A is a stable matrix $\Rightarrow A^T P + PA = -I$ has a positive solution P

Since $\frac{\|g(x)\|}{\|x\|} \to 0$ as $\|x\| \to 0$

$\forall \varepsilon, \exists \delta$ such that if $\|x\| < \delta(\varepsilon)$ $\frac{\|g(x)\|}{\|x\|} \leq \varepsilon$

Thus $\|g(x)\| \leq \varepsilon \|x\|$ when $\|x\| < \delta(\varepsilon)$

$V = x^T P x$

$$
\begin{aligned}
\dot{V} &= \dot{x}^T P x + x^T P \dot{x} \\
&= (Ax + g(x))^T P x + x^T P (Ax + g(x)) \\
&= x^T (A^T P + PA) x + 2x^T P g(x) \\
&= -x^T x + 2x^T P g(x) \\
&\leq -x^T x + 2\|Px\| \cdot \|g(x)\| \\
&\leq -\|x\| + 2\varepsilon \|P\| \|x\|^2
\end{aligned}
$$

when $\varepsilon \leq \frac{1}{4\|P\|}$ and $\|x\| < \delta(\varepsilon)$

$$
\begin{aligned}
\dot{V} &\leq -\frac{1}{2}\|x\|^2 \\
&\leq -\frac{x^T P x}{2\lambda_{max}(P)} \\
&= -\frac{V}{2\lambda_{max}(P)} \\
&= -\mu V
\end{aligned}
$$

Therefore. $V(t) \leq e^{-\mu t} V(0) \Rightarrow \lim_{t \to \infty} V(t) = 0$

$\Rightarrow \lim_{t \to \infty} x(t) = 0$ when $\|x_0\| < \delta(\varepsilon)$ and $\varepsilon = \frac{1}{4\|P\|}$