# Math 16-811 - HW1

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## 1 Question 1

implemented code:

```
x = 1;
```

## 2 Question 2

### 2.a

$$A = \begin{pmatrix} 4 & 7 & 0 \\ 2 & 2 & -6 \\ 1 & 2 & 1 \end{pmatrix} \tag{1}$$

Let $L = I, A' = A$

| **L** | **A´** | **Operations** |
|---|---|---|
| $\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} 4 & 7 & 0 \\ 0 & -\frac{3}{2} & -6 \\ 0 & \frac{1}{4} & 1 \end{pmatrix}$ | $R_2 = R_2 - \frac{1}{2}R_1$<br>$R_3 = R_3 - \frac{1}{4}R_1$ |
| $\begin{pmatrix} 1 & 0 & 0 \\ \frac{1}{2} & 1 & 0 \\ \frac{1}{4} & -\frac{1}{6} & 1 \end{pmatrix}$ | $\begin{pmatrix} 4 & 7 & 0 \\ 0 & -\frac{3}{2} & -6 \\ 0 & 0 & 0 \end{pmatrix}$ | $R_3 = R_3 + \frac{1}{6}R_1$ |

$D = \begin{pmatrix} 4 & 0 & 0 \\ 0 & -\frac{3}{2} & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$Q = \begin{pmatrix} 1 & \frac{7}{4} & 0 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{pmatrix}$

SVD Decomposition

$u = \begin{pmatrix} -0.8478 & 0.4286 & -0.3123 \\ -0.4908 & -0.8571 & 0.1562 \\ -0.2008 & 0.2857 & 0.9370 \end{pmatrix}$

$$s = \begin{pmatrix} 9.0554 & 0 & 0 \\ 0 & 5.7446 & 0 \\ 0 & 0 & 0.0000 \end{pmatrix}$$

$$v = \begin{pmatrix} -0.5051 & 0.0497 & 0.8616 \\ -0.8081 & 0.3233 & -0.4924 \\ 0.3030 & 0.9450 & 0.1231 \end{pmatrix}$$

## 2.b

Let $L = I, A' = A$

| **L** | **A´** | **Operations** |
|---|---|---|
| $\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 4 & 8 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 4 & -1 & 0 \\ 0 & -2 & 0 & 2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$ | $R_2 = R_2 - \frac{1}{2}R_1$ |
| $\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | $\begin{pmatrix} 4 & 8 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & -1 \end{pmatrix}$ | $R_3 = R_3 + R_2$<br>$R_4 = R_3 - \frac{1}{2}R_2$ |
| $\begin{pmatrix} 1 & 0 & 0 & 0 \\ \frac{1}{2} & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -\frac{1}{2} & 0 & 1 \\ 0 & 0 & \frac{2}{3} & \frac{1}{2} \end{pmatrix}$ | $\begin{pmatrix} 4 & 8 & 0 & 0 \\ 0 & -4 & -2 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$ | $R_5 = R_5 + \frac{2}{3}R_3$<br>$R_5 = R_5 + \frac{1}{2}R_4$ |

$$D = \begin{pmatrix} 4 & 0 & 0 & 0 \\ 0 & -4 & 0 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & 2 & 0 & 0 \\ 0 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

SVD Decomposition

$$u = \begin{pmatrix} -0.9005 & -0.0155 & 0.3562 & -0.1617 & -0.1895 \\ -0.0891 & -0.7311 & 0.1847 & 0.5290 & 0.3789 \\ -0.3787 & 0.0340 & -0.7352 & -0.1479 & 0.5413 \\ 0.1942 & -0.3642 & 0.2836 & -0.8023 & 0.3248 \\ 0.0078 & 0.5757 & 0.4670 & 0.1686 & 0.6496 \end{pmatrix}$$

$$s = \begin{pmatrix} 9.8844 & 0 & 0 & 0 \\ 0 & 3.3506 & 0 & 0 \\ 0 & 0 & 2.3134 & 0 \\ 0 & 0 & 0 & 1.9288 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$v = \begin{pmatrix} -0.3824 & -0.4549 & 0.7755 & 0.2132 \\ -0.9214 & 0.2210 & -0.2847 & -0.1456 \\ 0.0579 & 0.7699 & 0.5619 & -0.2970 \\ 0.0385 & -0.3892 & 0.0434 & -0.9193 \end{pmatrix}$$

