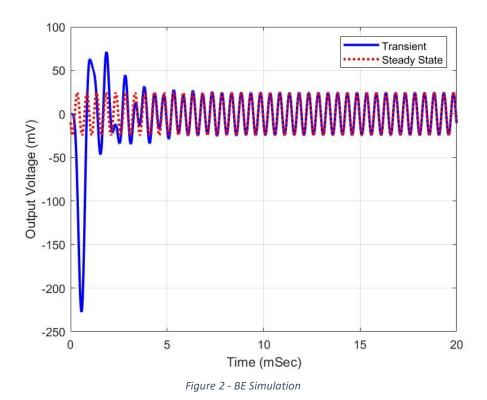
## **ECSE 597 Report**

## Q1

```
function [tpoints, y]= BE_method(tEnd,h, outNode)
% This function uses BACKWARD EULER method to compute the transient reponse
% of the circuit.
%Inputs: 1. tEnd: The simulation starts at time = 0s and ends at time =
                   tEND s.
         2. h: length of step size.
        3. outNode: is the node for which the transient is required.
%Output: 1. tpoints: list of time points.
         2. y: is the transient response at output node.
%Note: The function stub provided above is just an example. You can modify the
      in function in any fashion.
global elementList
tpoints = 0:h:tEnd;
Gmat = makeGmatrix;
Cmat = makeCmatrix;
out_NodeNumber = getNodeNumber(outNode) ;
[row,~] = size(Gmat);
X_n = zeros(row);
% x_n+1 = x_n + hx_n+1_dot
x_n+1_dot = (x_n+1 - x_n)/h
% Gx_n+1 + C
for I=1:length(tpoints)
    % make Bvector at time tpoints(I)
    Btr = makeBt(tpoints(I));
    % you can write your code here
    X_n = inv((Gmat+Cmat/h))*(Btr + (Cmat/h)*X_n);
    y(I) = X_n(out_NodeNumber);
end
end
```

Figure 1 - Backward Euler



As shown in Figure 2, the transient response gradually converges to the steady state response, which is as expected.

```
function [tpoints,y]= Trapezoidal_method(tEnd,h, outNode)
% This function uses Trapezoidal method to compute the transient reponse
% of the circuit.
%Inputs: 1. tEnd: The simulation starts at time = 0s and ends at
                  time = tEND s.
         2. h: length of step size.
         3. outNode: is the node for which the transient is required.
%Output: 1.y: is the transient response at output Node.
%Note: The function stub provided above is just an example. You can modify the
       in function in any fashion.
global elementList
out_NodeNumber = getNodeNumber(outNode) ;
tpoints = 0:h:tEnd;
Gmat = makeGmatrix;
Cmat = makeCmatrix;
[row, \sim] = size(Gmat);
X_n = zeros(row);
y(1) = 0;
for I=2:(length(tpoints))
   % make Bvector at time tpoints(I)
    Btr = makeBt(tpoints(I-1));
    Btr_1 = makeBt(tpoints(I));
    % you can write your code here
    X_n = inv((Gmat+(2/h)*Cmat))*(((2/h)*Cmat-Gmat)*X_n+Btr + Btr_1);
    y(I) = X_n(out_NodeNumber);
end
end
```

