

W6 - KNN

$$y = f(x) + \epsilon, \epsilon \sim N(0, \sigma^2)$$

$$\hat{y} = f(x), f(x_i) = \frac{1}{k} \sum_{j \in k} t(x_j) + \epsilon_i$$

$$\text{Bias}^2 = (t(x) - \frac{1}{k} \sum_{j \in k} t(x_j))^2$$

$$\text{Var}(\hat{y}) = E[(\frac{1}{k} \sum_{j \in k} \epsilon_j)^2] = \frac{\sigma^2}{k}$$

$$\text{Euclidean}(L^2) = \sqrt{\sum_{i=1}^d (a_i - b_i)^2}$$

$$\text{Manhattan}(L1) = \sum_{i=1}^d |a_i - b_i|$$

Sets to consider in exhaustive search	Sets to consider in naive search	Sets to consider in sequential forward (example)
{1, 2, 3, 4, 5, 6, 7, 8, 9}	{1, 2, 3, 4, 5, 6, 7, 8, 9}	{1, 2, 3, 4, 5, 6, 7, 8, 9}
→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19
→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19	→ 10, 11, 12, 13, 14, 15, 16, 17, 18, 19

W7 - Decision trees and ensembles

$$\text{Gini index} = \sum_{k=1}^K p_{mk} (1 - p_{mk})$$

p_{mk} is the proportion of training samples in k belonging to class K .

$$\text{Entropy} = - \sum_{k=1}^K p_{mk} \log_2 p_{mk}$$

$$\text{Conditional entropy: } E(S|x) = \sum_{v \in S} \frac{|S_v|}{|S|} E(S_v)$$

$$\text{Information gain: } G(S, x) = E(S) - E(S|x)$$

name	predator	milk	domestic	feathers	toothed	category
0	Paraphita	1	0	0	0	fish
1	herring	1	0	0	1	fish
2	sole	0	0	0	0	fish
3	daddock	0	0	0	0	fish
4	skimmer	1	0	0	1	0
5	parakeet	0	0	1	1	bird
6	catfish	1	0	0	0	fish
7	wren	0	0	0	1	bird
8	vulture	1	0	0	1	bird
9	flamingo	0	0	1	0	bird

First, we compute the entropy for the entire set:

$$E(S) = -\frac{5}{10} \log_2 \frac{5}{10} - \frac{5}{10} \log_2 \frac{5}{10} = 1$$

Note that entropy is maximized ($E = 1$) when the data is evenly divided between the two classes.

Now, we will compute the conditional entropy and the information gain for each of the features.

predator:

Considering the predator = 0 branch:

$$S_0 = \{3 \text{ bird, } 2 \text{ fish}\}, |S_0| = 5$$

$$E(S_0) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.97095$$

Considering the predator = 1 branch:

$$S_1 = \{2 \text{ bird, } 5 \text{ fish}\}, |S_1| = 5$$

$$E(S_1) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97095$$

Conditional entropy and information gain of splitting on predator:

$$E(S|\text{predator}) = \frac{1}{10} E(S_0) + \frac{9}{10} E(S_1) = 0.97095$$

$$G(S, \text{predator}) = E(S) - E(S|\text{predator}) = 1 - 0.97095 = 0.02905$$

Feature	Conditional entropy	Information gain
predator	$E(S \text{predator}) = 0.971$	$G(S, \text{predator}) = 0.029$
milk	$E(S \text{milk}) = 1.000$	$G(S, \text{milk}) = 0.000$
domestic	$E(S \text{domestic}) = 0.892$	$G(S, \text{domestic}) = 0.108$
feathers	$E(S \text{feathers}) = 0.898$	$G(S, \text{feathers}) = 0.100$
toothed	$E(S \text{toothed}) = 0.000$	$G(S, \text{toothed}) = 1.000$

$n \uparrow \rightarrow$ more complex

① Decision tree by depth. $\text{depth} \uparrow \rightarrow \text{bias} \downarrow, \text{Var} \uparrow$

② Decision tree by pruning parameter

$\alpha \uparrow \rightarrow \text{bias} \uparrow, \text{Var} \uparrow \rightarrow \text{less complex}$

③ Decision tree (A) vs. Bagged trees (B)

$$\text{Bias}(A) = \text{Bias}(B); \text{Var}(A) > \text{Var}(B)$$

④ Bagged trees by number of estimators

Ensemble $\uparrow \rightarrow \text{Var} \downarrow$

⑤ Bagged trees (A) vs. Random forest (B)

$$\text{Var}(A) > \text{Var}(B)$$

⑥ Decision tree (A) vs. AdaBoost (B)

$$\text{Bias}(A) > \text{Bias}(B); \text{Var}(A) > \text{Var}(B)$$

⑦ AdaBoost by number of iterations

iteration $\uparrow \rightarrow \text{Bias} \downarrow, \text{Var} \downarrow$

Bagged trees: highly correlated;

Random forest: much less correlated

AdaBoost: output will be a weighted

average of the predictions of all three trees

W8 - Support vector classifiers and kernels

$$\underset{w, b}{\text{minimize}} \frac{1}{2} \sum_{j=1}^P w_j^2 + C \sum_{i=1}^n \xi_i$$

Subject to $y_i(w \cdot \sum_{j=1}^P w_j x_{ij}) \geq 1 - \xi_i, \forall i$

$$E; \exists \alpha, \forall i$$

$\hookrightarrow \text{Bias} \downarrow, \text{Var} \downarrow$

PBF parameter: $E(x, z) = \exp(-\frac{|x-z|^2}{\sigma^2})$

σ - bandwidth $= \exp(-r ||x-z||^2)$

$\sigma \uparrow \rightarrow r \downarrow \rightarrow \text{Bias} \uparrow, \text{Var} \downarrow \rightarrow \text{less complex}$

