Pseudo Code for Repair Algorithm

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1 Introduction

In this section, we define some important definitions and algorithms....

Definition 1.1. (Mixed Integer Programming)
In this report we consider a generic mixed-integer programming problem (MIP) in the following form

$$\begin{aligned} \text{(MIP)} \quad & \min \ c^T x \\ s.t. \quad & Ax \geq b \\ & x_j \in \mathbb{Z} \quad \forall j \in \mathcal{I} \\ & x_j \in \mathbb{R} \quad \forall j \in \mathcal{N} \setminus \mathcal{I} \end{aligned}$$

where the vector $b \in \mathbb{R}^m$ and the vector $c \in \mathbb{R}^n$ are input vectors. A is a input matrix of size $m \times n$, the variable input set $\mathcal{I} \subseteq \mathcal{N} = \{1, 2, \ldots, n\}$. We denote $\mathcal{P}$ for this problem, which called a mixed-integer programming problem (MIP) with minimize objective function $c^T x$ subject to the constraints $Ax \geq b$. Besides, some variables are restricted to integer values while the else of are restricted to real value. S is a set of feasible solution if S satisfy all the constraints in the problem. A vector s^* with $s^* \in S$ is called *optimal solution* when $c^T x_{s^*} \leq c^T x_s$ for $\forall s \in S$. When all of the variables are restricted to integer, the problem is called *pureintegerlinearprogram*(IP) for $\mathcal{I} = \mathcal{N}$. If there is no integrality constraint, the program is called *linearprogram*

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Definition 1.2. (LP-relaxation)
Lp *relaxation* is obtained by removing all integrity constraints $\mathcal{I} \leftarrow \emptyset$. LP-*relaxation* is the foundation of LP-based branch-and-bound technology. As the searching space is increase by removing integrity restrictions, the optimal solution in MIP problem could not better than LP-*relaxation*, which is $s^*_{MIP} \geq s^*_{LP}$. This means the optimal solution found in LP problem could provide a lower or prime bound for MIP problem.

2 Input

- A MIP problem $\mathcal{P}^0$ with n variables x,constraint set C^0 with an optimal solution s^0, where s^0 is a n-vector.
- A MIP problem $\mathcal{P}^1$ with n variables x, constraint set C^1, such that $C^0 \subsetneq C^1$.

3 Output

- An optimal solution s^1 to $\mathcal{P}^1$, where s^1 is a n-vector too.

4 Pseudo Code

Algorithm 1: Solving Problem with Reoptimization

Input: k, $\mathcal{P}^0, s^0$ and $\mathcal{P}^1$ where $C^0 \subsetneq C^1$
Output: optimal solution s^* to $\mathcal{P}^1$

begin
  if s^0 *is feasible to* $\mathcal{P}^1$ **then**
    return s^0
  else
    $\alpha \longleftarrow k \times c^T s^0$
    create new variables y where $y_j \in \mathbb{Z}, \forall j \in \mathcal{I}$ `// add new variables`
    for j *in* $\mathcal{I}$ **do**
      $\mathcal{P}^2 \longleftarrow$ add constraints to $\mathcal{P}^1 : y_j \geq x_j - s^0_j$ `// add new constraints`
      $\mathcal{P}^2 \longleftarrow$ add constraints to $\mathcal{P}^2 : y_j \geq s^0_j - x_j$
    end
    add penalty function to $\mathcal{P}^2 : \alpha \times \sum_j y_j,\ j \in \mathcal{I}$ `// add penalty function`
    $r \longleftarrow 1$
    for $\alpha \geq 0$ **do**
      $s^r \longleftarrow$ solve $\mathcal{P}^2_r$ (r-th iteration) to optimal or stop criterion with reoptimization
      $\alpha \longleftarrow \alpha - \dfrac{\alpha}{k}$
    end
    $s^* \longleftarrow s^r$
    return s^*
  end
end