MFE 230E

Empirical Methods
Assignment 7

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1. (a) From our three measures of volatilites, we can see that the annualized volatility with daily returns is essentially the same as the annualized volatility from monthly volatilities calculated with daily returns. Although our results are slightly different, this is due to the fact that there were 13 leap years and the formula to annualize daily volatilities (multiplying by $\sqrt{252}$) does not account for the extra leap year days. The annualized monthly volatilities from monthly returns is slightly different due to the serial correlation from daily returns.
We also plotted annual volatilites calculated with the three methods to see how the volatilites of the three assets changed over time.

Volatility	SP500	GE	IBM
Annualized Daily	0.16818	0.28592	0.25863
Annualized Monthly	0.15268	0.27312	0.24681
Annualized Monthly (Daily Data)	0.16827	0.28607	0.25877

Table 1: Annualized Volatilites

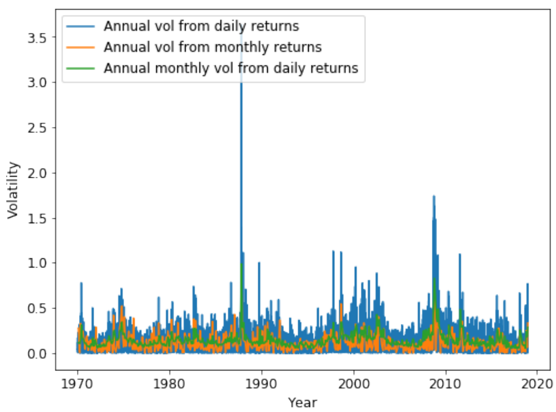


Figure 1: Plot of S&P 500 Volatilites

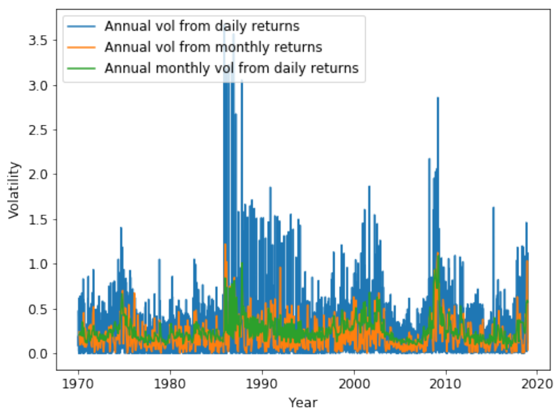


Figure 2: Plot of GE Volatilites

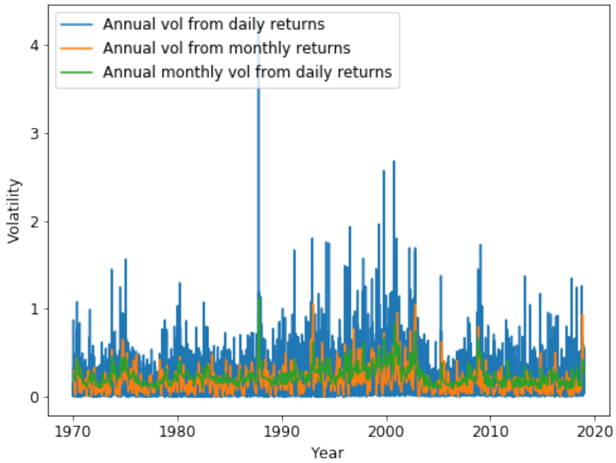


Figure 3: Plot of IBM Volatilites

(b) We can see that the constants for all the models are close to zero and are small compared to the ϕ coefficient. This means that volatility at time t is affected quite a bit by the volatility at t-1, meaning volatility is persistent. This property is also visible from the graphs from part (a) which is independent of the measurement methodology of volatility we choose (calculating annual volatility with daily data, monthly data, or annualizing monthly volatilities calculated with daily data) and applies to S&P 500, GE, and IBM.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.392
Model:                            OLS   Adj. R-squared:                  0.391
Method:                 Least Squares   F-statistic:                     377.0
Date:                Wed, 15 May 2019   Prob (F-statistic):           3.44e-65
Time:                        20:02:41   Log-Likelihood:                 1493.4
No. Observations:                 587   AIC:                            -2983.
Df Residuals:                     585   BIC:                            -2974.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0157      0.002     10.029      0.000       0.013       0.019
x1             0.6274      0.032     19.416      0.000       0.564       0.691
==============================================================================
Omnibus:                      609.702   Durbin-Watson:                   2.226
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            59397.335
Skew:                           4.444   Prob(JB):                         0.00
Kurtosis:                      51.472   Cond. No.                         41.2
==============================================================================
```

Figure 4: AR(1) estimation for S&P 500

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.308
Model:                            OLS   Adj. R-squared:                  0.307
Method:                 Least Squares   F-statistic:                     260.4
Date:                Wed, 15 May 2019   Prob (F-statistic):           1.01e-48
Time:                        20:02:44   Log-Likelihood:                 1168.0
No. Observations:                 587   AIC:                            -2332.
Df Residuals:                     585   BIC:                            -2323.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0321      0.003     11.295      0.000       0.027       0.038
x1             0.5575      0.035     16.136      0.000       0.490       0.625
==============================================================================
Omnibus:                      268.443   Durbin-Watson:                   2.351
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1855.617
Skew:                           1.897   Prob(JB):                         0.00
Kurtosis:                      10.841   Cond. No.                         25.4
==============================================================================
```

Figure 5: AR(1) estimation for GE

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.278
Model:                            OLS   Adj. R-squared:                  0.277
Method:                 Least Squares   F-statistic:                     225.0
Date:                Wed, 15 May 2019   Prob (F-statistic):           2.84e-43
Time:                        20:02:46   Log-Likelihood:                 1270.8
No. Observations:                 587   AIC:                            -2538.
Df Residuals:                     585   BIC:                            -2529.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0318      0.003     12.104      0.000       0.027       0.037
x1             0.5272      0.035     15.001      0.000       0.458       0.596
==============================================================================
Omnibus:                      354.723   Durbin-Watson:                   2.252
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5329.383
Skew:                           2.377   Prob(JB):                         0.00
Kurtosis:                      16.975   Cond. No.                         30.8
==============================================================================
```

Figure 6: AR(1) estimation for IBM

(c) We can see from the plots below that overall the AR(1) model can predict the volatility for S&P 500, GE, and IBM pretty well. The only concern is that the predicted values are overall smoother than the actual volatilities.

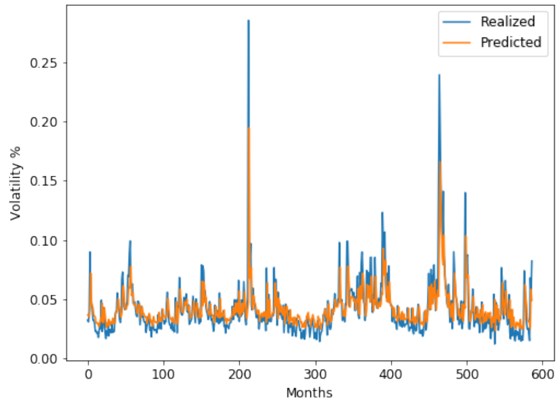


Figure 7: Realized vs Predicted volatility for S&P 500

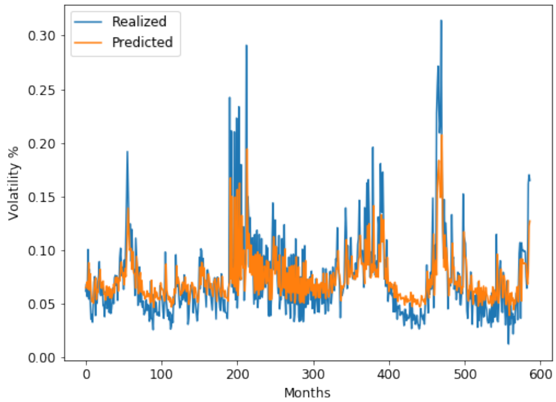


Figure 8: Realized vs Predicted volatility for GE

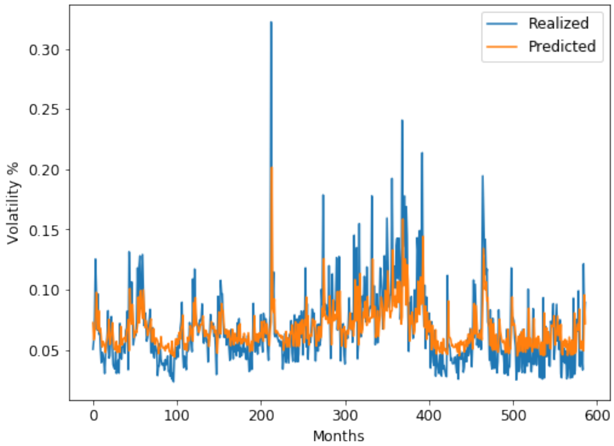


Figure 9: Realized vs Predicted volatility for IBM

(d) For Mean Square Error we have the following
MSE for S&P 500 is 0.00036.
MSE for GE is 0.00109.
MSE for IBM is 0.00077.

Another more intuitive performance measure is RMSE. We have
RMSE for S&P 500 is 0.019.
RMSE for GE is 0.033.
RMSE for IBM is 0.028.

Hence, by comparing MSE, the AR(1) model makes the best prediction for S&P 500 volatility.

It can also be noted from the RMSE, that the error is low in absolute terms. For example, for S&P 500 we have a RMSE of 1.9% for a notional of around 16% which is low.

2. (a) Now we have the following GARCH model

$$
\begin{aligned}
r &= \mu_t + \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t^2 &= \omega + \alpha \epsilon_{t-1}^2 + \beta \sigma_{t-1}^2 \\
e_t &\sim N(0,1)
\end{aligned}
$$

For S&P 500, we have $\mu = 6.46e-03$, $\omega = 1.91e-04$, $\alpha = 0.10$, and $\beta = 0.80$.
For GE, we have $\mu = 0.01$, $\omega = 2.40e-04$, $\alpha = 0.14$, and $\beta = 0.83$.
For IBM, we have $\mu = 7.12e-03$, $\omega = 4.09e-04$, $\alpha = 0.09$, and $\beta = 0.83$.

Given standard error from the calculations, we know that α's and β's are significant, especially the β's. Notice for all 3 stocks. β's as big as 0.8 mean that there is a lot of persistence in volatility: when we observe a large volatility, we are very likely to observe another one following it.

(b) We can see that although the conditional volatility from GARCH cannot catch the extreme values, it can predict the persistent volatility after the spikes very well.

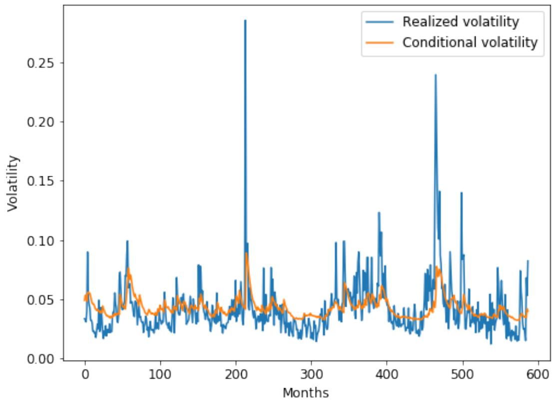


Figure 10: Realized vs Conditional volatility for S&P 500

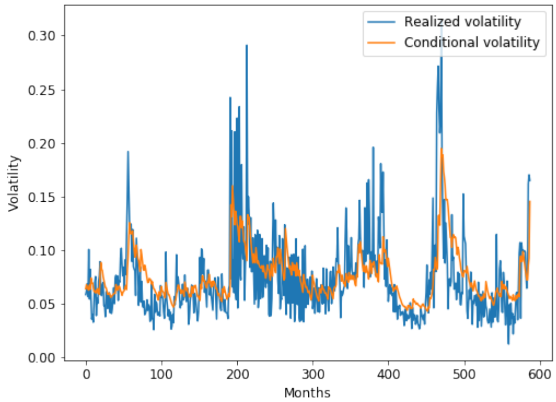


Figure 11: Realized vs Conditional volatility for GE

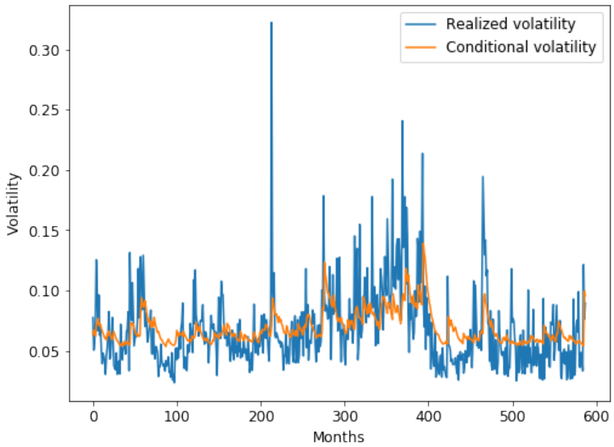


Figure 12: Realized vs Conditional volatility for IBM