Figure 3 - Trapezoidal Rule

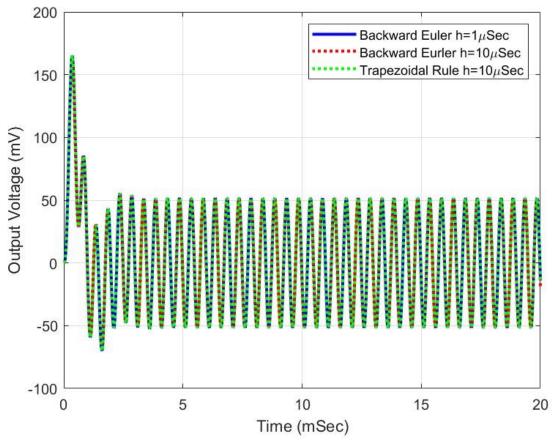


Figure 4 - TR Simulation

As shown in Figure 4, it can be deduced that both BE and TR methods can generates the same results and are both stable in most cases.

```
function [tpoints, y]= FE_method(tEnd,h, outNode)
\ensuremath{\text{\%}} This function uses FORWARD EULER method to compute the transient reponse
% of the circuit.
%Inputs: 1. tEnd: The simulation starts at time = 0s and ends at
                   time = tEND s.
%
         2. h: length of step size.
         3. outNode: is the node for which the transient is required.
%Output: 1.y: is the transient response at outNode.
%
%
%Note: The function stub provided above is just an example. You can modify the
% in function in any fashion.
global elementList
out_NodeNumber = getNodeNumber(outNode) ;
tpoints = 0:h:tEnd;
Gmat = makeGmatrix;
Cmat = makeCmatrix;
[row, \sim] = size(Gmat);
X_n = zeros(row);
y(1) = 0;
for I=2:(length(tpoints))
    % make Bvector at time tpoints(I)
    Btr = makeBt(tpoints(I-1));
    X_n = inv((Cmat/h))*(Btr-(Gmat-Cmat/h)*X_n);
    % you can write your code here
    y(I) = X_n(out_NodeNumber);
end
end
```

Figure 6 - Forward Euler

```
Gmat = makeGmatrix;
Cmat = makeCmatrix;

65
Cmat = makeCmatrix;

66

67
poles = eig((-1) * Cmat\Gmat)

68
```

Figure 5 - Pole Calculation

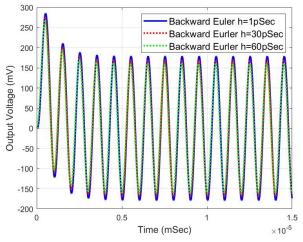


Figure 10 - BE Simulation Q3

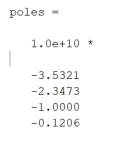


Figure 9 - Pole Values

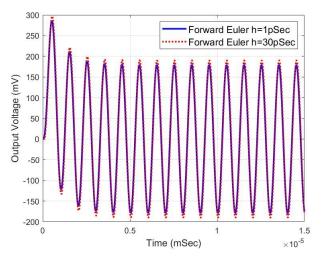


Figure 8 - FE Simulation (1 ps & 30 ps)

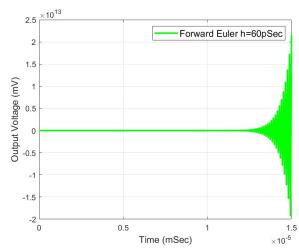


Figure 7 - FE Simulation (60 ps)

As shown in Figure 7 and 9, FE method is stable when step size h is 1 ps and 30 ps, and the results are the same as the BE method.

According to the property of FE method mentioned in class, the value of poles and the step size need to satisfy the condition of  $-2 < h\lambda < 0$  to have a stable simulation. By examine the poles with the eigen value function as shown in Figure 6, where the values are shown in Figure 8, the step size needs to be considered for the most negative pole of -3.5321e10. In this case, the step size h needs to be smaller than 56.624 ps to have a stable simulation for FE. This is verified in Figure 10, where the simulation diverges when the step size is 60 ps.

```
1 -
       function [D,S] = sens_perturbation_method(fpoints,eleNames,outNode)
 2
       % find elements
 3
       global elementList
 4
       flag = 0
 5
       ori_Gmat = makeGmatrix;
 6
       ori Cmat = makeCmatrix;
 7
       out_NodeNumber = getNodeNumber(outNode);
 8
       ori_r = fsolve( fpoints ,outNode, ori_Gmat, ori_Cmat);
 9 -
       for m = 1:length(eleNames)
10
           element = eleNames(m);
11
           Gmat = ori Gmat;
           Cmat = ori_Cmat;
12
13
           flag = 0;
14 🗀
           for I=1:elementList.Resistors.numElements
15
               name = elementList.Resistors.Name(I);
16
               if strcmp(name,element)
17
                    nodes = elementList.Resistors.nodeNumbers(I,:);
18
                    g = 1/( elementList.Resistors.value(I));
19
                    g delta = (1.0000001/( elementList.Resistors.value(I)));
20
                    delta_lamda = 0.0000001/( elementList.Resistors.value(I));
21
                    lamda = g;
22
                    if(nodes(1)\sim=0) \&\& (nodes(2)\sim=0)
23
                        Gmat(nodes(1), nodes(1)) = Gmat(nodes(1), nodes(1)) - g + g_delta;
24
                        Gmat(nodes(1),nodes(2)) = Gmat(nodes(1),nodes(2)) + g - g delta;
25
                        Gmat(nodes(2),nodes(1)) = Gmat(nodes(2),nodes(1)) + g - g_delta;
26
                        Gmat(nodes(2),nodes(2)) = Gmat(nodes(2),nodes(2)) - g + g_delta;
27
                    elseif (nodes(1)==0) && (nodes(2)\sim=0)
28
29
                        Gmat(nodes(2),nodes(2)) = Gmat(nodes(2),nodes(2)) - g + g delta;
30
31
                    elseif (nodes(1)\sim=0) && (nodes(2)==0)
32
                        Gmat(nodes(1), nodes(1)) = Gmat(nodes(1), nodes(1)) - g + g delta;
33
                    end
34
                    flag = 1;
35
                    break;
36
               end
37
38 -
           end
```

