This document provide extra information about problem 3.

How to use this document:**You can read this document. However, when you write your own proof, you must set it aside.**

Other Materials that might help:

1. A proof that uses interesting metaphor: "humbel" and "cocky".Link.

4: Larger Cardinality Always Exists(10 pts **)**

Show that $|A| < |\mathcal{P}(A)|$, for any set A, where $\mathcal{P}(A)$ denotes the powerset of A.

[Hint: Suppose an onto (or bijection) function f existed between A and $\mathcal{P}(A)$. Let

$$T = \{a \in A | a \notin f(a)\}$$

and show that no element a can exist for which $f(a) = T$. If that happens, will T contains a?]

Fun fact: The argument goes somewhat like the The Paradox of the Barber(https://www.youtube.com/watch?v=nI-MMJiFpqo).

Proof. Let T be as mentioned above. Let's assume the opposite and let $f(a) = T$. The we will have the following paradox:

1. a can not be in T. If $a \in T$, then $a \in f(a)$, then by T's definition, $a \notin T$.
2. a can not be not in T. If $a \notin T$, then $a \in f(a)$, then by T's definition, $a \in T$.

Both cases results in contradiction. Therefore, the assumption $f(a) = T$ for some a fails, which means T does not have a preimage under f. Hence f is not an onto function. ☐