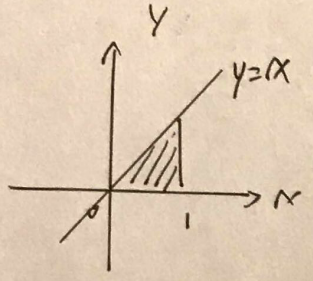
HW 2 Xiang Xu

(1.)

Y	Frequency	P(Y)
1	1	1/9
2	1	1/9
3	2	2/9
4	1	1/9
6	1	1/9
7	1	1/9
8	2	2/9

$$E(Y) = \frac{14}{3}$$

(2.)



$$E(XY) = \int_0^1 dx \int_0^x dy \; 12y^2 \cdot xy$$

$$= \frac{1}{2}$$

(3.) $(X_1 - 2X_2 + X_3)^2 = X_1^2 - 4X_1X_2 + 4X_2^2 + 2X_1X_3 - 4X_2X_3 + X_3^2$

Thus $E[(X_1 - 2X_2 + X_3)^2] = E(X_1^2) - 4E(X_1)E(X_2) + 2E(X_1)E(X_3)$

$$- 4E(X_2)E(X_3) + 4E(X_2^2) + E(X_3)^2$$

$X_i \sim Uni(0,1) \Rightarrow E(X_i) = \frac{1}{2}$, $E(X_i^2) = \frac{1}{3}$

$\therefore E[(X_1 - 2X_2 + X_3)^2] = \frac{1}{2}$

(4.)

$$E(Y) = \int_0^\infty e^{-x} e^{\frac{1}{4}x} dx = \int_0^\infty e^{-\frac{1}{4}x} dx = 4$$

(5) $X=1\ 2\ 3\ 4\ 5\ 6$

$Y=3\ 9\ 19\ 33\ 51\ 73$

$Y = g(X) = 2X^2+1$, $P(X)=P(Y)=\frac{1}{6}$

$E(Y)=\frac{1}{6}(3+9+19+33+51+73)=\frac{44}{3}$

(6.) $E(Y^2)=E\left((2X+1)^2\right)=4E(X^2)+4E(X)+1$

$E(X)=\frac{1}{2}$, $E(X^2)=\frac{1}{6}$

$E(Y^2)=4\times\frac{1}{2}+4\times\frac{1}{6}+1=3$

(7) $(ax+b)^n=\sum_{i=1}^{n}\binom{n}{i}a^{n-i}b^i x^{n-i}$

$E((ax+b)^n)=\sum_{i=0}^{n}\binom{n}{i}a^{n-i}b^i E(x^{n-i})$

(8) defective np X: # defective

good $n\cdot(1-p)$ Y: # good.

$n=20$, $p=5\%$. $X=1$, $Y=19$

$E(X-Y)=-18$