.C += alpha*P_C

    def assemble_flat_Hessian(self):
        n,r = self.B.shape
        dim = n*r
        H_BB = np.zeros((dim,dim))
        H_BC = np.zeros((dim,dim))
        H_CB = np.zeros((dim,dim))
        H_CC = np.zeros((dim,dim))
        A_BC = self.A - np.matmul(self.B,self.C)
        G_had_A_BC = np.multiply(self.G,A_BC)
        for l in range(n):
            for m in range(r):
                for p in range(n):
                    for q in range(r):

                        H_BB[l + n*m][p + n*q] += self.mu*(l==p)*(m==q)
                        H_BC[l + n*m][p + n*q] +=
2*self.G[l,p]*self.B[l,q]*self.C[m,p]\
                            -2*G_had_A_BC[l,p]*(m == q)
                        H_CB[l + n*m][p + n*q] +=
2*self.G[p,l]*self.B[p,m]*self.C[q,l]\
                            - 2*G_had_A_BC[p,l]*(m == q)
                        H_CC[l + n*m][p + n*q] += self.mu*(l==p)*(m==q)
                        for j in range(n):
                            H_BB[l + n*m][p + n*q] += 2*self.G[l,j]*
(l==p)\
                            *self.C[q,j]*self.C[m,j]
                            H_CC[l + n*m][p + n*q] += 2*self.G[j,l]*
(l==p)\
                            *self.B[j,m]*self.B[j,q]
        H = np.bmat([[H_BB,H_BC],[H_CB,H_CC]])
        return H

    def assemble_flat_gradient(self):
        n,r = self.B.shape
        dim = n*r
        flat_grad_B = np.zeros(dim)
        flat_grad_C = np.zeros(dim)
        A_BC = self.A - np.matmul(self.B,self.C)
        G_had_A_BC = np.multiply(self.G,A_BC)
        for l in range(n):
            for m in range(r):
```

```
                flat_grad_B[l+n*m] += self.mu*self.B[l,m]
                flat_grad_C[l+n*m] += self.mu*self.C[m,l]
                for j in range(n):
                    flat_grad_B[l+n*m] += -2*G_had_A_BC[l,j]*self.C[m,j]
                    flat_grad_C[l+n*m] += -2*G_had_A_BC[j,l]*self.B[j,m]
        g = np.concatenate([flat_grad_B,flat_grad_C])
        return g

    def split_and_reshape(self,p):
        temp_B,temp_C = np.split(p,2)
        p_B = temp_B.reshape((self.n,self.r))
        p_C = temp_C.reshape((self.n,self.r))
        return p_B, p_C.T

    def predicted_reduction(self,p):
        g = self.gradient()
        gp = np.sum(np.multiply(g[0],p[0]))
        pHp = self.Hessian_inner(X = p[0],Y=p[1])
        return -gp -0.5*pHp

    def tensor_norm(self,p):
        p_norm_2 = np.sum(np.multiply(p[0],p[0])) +np.sum(
np.multiply(p[1],p[1]))
        return np.sqrt(p_norm_2)

    def tensor_rescale(self,p,alpha):
        return [alpha*p[0],alpha*p[1]]

    def assemble_tensor_Hessian(self):
        n,r = self.B.shape
        dim = n*r
        H = np.zeros((2*n,r,2*n,r))
        A_BC = self.A - np.matmul(self.B,self.C)
        G_had_A_BC = np.multiply(self.G,A_BC)
        for l in range(n):
            for m in range(r):
                for p in range(n):
                    for q in range(r):
                        #HBB
                        H[l,m,p,q] += self.mu*(l==p)*(m==q)
                        #HBC
                        H[n+l,m,p,q] +=
2*self.G[l,p]*self.B[l,q]*self.C[m,p]\
                            -2*G_had_A_BC[l,p]*(m == q)
                        #HCB
                        H[l,m,n+p,q] +=
2*self.G[p,l]*self.B[p,m]*self.C[q,l]\
                            - 2*G_had_A_BC[p,l]*(m == q)
                        #HCC
                        H[n+l,m,n+p,q] += self.mu*(l==p)*(m==q)
                        for j in range(n):
                            #HBB
                            H[l,m,p,q] += 2*self.G[l,j]*(l==p)\
                            *self.C[q,j]*self.C[m,j]
                            #HCC
                            H[n+l,m,n+p,q] += 2*self.G[j,l]*(l==p)\
                            *self.B[j,m]*self.B[j,q]
```

