HW4 Xiang XU

1. MLE of P

$n=70$. $\hat{p} = \frac{12}{70} = 0.1714$

$x_i \overset{iid}{\sim} Bern(p)$

$f_X(x; p) = \prod_{i=1}^{70} p^{x_i} (1-p)^{1-x_i}$

$= p^{\sum^{70} x_i} (1-p)^{\sum^{70} (1-x_i)}$

$\ell(p, x) = \sum^{70} x_i \log p + (70 - \sum^{70} x_i) \log(1-p)$

thus $\hat{p} = \frac{\sum x_i}{70} = \bar{x} = 0.1714$

2. $x_1 \cdots x_n \overset{iid}{\sim} Bern(\theta)$

if all obs are 1. $L(\theta; 1) = \theta^{\sum 1} = \theta^n$

$\ell(\theta; 1) = n \log \theta$ #

$L(\theta; 1) = \theta \uparrow$

$\frac{\partial \ell}{\partial \theta} = \frac{n}{\theta} = 0 \rightarrow n=0$. which impossible.

Same for obs are 0.

3. $x_1 \cdots x_n \sim Po(\lambda)$

① $f(x_1, \ldots, x_n | \lambda) = \frac{e^{-\lambda} \lambda^{x_1}}{x_1!} \cdots \frac{e^{-\lambda} \lambda^{x_n}}{x_n!} = \frac{e^{-n\lambda} \cdot \lambda^{\sum x_i}}{x_1! \cdots x_n!}$

$\ln f = -n\lambda + (\ln \lambda) \sum x_i - \ln(\prod x_i!)$

$\frac{d \ln f}{d \lambda} = -n + \frac{\sum x_i}{\lambda} = 0 \Rightarrow \hat{\lambda} = \frac{\sum x_i}{n}$

② if all obs $= 0$. $x_1 = \cdots = x_n = 0$

$\ln f = \ell(\lambda; x) = \ell(\lambda, 0) = \log e^{-n\lambda} = -n\lambda$

$\frac{\partial \ln f}{\partial \lambda} = -n = 0 \rightarrow n=0$ impossible.

so MLE, in this case, doesn't exist.

4. $x_1 \cdots x_n \overset{iid}{\sim} N(\mu, \sigma^2)$

$$-\frac{n}{2}\ln(2\pi) - \frac{n}{2}\ln\sigma^2 - \frac{1}{2\sigma^2}\Sigma(x_i-\mu)^2$$

$$\frac{\partial l}{\partial \sigma^2} = \frac{\frac{n}{2}}{\sigma^2} + \frac{4\sigma \cdot \Sigma(x_i-\mu)^2}{4\sigma^4} = 0.$$

thus $\hat{\sigma}^2 = \frac{1}{n}\sum^{n}(x_i-\mu)^2$, with μ know.