```
Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.250
Model:                            OLS   Adj. R-squared:                  0.249
Method:                 Least Squares   F-statistic:                     195.5
Date:                Wed, 15 May 2019   Prob (F-statistic):           1.53e-38
Time:                        20:02:46   Log-Likelihood:                 1434.8
No. Observations:                 588   AIC:                            -2866.
Df Residuals:                     586   BIC:                            -2857.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0189      0.004     -4.256      0.000      -0.028      -0.010
x1             1.4221      0.102     13.982      0.000       1.222       1.622
==============================================================================
Omnibus:                      629.808   Durbin-Watson:                   1.170
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            46605.888
Skew:                           4.824   Prob(JB):                         0.00
Kurtosis:                      45.535   Cond. No.                         117.
==============================================================================
```

Figure 13: Regressing Conditional and Realized Volatilites for S&P 500

```
Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.293
Model:                            OLS   Adj. R-squared:                  0.292
Method:                 Least Squares   F-statistic:                     243.0
Date:                Wed, 15 May 2019   Prob (F-statistic):           4.36e-46
Time:                        20:02:47   Log-Likelihood:                 1164.2
No. Observations:                 588   AIC:                            -2324.
Df Residuals:                     586   BIC:                            -2316.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0040      0.005      0.877      0.381      -0.005       0.013
x1             0.9064      0.058     15.589      0.000       0.792       1.021
==============================================================================
Omnibus:                      250.201   Durbin-Watson:                   1.436
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1427.327
Skew:                           1.810   Prob(JB):                    1.15e-310
Kurtosis:                       9.719   Cond. No.                         42.4
==============================================================================
```

Figure 14: Regressing Conditional and Realized Volatilites for GE

```
Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.258
Model:                            OLS   Adj. R-squared:                  0.257
Method:                 Least Squares   F-statistic:                     204.2
Date:                Wed, 15 May 2019   Prob (F-statistic):           5.74e-40
Time:                        20:02:47   Log-Likelihood:                 1265.6
No. Observations:                 588   AIC:                            -2527.
Df Residuals:                     586   BIC:                            -2519.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0174      0.006     -2.889      0.004      -0.029      -0.006
x1             1.2202      0.085     14.292      0.000       1.053       1.388
==============================================================================
Omnibus:                      349.718   Durbin-Watson:                   1.460
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4941.665
Skew:                           2.348   Prob(JB):                         0.00
Kurtosis:                      16.403   Cond. No.                         73.9
==============================================================================
```

Figure 15: Regressing Conditional and Realized Volatilites for IBM

(c) RMSE for S&P 500 is 0.021.
RMSE for GE is 0.033.
RMSE for IBM is 0.028.

Looking at the RMSE, we would say that GARCH performs well in predicting volatility for all 3 assets.

3. (a) Now we have the following GJR-GARCH model

$$
\begin{aligned}
r &= \mu_t + \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t^2 &= \omega + (\alpha + \gamma I_{t-1})\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \\
e_t &\sim N(0,1)
\end{aligned}
$$

For S&P 500, we have $\mu = 5.56e-03$, $\omega = 1.28e-04$, $\alpha = 0.056$, $\gamma = 0.1160$, and $\beta = 0.822$.
For GE, we have $\mu = 0.0135$, $\omega = 2.38e-04$, $\alpha = 0.104$, $\gamma = 0.070$, and $\beta = 0.834$.
For IBM, we have $\mu = 6.60e-03$, $\omega = 5.10e-04$, $\alpha = 0.055$, $\gamma = 0.098$, and $\beta = 0.796$.

(b) We can see that although the conditional volatility from GJR-GARCH cannot catch the extreme values, it can predict the persistent volatility spikes very well.

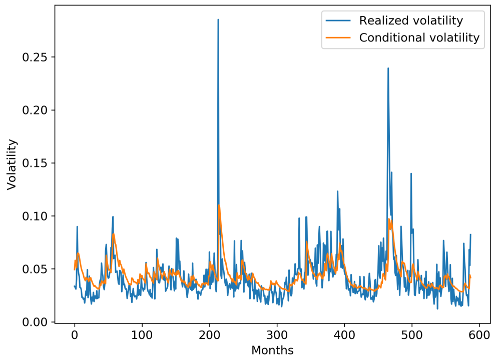


Figure 16: Realized vs Conditional volatility for S&P 500

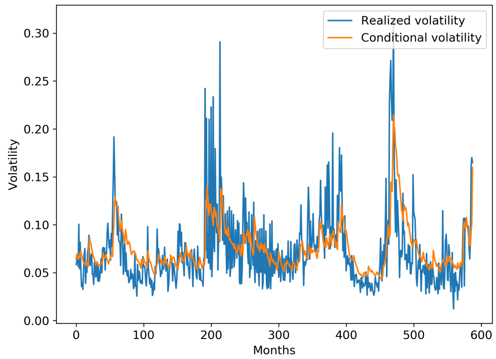


Figure 17: Realized vs Conditional volatility for GE

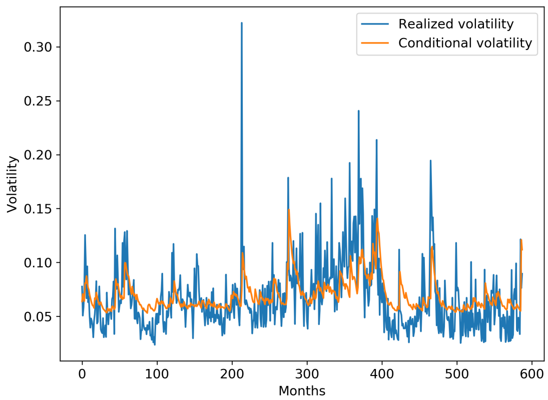


Figure 18: Realized vs Conditional volatility for IBM

```
Regression
                    OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.276
Model:                            OLS   Adj. R-squared:                  0.275
Method:                 Least Squares   F-statistic:                     223.8
Date:                Wed, 15 May 2019   Prob (F-statistic):           4.37e-43
Time:                        21:19:31   Log-Likelihood:                 1445.2
No. Observations:                 588   AIC:                            -2886.
Df Residuals:                     586   BIC:                            -2878.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0020      0.003     -0.664      0.507      -0.008       0.004
x1             1.0222      0.068     14.959      0.000       0.888       1.156
==============================================================================
Omnibus:                      644.223   Durbin-Watson:                   1.204
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            53325.423
Skew:                           4.971   Prob(JB):                         0.00
Kurtosis:                      48.582   Cond. No.                         80.0
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
MSE: 0.02074760329089548
RMSE: 0.144040283569894
MAE: 0.01258171241269164
```

Figure 19: Regressing Conditional and Realized Volatilites for S&P 500

```
Regression
                    OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.327
Model:                            OLS   Adj. R-squared:                  0.326
Method:                 Least Squares   F-statistic:                     284.5
Date:                Wed, 15 May 2019   Prob (F-statistic):           2.53e-52
Time:                        21:19:59   Log-Likelihood:                 1178.6
No. Observations:                 588   AIC:                            -2353.
Df Residuals:                     586   BIC:                            -2344.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0017      0.004      0.390      0.697      -0.007       0.010
x1             0.9399      0.056     16.867      0.000       0.830       1.049
==============================================================================
Omnibus:                      259.460   Durbin-Watson:                   1.482
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1615.945
Skew:                           1.856   Prob(JB):                         0.00
Kurtosis:                      10.224   Cond. No.                         41.6
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
MSE: 0.03275639135296147
RMSE: 0.18098726848306615
MAE: 0.022813610643813383
```

Figure 20: Regressing Conditional and Realized Volatilites for GE

```
Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.262
Model:                            OLS   Adj. R-squared:                  0.260
Method:                 Least Squares   F-statistic:                     207.7
Date:                Wed, 15 May 2019   Prob (F-statistic):           1.60e-40
Time:                        21:20:09   Log-Likelihood:                 1266.9
No. Observations:                 588   AIC:                            -2530.
Df Residuals:                     586   BIC:                            -2521.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0107      0.006     -1.930      0.054      -0.022       0.000
x1             1.1232      0.078     14.411      0.000       0.970       1.276
==============================================================================
Omnibus:                      341.978   Durbin-Watson:                   1.473
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4617.936
Skew:                           2.294   Prob(JB):                         0.00
Kurtosis:                      15.940   Cond. No.                         67.6
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
MSE: 0.02819623125759392
RMSE: 0.16791733459531186
MAE: 0.0201179591229439
```

Figure 21: Regressing Conditional and Realized Volatilites for IBM

(c) RMSE for S&P 500 is 0.021.
RMSE for GE is 0.033.
RMSE for IBM is 0.028.

Looking at the RMSE, we would say that GJR-GARCH also performs well in predicting volatility for all 3 assets.

4. Table of RMSE and MAE of the AR(1), GARCH, and GJR-GARCH:

Error Type	SP500	GE	IBM
AR(1) RMSE	0.019	0.033	0.028
GARCH RMSE	0.021	0.033	0.028
GJR-GARCH RMSE	0.021	0.033	0.028
AR(1) MAE	0.011	0.022	0.019
GARCH MAE	0.013	0.023	0.020
GJR-GARCH MAE	0.013	0.023	0.020

Table 2: Comparing RMSE and MAE of the models

From the table above, we can see that the results for all 3 models are very similar. The AR(1) model tends to produce sometimes slightly smaller RMSE and MAE. AR(1) and GARCH seem like reasonable options as they are the most parsimonious (being AR(1) the simplest). GARCH gives the flexibility of also modeling mean returns as a time series while accounting for persistence in volatility. However, for this particular setting, we could choose any model as results would be very similar in practice.

Assignment 7

May 16, 2019

In [1]:
```
import numpy as np
import statsmodels.api as sm
import pandas as pd
from datetime import datetime
import copy
from statsmodels.stats.sandwich_covariance import cov_hac as cov
import matplotlib.pyplot as plt
from scipy.stats import chi2
from arch import arch_model
from pandas.plotting import register_matplotlib_converters
register_matplotlib_converters()
```

In [2]:
```
def OLS(x, y, addcon=True, cov_type=None, sig_level=.05, summary=0, cov_kwds = None):
    """Wrapper for statsmodels OLS regression
    """
    if addcon:
        X = sm.add_constant(x)
    else:
        X = x
    if cov_type==None:
        ols_results = sm.OLS(y,X).fit(cov_type='nonrobust')
    else:
        ols_results = sm.OLS(y,X).fit(cov_type=cov_type, cov_kwds=cov_kwds)