```
        return H

    def assemble_tensor_gradient(self):
        n,r = self.B.shape
        gradient = np.zeros((2*n,r))
        A_BC = self.A - np.matmul(self.B,self.C)
        G_had_A_BC = np.multiply(self.G,A_BC)
        for l in range(n):
            for m in range(r):
                #G_B
                gradient[l,m] += self.mu*self.B[l,m]
                #G_C
                gradient[n+l,m] += self.mu*self.C[m,l]
                for j in range(n):
                    #G_B
                    gradient[l,m] += -2*G_had_A_BC[l,j]*self.C[m,j]
                    #G_C
                    gradient[n+l,m] += -2*G_had_A_BC[j,l]*self.B[j,m]
        return gradient

    def split_p(self,p):
        p_B,p_CT = np.split(p,2)
        return p_B, p_CT.T

    def Newton_direction_tensor(self):
        g = self.assemble_tensor_gradient()
        H = self.assemble_tensor_Hessian()
        p = np.linalg.tensorsolve(H,-g)
        return self.split_p(p)
```

Now I need to generate the data matrix A, sparse observational pattern G and initialize the control variables B and C

Because it doesn't really matter I will take $A \sim \mathcal{N}(0, \sigma^2)^{n \times n}$

I take B and C to be a pair of low rank recovery of identity (i.e. they are both identity padded with extra zeros in the strictly rectuangular rows or columns see `numpy.matlib.eye` for implementation). And $B * C = I_r \in \mathbb{R}^{n \times n}$, identity up to the r^{th} diagonal and then all zeros.

In [2]:
```
n = 8
r = 4
mean = 0.0
sigma = 1.0
from numpy.matlib import eye
B = eye(n=n,M=r)
C = eye(n=r,M=n)


A = np.matmul(B,C)

B += np.random.normal(loc=mean,scale=sigma,size=(n,r))
C += np.random.normal(loc=mean,scale=sigma,size=(r,n))
G = np.multiply(np.random.randint(2,size=
(n,n)),np.random.randint(2,size=(n,n)))
```

In [3]:
```
def alternating_minimization(low_rank_matrix_handler,tolerance = 1e-8,ma
x_iters =10000,
                      c_armijo = 1e-4,rho_armijo =0.9,verbose = False,
                      back_track = False,min_length = 1e-3):
    lagr = low_rank_matrix_handler
    grad_norm0_sq = la.norm(lagr.gradient()[0])**2 + la.norm(lagr.gradie
nt()[1])**2
    grad_norm0 = np.sqrt(grad_norm0_sq)
    grad_norm = grad_norm0
    iters = 0

    while iters <= max_iters:
        for i in range(2):
            partial_grad_norm0 = la.norm(lagr.gradient()[i])
            partial_grad_norm = partial_grad_norm0
            while partial_grad_norm > tolerance*partial_grad_norm0:
                cost = lagr.objective()
                P = lagr.gradient()
                alpha = 0.1
                if back_track:
                    cost_old = cost
                    alpha = 1.
                    while alpha > min_length:
                        B_test = lagr.B -alpha*P[0]
                        C_test = lagr.C -alpha*P[1]
                        if i ==0:
                            cost = lagr.objective(B_test)
                        elif i == 1:
                            cost = lagr.objective(C_test)
                        if cost < cost_old +c_armijo*alpha*grad_norm :
                            break
                        else:
                            alpha *= rho_armijo
                if i == 0:
                    lagr.B -= alpha*P[0]
                elif i == 1:
                    lagr.C -= alpha*P[1]
                partial_grad_norm = la.norm(P[i])
                grad_norm = lagr.tensor_norm(P)
                if (grad_norm < tolerance*grad_norm0):
```

