Xiang XU

03/28 sufficiency.

① Bern(p) $T=\Sigma x_i$

$$P\{X=x\}=Pr\{X_1=x_1 \cdots X_n=x_n\}$$

$$=p^{x_1}(1-p)^{1-x_1}\cdots p^{x_n}(1-p)^{1-x_n}$$

$$=p^{\Sigma x_i}(1-p)^{n-\Sigma x_i}=p^T(1-p)^{n-T}$$

② Geom(p) $T=\Sigma x_i$

$$P\{X=x\}=Pr\{X_1=x_1,\cdots,X_n=x_n\}$$

$$=\prod_{i=1}^{n}p(1-p)^{x_i-1}$$

$$=p^n(1-p)^{\Sigma x_i-n}$$

$$=p^n(1-p)^{T-n}$$

③ NegBinom(r,p) $T=\Sigma x_i$

$f(r,p,x)=\binom{x+r-1}{x}(1-p)^r p^x$ for $x\in\{1,2,\cdots,n\}$. # of success.

$$\mathcal{L}(r,p,x)=\prod_{i=1}^{n}f(r,p,x)=\prod_{i=1}^{n}\binom{x+r-1}{x}(1-p)^r p^x$$

$$=\prod_{i=1}^{n}\binom{x+r-1}{x}(1-p)^{n\cdot r}p^{n\Sigma x_i}$$

$$=\prod_{i=1}^{n}\binom{x+r-1}{x}(1-p)^{nr}p^{nT}$$

④ Gamma(α, β) $T = \Sigma x_i$, α is known.

$$f(\alpha, \beta, x) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$\mathcal{L}(\alpha, \beta, x) = \prod_{i=1}^{n} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$$

$$= \left(\frac{1}{\Gamma(\alpha)} \beta^\alpha\right)^n \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} e^{-\frac{1}{\beta}\Sigma x_i}$$

$$= \left(\frac{1}{\Gamma(\alpha)} \beta^\alpha\right)^n \cdot \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} e^{-\frac{T}{\beta}}$$, where α is known

⑤ Gamm(α, β) , $T = \Sigma x_i$, β is known.

same as in ④

$$\mathcal{L}(\alpha, \beta, x) = \left(\frac{1}{\Gamma(\alpha)} \beta^\alpha\right)^n \left(\prod_{i=1}^{n} x_i\right)^{\alpha-1} e^{-\frac{1}{\beta}\Sigma x_i}$$, where β is known.