    ### print out the OLS estimation results
    if summary==1:
        print(ols_results.summary())

    ols_cov_mat  = cov(ols_results)
    ols_beta_hat = ols_results.params # beta_hat
    ols_resids   = ols_results.resid  # resids
    ols_se       = ols_results.bse
    ols_pvalues  = ols_results.pvalues

    return ols_beta_hat, ols_resids, ols_se, ols_cov_mat, ols_pvalues

def latex_table(df, caption="", label="", index=False):
```

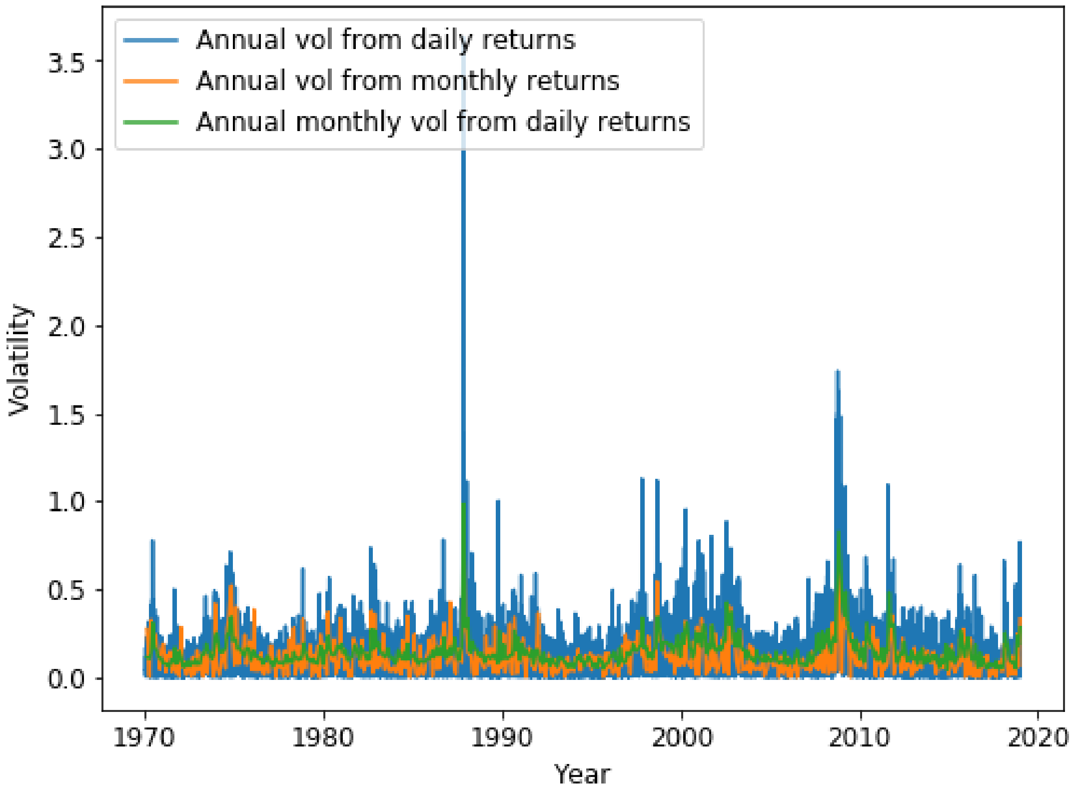
```
        return "\\begin{table}[H]\n\centering\n"+df.to_latex(index=index)+"\caption{"+captio
               \n\end{table}"

In [3]: def question_1_wrapper(data_name, print_OLS):
            '''
            data_name can be either 'SP', 'GE' or 'IBM'
            '''

            # Data
            data = pd.read_csv(data_name+'.csv')
            data['Date'] = pd.to_datetime(data['Date']).dt.date
            data = data.set_index('Date')
            data = data.loc[data.index.astype(str) <= '2018-12-31'] # Every year and month are c

            # Daily continuous returns (not scaled)
            Daily_Returns = []
            for i in range(len(data['Adj Close'])-1):
                Daily_Returns.append(np.log(data['Adj Close'][i+1]/data['Adj Close'][i]))
            data = data.iloc[1:]
            data['Daily Returns'] = Daily_Returns
            data = data.reset_index()

            # Annualized Volatility with Daily Data (Monthly)
            RV_annual_from_daily_monthly = []
            RV_List = [] # Daily returns in the month
            cur_month = data['Date'][0].month
            for i in range(len(data['Date'])):
                if data['Date'][i].month == cur_month and i < (len(data['Date'])-1):
                    RV_List.append(data['Daily Returns'][i])
                else:
                    if i == (len(data['Date']) - 1):
                        RV_List.append(data['Daily Returns'][i])
                    RV_annual_from_daily_monthly.append(np.sqrt(np.sum((np.array(RV_List))**2.0)
                    RV_List = []
                    cur_month = data['Date'][i].month
                    RV_List.append(data['Daily Returns'][i])
            RV_annual_from_daily_monthly_var = np.array(RV_annual_from_daily_monthly)**2
            RV_annual_from_daily_monthly = np.array(RV_annual_from_daily_monthly)
            RV_annual_from_daily_monthly_annualized = RV_annual_from_daily_monthly * np.sqrt(12)

            # Realized monthly volatility from daily data
            Monthly_Returns = []
            RV_monthly_from_daily = []
            RV_List = [] # Daily returns in the month
            cur_month = data['Date'][0].month
            Month_List = []
            for i in range(len(data['Date'])):
                if data['Date'][i].month == cur_month and i < (len(data['Date'])-1):
```

```
        RV_List.append(data['Daily Returns'][i])
    else:
        if i == (len(data['Date']) - 1):
            RV_List.append(data['Daily Returns'][i])
        return_for_the_month = np.sum(np.array(RV_List))
        Monthly_Returns.append(return_for_the_month)
        RV_monthly_from_daily.append(np.sqrt(return_for_the_month**2.0))
        RV_List = []
        Month_List.append(data['Date'][i])
        cur_month = data['Date'][i].month
        RV_List.append(data['Daily Returns'][i])
RV_monthly_from_daily_var = np.array(RV_monthly_from_daily)**2
RV_monthly_from_daily = np.array(RV_monthly_from_daily) * np.sqrt(12)

# Realized volatility with daily returns
RV_annual_from_daily_var = np.array((data['Daily Returns'])**2)
RV_annual_from_daily = np.sqrt(np.array((data['Daily Returns'])**2)) * np.sqrt(252)

# Output
Daily_Returns = np.array(Daily_Returns)
Monthly_Returns = np.array(Monthly_Returns)
RV_annual_from_daily = np.array(RV_annual_from_daily)
RV_annual_from_monthly = np.array(RV_monthly_from_daily)
RV_annual_from_monthly_daily = np.array(RV_annual_from_daily_monthly)
vol = RV_annual_from_monthly_daily

# Comparisson
plt.figure(figsize=(8,6))
plt.rcParams.update({'font.size': 12})
plt.plot(data['Date'], RV_annual_from_daily, label='Annual vol from daily returns')
plt.plot(Month_List, RV_annual_from_monthly, label='Annual vol from monthly returns'
plt.plot(Month_List, RV_annual_from_daily_monthly_annualized, label='Annual monthly
plt.xlabel('Year')
plt.ylabel('Volatility')
plt.legend(loc='upper left')
plt.show()

# Fit AR(1)
x = vol[0:len(vol)-1]
y = vol[1:len(vol)]
ols_beta_hat, ols_resids, ols_se, ols_cov_mat, ols_pvalues = OLS(x, y, addcon = True

# Forecast
y_hat = x*ols_beta_hat[1] + ols_beta_hat[0]
plt.figure(figsize=(8,6))
plt.rcParams.update({'font.size': 12})
plt.plot(y, label='Realized')
plt.plot(y_hat, label='Predicted')
```

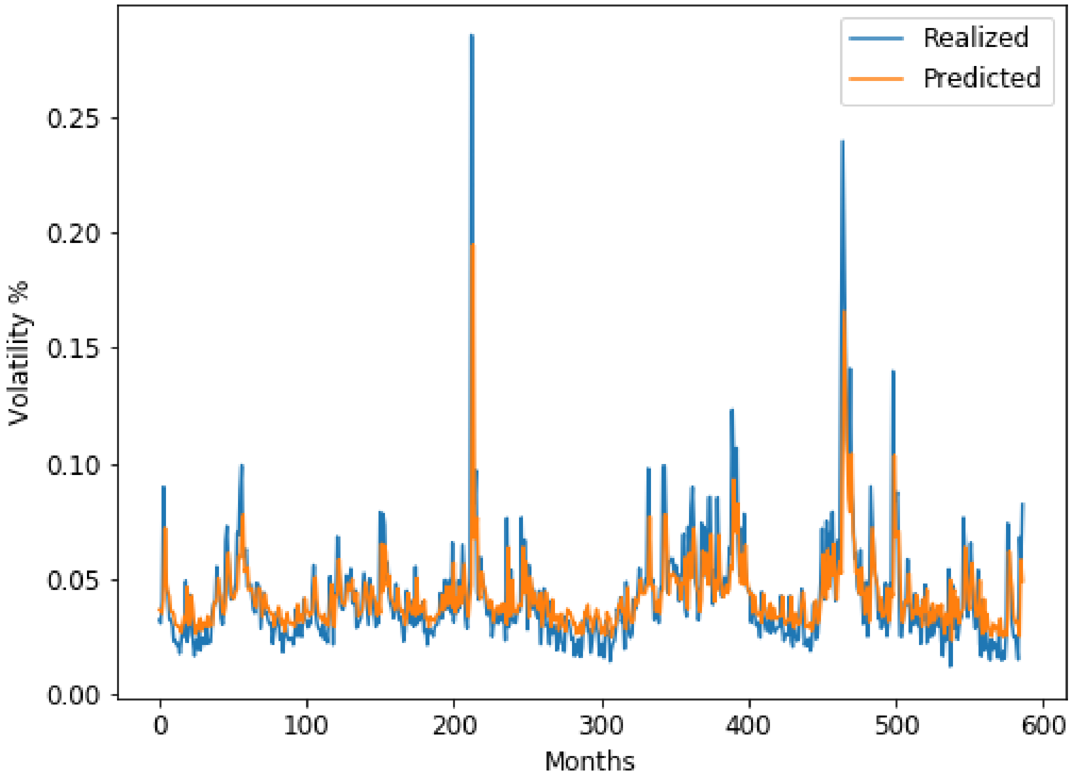
```
    plt.legend()
    plt.xlabel('Months')
    plt.ylabel('Volatility %')
    plt.show()

