Tentative Syllabus: MATH-COMP 309/409

Numerical Methods/Advanced Numerical Analysis

Spring 2023, 3 Credit Hours

*Preparing people to lead extraordinary lives*

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## ★ Course Information and Resources

**Pre-requisite:** MATH 212 & 264 (or MATH 266). A large part of this course will be about analysis of numerical methods for solving/approximating linear equations and ordinary differential equations. Many investigated methods are matrix-based.

For students in COMP 309/409, a programming pre-requisite is also needed, as part of the course expectation is to realize some of the fundamental algorithms covered in this course, with less asked on proofs compared with their counterparts in MATH courses. Python is most likely chosen for this purpose.

**Text:** There is no mandatory text, as there are many great lecture notes online regarding this subject. For each section, the professor will post some pre-selected notes for supplementary reading. Meanwhile, the following texts are standard for this course and can be used as references or more extensive reading:

- *Numerical Analysis (10th ed)* by Richard L. Burden, J. Douglas Faires, Annette M. Burden, Cengage.
- *Numerical Analysis (3rd ed)* by Timothy Sauer (2018), Pearson.

**Course Summary:** Numerical Analysis (NA) is a subject on the theory and applications of numerical approximation of functions, derivatives, integrals, differential equations, etc. It is the sibling, as a discrete theory, of the continuous theory of such objects developed in Calculus and its sequels (including Math 264 ODE). The goals of NA are two-fold: (i) to develop algorithms that can be implemented by computers to realize these approximations; (ii) to theoretically establish the convergence, stability and error estimates of such developed algorithms.

These two parts are equally important and stand as two pillars to the whole subject. While the first goal ensures that a script can practically 'run' on computers, it is the second goal that guarantees meaningful results from the execution of codes. Without either one, talking about the other is rather 'hypothetical'. This is more true nowadays than decades ago when NA was first developed.

That said, the main focus of this course is the **second goal** – theoretical analysis of the important elements in NA. The first goal will be touched throughout the semester to put the *a priori* analysis into context. We will mostly look at various introductory numerical methods and try to understand why and how well (not so, respectively) they solve the tasks we want to solve.

After taking this course, students should have a basic grasp of some classic methods used to approximate functions, eigenvalues, derivatives, solutions of linear/nonlinear/differential systems. More specifically, we will cover the following topics (using chapters in Burden-Faires-Burden as illustration):

- Preliminaries (Chapter 1)
- Polynomial Approximation (Chapter 3 & 8)
- Numerical Differentiation and Integration (Chapter 4)
- Solving Linear System – Matrix Factorization (Chapter 6, 7, & 9)
- Solving Non-linear System (Chapter 2 & 10)
- Solving Ordinary Differential Equations – IVP vs BVP (Chapter 5 & 10)

## ★ Assessment of Learning

- Weekly Written Homework (40%). Doing homework is a **big** part of the learning itself for this course. After covering the most essential sections, some others will be left in homework for you to explore.

  Due to the nature of this joint course between undergraduate and graduate students, the latter group will have a little bit extra and 'harder' problems in their weekly homework as well as in exams.

- Midterm (20%).

  The Midterm will be held tentatively in early- to mid-March. The exact date will be announced at least 2 weeks beforehand.

- Project (10%).

  - Undergraduate students: it will be homework assignment on materials presented by graduate students at the end of the semester.
  - Graduate students: you will be assigned with some materials (advanced topics in NA) to read, at least by the beginning of April, and to present in class during the last two weeks of the semester. The work involves reading, preparing notes, and presentations. Undergraduate students may participate to earn extra credit.

- Final (25%).

- Slack and class attendance/participation (5%).

  We will use Slack for out of class communication (you will likely use it a lot with other members of the class). You should actively join the Slack discussions. In particular, I want to point out that explaining something to others is A LOT harder than just knowing how to do it, and it significantly deepens one's knowledge about the topic. My goal is to make it a valuable portion of your learning in this class.

**A bit about me:** I am an Assistant Professor at the Department of Mathematics and Statistics. My research area is Applied Mathematics, where I study both qualitative (analytic) and quantitative (numerical) aspects of applied problems. Topics in this course form a significant port of my research foundation. Due to this nature, there are many projects I can initiate that are suitable for student research. During the semester, if a student shows great interest and promising potential in this course, I am happy to further discuss some of these projects to begin after the conclusion of the course, such as in Summer 2023, or in academic year 2023-24.