Interdisciplinary Centre
for Scientific Computing (IWR)
Ruprecht–Karls–Universität Heidelberg

Prof. Dr. Vincent Heuveline
M.Sc. Simon Gawlok
M.Sc. Saskia Haupt

Exercise Sheet 1

October 26th, 2017 – November 2nd, 2017

Uncertainty Quantification 1 (Winter Term 2017)

Exercise 1. Conditioning I (2+2 = 4 Points)

Investigate the conditioning of the following arithmetic operations:

a.
$$f(x_1, x_2) = \frac{x_1}{x_2} \quad (x_2 \neq 0)$$

b.
$$f(x_1, x_2) = x_1^{x_2} \quad (x_1 > 0)$$

Are the simple operations
$$f(x) = \frac{1}{x} \quad \text{and} \quad f(x) = \sqrt{x}$$
well-conditioned?

Exercise 2. Conditioning II (2+4 = 6 Points)

Consider the function
$$f(x) = \frac{1 - \cos(x)}{x}.$$

a. Determine the values of x, for which the evaluation of $f(x)$ is well- and ill-conditioned, respectively.

b. Derive an algorithm for the evaluation of $f(x)$, which is stable for $|x| \ll 1$. Assume, that $\cos(x)$ is computed with machine precision.
Hint: The representation of f can be transformed with the aid of calculation rules for trigonometric functions.

Practical Exercise 3. Machine precision and stable algorithm (2+3 = 5 Points)

a. Determine the machine precision of the used computer with the aid of a test program.
Hint: The result depends on your computer *and* the used programming language.

b. Write a program for the calculation of the values of the exponential function e^x with the aid of its Taylor sums

$$T_n(x) = \sum_{k=0}^{n} \frac{x^k}{k!}.$$

For $n \in [0, 20]$, plot the *relative error* for the arguments $x \in \{10, 1, -1, -10\}$. Explain, why the results are bad for negative arguments. Describe a modification of the program, such that the results are equally good for negative *and* positive arguments.

Submission: until November 2$^{\text{nd}}$, 2017, 11 a.m. (EMCL, INF 205, 1$^{\text{st}}$ floor, room 1/214 *or* room 1/232)