    # Error
    mse = np.mean((y - y_hat)**2)
    rmse = np.sqrt(mse)
    mae = np.mean(np.abs(y-y_hat))
    print('MSE:', mse)
    print('RMSE:', rmse)
    print('MAE:', mae)
    print('Annualized Volatility with Daily Returns:', np.sqrt(np.mean(RV_annual_from_da
    print('Annualized Volatility with Monthly Returns:', np.sqrt(np.mean(RV_monthly_from
    print('Annualized Volatility with Monthly Vol and Daily Data:', np.sqrt(np.mean(RV_a


    return Monthly_Returns, RV_annual_from_monthly_daily
```

0.1 Problem 1

```
In [4]: SP_returns, SP_vol = question_1_wrapper('SP', 1)
```

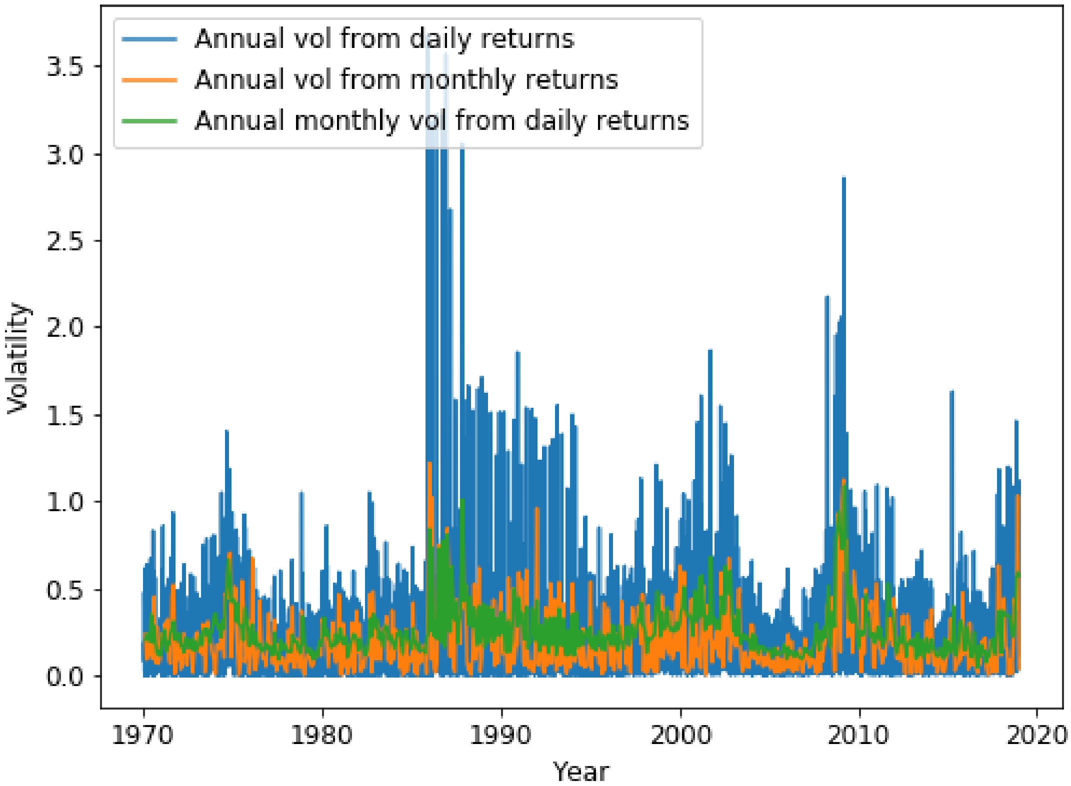


```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.392
Model:                            OLS   Adj. R-squared:                  0.391
Method:                 Least Squares   F-statistic:                     377.0
Date:                Wed, 15 May 2019   Prob (F-statistic):           3.44e-65
Time:                        22:37:28   Log-Likelihood:                 1493.4
No. Observations:                 587   AIC:                            -2983.
Df Residuals:                     585   BIC:                            -2974.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0157      0.002     10.029      0.000       0.013       0.019
x1             0.6274      0.032     19.416      0.000       0.564       0.691
==============================================================================
Omnibus:                      609.702   Durbin-Watson:                   2.226
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            59397.335
Skew:                           4.444   Prob(JB):                         0.00
Kurtosis:                      51.472   Cond. No.                         41.2
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```



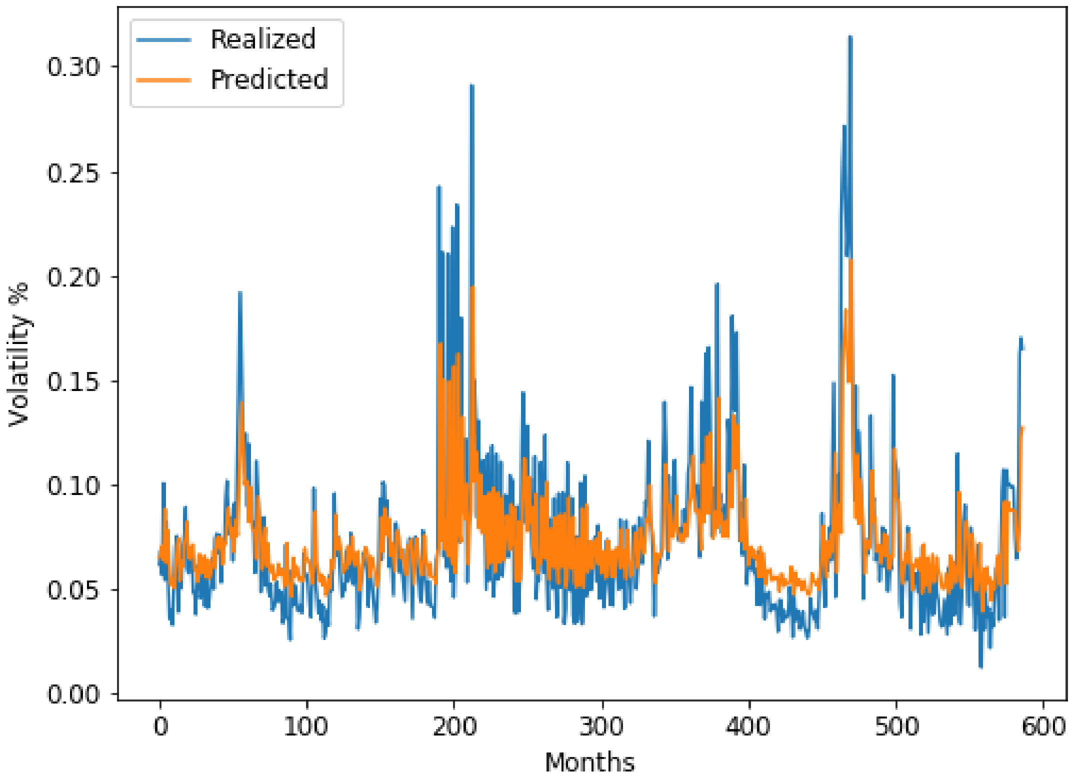

```
MSE: 0.00036122005150237944
RMSE: 0.019005789946813036
MAE: 0.010968749039436572
Annualized Volatility with Daily Returns: 0.16818403557750747
Annualized Volatility with Monthly Returns: 0.15267557334233467
Annualized Volatility with Monthly Vol and Daily Data: 0.16827254453869964
```

```
In [5]: GE_returns, GE_vol = question_1_wrapper('GE', 1)
```



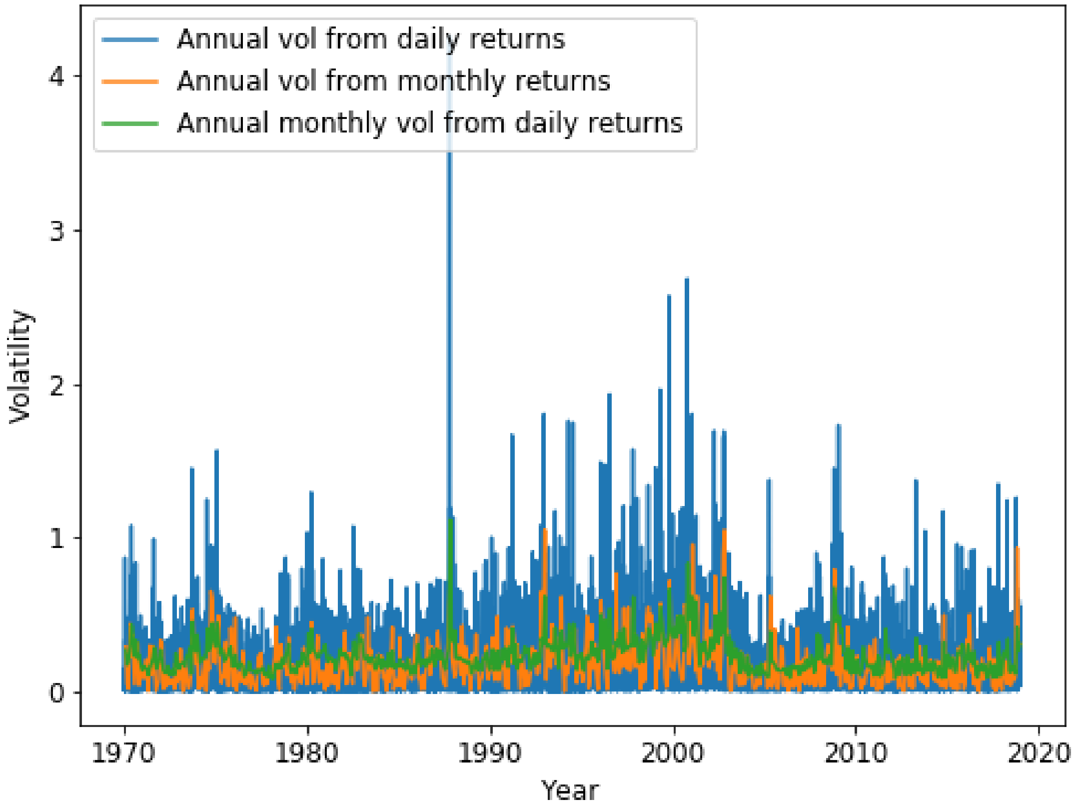

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.308
Model:                            OLS   Adj. R-squared:                  0.307
Method:                 Least Squares   F-statistic:                     260.4
Date:                Wed, 15 May 2019   Prob (F-statistic):           1.01e-48
Time:                        22:37:31   Log-Likelihood:                 1168.0
No. Observations:                 587   AIC:                            -2332.
Df Residuals:                     585   BIC:                            -2323.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0321      0.003     11.295      0.000       0.027       0.038
x1             0.5575      0.035     16.136      0.000       0.490       0.625
==============================================================================
Omnibus:                      268.443   Durbin-Watson:                   2.351
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1855.617
Skew:                           1.897   Prob(JB):                         0.00
Kurtosis:                      10.841   Cond. No.                         25.4
```

