Reading:

1: Trivial Equivalent Relation (10 pts) *

Consider all the binary strings of length 3, or S=$\{0,1\}^3$ If we say for $s,t \in S$, sRt if and only if $s = t$. Then what are all the equivalent classes?

2: Binary String Equivalence (10 pts) **

Let $S = \{0,1\}^4$, that is, all the binary strings of length 4. Let relation R be defined as for $s,t \in S$, sRt if and only if s and t has the same number of zeros.

(a) Show R is an equivalent relation.

(b) List all the equivalent classes. Please recall that a equivalent class is a subset of S, and the equivalent classes form a partition of S.

3: Binary String Equivalence II (10 pts) **

Let $S = \{0,1\}^4$,that is, all the binary strings of length 4. For s,t be a string in S, we define sRt if and only t can be obtained from s by rotatation. For example, we can get 1011 by rotating 0111 to the right.

(a) Show R is an equivalent relation.

(b) List all the equivalent classes.

4: Counting Partisions or Equivalent Class (10 pts) ***

Let $p(n)$ denote the number of different equivalence relations on a set with n elements (or equivalently, the number of partitions of a set with n elements). Show that $p(n)$ satisfies the recurrence relation

$$p(n) = \sum_{0}^{n-1} \binom{n-1}{j} p(n-1-j),$$

and the initial condition p(0) = 1.
Hint: Use recursive thinking. Consider the last element a_n, and the rest of $n-1$ elements. Is a_n going to be in the same partition with any of the rest of $n-1$ elements? How many?

5: Seven Bridges of Königsberg

Read/watch the following materials and write a report on Euler's analysis and rewrite his proof of his following theorem.

1. Seven Bridges of Königsberg
2. Euler Paths & the 7 Bridges of Konigsberg

Theorem 1. Euler's Theorem: A connected graph has an Euler cycle if and only if every vertex has even degree.

You might want to read the Wiki page for Eulerian Path for more information.

In case you can not open the link, here are the list of links:

1. https://en.wikipedia.org/wiki/Seven_Bridges_of_K%C3%B6nigsberg
2. https://www.youtube.com/watch?v=dSK5jTEe-AM
3. https://en.wikipedia.org/wiki/Eulerian_path