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Problem 1 & Problem 2.

Objective : $J = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} (a_{ij} - (e_i^T B)(C e_j))^2 + \frac{\mu}{2}(\|B\|_F^2 + \|C\|_F^2)$

$$= \|G \circ (A - BC)\|_F^2 + \frac{\mu}{2} (\|B\|_F^2 + \|C\|_F^2).$$

$$\frac{\partial J}{\partial B} = -2(G \circ (A - BC)) \cdot C^T + \mu B$$

$$\frac{\partial J}{\partial C} = -2 B^T (G \circ (A - BC)) + \mu C.$$

if we let $B = \begin{pmatrix} b_1^T \\ b_2^T \\ \vdots \\ b_n^T \end{pmatrix}$, $C = (C_1, C_2, \cdots C_n)$

~~Then~~ if we let $J(B, C) = J(x)$,

where $x = \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \\ C_1 \\ \vdots \\ C_n \end{pmatrix}$ is a $2nr \times 1$ vector,

Then
$$\frac{\partial J}{\partial x_{b_i}} = \left(2 \sum_{k=1}^{n} g_{ik} C_k C_k^T + \mu I\right) b_i - \sum_{k=1}^{n} g_{ik} a_{ik} C_k$$
$$\frac{\partial J}{\partial x_{C_i}} = \left(2 \sum_{k=1}^{n} g_{ki} b_k b_k^T + \mu I\right) C_i - \sum_{k=1}^{n} g_{ki} a_{ki} b_k$$

For Hessian:

$$\frac{\partial^2 J}{\partial x_{b_i} \partial x_{b_j}} = \frac{\partial^2 J}{\partial x_{C_i} \partial x_{C_j}} = 0 \quad (i \neq j), \quad \frac{\partial^2 J}{\partial x_{b_i}^2} = 2 \sum_{k=1}^{n} g_{ik} C_k C_k^T + \mu I$$

$$\frac{\partial^2 J}{\partial x_{C_i}^2} = 2 \sum_{k=1}^{n} g_{ki} b_k b_k^T + \mu I, \quad \frac{\partial^2 J}{\partial x_{b_i} x_{C_j}} = 2 g_{ij} (b_i^T C_j - a_{ij}) I + b_i C_j^T.$$

For Gradient descend, just update $B = B - \varepsilon \frac{\partial J}{\partial B}$, $C = C - \varepsilon \frac{\partial J}{\partial C}$,

For Alternating ~~of~~ minimization, solve $\frac{\partial J}{\partial x_{b_i}} = 0$ & $\frac{\partial J}{\partial x_{C_i}} = 0$ alternatively.

For Trust Region, use Hessian and gradient to do some computing.