Pseudo Code for Repair Algorithm

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1 Introduction

Definition 1.1. (Mixed Integer Programming)
In this report, we denote the $Mixed\,Integer\,programming$ (MIP) problem in the follow form

$$(\text{MIP}) \quad \min\{\, c^T x : Ax \leq b, x \in \mathbb{Z}^l \,\text{x}\, \mathbb{R}^k\}$$

where $m, n \in \mathbb{N}$, and $n = l + k$, A is a matrix for $A \in \mathbb{R}^{m\,\text{x}\,n}$, and the vector $b \in \mathbb{R}^m$, the vector $c \in \mathbb{R}^n$.

S is a set of $feasible\,solution$ if S satisfy all the constraints in the problem. If $s \in S$, s is called a feasible solution to the problem $\mathcal{Z}_{MIP}$. The MIP is called infeasible when S is empty, other wise, the MIP is called feasible. The feasible solution s^* is called optimal when $s^* \in S$ and $c^T x_{s^*} \leq c^T x_s$ for $\forall s \in S$.

Definition 1.2. (Linear Programming and LP-relaxation)
With the definition of MIP in 1.1, there is a special case of MIP when all variables are continuous, which is called $Linear Programming$ (LP) problem

$$\mathcal{Z}_{LP} = \min\{\, c^T x : Ax \leq b, x \in \mathbb{R}^k\}$$

A LP can also be obtained by removing all integrity constraints: $x_i \in x$ where $i \in n \setminus l$, this is called LP-*relaxation*. LP-*relaxation* is the foundation of LP-based branch-and-bound technology. As the searching space is increase by removing integrity restrictions, the optimal solution in MIP could not better than LP-*relaxation*, which is $s^*_{MIP} \geq s^*_{LP}$. This means the optimal solution found in LP could provide a lower or prime bound for MIP.

Definition 1.3. (Binary Programs)
In the definition 1.1, When all variables are integer

$$\mathcal{Z}_{IP} = \min\{\, c^T x : Ax \leq b, x \in \mathbb{Z}^l\}$$

The problem is call pure Integer Programming (IP) problem. And if we denote $\mathbb{B}$ be a set of binary values where $\mathbb{B} = \{0, 1\}$ for $\forall x \in \mathbb{B}^l$, then we call this problem a Binary Programming problem. This report we focus on Binary Programs.
Question: the two terminologies $Binaryprogrammingproblem$ and $pseudo - Booleanproblem$ is same? or not?

2 Input

- A problem $\mathcal{P}^0 = min\{c^T x \,|\, Ax \geq b,\, x_i \in \mathbb{B}^n, \mathbb{B} = \{0,1\}\}$ with n binary variables x, objective function $O^0 = c^T x$ (min), constraint set C^0 with an optimal solution s^0.
- A problem $\mathcal{P}^1$ with n binary variables x, objective function $O^1 = O^0 = c^T x$ (min), constraint set C^1, such that $C^0 \subsetneq C^1$.

3 Output

- An optimal solution s^1 to $\mathcal{P}^1$.

4 Pseudo Code

Algorithm 1: Solving Problem with Reoptimization

Data: $r, \mathcal{P}^0, s^0$ and $\mathcal{P}^1$ where $C^0 \subsetneq C^1$, $O^0 = O^1$
Result: optimal solution s^1 to $\mathcal{P}^1$
begin
  if s^0 *is feasible to*$\mathcal{P}^1$ **then**
    return s^0
  else
    $C^+ \longleftarrow C^1 - C^0$
    for $c \in C^+$ **do**
      add v_i to the set V where it's coefficient $c_i \neq 0$
    end
    $y^0 = c^T x$ for $x \in s^0$
    $y^p = c^T x$ for $x \in s^0 \cap V^+$
    $\Delta y = |Y^0 - Y^p|$
    if $\Delta y / y^0 \geq r$ *solve* $\mathcal{P}^1$ *from scratch*
     then
      solve it with reoptimization
      $p \longleftarrow (|y^0| + |y^p|)$
      $O^p \longleftarrow p * \sum x$ for $x \in V^+$
      $\mathcal{P}^* \longleftarrow O^p + C1$
      $V^+ \longleftarrow forvinC^+ wherec$
    else

    end
    $V \longleftarrow U$
    $S \longleftarrow \emptyset$
    for $x \in X$ **do**
      $NbSuccInS(x) \longleftarrow 0$
      $NbPredInMin(x) \longleftarrow 0$
      $NbPredNotInMin(x) \longleftarrow |ImPred(x)|$
    end
    for $x \in X$ **do**
      if $NbPredInMin(x) = 0$ **and** $NbPredNotInMin(x) = 0$ **then**
        $AppendToMin(x)$
      end
    end
    while $S \neq \emptyset$ **do**
      remove x from the list of T of maximal index
      while $|S \cap ImSucc(x)| \neq |S|$ **do**
        for $y \in S - ImSucc(x)$ **do**
          { remove from V all the arcs zy : }
          for $z \in ImPred(y) \cap Min$ **do**
            remove the arc zy from V
            $NbSuccInS(z) \longleftarrow NbSuccInS(z) - 1$
            move z in T to the list preceding its present list
            {i.e. If $z \in T[k]$, move z from $T[k]$ to $T[k-1]$}
          end
          $NbPredInMin(y) \longleftarrow 0$
          $NbPredNotInMin(y) \longleftarrow 0$
          $S \longleftarrow S - \{y\}$
          $AppendToMin(y)$
        end
      end
      $RemoveFromMin(x)$

