Pseudo Code for Repair Algorithm

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1 Input

- A problem $P^0 = min\{c^T x \mid Ax \geq b,\, x_i \in \{0,1\},\, \forall i \in \mathcal{I},\, \mathcal{I} \subseteq N = \{1, 2, \ldots, n\}\}$ with n binary variables x, objective function $c^T x$ (min), constraint set C^0 with an optimal solution x^0.
- A problem P^1 with n binary variables x, objective function cx (min), constraint set C^1, such that $C^0 \subsetneq C^1$.

2 Output

- An optimal solution x^1 to P^1.

3 General Idea

We propose to solve P^1 by reusing the optimal solution x^0 to P^0. In order to achieve this, we define a new problem Q with constraint set C^1 and objective function

$$\min cx + \alpha|x - x^0|,$$

where $|x - x^0| = \sum_{i=0}^{n-1} |x_i - x_i^0|$, and α is a *penalty* term for deviating from the input solution x^0. This would tentatively help the search for a good solution to P^1. However, unless x^0 is feasible for P^1, an optimal solution to Q will in general not be optimal for P^1.

To remedy thiss problem, we will instead solve a sequence of problems $Q^0, Q^1, \ldots,$ where the penalty factor α will gradually decrease until it reaches 0, say at iteration k, in which case $Q^k = P^1$. This sequence of problems can be efficiently solved using a technique called *reoptimisation*, which is implemented in the MIP solver SCIP.

4 Pseudo Code

Algorithm 1: Restriction

Data: $G = (X, U)$ such that G^{tc} is an order.

Result: $G' = (X, V)$ with $V \subseteq U$ such that G'^{tc} is an interval order.

```
begin
    $V \longleftarrow U$
    $S \longleftarrow \emptyset$
    for $x \in X$ do
        $NbSuccInS(x) \longleftarrow 0$
        $NbPredInMin(x) \longleftarrow 0$
        $NbPredNotInMin(x) \longleftarrow |ImPred(x)|$
    end
    for $x \in X$ do
        if $NbPredInMin(x) = 0$ and $NbPredNotInMin(x) = 0$ then
            $AppendToMin(x)$
        end
    end
    while $S \neq \emptyset$ do
        remove $x$ from the list of $T$ of maximal index
        while $|S \cap ImSucc(x)| \neq |S|$ do
            for $y \in S - ImSucc(x)$ do
                { remove from $V$ all the arcs $zy$ : }
                for $z \in ImPred(y) \cap Min$ do
                    remove the arc $zy$ from $V$
                    $NbSuccInS(z) \longleftarrow NbSuccInS(z) - 1$
                    move $z$ in $T$ to the list preceding its present list
                    {i.e. If $z \in T[k]$, move $z$ from $T[k]$ to $T[k-1]$}
                end
                $NbPredInMin(y) \longleftarrow 0$
                $NbPredNotInMin(y) \longleftarrow 0$
                $S \longleftarrow S - \{y\}$
                $AppendToMin(y)$
            end
        end
        $RemoveFromMin(x)$
    end
end
```

Algorithm 2: How to write algorithms

Result: Write here the result

initialization;
while *While condition* **do**
    instructions;
    if *condition* **then**
        instructions1;
        instructions2;
    else
        instructions3;
    end
end

5 MIP

5.1 Mixed Integer Programming

Definition 5.1. (Mixed Integer Programming)
Let $m, n \in \mathbb{N}$, The given matrix $A \in \mathbb{R}^{m \times n}$, vectors $b \in \mathbb{R}^m$, and the vector $c \in \mathbb{R}^n$, and a set $\mathcal{I} \subseteq N = \{1, \dots, n\}$. the problem

$$\begin{aligned} \text{(MIP)} \quad & c^* = \min c^T x \\ & s.t. \quad Ax \geq b \\ & \qquad\; x_i \in \mathbb{Z}_{\geq 0} \quad \forall_i \in \mathcal{I} \\ & \qquad\; x_j \in \mathbb{R}_{\geq 0} \quad \forall_i \in \mathcal{N} \setminus \mathcal{I} \end{aligned}$$

is called *mixedintegerprogram* with the objective function $c^T x$ and constraints $A_i x \geq b_i$ for all $i = 1, \dots, m$.