```
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```



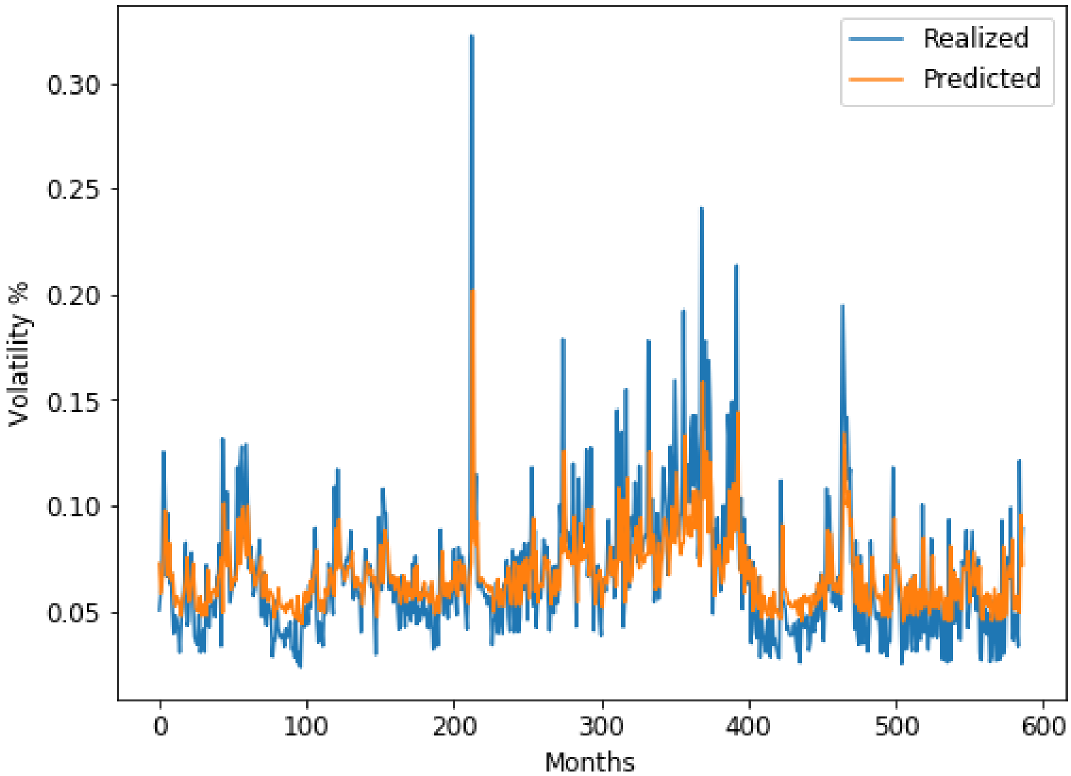

```
MSE: 0.0010943459149488828
RMSE: 0.033080899548665284
MAE: 0.02199152059292526
Annualized Volatility with Daily Returns: 0.2859189536300444
Annualized Volatility with Monthly Returns: 0.2731223941114715
Annualized Volatility with Monthly Vol and Daily Data: 0.2860694220706669
```

```
In [6]: IBM_returns, IBM_vol = question_1_wrapper('IBM', 1)
```




```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.278
Model:                            OLS   Adj. R-squared:                  0.277
Method:                 Least Squares   F-statistic:                     225.0
Date:                Wed, 15 May 2019   Prob (F-statistic):           2.84e-43
Time:                        22:37:33   Log-Likelihood:                 1270.8
No. Observations:                 587   AIC:                            -2538.
Df Residuals:                     585   BIC:                            -2529.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0318      0.003     12.104      0.000       0.027       0.037
x1             0.5272      0.035     15.001      0.000       0.458       0.596
==============================================================================
Omnibus:                      354.723   Durbin-Watson:                   2.252
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             5329.383
Skew:                           2.377   Prob(JB):                         0.00
```

```
Kurtosis:                      16.975   Cond. No.                        30.8
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```



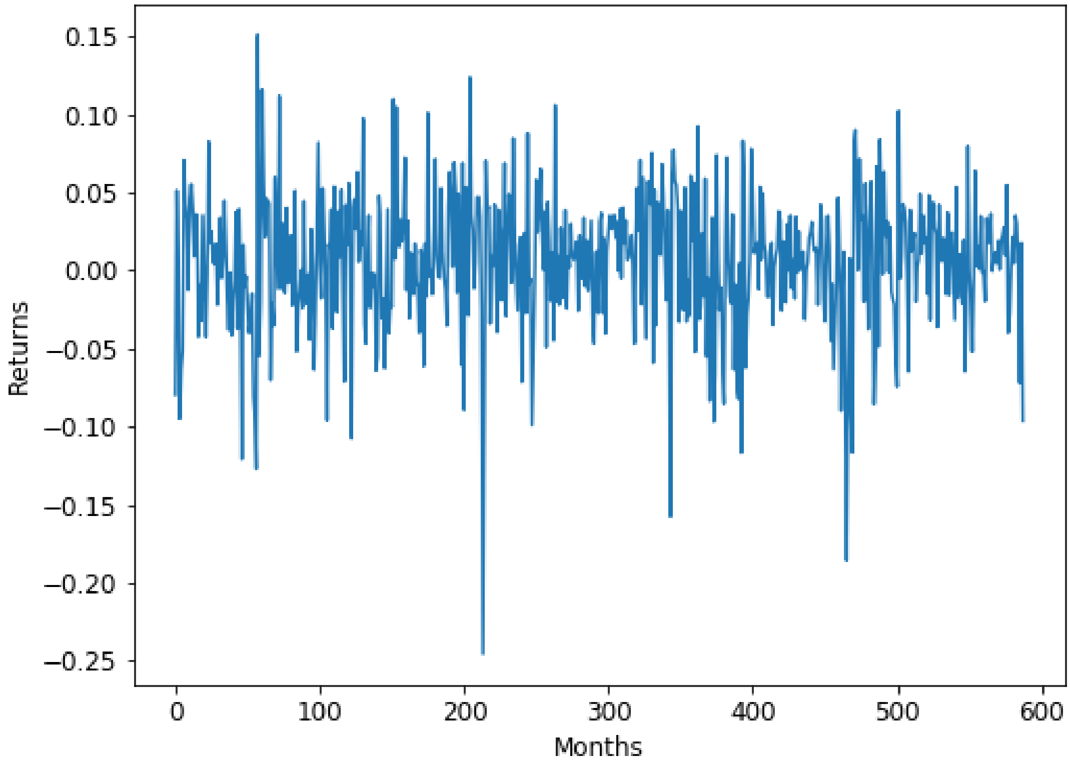

```
MSE: 0.0007711503591662113
RMSE: 0.027769594148388473
MAE: 0.018829690059106226
Annualized Volatility with Daily Returns: 0.25863424149889247
Annualized Volatility with Monthly Returns: 0.2468062143866231
Annualized Volatility with Monthly Vol and Daily Data: 0.25877035101704726
```

0.2 Problem 2

```
In [7]: def question_2_3_wrapper(returns, realized_vol, p=1, o=0, q=1):
            '''
            p (int, optional)  Lag order of the symmetric innovation
            o (int, optional)  Lag order of the asymmetric innovation
```

```
    q (int, optional)  Lag order of lagged volatility or equivalent

    For GARCH: [p, o, q] = [1, 0, 1]
    For GJR-GARCH: [p, o, q] = [1, 1, 1]
    '''

    # Plot returns
    plt.figure(figsize=(8,6))
    plt.rcParams.update({'font.size': 12})
    plt.plot(returns)
    plt.xlabel('Months')
    plt.ylabel('Returns')
    plt.show()

    # GARCH (part a)
    GARCH_model = arch_model(returns, p=p, o=o, q=q)
    Res = GARCH_model.fit(update_freq = 5)
    conditional_vol = Res.conditional_volatility
    print('\n\n\n\n\n GARCH')
    print(Res)

    # Regression (part b)
    print('\n\n\n\n\n Regression')
    vol_beta, vol_resids, _, _, _ = OLS(conditional_vol, realized_vol, summary = 1)

    # Plot
    plt.figure(figsize=(8,6))
    plt.rcParams.update({'font.size': 12})
    plt.plot(realized_vol, label='Realized volatility')
    plt.plot(conditional_vol, label='Conditional volatility')
    plt.legend()
    plt.xlabel('Months')
    plt.ylabel('Volatility')

    # Error
    mse = np.mean((realized_vol - conditional_vol)**2)
    rmse = np.sqrt(mse)
    mae = np.mean(np.abs(realized_vol-conditional_vol))
    print('MSE:', mse)
    print('RMSE:', rmse)
    print('MAE:', mae)
```

```
In [8]: question_2_3_wrapper(SP_returns, SP_vol)
```




```
Positive directional derivative for linesearch    (Exit mode 8)
            Current function value: -1028.5391663334103
            Iterations: 6
            Function evaluations: 18
            Gradient evaluations: 2
```

```
 GARCH
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                      y   R-squared:                      -0.000
Mean Model:             Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                      GARCH   Log-Likelihood:                1028.54
Distribution:                  Normal   AIC:                          -2049.08
Method:            Maximum Likelihood   BIC:                          -2031.57
                                        No. Observations:                  588
Date:                Wed, May 15 2019   Df Residuals:                      584
Time:                        22:37:34   Df Model:                            4
                                 Mean Model
```

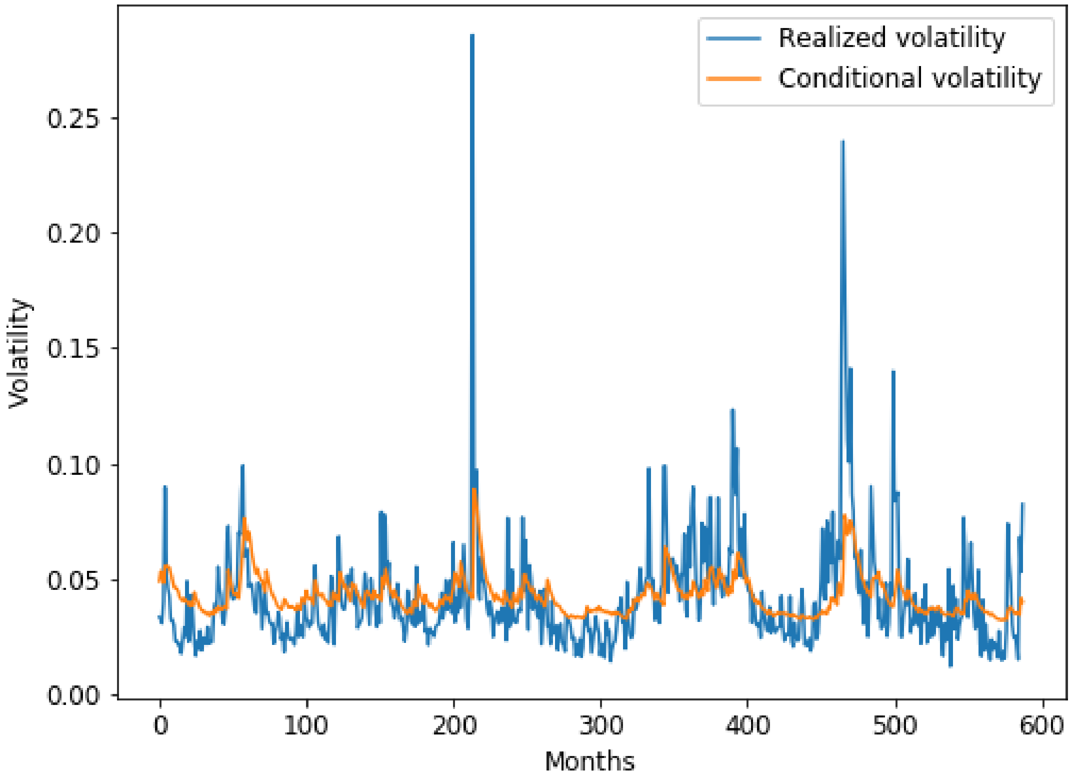
```
==============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
mu         6.4621e-03  2.013e-03      3.210  1.329e-03 [2.516e-03,1.041e-02]
                               Volatility Model
===============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-------------------------------------------------------------------------------
omega      1.9111e-04  1.008e-03      0.190      0.850 [-1.784e-03,2.166e-03]
alpha[1]       0.1000  2.725e-02      3.669  2.433e-04    [4.658e-02,  0.153]
beta[1]        0.8000      0.553      1.446      0.148      [ -0.285,  1.885]
===============================================================================

