

1.

$$u^1 = x^1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$u^2 = x^2 - \frac{\langle u^1, x^2 \rangle}{\|u^1\|^2} u^1 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \\ -\frac{1}{2} \end{bmatrix}$$

$$\begin{aligned} u^3 &= x^3 - \frac{\langle u^1, x^3 \rangle}{\|u^1\|^2} u^1 - \frac{\langle u^2, x^3 \rangle}{\|u^2\|^2} u^2 \\ &= \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \end{bmatrix} - \frac{1}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix} - \frac{\frac{1}{2}}{\frac{3}{2}} \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \\ -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ 1 \\ -\frac{1}{3} \end{bmatrix} \end{aligned}$$

Normalization:

$$u^1 = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$$

$$u^2 = \frac{\sqrt{6}}{3} \begin{bmatrix} -\frac{1}{2} \\ 1 \\ 0 \\ -\frac{1}{2} \end{bmatrix}$$

$$u^3 = \frac{\sqrt{3}}{2} \begin{bmatrix} -\frac{1}{3} \\ \frac{1}{3} \\ 1 \\ -\frac{1}{3} \end{bmatrix}$$