Hypothesis test.

1. $X \sim Exp(\lambda)$

$f(x) = \lambda e^{-\lambda x}$, $F(x) = 1 - e^{-\lambda x}$

$H_0: \lambda \geq 1$ $\quad$ $H_1: \lambda < 1$

① $B(\lambda) = P(X \geq 1 \mid \lambda) = 1 - P(X < 1) = e^{-\lambda x}$

② $\alpha = \sup\limits_{\lambda \geq 1} B(\lambda) = e^{-1} = 0.3678$

2. binomial dist° , 2-tail problem

$f(x) = \binom{n}{y} p^y (1-p)^{n-y}$

$n = 20$

thus $f(y) = \binom{20}{y} p^y (1-p)^{20-y}$

$H_0: p = 0.2$ $\quad$ $H_1: p \neq 0.2$

① $B(p) = P(y \geq 7 \mid p) + P(y < 1 \mid p)$

$= 1 - \sum_{0}^{6} \binom{20}{y} p^y (1-p)^{20-y} + \sum_{0}^{1} \binom{70}{7} p^y (1-p)^{20-y}$

$B(p=0) = 1,$ $\quad$ $B(p=0.1) = 0.394$

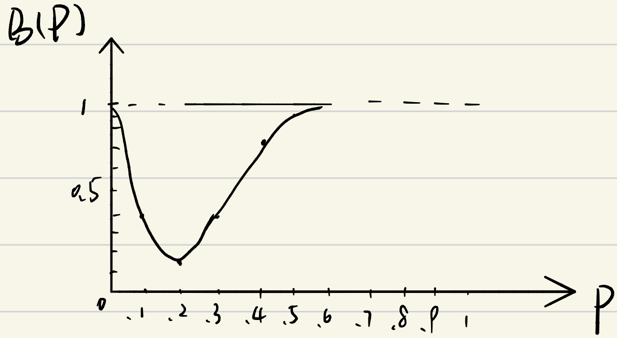
$B(p=0.2) = 0.1558$ $\quad$ $B(p=0.3) = 0.399$

$B(p=0.4) = 0.7505$ $\quad$ $B(p=0.5) = 0.9423$

$B(p=0.6) = 0.993$ $\quad$ $B(p=0.7) = 0.999$

$B(p=0.8) = 0.999$ $\quad$ $B(p=0.9) = 1$

$B(p=1) = 1$



② $\alpha = \sup_{p=0.2} B(p) = 0.1558$

$$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ \frac{(x-\mu)^2}{2\sigma^2} \right\}$$

$H_0: \mu = \mu_0$, vs. $H_1: \mu \neq \mu_0$

$$B(\mu) = P(T(x) > c) = P(|\bar{x} - \mu_0| > c)$$

$$= P\left(|Z| > \frac{\sqrt{n}\, c}{\sigma}\right) = 1 - \Phi\left(\frac{\sqrt{n}\, c}{\sigma}\right)$$

$\alpha = B(\mu) = 0.05$

$$c = \frac{\Phi^{-1}(0.975)}{\sqrt{n}} = \frac{1.96}{5} = 0.392$$

$$f(p, x) = \binom{9}{x} p^{x} (1-p)^{9-x}$$

$H_0: p = 0.4, \quad H_1: p \neq 0.4$

① $P(Y \le c_1 \mid p = 0.4) + P(Y \ge c_2 \mid p = 0.4) < 0.1$

thus. $c_1 = 1, \ c_2 = 7$

② $\beta(p) = P(Y \ge 7 \mid p) + P(Y \le 1 \mid p)$

$$= \sum_{0,1,7,8,9} \binom{9}{y} p^{y} (1-p)^{9-y}$$

$$\alpha = \sup_{p=0.4} \beta(p) = 0.0956$$