Covariance estimator: robust

WARNING: The optimizer did not indicate successful convergence. The message was
Positive directional derivative for linesearch. See convergence_flag.
```

Regression

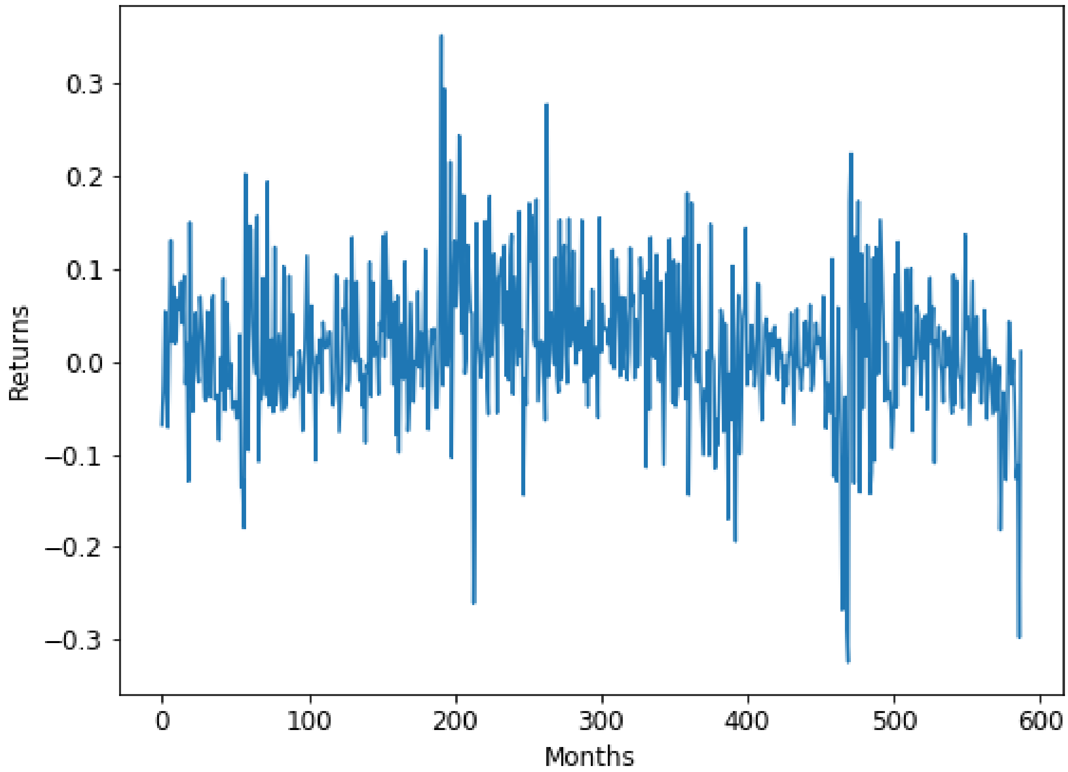
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.250
Model:                            OLS   Adj. R-squared:                  0.249
Method:                 Least Squares   F-statistic:                     195.5
Date:                Wed, 15 May 2019   Prob (F-statistic):           1.53e-38
Time:                        22:37:34   Log-Likelihood:                 1434.8
No. Observations:                 588   AIC:                            -2866.
Df Residuals:                     586   BIC:                            -2857.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0189      0.004     -4.256      0.000      -0.028      -0.010
x1             1.4221      0.102     13.982      0.000       1.222       1.622
==============================================================================
Omnibus:                      629.808   Durbin-Watson:                   1.170
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            46605.888
Skew:                           4.824   Prob(JB):                         0.00
Kurtosis:                      45.535   Cond. No.                         117.
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
MSE: 0.0004585009094400436
RMSE: 0.021412634341435983
MAE: 0.013028995258359091
```

```
C:\Users\conan\Anaconda3\lib\site-packages\arch\univariate\base.py:577: ConvergenceWarning:
The optimizer returned code 8. The message is:
Positive directional derivative for linesearch
See scipy.optimize.fmin_slsqp for code meaning.

  ConvergenceWarning)
```




```
In [9]: question_2_3_wrapper(GE_returns, GE_vol)
```



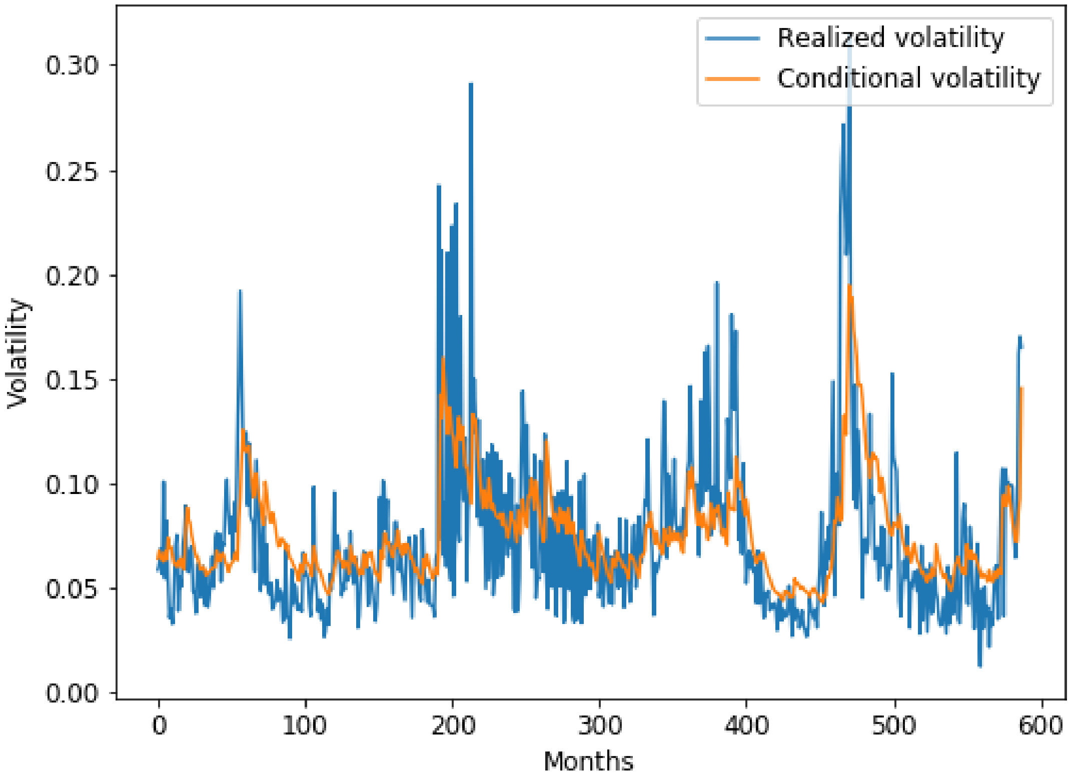

```
Iteration:      5,   Func. Count:     44,   Neg. LLF: -709.5464610335621
Iteration:     10,   Func. Count:     76,   Neg. LLF: -709.5994262569297
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -709.5994262567933
            Iterations: 10
            Function evaluations: 76
            Gradient evaluations: 10
```

```
 GARCH
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                      y   R-squared:                      -0.000
Mean Model:             Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                      GARCH   Log-Likelihood:                709.599
Distribution:                  Normal   AIC:                          -1411.20
Method:            Maximum Likelihood   BIC:                          -1393.69
                                        No. Observations:                  588
```

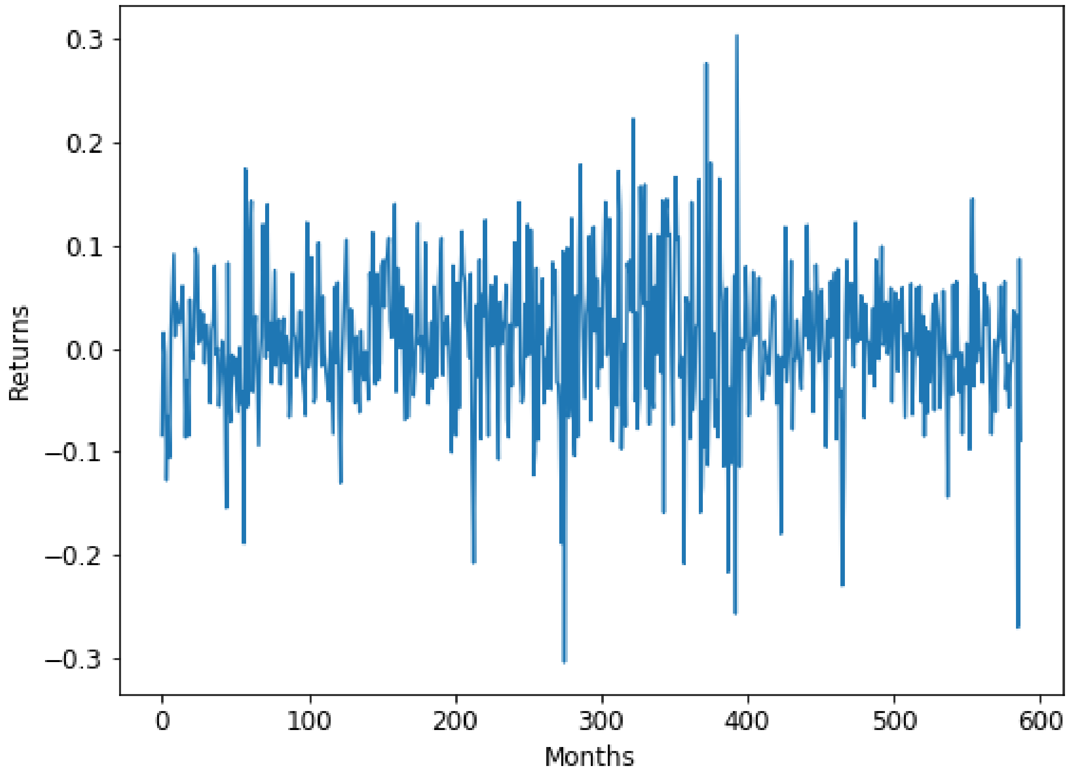
```
Date:                Wed, May 15 2019   Df Residuals:                     584
Time:                        22:37:35   Df Model:                           4
                               Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0145  2.779e-03      5.233  1.672e-07 [9.096e-03,1.999e-02]
                            Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      2.4052e-04  1.141e-04      2.108  3.499e-02 [1.694e-05,4.641e-04]
alpha[1]       0.1412  3.855e-02      3.661  2.509e-04   [6.559e-02,  0.217]
beta[1]        0.8291  3.571e-02     23.216 3.103e-119      [  0.759,  0.899]
============================================================================

Covariance estimator: robust
```

```
 Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.293
Model:                            OLS   Adj. R-squared:                  0.292
Method:                 Least Squares   F-statistic:                     243.0
Date:                Wed, 15 May 2019   Prob (F-statistic):           4.36e-46
Time:                        22:37:35   Log-Likelihood:                 1164.2
No. Observations:                 588   AIC:                            -2324.
Df Residuals:                     586   BIC:                            -2316.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0040      0.005      0.877      0.381      -0.005       0.013
x1             0.9064      0.058     15.589      0.000       0.792       1.021
==============================================================================
Omnibus:                      250.201   Durbin-Watson:                   1.436
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1427.327
Skew:                           1.810   Prob(JB):                    1.15e-310
Kurtosis:                       9.719   Cond. No.                         42.4
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
MSE: 0.0011303178725781107
RMSE: 0.03362020036493106
MAE: 0.023377654172692438
```




```
In [10]: question_2_3_wrapper(IBM_returns, IBM_vol)
```

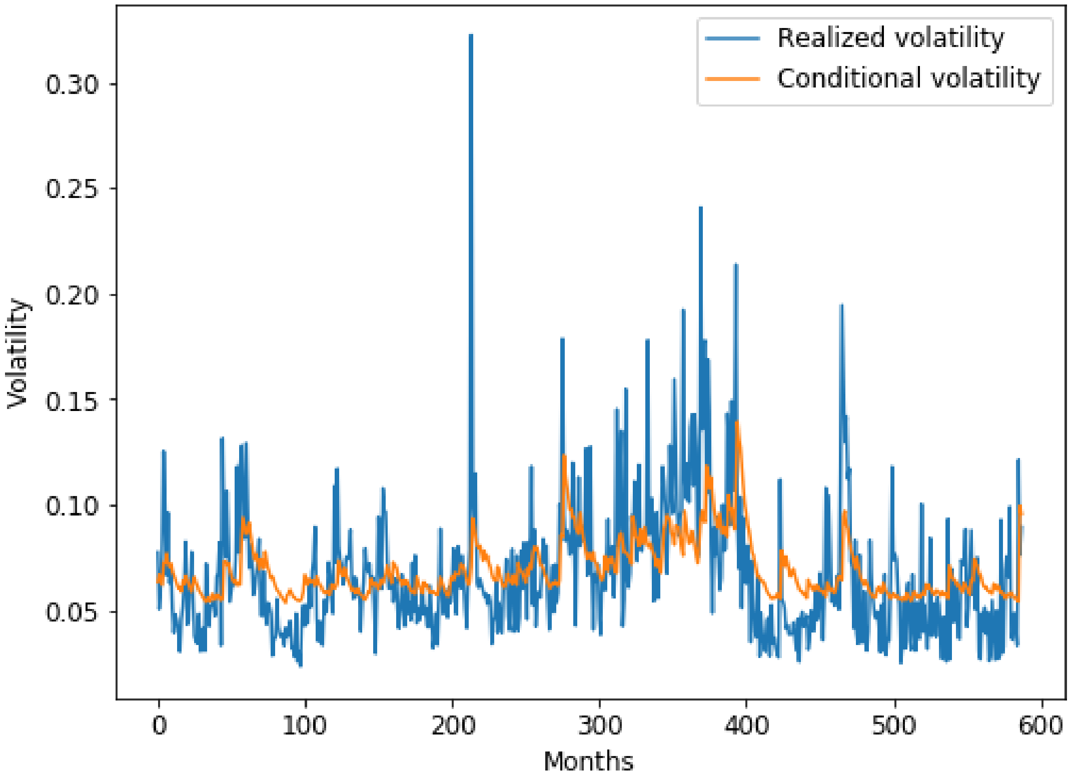



```
Iteration:      5,   Func. Count:     40,   Neg. LLF: -744.0163485046576
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -744.0373348127415
            Iterations: 9
            Function evaluations: 69
            Gradient evaluations: 9
```

```
 GARCH
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:                      y   R-squared:                      -0.000
Mean Model:             Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                      GARCH   Log-Likelihood:                744.037
Distribution:                  Normal   AIC:                          -1480.07
Method:            Maximum Likelihood   BIC:                          -1462.57
                                        No. Observations:                  588
Date:                Wed, May 15 2019   Df Residuals:                      584
```

