Math 16-811 - HW1

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1 Question 1

implemented code:

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x = 1;
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2 Question 2

2.a

$$A = \begin{pmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{pmatrix} \tag{1}$$

Let $L = I, A' = A$

L	**A´**	**Operations**
$\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 4 & 7 & 0 \\ 0 & -\frac{3}{2} & -6 \\ 0 & \frac{1}{4} & 1 \end{pmatrix}$	$R_2 = R_2 - \frac{1}{2}R_1$ $R_3 = R_3 - \frac{1}{4}R_1$
$\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & -\frac{1}{6} & 1 \end{pmatrix}$	$\begin{pmatrix} 4 & 7 & 0 \\ 0 & -\frac{3}{2} & -6 \\ 0 & 0 & 0 \end{pmatrix}$	$R_3 = R_3 + \frac{1}{6}R_1$

$D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$Q = \begin{pmatrix} 1 & \frac{7}{4} & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

SVD Decomposition

$u = \begin{pmatrix} -0.8478 & 0.4286 & -0.3123 \\ -0.4908 & -0.8571 & 0.1562 \\ -0.2008 & 0.2857 & 0.9370 \end{pmatrix}$

$$s = \begin{pmatrix} 9.0554 & 0 & 0 \\ 0 & 5.7446 & 0 \\ 0 & 0 & 0.0000 \end{pmatrix}$$

$$v = \begin{pmatrix} -0.5051 & 0.0497 & 0.8616 \\ -0.8081 & 0.3233 & -0.4924 \\ 0.3030 & 0.9450 & 0.1231 \end{pmatrix}$$

2.b

Let $L = I, A' = A$

L	**A´**	**Operations**
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 4 & 8 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 4 & -1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$	$R_2 = R_2 - \frac{1}{2}R_1$
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 4 & 8 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$	$R_3 = R_3 + R_2$ $R_4 = R_3 - \frac{1}{2}R_2$
$\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & \frac{2}{3} & \frac{1}{2} \end{pmatrix}$	$\begin{pmatrix} 4 & 8 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$	$R_5 = R_5 + \frac{2}{3}R_3$ $R_5 = R_5 + \frac{1}{2}R_4$

$$D = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

SVD Decomposition

$$u = \begin{pmatrix} -0.9005 & -0.0155 & 0.3562 & -0.1617 & -0.1895 \\ -0.0891 & -0.7311 & 0.1847 & 0.5290 & 0.3789 \\ -0.3787 & 0.0340 & -0.7352 & -0.1479 & 0.5413 \\ 0.1942 & -0.3642 & 0.2836 & -0.8023 & 0.3248 \\ 0.0078 & 0.5757 & 0.4670 & 0.1686 & 0.6496 \end{pmatrix}$$

$$s = \begin{pmatrix} 9.8844 & 0 & 0 & 0 \\ 0 & 3.3506 & 0 & 0 \\ 0 & 0 & 2.3134 & 0 \\ 0 & 0 & 0 & 1.9288 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$v = \begin{pmatrix} -0.3824 & -0.4549 & 0.7755 & 0.2132 \\ -0.9214 & 0.2210 & -0.2847 & -0.1456 \\ 0.0579 & 0.7699 & 0.5619 & -0.2970 \\ 0.0385 & -0.3892 & 0.0434 & -0.9193 \end{pmatrix}$$

2.c

Let $L = I, A' = A$

L	**A´**	**Operations**
$\begin{pmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 2 & 2 & 5 \\ 0 & -1 & -\frac{5}{2} \\ 0 & 0 & \frac{5}{2} \end{pmatrix}$	$R_2 = R_2 - \frac{3}{2}R_1$ $R_3 = R_3 - \frac{1}{2}R_1$

$$D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \frac{5}{2} \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & 1 & \frac{5}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

SVD Decomposition

$$u = \begin{pmatrix} -0.5859 & -0.0444 & -0.8091 \\ -0.6231 & -0.6138 & 0.4849 \\ -0.5182 & 0.7882 & 0.3319 \end{pmatrix}$$

$$s = \begin{pmatrix} 9.7910 & 0 & 0 \\ 0 & 1.4162 & 0 \\ 0 & 0 & 0.3606 \end{pmatrix}$$

$$v = \begin{pmatrix} -0.3635 & -0.8063 & 0.4666 \\ -0.2999 & -0.3729 & -0.8781 \\ -0.8820 & 0.4591 & 0.1062 \end{pmatrix}$$

