Assignment 5

Robotics 811, Fall 2015

DUE: Thursday, November 12, 2015

1. Consider a plane curve $y(x)$ over the interval $[x_0, x_1]$, with specified endpoints $y_0 = y(x_0)$ and $y_1 = y(x_1)$. Assume that $y_0 > 0$ and $y_1 > 0$ and that $y(x) \geq 0$ for $x_0 \leq x \leq x_1$. Now imagine rotating the curve about the x-axis to obtain a surface of revolution. Find the C^2 curve $y(x)$ with specified endpoints that minimizes the surface area of this surface of revolution.

   [Hint: This problem explores further some of the limitations of the Calculus of Variations. Depending on the endpoint conditions there may or may not be a C^2 solution. How do the endpoint conditions matter? The optimal "curve" may sometimes not be C^2. What does the curve look like in those cases?]

2. Show that the shortest curve between two points on a sphere is an arc of a great circle.

   [Hints: Use spherical (u, v) coordinates, where $x = R \sin v \cos u$, $y = R \sin v \sin u$, $z = R \cos v$, with R the radius of the sphere. Cast 3D arclength $\sqrt{dx^2 + dy^2 + dz^2}$ into (u, v) space, and parametrize the curve in terms of the coordinate u.

   Observe that the integrand in the expression for arclength does not depend on u.

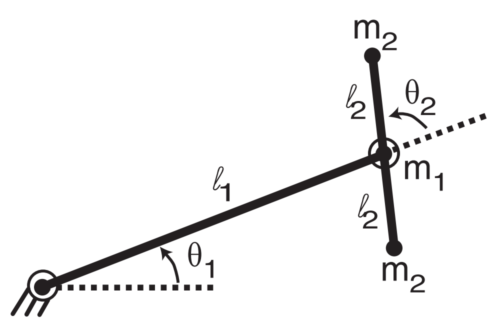
   You may find the following identity useful:

$$\int \frac{a\, dw}{\sqrt{\sin^4 w - a^2 \sin^2 w}} = -\sin^{-1}\left(\frac{\cot w}{\sqrt{\frac{1}{a^2} - 1}}\right) + k,$$

   where a and k are appropriate constants.]

3. In the brachistochrone problem, suppose the right endpoint is constrained only to touch some curve given implicitly by an equation of the form $g(x, y) = 0$. Show that the optimizing curve $y(x)$ must touch the iso-contour $g(x, y) = 0$ orthogonally.

4. (a) Using Lagrangian Dynamics, derive the relationship between joint torques and the angular state (angles, velocities, and accelerations) of the following balanced manipulator:



   There is no gravity (in practice, gravity is perpendicular to the sheet of the paper).
   Legend: All of link #1's mass, m_1, is concentrated at distance ℓ_1 from its rotational joint (which is attached to the ground). In turn, link #2 rotates around this distal point, with two masses, m_2, located symmetrically, each at distance ℓ_2, from the joint. In practice, these two masses might constitute one counter-balanced end-effector or two different but equally weighted end-effectors. — This is a variation of a basic Scara-type robot arm, often used in industrial assembly, for instance by SONY.

   (b) When $\ddot{\theta}_2 = 0$, explain the terms relating $\ddot{\theta}_1$ to τ_1.