Algorithm 2: How to write algorithms

```
Result: Write here the result
initialization;
while While condition do
    instructions;
    if condition then
        instructions1;
        instructions2;
    else
        instructions3;
    end
end
```

5 MIP

5.1 Mixed Integer Programming

Definition 5.1. (Mixed Integer Programming)
Let $m, n \in \mathbb{N}$, The given matrix $A \in \mathbb{R}^{m \times n}$, vectors $b \in \mathbb{R}^m$, and the vector $c \in \mathbb{R}^n$, and a set $\mathcal{I} \subseteq N = \{1, \ldots, n\}$. the problem

$$\begin{aligned} \text{(MIP)} \quad & c^* = \min c^T x \\ & s.t. \quad Ax \geq b \\ & \qquad x_i \in \mathbb{Z}_{\geq 0} \quad \forall_i \in \mathcal{I} \\ & \qquad x_j \in \mathbb{R}_{\geq 0} \quad \forall_i \in \mathcal{N} \setminus \mathcal{I} \end{aligned}$$

is called *mixedintegerprogram* with the objective function $c^T x$ and constraints $A_i x \geq b_i$ for all $i = 1, \ldots, m$.

A vector $x \in X_{MIP} = \{x \in \mathbb{R}_{\geq} 0^n \mid A_x \geq b,\, x_i \in \mathbb{Z}_{\geq 0} \, \forall i \in \mathcal{I}\}$ is called $feasiable solution$ and X_{MIP} the set of feasible solutions. A feasible x^* is called optimal, if x^* satisfies $c^* = c^T x^*$.

Common special cases of MIPs are *linearprograms* (LPs) for $\mathcal{I} = \emptyset$ and *integerprogram* (IPs) for $\mathcal{I} = N$. Additional, an integer variable bounded by 0 and 1 is called *binaryvariable.* let $\mathcal{B} \subset \mathcal{I}$ denote the set of binary variables. An integer program with $\mathcal{B} = N$ is called *binaryprogram* (BP) or *mixedbinaryprogram* (MBP), if $\mathcal{B} = \mathcal{I} \subsetneq N$.

A lower or dual bound on a MIP can be computed by neglecting the intergrality constraints. the so-obtained problem is called the *LP-relaxation* of the MIP.

Definition 5.2. (LP-relaxation)

Given a MIP as introduced in Definition **??**. The LP-*relaxation* is defined as

$$\begin{aligned} \text{(MIP)} \quad & c^* = \min c^T x \\ & s.t. \quad Ax \geq b \\ & \qquad\quad x \in \mathbb{R}^n_{\geq 0} \end{aligned}$$

Analogous to X_{MIP} we can define X_{LP} as the set of feasible solutions of the LP-relaxation. A feasible solution $x^*_{LP} \in X_{LP}$ is called LP-*optimal* if $c^*_{LP} = c^T x^*_{LP}$. In general solving MIPs is NP-hard. One common method for solving MIPs is LP-$based branch-and-bound$. This method splits the problem into smaller subproblems, and procedure is repeated on these subproblems. At any point a global upper or primal bound is given by the best known solution, if existent, and a local lower bound or dual bound is given by the respective LP-relaxation.

5.2 Pseudo-Boolean optimization

In the section 2.4 we will present a special kind of a binary problem, a so-called $pseudo-Boolean problem$. For this purpose we introduce the basic definition of a $pseudo-Boolean problem$ in this section.For more detail we refer to Hammer and Rubin and Boros and Hammer and the references therein.

Let us denote by $\mathbb{B} = \{0, 1\}$ the set of binary values and let $N = \{1, \ldots, n\}$ be an index set, Reflecting to Boros and Hammer we consider functions in n binary variables $x_1, x_2 \ldots, x_n$ and denote the binary vector by $(x_1, x_2, \ldots, x_n in \mathbb{B}^n)$. A function $f : \mathbb{B}^n \to \mathbb{R}$ of the form

$$f(x_1, x_2, \ldots, x_n) = \sum_{S \subseteq N} C_S \prod_{i \in S} x_i$$

where $C_s \in \mathbb{R}$ for each $S \subseteq N$, is called $pseudo-Boolean function$. see e.g., Hannmer et al. The degree of the function $deg(f)$ is given by the seize of the largest set $S \subseteq N$ which$C_S \neq 0$.A pseudo-Boolean function f is called linear, quadratic, cubic etc. if $deg(f) \leq 1, 2, 3$ etc. repectively. Liu and Truszczynski defined a $pseudo-Boolean constraint$ as an integer inequality of the form

$$\sum_{i=1}^{n} a_i x_i \geq b$$

with $a_i, b in \mathbb{Z}$ and $xi \in \mathbb{B}$ for all $i in [n]$. A binary problem defined by a pseudo-Boolean objective function and a set of pseudo-Boolean constraints is called $pseudo-Boolean\, optimization\, problem$.