3 Question 3

3.a

the following row reduce shows that b is in the column space of A $\begin{pmatrix} 2 & 1 & 3 & 5 \\ 2 & 1 & 2 & -5 \\ 5 & 5 & 5 & 0 \end{pmatrix}$

$R_2 = R_2 - R_1$
$R_2 = -R_2$

$$\begin{pmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & 1 & 10 \\ 5 & 5 & 5 & 0 \end{pmatrix}$$

$R_1 = R_1 - 3R_2$

$$\begin{pmatrix} 2 & 1 & 0 & -25 \\ 0 & 0 & 1 & 10 \\ 5 & 5 & 5 & 0 \end{pmatrix}$$

$R_3 = R_3 - 5R_1$

$R_3 = R_3 - 5R_2$

$R_3 = R_3 / -5$ $R_1 = R1 - 2R3$ $\begin{pmatrix} 0 & 1 & 0 & -21 \\ 0 & 0 & 1 & 10 \\ 1 & 0 & 0 & -2 \end{pmatrix}$

the last matrix is a consistent system, this shows that b is in column space of A. Therefore the $x = V\sigma$ is the only solution

SVD Decomposition

$$u = \begin{pmatrix} -0.3635 & 0.8063 & 0.4666 \\ -0.2999 & 0.3729 & -0.8781 \\ -0.8820 & -0.4591 & 0.1062 \end{pmatrix}$$

$$s = \begin{pmatrix} 9.7910 & 0 & 0 \\ 0 & 1.4162 & 0 \\ 0 & 0 & 0.3606 \end{pmatrix}$$

$$v = \begin{pmatrix} -0.5859 & 0.0444 & -0.8091 \\ -0.5182 & -0.7882 & 0.3319 \\ -0.6231 & 0.6138 & 0.4849 \end{pmatrix}$$

3.b

SVD Decomposition

$$u = \begin{pmatrix} -0.8478 & 0.4286 & -0.3123 \\ -0.4908 & -0.8571 & 0.1562 \\ -0.2008 & 0.2857 & 0.9370 \end{pmatrix}$$

$$s = \begin{pmatrix} 9.0554 & 0 & 0 \\ 0 & 5.7446 & 0 \\ 0 & 0 & 0.0000 \end{pmatrix}$$

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3.c

SVD Decomposition

$$u = \begin{pmatrix} -0.8478 & 0.4286 & -0.3123 \\ -0.4908 & -0.8571 & 0.1562 \\ -0.2008 & 0.2857 & 0.9370 \end{pmatrix}$$

$$s = \begin{pmatrix} 9.0554 & 0 & 0 \\ 0 & 5.7446 & 0 \\ 0 & 0 & 0.0000 \end{pmatrix}$$

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4 Question 4

4.1 4.d

While the property of an idempotent matrix is AA = A. The following is a proof that any element in the uu^T element will lead to itself.

$$Letuu^t = A = \begin{pmatrix} {p_1}^2 & p_1p_2 & p_1p_3 & \dots & p_1p_n \\ p_1p_2 & {p_2}^2 & p_2p_3 & \dots & p_2p_n \\ \dots & \ddots & \ddots & \ddots & \dots \\ p_1p_n & p_np_2 & p_np_3 & \dots & p_np_n \end{pmatrix} \tag{2}$$

The following is the multiplication for the first element.

$$AA(1,1) = {u_1}^2 + (u_1u_2)^2 + (u_1u_3)^2 + \cdots + (u_1u_n)^2 = {u_1}^2 \sum_{i=1}^{n} {u_i}^2 \tag{3}$$

since we know the length of the unit vector is $= 1 = \sqrt{\sum_{i=1}^{n} {u_i}^2} = 1 = \sum_{i=1}^{n} {u_i}^2$. By subing it into the equations, we get $= {u_1}^2$ which is the same as the original element. Because of the symmetric property of matrix. All of the elements are follow similar construct and can be prove using the same method as above. Therefore $A^2 = AA = A$

5 Question 5

This work was based on work titled Least-Squares Fitting of Two 3-D Point Sets by Arun.