hw1_5.1&5.2

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Contents

5.1

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(a) Geometric distribution

(b) Find P (T > 3).

$$P(T > 3) = 1 - P(T <= 2)$$

$$P(T > 3) = 1 - (1/6 + 5/6 * 1/6)$$

$$P(T > 3) = 0.694$$

(c)

$$P(T > 6|T > 3) = \frac{P(T > 6) \cap P(T > 3)}{P(T > 3)}$$

$$P(T > 6|T > 3) = \frac{P(T > 6)}{P(T > 3)}$$

$$P(T > 6|T > 3) = \frac{0.402}{0.694}$$

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(a) Hyper-geometric distribution So

$$P(X = k) = \frac{\binom{n_2}{k}\binom{N-n_2}{n_1-k}}{\binom{N}{n_2}}$$

(b) Fixing $k = n_{12}$, we can find the maximum for N by seeing when

$$\frac{h(N+1, n_1, n_2, n_{12})}{h(N, n_1, n_2, n_{12})}$$

goes from positive to negative. This occurs when $N = \frac{n_1 n_2}{n_{12}}$.

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The probability of missing at most one call is composed of two parts. The probability of missing 0 call plus the probability of missing 1 call.

In this problem, lambda can be calculated as 0.01*300= 3 (miss / 5 minutes)

```
# Manual calculation
# Set e and lambda
e <- exp(1)
lda <- 0.01 * 300
# Chance of zero calls
c0 <- ((lda^0)*(e^-3)) / factorial(0)
# Chance of one call
c1 <- ((lda^1)*(e^-3)) / factorial(1)
# Add the two together
c0 + c1
```

```
## [1] 0.1991483
```

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(a)

```
# Lambda
lda <-1.2 #on average, 1.2 raisins in each cookie
c0 <- exp(-lda)
c0
```

```
## [1] 0.3011942
```

(b)

```
lda <- 400/500
k <- 2
c0 <- lda^k/factorial(k)*exp(-lda)
c0
```

```
## [1] 0.1437853
```

(c)

$$P(X >= 2) = 1 - P(X <= 1) = 1 - F_x(1)$$

p=1/500 n=1000

```r
1- ppois(1, lambda = 2)
```

```
## [1] 0.5939942
```

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lambda = 100*0.05 = 5 Paying the meter each time is 10*100=10dollars No paying estimated cost can be calculated by poisson distribution

```r
a1 <- 2 * dpois(1, lambda=5)
b<-0
for (i in 2:100) {
  b1<-(2+5*(i-2))*dpois(i,lambda=5)
  b <-b+b1
  b
}
b
```

```
## [1] 17.15497
```

```r
total <- a1 +b
total
```

```
## [1] 17.22235
```

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```r
lambda <- 0.001 *100
p <- 1- dpois(0, lambda = lambda)
p
```

```
## [1] 0.09516258
```

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The probability of not showing up is 4%. And lambda equals 4% time 100 passengers. We want to find out probability of P(x>2)=1-P(x=0)-P(X=1)

```r
lambda <- 0.04*100
p<- 1 - dpois(0, lambda)-dpois(1,lambda)
p
```

```
## [1] 0.9084218
```

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(a)

```r
p = 5*(5/20)*(15/20)^4
p
```

```
## [1] 0.3955078
```

(b)

```r
q <- dhyper(x=1, m=5, n=15, k=5, log = FALSE)
q
```

```
## [1] 0.440209
```

5.2

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(a)

$$F_Y(y) = P(U + 2 \leq y) = P(U \leq y - 2) = F_U(y - 2) = \frac{y - 2 - 0}{1 - 0} = y - 2$$

with $y \in [2, 3]$ $f_Y(y) = F_Y^{'}(y) = 1$ when $2 < y < 3$ $f_Y(y) = F_Y^{'}(y) = 0$ when otherwise

(b) $Y = U^3$, with $Y \in [0, 1]$ $F_Y(y) = P(U^3 \leq y) = P(U \leq y^{\frac{1}{3}}) = F_U(y^{\frac{1}{3}}) = y^{\frac{1}{3}}$ when $y \in [0, 1]$ Therefore, $f_Y(y) = F_Y^{'}(y) = \frac{1}{3}y^{-\frac{2}{3}}$ on [0,1]

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(a) when $0 \leq x \leq 1$: $f(x) = 2sin(\frac{\pi}{2}x)cos(\frac{\pi}{2}x)\frac{\pi}{2} = \frac{\pi}{2}sin(\pi x)$ otherwise: $f(x) = 0$

(b) $P(\bar{x} < \frac{1}{4}) = P(0 < \bar{x} < \frac{1}{4})$ Integrating $\frac{\pi}{2}sin(\pi x)$ from 0 to 1/4 we will get 0.1464.

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$F_Y(y) = P(Y \leq y) = P(F(X) \leq y) = P(\bar{x} \leq F^{-1}(y)) = F(F^{-1}(y)) = y$ on [0,1]

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$F_Y(y) = P(e^X \leq y) = P(X \leq lny) = \phi(lny)$

$$f_Y(y) = F_Y^{'}(y) = f_X(lny)(lny)^{'} = \frac{1}{\sqrt(2\pi)\sigma y}e^{-\frac{(lny-\mu)^2}{2\sigma^2}}$$

when y>0

$$f_Y(y) = F_Y^{'}(y) = f_X(lny)(lny)^{'} = 0$$

when $y \leq 0$