### 2.c

Let $L = I, A' = A$

| **L** | **A´** | **Operations** |
|---|---|---|
| $\begin{pmatrix} 1 & 0 & 0 \\ \frac{3}{2} & 1 & 0 \\ \frac{1}{2} & 0 & 1 \end{pmatrix}$ | $\begin{pmatrix} 2 & 2 & 5 \\ 0 & -1 & -\frac{5}{2} \\ 0 & 0 & \frac{5}{2} \end{pmatrix}$ | $R_2 = R_2 - \frac{3}{2}R_1$ $R_3 = R_3 - \frac{1}{2}R_1$ |

$D = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & \frac{5}{2} \end{pmatrix}$

$Q = \begin{pmatrix} 1 & 1 & \frac{5}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

SVD Decomposition

$u = \begin{pmatrix} -0.5859 & -0.0444 & -0.8091 \\ -0.6231 & -0.6138 & 0.4849 \\ -0.5182 & 0.7882 & 0.3319 \end{pmatrix}$

$$s = \begin{pmatrix} 9.7910 & 0 & 0 \\ 0 & 1.4162 & 0 \\ 0 & 0 & 0.3606 \end{pmatrix}$$
$$v = \begin{pmatrix} -0.3635 & -0.8063 & 0.4666 \\ -0.2999 & -0.3729 & -0.8781 \\ -0.8820 & 0.4591 & 0.1062 \end{pmatrix}$$

# 3 Question 3

### 3.a

the following row reduce shows that b is in the column space of A $\begin{pmatrix} 2 & 1 & 3 & 5 \\ 2 & 1 & 2 & -5 \\ 5 & 5 & 5 & 0 \end{pmatrix}$

$R_2 = R_2 - R_1$
$R_2 = -R_2$
$\begin{pmatrix} 2 & 1 & 3 & 5 \\ 0 & 0 & 1 & 10 \\ 5 & 5 & 5 & 0 \end{pmatrix}$

$R_1 = R_1 - 3R_2$
$$\begin{pmatrix} 2 & 1 & 0 & -25 \\ 0 & 0 & 1 & 10 \\ 5 & 5 & 5 & 0 \end{pmatrix}$$
$R_3 = R_3 - 5R_1$
$R_3 = R_3 - 5R_2$
$R_3 = R_3 / -5$ $R_1 = R1 - 2R3$ $\begin{pmatrix} 0 & 1 & 0 & -21 \\ 0 & 0 & 1 & 10 \\ 1 & 0 & 0 & -2 \end{pmatrix}$
the last matrix is a consistent system, this shows that $b$ is in column space of $A$. Therefore the $x = V\sigma$ is the only solution

SVD Decomposition
$$u = \begin{pmatrix} -0.3635 & 0.8063 & 0.4666 \\ -0.2999 & 0.3729 & -0.8781 \\ -0.8820 & -0.4591 & 0.1062 \end{pmatrix}$$
$$s = \begin{pmatrix} 9.7910 & 0 & 0 \\ 0 & 1.4162 & 0 \\ 0 & 0 & 0.3606 \end{pmatrix}$$
$$v = \begin{pmatrix} -0.5859 & 0.0444 & -0.8091 \\ -0.5182 & -0.7882 & 0.3319 \\ -0.6231 & 0.6138 & 0.4849 \end{pmatrix}$$

## 3.b

SVD Decomposition
$$u = \begin{pmatrix} -0.8478 & 0.4286 & -0.3123 \\ -0.4908 & -0.8571 & 0.1562 \\ -0.2008 & 0.2857 & 0.9370 \end{pmatrix}$$
$$s = \begin{pmatrix} 9.0554 & 0 & 0 \\ 0 & 5.7446 & 0 \\ 0 & 0 & 0.0000 \end{pmatrix}$$
$$v = \begin{pmatrix} -0.5051 & 0.0497 & 0.8616 \\ -0.8081 & 0.3233 & -0.4924 \\ 0.3030 & 0.9450 & 0.1231 \end{pmatrix}$$

## 3.c

SVD Decomposition
$$u = \begin{pmatrix} -0.8478 & 0.4286 & -0.3123 \\ -0.4908 & -0.8571 & 0.1562 \\ -0.2008 & 0.2857 & 0.9370 \end{pmatrix}$$
$$s = \begin{pmatrix} 9.0554 & 0 & 0 \\ 0 & 5.7446 & 0 \\ 0 & 0 & 0.0000 \end{pmatrix}$$

$$v = \begin{pmatrix} -0.5051 & 0.0497 & 0.8616 \\ -0.8081 & 0.3233 & -0.4924 \\ 0.3030 & 0.9450 & 0.1231 \end{pmatrix}$$

# 4 Question 4

# 5 Question 5