A vector $x \in X_{MIP} = \{x \in \mathbb{R}_{\geq} 0^n \mid A_x \geq b,\, x_i \in \mathbb{Z}_{\geq 0}\, \forall i \in \mathcal{I}\}$ is called $feasiablesolution$ and X_{MIP} the set of feasible solutions. A feasible x^* is called optimal, if x^* satisfies $c^* = c^T x^*$.

Common special cases of MIPs are *linearprograms* (LPs) for $\mathcal{I} = \emptyset$ and *integerprogram* (IPs) for $\mathcal{I} = N$. Additional, an integer variable bounded by 0 and 1 is called *binaryvariable.* let $\mathcal{B} \subset \mathcal{I}$ denote the set of binary variables. An integer program with $\mathcal{B} = N$ is called *binaryprogram* (BP) or *mixedbinaryprogram* (MBP), if $\mathcal{B} = \mathcal{I} \subsetneq N$.

A lower or dual bound on a MIP can be computed by neglecting the intergrality constraints. the so-obtained problem is called the *LP-relaxation* of the MIP.

Definition 5.2. (LP-relaxation)

Given a MIP as introduced in Definition 5.1. The LP-*relaxation* is defined as

$$\begin{aligned} \text{(MIP)} \quad & c^* = \min c^T x \\ & s.t. \quad Ax \geq b \\ & \qquad\;\; x \in \mathbb{R}^n_{\geq 0} \end{aligned}$$

Analogous to X_{MIP} we can define X_{LP} as the set of feasible solutions of the LP-relaxation. A feasible solution $x^*_{LP} \in X_{LP}$ is called LP-*optimal* if $c^*_{LP} = c^T x^*_{LP}$. In general solving MIPs is NP-hard. One common method for solving MIPs is LP-$basedbranch - and - bound$. This method splits the problem into smaller subproblems, and procedure is repeated on these subproblems. At any point a global upper or primal bound is given by the best known solution, if existent, and a local lower bound or dual bound is given by the respective LP-relaxation.

5.2 Pseudo-Boolean optimization

In the section 2.4 we will present a special kind of a binary problem, a so-called $pseudo{-}Booleanproblem$. For this purpose we introduce the basic definition of a $pseudo-Booleanproblem$ in this section.For more detail we refer to Hammer and Rubin and Boros and Hammer and the references therein.

Let us denote by $\mathbb{B} = \{0, 1\}$ the set of binary values and let $N = \{1, \dots, n\}$ be an index set, Reflecting to Boros and Hammer we consider functions in n binary variables $x_1, x_2 \dots, x_n$ and denote the binary vector by $(x_1, x_2, \dots, x_n in \mathbb{B}^n)$. A function $f : \mathbb{B}^n \to \mathbb{R}$ of the form

$$f(x_1, x_2, \dots, x_n) = \sum_{S \subseteq N} C_S \prod_{i \in S} x_i$$

where $C_s \in \mathbb{R}$ for each $S \subseteq N$, is called $pseudo - Booleanfunction$. see e.g., Hannmer et al. The degree of the function $deg(f)$ is given by the seize of the largest set $S \subseteq N$ which$C_S \neq 0$.A pseudo-Boolean function f is called linear, quadratic, cubic etc. if $deg(f) \leq 1, 2, 3$ etc. repectively. Liu and Truszczynski defined a $pseudo - Booleanconstraint$ as an integer inequality of the form

$$\sum_{i=1}^{n} a_i x_i \geq b$$

with $a_i, b in \mathbb{Z}$ and $xi \in \mathbb{B}$ for all $iin[n]$. A binary problem defined by a pseudo-Boolean objective function and a set of pseudo-Boolean constraints is called $pseudo - Boolean\, optimization\, problem$.