Figure 11 - Perturbation Method (Part A)

```
39
           if flag == 0
40
               for I=1:elementList.Capacitors.numElements
41
                    name = elementList.Capacitors.Name(I);
42
                    if strcmp(name,element)
43
                        nodes = elementList.Capacitors.nodeNumbers(I,:);
44
                        c= elementList.Capacitors.value(I);
                        c_delta= elementList.Capacitors.value(I)*1.0000001;
45
46
                        delta_lamda = elementList.Capacitors.value(I)*0.0000001;
47
                        lamda = c;
48
                        if(nodes(1)\sim=0) \&\& (nodes(2)\sim=0)
49
                            Cmat(nodes(1), nodes(1)) = Cmat(nodes(1), nodes(1)) - c + c_delta;
50
                            Cmat(nodes(1),nodes(2)) = Cmat(nodes(1),nodes(2)) + c - c_delta;
51
                            Cmat(nodes(2),nodes(1)) = Cmat(nodes(2),nodes(1)) + c - c_delta;
52
                            Cmat(nodes(2),nodes(2)) = Cmat(nodes(2),nodes(2)) - c + c_delta;
53
                        elseif (nodes(1)==0) \&\& (nodes(2)\sim=0)
54
55
                            Cmat(nodes(2),nodes(2)) = Cmat(nodes(2),nodes(2)) - c + c_delta;
56
57
                        elseif (nodes(1)\sim=0) && (nodes(2)==0)
58
                            Cmat(nodes(1),nodes(1)) = Cmat(nodes(1),nodes(1)) - c + c_delta;
59
                        end
60
                        break;
61
                    end
62
               end
           end
63
64
65
           % now we have the changed G and C
66
67
           delta_r = fsolve( fpoints ,outNode, Gmat, Cmat) - ori_r;
68
           D(:,m) = delta_r/delta_lamda;
69
           for I = 1:length(fpoints)
70
               S(I,m) = D(I,m)*(lamda/ori_r(I));
71
           end
72
       end
73
74
       end
75
```

Figure 12 - Perturbation (Part B)

```
1 -
       function [D,S] = sens_differentiation_method(fpoints,eleNames,out)
 2
       global elementList
 3
       flag = 0;
 4
       ori_Gmat = makeGmatrix;
 5
       ori Cmat = makeCmatrix;
 6
       f_r_v = f_solve_vector(fpoints, ori_Gmat, ori_Cmat);
 7
       out_NodeNumber = getNodeNumber(out) ;
 8
       lamda = 0;
 9
       for m = 1:length(eleNames)
10
           delta_Gmat = zeros(size(ori_Gmat));
11
           delta_Cmat = zeros(size(ori_Cmat));
12
           element = eleNames(m);
13
           flag = 0;
14 🗀
           for I=1:elementList.Resistors.numElements
15
                name = elementList.Resistors.Name(I);
16
                lamda = 1/elementList.Resistors.value(I);
17
                if strcmp(name,element)
18
                    nodes = elementList.Resistors.nodeNumbers(I,:);
19
                    if(nodes(1) \sim = 0) & (nodes(2) \sim = 0)
20
                        delta_Gmat(nodes(1), nodes(1)) = 1;
21
                        delta Gmat(nodes(1),nodes(2)) = -1;
22
                        delta_Gmat(nodes(2),nodes(1)) = -1;
23
                        delta Gmat(nodes(2),nodes(2)) = 1;
24
                    elseif (nodes(1)==0) && (nodes(2)\sim=0)
25
26
                        delta_Gmat(nodes(2), nodes(2)) = 1;
27
28
                    elseif (nodes(1)\sim=0) && (nodes(2)==0)
29
                        delta_Gmat(nodes(1), nodes(1)) = 1;
30
                    end
31
                    flag = 1;
32
                    break;
33
                end
34
35 -
           end
```

