Applied Matrix Theory

ECE-GY 5253 Midterm

Fall 2023

Due: Saturday, November 4, 11 am (US Eastern Time)

Problem 1 (30 pts)

Transform the following matrix into one of the canonical forms.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 1 & 2 \end{bmatrix}$$

Problem 2 (30 pts)

Are the following statements true or false ? If true, provide a proof; if false, give a counter-example.

(a) Let $M \in \mathbb{R}^{n\times m}$ and $r = rank(M)$. There exist $A \in \mathbb{R}^{n\times r}$ and $B \in \mathbb{R}^{r\times m}$ such that $M = AB$.

(b) Let $A \in \mathbb{R}^{n\times n}$ be a symmetric matrix. A is orthogonal if and only if its eigenvalues all have the absolute value of one.

(c) If $A \in \mathbb{R}^{n\times n}$, then the sum of the n eigenvalues of A is the trace of A and the product of the n eigenvalues is the determinant of A.

Problem 3 (40 pts)

Let $A \in \mathbb{R}^{n\times n}$. Suppose that $u \in \mathbb{R}^n$ ($v \in \mathbb{R}^n$) is nonzero right (left) eigenvector corresponding to the eigenvalue $\alpha \in \mathbb{C}$ ($\beta \in \mathbb{C}$), i.e.

$$Au = \alpha u, \quad v^T A = \beta v^T.$$

Prove that

1. If $\alpha \neq \beta$, then $v^T u = 0$.
2. If $\alpha = \beta$ and $v^T u \neq 0$, then there exists an invertible matrix $T = [u, T_1]$ with $(T^{-1})^T = [v/(u^T v), M_1]$, where $T_1, M_1 \in \mathbb{R}^{n\times(n-1)}$, such that

$$A = T \begin{bmatrix} \alpha & 0 \\ 0 & N \end{bmatrix} T^{-1}, \quad N \in \mathbb{R}^{(n-1)\times(n-1)}.$$