```
                    return iters
                if verbose:
                    alt_string = ['B','C']
                    if iters == 0:
                        print "\n{0:5} {1:5} {2:15} {3:15} {4:20} {5:1
5}".format(
                                "Iter",'Alt' , "cost", "||g||L2","partial
 \|g\|L2", "alpha" )
                    else:
                        print "\n{0:<5d} {1:<5} {2:<15e} {3:<15e} {4:<20
e} {5:<15e}".format(
                                iters, alt_string[i], cost, grad_norm, par
tial_grad_norm, alpha )
                converged = (grad_norm < tolerance*grad_norm0)
                if converged:
                    return converged, iters
                iters += 1
    return converged, iters
```

In [4]:
```
B = 1e-1*np.ones((n,r))
C = 1e-1*np.ones((r,n))

lagr = low_rank_matrix_handler(A,B,C,G)


g = lagr.gradient()

alternating_minimization(lagr,back_track = False,verbose=True)
```

Iter	Alt	cost	||g||L2	partial |g|L2	alpha
1	B	1.219645e+00	8.072305e-01	5.401241e-01	1.000000e-01
2	B	1.192064e+00	7.729692e-01	4.811701e-01	1.000000e-01
3	B	1.170175e+00	7.467592e-01	4.286594e-01	1.000000e-01
4	B	1.152802e+00	7.271402e-01	3.818867e-01	1.000000e-01
5	B	1.139014e+00	7.128260e-01	3.402240e-01	1.000000e-01
6	B	1.128070e+00	7.027104e-01	3.031125e-01	1.000000e-01
7	B	1.119384e+00	6.958636e-01	2.700541e-01	1.000000e-01
8	B	1.112488e+00	6.915209e-01	2.406056e-01	1.000000e-01
9	B	1.107015e+00	6.890646e-01	2.143723e-01	1.000000e-01
10	B	1.102670e+00	6.880039e-01	1.910026e-01	1.000000e-01
11	B	1.099220e+00	6.879545e-01	1.701836e-01	1.000000e-01
12	B	1.096482e+00	6.886191e-01	1.516365e-01	1.000000e-01
13	B	1.094308e+00	6.897708e-01	1.351131e-01	1.000000e-01
14	B	1.092582e+00	6.912378e-01	1.203922e-01	1.000000e-01
15	B	1.091211e+00	6.928922e-01	1.072770e-01	1.000000e-01
16	B	1.090123e+00	6.946397e-01	9.559218e-02	1.000000e-01
17	B	1.089259e+00	6.964121e-01	8.518145e-02	1.000000e-01
18	B	1.088573e+00	6.981609e-01	7.590576e-02	1.000000e-01

```
19    B     1.088028e+00    6.998529e-01    6.764121e-02          1.0000
00e-01