Figure 13 - Differentiation Method (Part A)

```
36
           if flag == 0
37 =
                for I=1:elementList.Capacitors.numElements
38
                    name = elementList.Capacitors.Name(I);
39
                    lamda = elementList.Capacitors.value(I);
40
                    if strcmp(name,element)
41
                        nodes = elementList.Capacitors.nodeNumbers(I,:);
42
                        c= elementList.Capacitors.value(I);
43
                        c_delta= elementList.Capacitors.value(I)*1.01;
44
                        if(nodes(1) \sim = 0) && (nodes(2) \sim = 0)
45
                            delta_Cmat(nodes(1),nodes(1)) = 1;
46
                            delta_Cmat(nodes(1),nodes(2)) = -1;
47
                            delta Cmat(nodes(2),nodes(1)) = -1;
48
                            delta_Cmat(nodes(2),nodes(2)) = 1;
49
                        elseif (nodes(1)==0) && (nodes(2)\sim=0)
50
51
                            delta_Cmat(nodes(2),nodes(2)) = 1;
52
53
                        elseif (nodes(1)\sim=0) && (nodes(2)==0)
54
                            delta_Cmat(nodes(1), nodes(1)) = 1;
55
                        end
                        break;
56
57
                    end
58
                end
59
           end
60
61
           % now we have the changed delta_G and delta_C -> dA/dlamda = delta_G +
           % jw*delta_C
62
           % loop over all frequency, select outnode
63
64
65 =
            for I = 1:length(fpoints)
66
                A = (ori_Gmat+2*pi*fpoints(I)*1i*ori_Cmat);
67
                delta_abs = A\((-1)*(delta_Gmat+2*pi*fpoints(I)*1i*delta_Cmat)*transpose(f_r_v(I,:)));
68
                D(I,m) = delta_abs(out_NodeNumber);
69
                S(I,m) = D(I,m)*(lamda/f_r_v(I,out_NodeNumber));
70
            end
71 -
       end
```

Figure 14 - Differentiation Method (Part B)

```
1 -
       function [D,S] = sens_adjoint_method(fpoints,eleNames,out)
 2
       global elementList
 3
       flag = 0;
 4
       ori_Gmat = makeGmatrix;
 5
       ori_Cmat = makeCmatrix;
 6
       f_r_v = f_solve_vector(fpoints, ori_Gmat, ori_Cmat);
 7
       out_NodeNumber = getNodeNumber(out) ;
 8
       lamda = 0;
 9
       d = zeros(length(ori_Gmat));
10
       d(out_NodeNumber) = 1;
11
12 -
       for I = 1:length(fpoints)
13
           A = (ori_Gmat+2*pi*fpoints(I)*1i*ori_Cmat);
14
            A f(I,:,:) = d/A; % precalculate Xat for all frequency
15
16
17 🗀
       for m = 1:length(eleNames)
18
           delta_Gmat = zeros(size(ori_Gmat));
19
           delta_Cmat = zeros(size(ori_Cmat));
20
           element = eleNames(m);
21
           flag = 0;
22
           for I=1:elementList.Resistors.numElements
23
24
                % access nodes Numbers of the Resistor
25
                %nodes of a I^{th} element are located in Row I of the nodeNumbers
26
                %field
27
                name = elementList.Resistors.Name(I);
28
                lamda = 1/elementList.Resistors.value(I);
29
                if strcmp(name,element)
30
                    nodes = elementList.Resistors.nodeNumbers(I,:);
31
32
                    % get the conductance for the resisitor
33
                    % resistance is stored in the field named value
34
                    if(nodes(1) \sim = 0) && (nodes(2) \sim = 0)
35
                        delta_Gmat(nodes(1), nodes(1)) = 1;
36
                        delta_Gmat(nodes(1),nodes(2)) = -1;
37
                        delta_Gmat(nodes(2),nodes(1)) = -1;
38
                        delta_Gmat(nodes(2), nodes(2)) = 1;
39
                    elseif (nodes(1)==0) && (nodes(2)\sim=0)
40
41
                        delta Gmat(nodes(2), nodes(2)) = 1;
42
43
                    elseif (nodes(1)\sim=0) && (nodes(2)==0)
44
                        delta_Gmat(nodes(1), nodes(1)) = 1;
45
                    end
46
                    flag = 1;
47
                    break;
                end
48
49
           end
```

