ECE-GY 5253: Applied Matrix Theory — Fall 2023 Final Exam

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Due: December 19, 2pm (EST)

Problem 1. *(30 pts)*

Given the matrix A

$$A = \begin{bmatrix} 3 & -1 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

1.1 Find the Jordan form of the matrix A, and give the corresponding transformation matrix P. Show all your steps.

1.2 Compute the solution of the differential equation $\dot{x}(t) = Ax(t)$ with the initial condition $x(0) = [0, 2, 1]^T$. Show all your steps.

Problem 2. *(40 pts)*

Given the matrix A

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

2.1 Perform the QR factorization of matrix A using the Gram-Schmidt process. That is, $A = QR$ where Q is an orthogonal matrix and R is an upper triangular matrix. Show all your steps.

2.2 Find the singular value decomposition (SVD) of A. Show all your steps.

Problem 3. *(30 pts)*

Consider a nonsingular matrix $S \in \mathbb{R}^{n\times n}$ and two vectors $x, y \in \mathbb{R}^n$. Please prove the following statements.

3.1 $S + xy^T$ is nonsingular if and only if $1 + y^T S^{-1} x \neq 0$.

3.2 Suppose $1 + y^T S^{-1} x \neq 0$, show that $(S + xy^T)^{-1} = S^{-1} - \frac{S^{-1}xy^TS^{-1}}{1+y^TS^{-1}x}$.