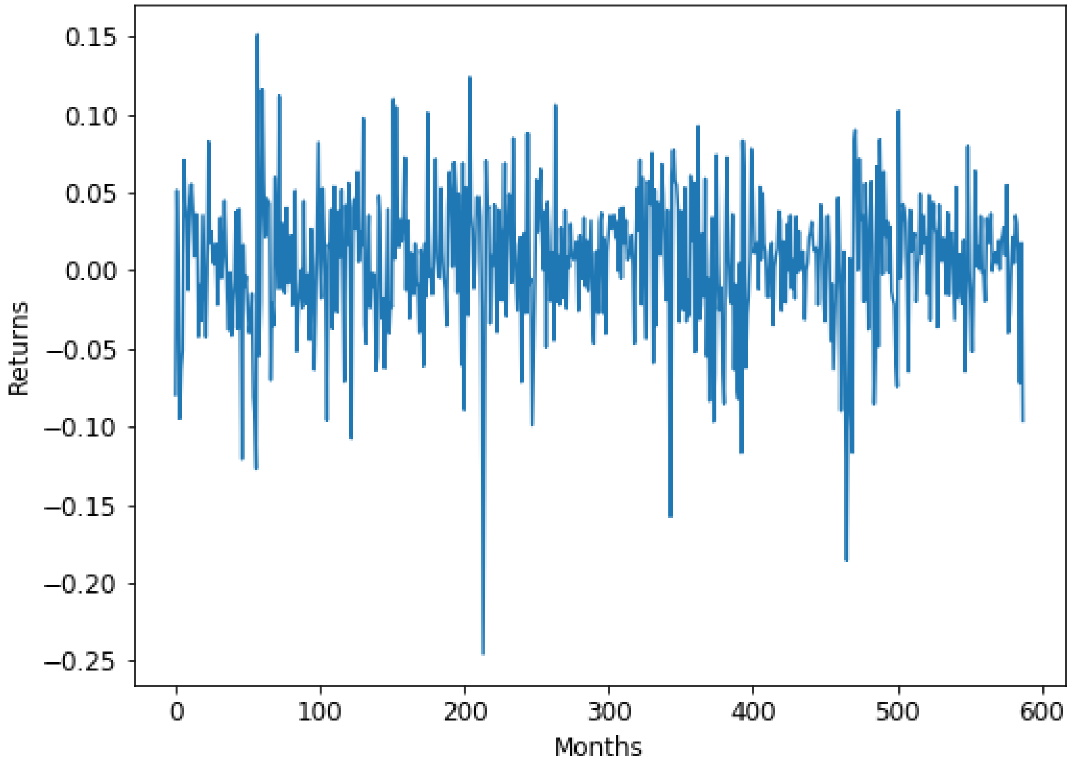
```
Time:                        22:37:35   Df Model:                            4
                                    Mean Model
==========================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
--------------------------------------------------------------------------
mu         7.1227e-03  2.619e-03      2.720  6.529e-03 [1.990e-03,1.226e-02]
                                  Volatility Model
===========================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
---------------------------------------------------------------------------
omega      4.0907e-04  2.831e-04      1.445      0.148 [-1.457e-04,9.638e-04]
alpha[1]       0.0920  4.769e-02      1.928  5.381e-02   [-1.506e-03,  0.185]
beta[1]        0.8272  9.068e-02      9.122  7.353e-20      [  0.649,  1.005]
===========================================================================

Covariance estimator: robust




 Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.258
Model:                            OLS   Adj. R-squared:                  0.257
Method:                 Least Squares   F-statistic:                     204.2
Date:                Wed, 15 May 2019   Prob (F-statistic):           5.74e-40
Time:                        22:37:35   Log-Likelihood:                 1265.6
No. Observations:                 588   AIC:                            -2527.
Df Residuals:                     586   BIC:                            -2519.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0174      0.006     -2.889      0.004      -0.029      -0.006
x1             1.2202      0.085     14.292      0.000       1.053       1.388
==============================================================================
Omnibus:                      349.718   Durbin-Watson:                   1.460
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4941.665
Skew:                           2.348   Prob(JB):                         0.00
Kurtosis:                      16.403   Cond. No.                         73.9
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
MSE: 0.0008042179880348184
```

```
RMSE: 0.028358737419617582
MAE: 0.020117902305045335
```



0.3 Problem 3

```
In [11]: question_2_3_wrapper(SP_returns, SP_vol, p=1, o=1, q=1)
```



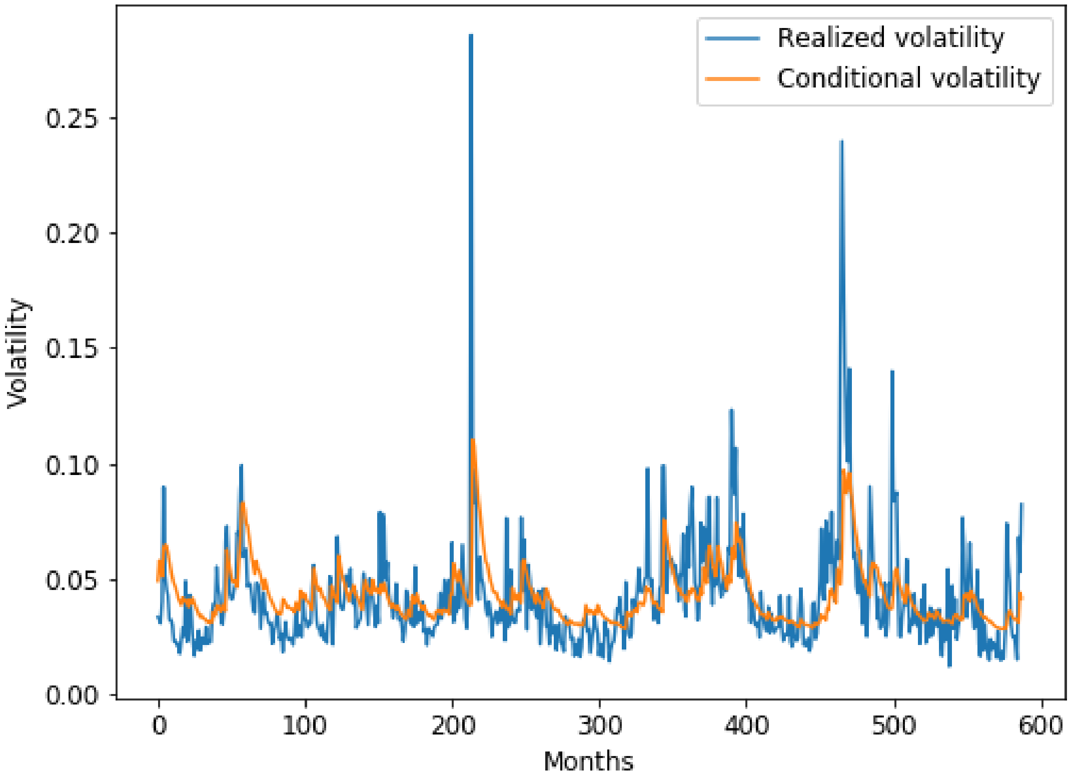

```
Iteration:      5,   Func. Count:     48,   Neg. LLF: -1033.2384143407485
Iteration:     10,   Func. Count:     88,   Neg. LLF: -1033.4340740534335
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -1033.4444615316143
            Iterations: 13
            Function evaluations: 109
            Gradient evaluations: 13
```

```
 GARCH
                   Constant Mean - GJR-GARCH Model Results
==============================================================================
Dep. Variable:                      y   R-squared:                      -0.000
Mean Model:             Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                  GJR-GARCH   Log-Likelihood:                1033.44
Distribution:                  Normal   AIC:                          -2056.89
Method:            Maximum Likelihood   BIC:                          -2035.01
                                        No. Observations:                  588
Date:                Wed, May 15 2019   Df Residuals:                      583
```

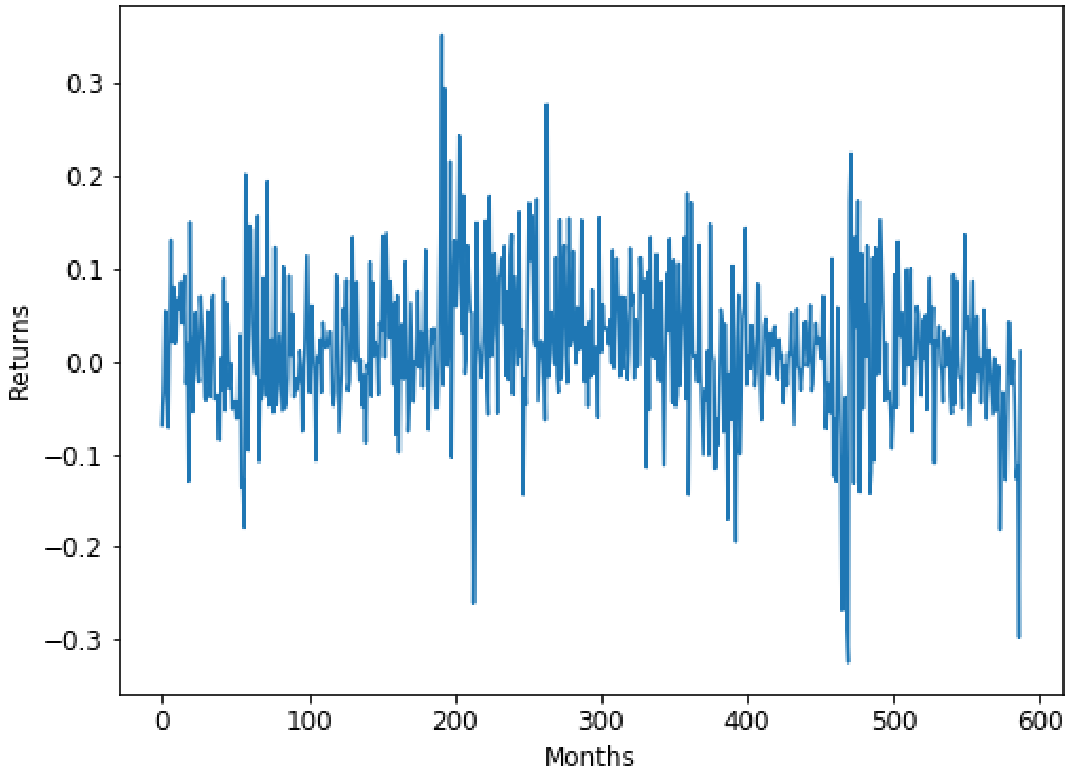
```
Time:                             22:37:36   Df Model:                            5
                                    Mean Model
==============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
------------------------------------------------------------------------------
mu          5.5600e-03  1.619e-03      3.434  5.941e-04 [2.387e-03,8.733e-03]
                                  Volatility Model
===============================================================================
                  coef    std err          t      P>|t|        95.0% Conf. Int.
-------------------------------------------------------------------------------
omega       1.2773e-04  1.286e-04      0.994      0.320 [-1.242e-04,3.797e-04]
alpha[1]        0.0557  8.823e-02      0.631      0.528      [ -0.117,  0.229]
gamma[1]        0.1156      0.134      0.860      0.390      [ -0.148,  0.379]
beta[1]         0.8216  6.827e-02     12.036  2.303e-33      [  0.688,  0.955]
===============================================================================

Covariance estimator: robust
```

```
 Regression
                               OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.276
Model:                            OLS   Adj. R-squared:                  0.275
Method:                 Least Squares   F-statistic:                     223.8
Date:                Wed, 15 May 2019   Prob (F-statistic):           4.36e-43
Time:                        22:37:36   Log-Likelihood:                 1445.2
No. Observations:                 588   AIC:                            -2886.
Df Residuals:                     586   BIC:                            -2878.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         -0.0020      0.003     -0.665      0.507      -0.008       0.004
x1             1.0222      0.068     14.959      0.000       0.888       1.156
==============================================================================
Omnibus:                      644.224   Durbin-Watson:                   1.204
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            53325.985
Skew:                           4.971   Prob(JB):                         0.00
Kurtosis:                      48.582   Cond. No.                         80.0
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
MSE: 0.0004304619546796192
RMSE: 0.020747577079736786
MAE: 0.01258169591706444
```




```
In [12]: question_2_3_wrapper(GE_returns, GE_vol, p=1, o=1, q=1)
```