20    B     1.087596e+00    7.014661e-01    6.027744e-02          1.0000
00e-01

21    B     1.087252e+00    7.029867e-01    5.371616e-02          1.0000
00e-01

22    B     1.086979e+00    7.044076e-01    4.786981e-02          1.0000
00e-01

23    B     1.086763e+00    7.057257e-01    4.266041e-02          1.0000
00e-01

24    B     1.086590e+00    7.069415e-01    3.801849e-02          1.0000
00e-01

25    B     1.086454e+00    7.080576e-01    3.388215e-02          1.0000
00e-01

26    B     1.086345e+00    7.090781e-01    3.019627e-02          1.0000
00e-01

27    B     1.086259e+00    7.100081e-01    2.691175e-02          1.0000
00e-01

28    B     1.086191e+00    7.108532e-01    2.398483e-02          1.0000
00e-01

29    B     1.086136e+00    7.116193e-01    2.137653e-02          1.0000
00e-01

30    B     1.086093e+00    7.123124e-01    1.905214e-02          1.0000
00e-01

31    B     1.086059e+00    7.129384e-01    1.698072e-02          1.0000
00e-01

32    B     1.086031e+00    7.135028e-01    1.513472e-02          1.0000
00e-01

33    B     1.086010e+00    7.140112e-01    1.348957e-02          1.0000
00e-01

34    B     1.085992e+00    7.144684e-01    1.202340e-02          1.0000
00e-01

35    B     1.085979e+00    7.148793e-01    1.071673e-02          1.0000
00e-01

36    B     1.085968e+00    7.152482e-01    9.552178e-03          1.0000
00e-01

37    B     1.085959e+00    7.155792e-01    8.514280e-03          1.0000
00e-01
```

```
38    B     1.085952e+00    7.158760e-01    7.589248e-03          1.0000
00e-01

39    B     1.085947e+00    7.161419e-01    6.764796e-03          1.0000
00e-01

40    B     1.085943e+00    7.163800e-01    6.029978e-03          1.0000
00e-01

41    B     1.085939e+00    7.165932e-01    5.375041e-03          1.0000
00e-01

42    B     1.085936e+00    7.167839e-01    4.791293e-03          1.0000
00e-01

43    B     1.085934e+00    7.169545e-01    4.270991e-03          1.0000
00e-01

44    B     1.085933e+00    7.171071e-01    3.807231e-03          1.0000
00e-01

45    B     1.085931e+00    7.172435e-01    3.393865e-03          1.0000
00e-01

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55    B     1.085926e+00    7.180159e-01    1.075949e-03          1.0000
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59	B	1.085926e+00	7.181495e-01	6.797441e-04	1.0000 00e-01
60	B	1.085926e+00	7.181744e-01	6.060289e-04	1.0000 00e-01
61	B	1.085926e+00	7.181967e-01	5.403123e-04	1.0000 00e-01
62	B	1.085926e+00	7.182165e-01	4.817259e-04	1.0000 00e-01
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66	B	1.085926e+00	7.182767e-01	3.044076e-04	1.0000 00e-01
67	B	1.085926e+00	7.182879e-01	2.714112e-04	1.0000 00e-01
68	B	1.085926e+00	7.182979e-01	2.419933e-04	1.0000 00e-01
69	B	1.085926e+00	7.183068e-01	2.157656e-04	1.0000 00e-01
70	B	1.085926e+00	7.183148e-01	1.923819e-04	1.0000 00e-01
71	B	1.085926e+00	7.183219e-01	1.715336e-04	1.0000 00e-01
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73	B	1.085926e+00	7.183339e-01	1.363731e-04	1.0000 00e-01
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652   C     7.500000e-01    2.614369e-08    1.121022e-08          1.0000
00e-01

653   C     7.500000e-01    2.526364e-08    8.968176e-09          1.0000
00e-01

654   C     7.500000e-01    2.468395e-08    7.174541e-09          1.0000
00e-01

655   C     7.500000e-01    2.430570e-08    5.739633e-09          1.0000
00e-01

656   C     7.500000e-01    2.406049e-08    4.591706e-09          1.0000
00e-01

657   C     7.500000e-01    2.390224e-08    3.673365e-09          1.0000
00e-01

658   C     7.500000e-01    2.380041e-08    2.938692e-09          1.0000
00e-01

659   C     7.500000e-01    2.373501e-08    2.350954e-09          1.0000
00e-01

660   C     7.500000e-01    2.369305e-08    1.880763e-09          1.0000
00e-01

661   C     7.500000e-01    2.366617e-08    1.504610e-09          1.0000
00e-01

662   C     7.500000e-01    2.364894e-08    1.203688e-09          1.0000
00e-01

663   C     7.500000e-01    2.363791e-08    9.629508e-10          1.0000
00e-01

664   C     7.500000e-01    2.363085e-08    7.703607e-10          1.0000
00e-01
```

```
665   C     7.500000e-01    2.362633e-08    6.162884e-10          1.0000
00e-01

