You should read Ch3 of the text, or Ch5 of the reference book Discrete Math and Its Application to make sure you are familiar with the basic definitions before you start working on this assignment.

1 Mathematical Induction

1: Cube Sum. (7th Ed P329 Q4, 10 pts) **

Let $P(n)$ be the statement that $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$ for positive integer n.

(a) What is the statement $P(1)$?

(b) Show that $P(1)$ is true, completing the basis step of the proof.

(c) What is the inductive hypothesis?

(d) What do you need to prove the inductive step?

(e) Complete the inductive step, identifying where you use the inductive hypothesis.

(f) Explain why these steps show that this formula is true whenever n is a positive integer.

2: Wrong Proof! (Text Page 24, 3.12. 10 pts)

Read carefully the following induction proof:

Assertion: $n(n+1)$ is an odd number for every n.

Proof. Suppose that this is true for $n-1$ in place of n; we prove it for n, using the induction hypothesis. We have $n(n+1) = (n-1)n + 2n$. Now here $(n-1)n$ is odd by the induction hypothesis, and $2n$ is even. Hence $n(n+1)$ is the sum of an odd number and an even number, which is odd. □

The assertion that we proved is obviously wrong for n = 10: $10 \cdot 11 = 110$ is even.

What is wrong with the proof? Give you answer in one short sentense.

3: Wrong Proof! (Text Page 24, 3.13. 10 pts)

Read carefully the following induction proof:

Assertion: If we have n lines in the plane, no two of which are parallel, then they all go through one point.

Proof. The assertion is true for one line (and also for 2, since we have assumed that no two lines are parallel). Suppose that it is true for any set of $n-1$ lines. We are going to prove that it is also true for n lines, using this induction hypothesis.

So consider a set of $S = \{a, b, c, d, \cdots\}$of n lines in the plane, no two of which are parallel. Delete the line c, then we are left with a set S' of $n-1$lines, and obviously no two of these are parallel. So we can apply the induction hypothesis and conclude that there is a point P such that all the lines in S' go through P . In particular, a and b go through P , and so P must be the point of intersection of a and b.

Now put c back and delete d, to get a set S'' of $n-1$ lines. Just as above, we can use the induction hypothesis to conclude that these lines go through the same point P' ; but just like above, P' must be the point of intersection of a and b. Thus $P' = P$. But then we see that c goes through P . The other lines also go through P (by the choice of P), and so all the n lines go through P . □

But the assertion we proved is clearly wrong; where is the error? Keep you answer short.

4: Postage Problem (10 pts ***)

Let $P(n)$ be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of this exercise outline a strong induction proof that $P(n)$ is true for $n \geq 18$.

(a) Show statements $P(18)$, $P(19)$, $P(20)$, and $P(21)$ are true, completing the basis step of the proof.

(b) What is the inductive hypothesis of the proof?

(c) What do you need to prove in the inductive step?

(d) Complete the inductive step for $k \geq 21$.

(e) Explain why these steps show that this statement is true whenever $n \geq 18$.

5: Line and Faces(10 pts ***)

Suppose that we draw n lines in the plane, in general position (no lines are parallel, no point belongs to more than two lines). The lines divide up the plane into a set of regions. Prove the following claim, for any positive integer n:

Claim: we can color these regions with two colors, such that adjacent regions (i.e. touching along an edge) never have the same color.

You can find hints here(https://mfleck.cs.illinois.edu/study-problems/easy-induction/easy-induction.html) if you get stuck.