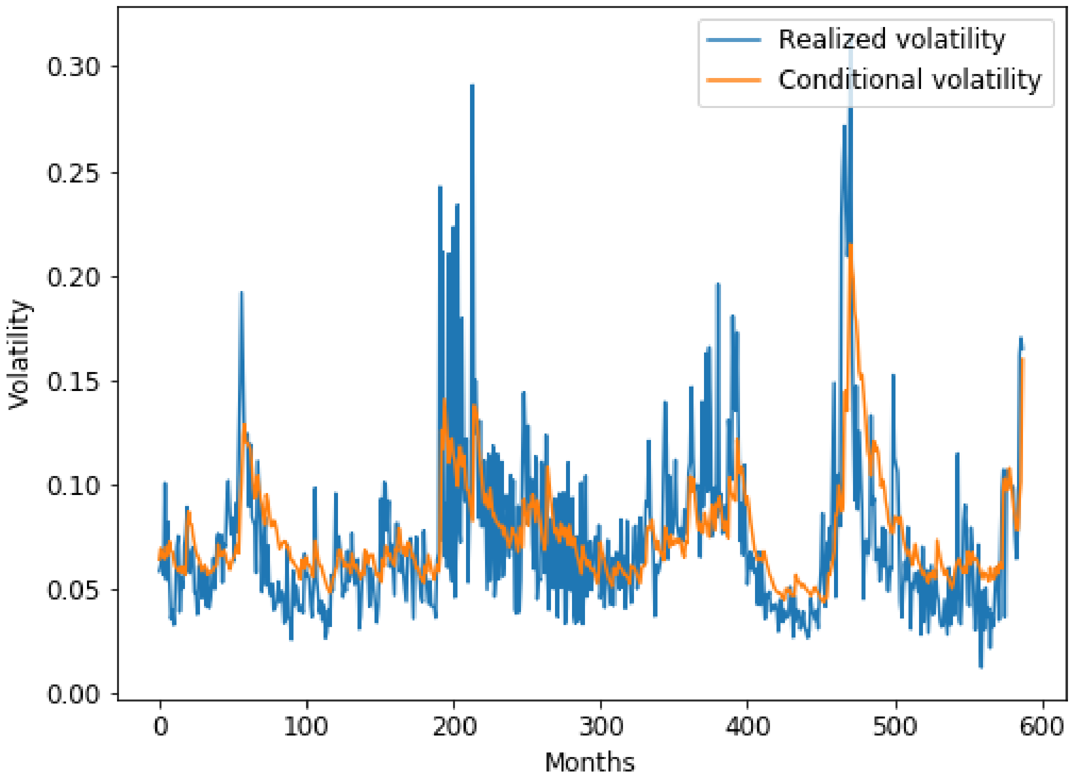

```
Iteration:      5,   Func. Count:     55,   Neg. LLF: -711.0202362292537
Iteration:     10,   Func. Count:     97,   Neg. LLF: -711.494028499884
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -711.508038708007
            Iterations: 14
            Function evaluations: 119
            Gradient evaluations: 13
```

```
 GARCH
                   Constant Mean - GJR-GARCH Model Results
==============================================================================
Dep. Variable:                      y   R-squared:                      -0.000
Mean Model:             Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                  GJR-GARCH   Log-Likelihood:                711.508
Distribution:                  Normal   AIC:                          -1413.02
Method:            Maximum Likelihood   BIC:                          -1391.13
                                        No. Observations:                  588
```

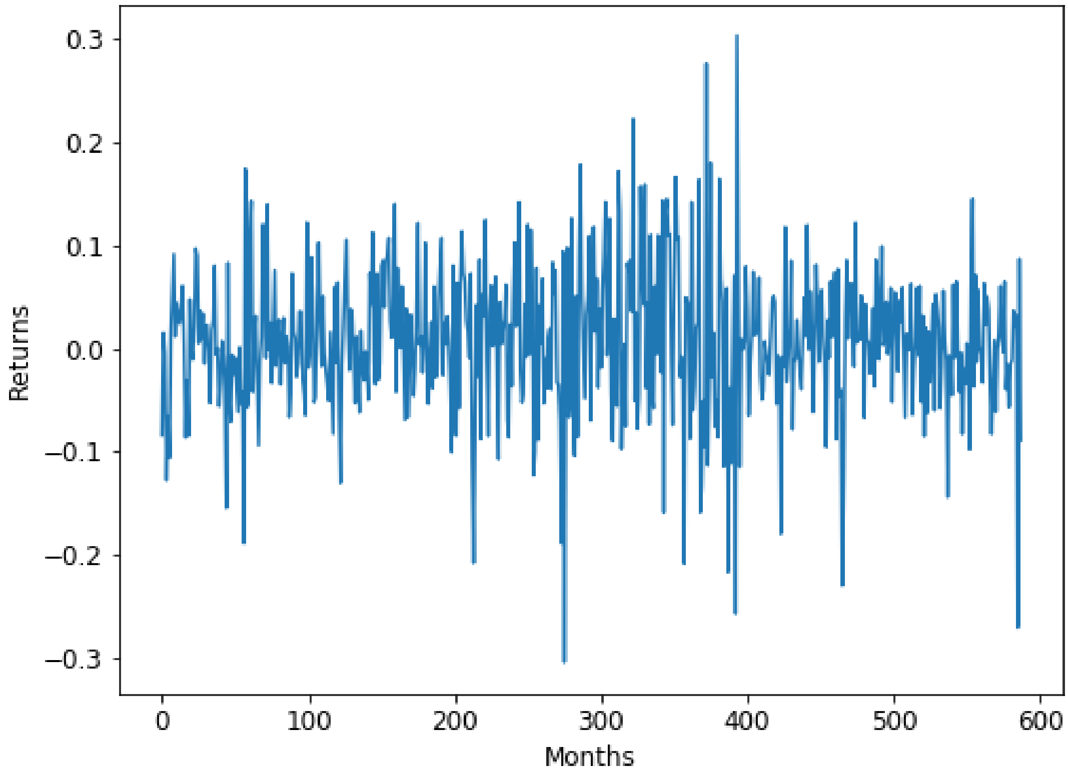
```
Date:                Wed, May 15 2019   Df Residuals:                     583
Time:                        22:37:37   Df Model:                           5
                                  Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0135  2.902e-03      4.666  3.075e-06 [7.853e-03,1.923e-02]
                               Volatility Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
omega      2.3804e-04  1.197e-04      1.989  4.671e-02 [3.466e-06,4.726e-04]
alpha[1]       0.1037  3.450e-02      3.005  2.656e-03   [3.606e-02,  0.171]
gamma[1]       0.0695  4.516e-02      1.540      0.124  [-1.898e-02,  0.158]
beta[1]        0.8340  3.398e-02     24.543 5.181e-133     [  0.767,  0.901]
============================================================================

Covariance estimator: robust
```

```
 Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.327
Model:                            OLS   Adj. R-squared:                  0.326
Method:                 Least Squares   F-statistic:                     284.5
Date:                Wed, 15 May 2019   Prob (F-statistic):           2.53e-52
Time:                        22:37:37   Log-Likelihood:                 1178.6
No. Observations:                 588   AIC:                            -2353.
Df Residuals:                     586   BIC:                            -2344.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.0017      0.004      0.390      0.697      -0.007       0.010
x1             0.9399      0.056     16.867      0.000       0.830       1.049
==============================================================================
Omnibus:                      259.460   Durbin-Watson:                   1.482
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             1615.945
Skew:                           1.856   Prob(JB):                         0.00
Kurtosis:                      10.224   Cond. No.                         41.6
==============================================================================

Warnings:
```

```
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
MSE: 0.001072981174468369
RMSE: 0.03275639135296147
MAE: 0.022813610643813383
```




```
In [13]: question_2_3_wrapper(IBM_returns, IBM_vol, p=1, o=1, q=1)
```




```
Iteration:      5,   Func. Count:     50,   Neg. LLF: -745.883298363608
Optimization terminated successfully.    (Exit mode 0)
            Current function value: -745.8882723590806
            Iterations: 8
            Function evaluations: 74
            Gradient evaluations: 8
```

```
 GARCH
                   Constant Mean - GJR-GARCH Model Results
==============================================================================
Dep. Variable:                      y   R-squared:                      -0.000
Mean Model:             Constant Mean   Adj. R-squared:                 -0.000
Vol Model:                  GJR-GARCH   Log-Likelihood:                745.888
Distribution:                  Normal   AIC:                          -1481.78
Method:            Maximum Likelihood   BIC:                          -1459.89
                                        No. Observations:                  588
Date:                Wed, May 15 2019   Df Residuals:                      583
```

```
Time:                        22:37:38   Df Model:                            5
                                    Mean Model
==============================================================================
                  coef    std err          t      P>|t|      95.0% Conf. Int.
------------------------------------------------------------------------------
mu          6.5981e-03  2.595e-03      2.543  1.099e-02 [1.513e-03,1.168e-02]
                                  Volatility Model
===============================================================================
                  coef    std err          t      P>|t|       95.0% Conf. Int.
-------------------------------------------------------------------------------
omega       5.0976e-04  3.418e-04      1.491      0.136 [-1.602e-04,1.180e-03]
alpha[1]        0.0552  3.123e-02      1.766  7.739e-02   [-6.056e-03,  0.116]
gamma[1]        0.0979  9.656e-02      1.014      0.310   [-9.132e-02,  0.287]
beta[1]         0.7961  9.053e-02      8.794  1.448e-18      [  0.619,  0.973]
===============================================================================

Covariance estimator: robust
```

```
 Regression
                            OLS Regression Results
==============================================================================
Dep. Variable:                      y   R-squared:                       0.262
Model:                            OLS   Adj. R-squared:                  0.260
Method:                 Least Squares   F-statistic:                     207.7
Date:                Wed, 15 May 2019   Prob (F-statistic):           1.60e-40
Time:                        22:37:38   Log-Likelihood:                 1266.9
No. Observations:                 588   AIC:                            -2530.
Df Residuals:                     586   BIC:                            -2521.
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                  coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          -0.0107      0.006     -1.930      0.054      -0.022       0.000
x1              1.1232      0.078     14.411      0.000       0.970       1.276
==============================================================================
Omnibus:                      341.978   Durbin-Watson:                   1.473
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             4617.936
Skew:                           2.294   Prob(JB):                         0.00
Kurtosis:                      15.940   Cond. No.                         67.6
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

```
MSE: 0.0007950274571317165
RMSE: 0.02819623125759392
MAE: 0.0201179591229439
```