666   C     7.500000e-01    2.362343e-08    4.930309e-10          1.0000
00e-01

667   C     7.500000e-01    2.362158e-08    3.944247e-10          1.0000
00e-01

668   C     7.500000e-01    2.362040e-08    3.155397e-10          1.0000
00e-01

669   C     7.500000e-01    2.361964e-08    2.524319e-10          1.0000
00e-01

670   C     7.500000e-01    2.361915e-08    2.019453e-10          1.0000
00e-01

671   C     7.500000e-01    2.361884e-08    1.615563e-10          1.0000
00e-01

672   C     7.500000e-01    2.361864e-08    1.292451e-10          1.0000
00e-01

673   C     7.500000e-01    2.361851e-08    1.033962e-10          1.0000
00e-01

674   C     7.500000e-01    2.361843e-08    8.271683e-11          1.0000
00e-01

675   C     7.500000e-01    2.361838e-08    6.617362e-11          1.0000
00e-01

676   C     7.500000e-01    2.361835e-08    5.293854e-11          1.0000
00e-01

677   C     7.500000e-01    2.361833e-08    4.235101e-11          1.0000
00e-01

678   C     7.500000e-01    2.361831e-08    3.388079e-11          1.0000
00e-01

679   C     7.500000e-01    2.361830e-08    2.710476e-11          1.0000
00e-01

680   C     7.500000e-01    2.361830e-08    2.168365e-11          1.0000
00e-01

681   C     7.500000e-01    2.361829e-08    1.734701e-11          1.0000
00e-01

682   C     7.500000e-01    2.361829e-08    1.387757e-11          1.0000
00e-01

683   C     7.500000e-01    2.361829e-08    1.110190e-11          1.0000
00e-01
```

```
684   C     7.500000e-01    2.361829e-08    8.881562e-12          1.0000
00e-01

685   C     7.500000e-01    2.361829e-08    7.105094e-12          1.0000
00e-01

686   C     7.500000e-01    2.361829e-08    5.684231e-12          1.0000
00e-01

687   C     7.500000e-01    2.361829e-08    4.547029e-12          1.0000
00e-01

688   C     7.500000e-01    2.361829e-08    3.637646e-12          1.0000
00e-01

689   C     7.500000e-01    2.361829e-08    2.910228e-12          1.0000
00e-01

690   C     7.500000e-01    2.361829e-08    2.328138e-12          1.0000
00e-01

691   B     7.500000e-01    2.361829e-08    2.361829e-08          1.0000
00e-01

692   B     7.500000e-01    1.889463e-08    1.889463e-08          1.0000
00e-01

693   B     7.500000e-01    1.511570e-08    1.511570e-08          1.0000
00e-01

694   B     7.500000e-01    1.209256e-08    1.209256e-08          1.0000
00e-01

695   B     7.500000e-01    9.674051e-09    9.674051e-09          1.0000
00e-01
```

Out[4]: 696

In [5]:
```
def Newton(low_rank_matrix_handler,tolerance = 1e-8,max_iters =10000,
                    c_armijo = 1e-4,rho_armijo =0.9,verbose = False,
                    back_track = False,min_length = 1e-3):
    lagr = low_rank_matrix_handler
    grad_norm0_sq = la.norm(lagr.gradient()[0])**2 + la.norm(lagr.gradie
nt()[1])**2
    grad_norm0 = np.sqrt(grad_norm0_sq)
    grad_norm = grad_norm0
    iters = 0