Figure 15 - Adjoint Method (Part A)

```
50
           if flag == 0
51 🗀
                for I=1:elementList.Capacitors.numElements
52
                    name = elementList.Capacitors.Name(I);
53
                    lamda = elementList.Capacitors.value(I);
54
                    if strcmp(name,element)
55
                        nodes = elementList.Capacitors.nodeNumbers(I,:);
56
                        c= elementList.Capacitors.value(I);
57
                        c_delta= elementList.Capacitors.value(I)*1.01;
58
                        if(nodes(1)\sim=0) && (nodes(2)\sim=0)
59
                            delta_Cmat(nodes(1), nodes(1)) = 1;
60
                            delta_Cmat(nodes(1),nodes(2)) = -1;
61
                            delta_Cmat(nodes(2),nodes(1)) = -1;
62
                            delta_Cmat(nodes(2),nodes(2)) = 1;
63
                        elseif (nodes(1)==0) && (nodes(2)\sim=0)
64
65
                            delta_Cmat(nodes(2),nodes(2)) = 1;
66
67
                        elseif (nodes(1) \sim = 0) && (nodes(2) = = 0)
68
                            delta_Cmat(nodes(1), nodes(1)) = 1;
69
                        end
70
                        break;
71
                    end
72
                end
73
           end
74
75 🗀
           for I = 1:length(fpoints)
                delta\_abs = A\_f(I,:,:)*(delta\_Gmat+2*pi*fpoints(I)*1i*delta\_Cmat)*transpose(f\_r\_v(I,:));
76
77
                D(I,m) = delta_abs;
                S(I,m) = D(I,m)*(lamda/f_r_v(I,out_NodeNumber));
78
79
           end
80
81
       end
```

Figure 16 - Adjoint Method (Part B)

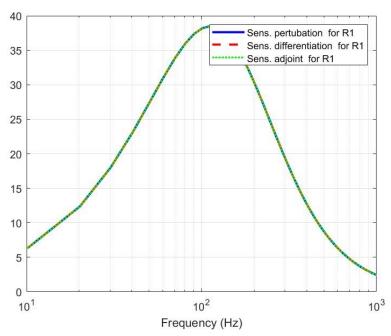


Figure 18 - R1 Absolute

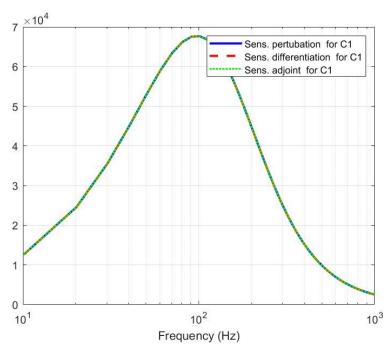


Figure 17 - C1 Absolute

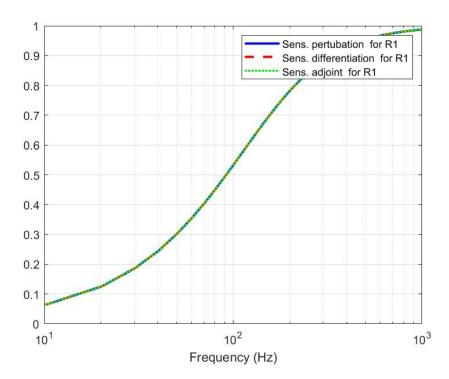


Figure 20 - R1 Relative

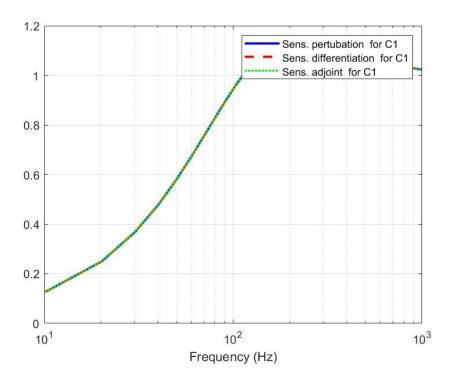


Figure 19 - C1 Relative