Xiang XU HW-MLE

1. Butterfly Mark

① $n=43$ to have 5

$\hat{p}_{MLE} = \frac{5}{43}$

② $\hat{p}_{MLE} = \frac{3}{58}$

2. $x_1 \sim x_n \sim U(0,\theta)$

$l(\theta|x) = \prod_{i=1}^{n} \frac{1}{\theta} = \theta^{-n}$

$\frac{\partial l(\theta|x)}{\partial \theta} = -\frac{n}{\theta} < 0$

thus $l(\theta|x) = \theta^{-n}$ is a $\downarrow$ function for $\theta \geq x_n$

thus $\arg\max(l(\theta|x)) = x_n$, $\hat{\theta} = x_n$

3. $x_1 \sim x_n \sim N(\mu,\sigma^2)$

MLE, 0.95 quantile, one-tail

MLE: $\mu + 1.64\sigma$

4. $x_1 \sim, x_n \sim N(\mu,\sigma^2)$

$MLE = P(X>2) = P(Z > \frac{2-\mu}{\sigma})$

$= 1 - \Phi(\frac{2-\mu}{\sigma})$

5. cauchy (θ)

$f(x;\theta) = \frac{1}{\pi} \cdot \frac{1}{1+(x-\theta)^2}$

$l(\theta;x) = -n\log\pi - \Sigma\log\left(1+(x_i-\theta)^2\right)$

$\frac{\partial l(\theta;x)}{\partial \theta} = 0$, using R: $\hat{\theta} = 0.407$

6. $\begin{cases} 20\text{ obs}^{\circ}: \bar{x}_{20} = 6 \\ 21\text{st obs}^{\circ}: x_{21} > 15 \end{cases}$

$\frac{1}{\mu^{20}} e^{-\frac{120}{\mu}} e^{-\frac{15}{\mu}}$

$= \frac{1}{\mu^{20}} \exp(-\frac{135}{\mu})$

$\hat{\mu} = \frac{135}{20} = 6.75$

7. $x_1 \sim x_n \sim Po(\lambda)$

$f(x|\lambda) = \frac{e^{-n\lambda} \lambda^{\Sigma x_i}}{x_1! \cdots x_n!}$

$\frac{\partial \ln f}{\partial \lambda} = -n + \frac{\Sigma x_i}{\lambda} = 0 \Rightarrow \hat{\lambda} = \frac{\Sigma x_i}{n}$

$\hat{sd} = \sqrt{\lambda} = \sqrt{\frac{\Sigma x_i}{n}}$

8. $x_1 \sim x_n \sim \exp(\beta)$

in exp dist° median $= \beta^{-1} \ln 2$

$l(\beta; x) = -n\ln(\beta) - \frac{1}{\beta}\Sigma x_i$

$\frac{\partial l}{\partial \beta} = 0 \Rightarrow \hat{\beta} = \frac{\Sigma x_i}{n}$

$\hat{\text{median}} = \ln 2 \sqrt{\frac{\Sigma x_i}{n}}$