    while grad_norm > tolerance*grad_norm0 and iters <= max_iters:
        cost = lagr.objective()
        g = lagr.assemble_flat_gradient()
        H = lagr.assemble_flat_Hessian()
        P = lagr.Newton_direction_tensor()
        alpha = 1.0
        if back_track:
            cost_old = cost
            alpha = 1.
            while alpha > min_length:
                B_test = lagr.B +alpha*P[0]
                C_test = lagr.C +alpha*P[1]
                cost = lagr.objective(B_test,C_test)
                if cost < cost_old +c_armijo*alpha*grad_norm :
                    break
                else:
                    alpha *= rho_armijo
        lagr.B += alpha*P[0]
        lagr.C += alpha*P[1]
        grad_norm = lagr.tensor_norm(P)
        if verbose:
            if iters == 0:
                print "\n{0:10} {1:15} {2:15} {3:15}".format(
                      "Iteration",  "cost", "||g||L2", "alpha" )
            else:
                print "\n{0:<10d} {1:<15e} {2:<15e} {3:<15e}".format(
                      iters,  cost, grad_norm, alpha )
        converged = (grad_norm < tolerance*grad_norm0)
        if converged:
            return converged, iters
        iters += 1
    return converged, iters
```

In [6]:
```
B = np.ones((n,r))
C = np.ones((r,n))

lagr = low_rank_matrix_handler(A,B,C,G)

Newton(lagr,back_track = False,verbose=True)
```

```
Iteration  cost            ||g||L2         alpha

1          2.728845e+01    5.305739e+00    1.000000e+00

2          3.351323e+00    1.003696e+01    1.000000e+00

3          1.151980e+03    9.298092e+00    1.000000e+00

4          9.234720e+00    7.417011e+00    1.000000e+00

5          4.058713e+01    2.748416e+00    1.000000e+00

6          1.070959e+01    1.622058e+00    1.000000e+00

7          3.297881e+00    2.959304e+00    1.000000e+00

8          1.710127e+00    7.327686e-01    1.000000e+00

9          1.135522e+00    8.775151e-01    1.000000e+00

10         7.506281e-01    3.544399e-02    1.000000e+00

11         7.500000e-01    3.188247e-06    1.000000e+00

12         7.500000e-01    2.320439e-18    1.000000e+00
```

Out[6]: (True, 12)

In [7]:
```
def trust_region(low_rank_matrix_handler,tolerance = 1e-3,max_iters =100
00,
                     delta_hat = 1.,delta_0_rel = 1.0, eta = 0.25,verbos
e = False):
    lagr = low_rank_matrix_handler
    grad_norm0 = lagr.tensor_norm(lagr.gradient())
    grad_norm = grad_norm0
    iters = 0
    delta_0 = delta_0_rel*delta_hat
    delta = delta_0
    while grad_norm > tolerance*grad_norm0 and iters <= max_iters:
        cost_0 = lagr.objective()
        P = lagr.Newton_direction_tensor()
        P_norm = lagr.tensor_norm(P)
        if P_norm > delta:
            scale = delta/P_norm
            P = lagr.tensor_rescale(P,scale)
            P_norm = delta
        predicted_reduction = lagr.predicted_reduction(P)
        B_test = lagr.B +P[0]
        C_test = lagr.C +P[1]
        cost_p = lagr.objective(B=B_test,C=C_test)
        actual_reduction = cost_0 - cost_p
        rho = actual_reduction/predicted_reduction
        if rho < 0.25:
            delta *= 0.25
        else:
            if (rho > 0.75) and (P_norm == delta):
                delta = min(2*delta,delta_hat)
        if rho > eta:
            lagr.B += delta*P[0]
            lagr.C += delta*P[1]
        grad_norm = lagr.tensor_norm(lagr.gradient())
        if verbose:
            if iters == 0:
                print "\n{0:4} {1:9} {2:9} {3:9} {4:9}".format(
                      "Iteration",  "cost", "||g||L2", "rho", "delta" )
            else:
                print "\n{0:<9d} {1:<9.1e} {2:<9.1e} {3:<9.1e} {4:<9.1
e}".format(
                      iters,  cost_0, grad_norm, rho, delta )
        converged = (grad_norm < tolerance*grad_norm0)
        if converged:
            return converged, iters
        if (rho ==np.nan):
            return False, iters
        iters += 1
    return iters
```