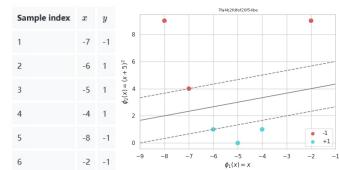
W9 - Neural networks

d. categorical_crossentropy Labels: [0, 1, 0, 1], [1, 0, 0], [0, 1, 0], indicating that these samples belong to class 2, class 0, and class 1, respectively.

b. mean_squared_error function Labels: 2, 0, 1, each representing the value of some continuous quantity.

a. sparse_categorical_crossentropy Labels: 2, 0, 1, indicating that these samples belong to class 2, class 0, and class 1, respectively.

c. binary_crossentropy Labels: 1, 0, 1, each representing a binary class label.



The slope of the line is 1/3, and the intercept is 14/3.

The hypothesis is defined by $\phi_1(x) = 1/3 \phi_2(x) + 14/3$, or equivalently $14 + 1/\phi_2(x) - 3\phi_1(x) = 0$ (any one of the versions of this expression).

Part 2: Express the objective of the dual problem

To find α_1 for the maximal margin classifier, we will solve the dual problem

$$\begin{aligned} \underset{\alpha}{\text{maximize}} \quad & \sum_{i=1}^n \sum_{j=1}^n y_i y_j \phi_1(x_i)^T \phi_1(x_j) \\ \text{s.t.} \quad & \sum_{i=1}^n \alpha_i y_i = 0 \end{aligned}$$

Notice this maximal margin classifier problem is slightly different from the support vector classifier problem. There is no slack variable for the maximal margin classifier.

We know that $\alpha_i = 0$ for any sample that is not a support vector – in this case, only $i = 1, 2$ will have non-zero values. So, we can write the expression

$$\sum_{i=1}^n \alpha_i y_i = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n y_i y_j \phi_1(x_i)^T \phi_1(x_j)$$

in terms of α_1 and α_2 , the support vectors, and x_1 and x_2 , their labels, and y_1 and y_2 . Notice that $\phi_1(x_1)$ and $\phi_1(x_2)$ are vectors. The others are scalars in this expression, and j are sample indices.

Then substitute the values of x_1 and x_2 , y_1 and y_2 , and ϕ_1 , and simplify. So the only unknowns are α_1 ($\alpha_2 = 0$) and α_1 ($\alpha_2 = 0$)

$$\frac{\partial L}{\partial \alpha_1} = 4\alpha_1 + \alpha_2 - \frac{1}{2} (\alpha_1 \phi_1(x_1)^T \phi_1(x_1) - 2\alpha_1 \phi_1(x_1)^T \phi_1(x_2) + \alpha_2 \phi_1(x_2)^T \phi_1(x_2))$$

$$\alpha_1 + \alpha_2 = \frac{1}{2} (\alpha_1^2 - 1) \Rightarrow \alpha_1^2 - 7\alpha_1 - 6 = 0 \Rightarrow \alpha_1 = 6, \alpha_2 = -6, \alpha_1 = 1, \alpha_2 = -1$$

$$\alpha_1 + \alpha_2 = \frac{1}{2} (\phi_1(x_1)^T \phi_1(x_2) + 92\alpha_1 + 37\alpha_2)$$

implies that $\alpha_1 = \alpha_2$.

In your expression for the objective in the previous part, substitute $\alpha_1 = \alpha$ and $\alpha_2 = \alpha$, and simplify. You should be able to express an expression in terms of α (α_1, α_2).

$$\alpha^2 = 2\alpha$$

Comment With $\alpha_1 = \alpha_2 = \alpha$, the expression

$$\alpha = \alpha - \frac{1}{2} (65\alpha^2 + 92\alpha + 37\alpha^2) = 2\alpha - 5\alpha^2$$

becomes

$$\alpha = \alpha - \frac{1}{2} (65\alpha^2 + 92\alpha + 37\alpha^2) = 2\alpha - 5\alpha^2$$

Part 4: Solve by setting the derivative to zero

Take the derivative of the expression from the previous part with respect to α :

$$\frac{\partial L}{\partial \alpha} = 2 - 10\alpha$$

Then, set it equal to zero. Solve to find the value of $\alpha_1 = \alpha_2 = \alpha$.

$$\alpha = 2/10 = 0.2$$

Find the optimal values of the coefficients of the separating hyperplane, using

$$w_j = \sum_{i=1}^n \alpha_i y_i \phi_j(x_i), j = 1, 2$$

where i is a sample index, and j indexes the columns (in the transformed space defined by $\phi(x)$).

$$w_1 = 0.2, w_2 = 0.4$$

Comment

$$\frac{d}{d\alpha} 2\alpha - 5\alpha^2 = 0 \rightarrow 2 = (2)(5)(\alpha) = 0 \rightarrow \alpha = -\frac{1}{5}$$

Then,

$$w_1 = \left(\frac{1}{5}\right)(-1)(-7) + \left(\frac{1}{5}\right)(1)(-6) = \frac{1}{5}$$

and

$$w_2 = \left(\frac{1}{5}\right)(-1)(4) + \left(\frac{1}{5}\right)(1)(1) = -\frac{3}{5}$$

Part 5: Find w_0 , compute:

$$w_0 = \frac{1}{2} \sum_{i=1,2} w_i \cdot (\langle w_i, w_2 \rangle, \phi(x_i))$$

where i is a sample index, and you use the w_1 and w_2 you found above.

$$w_0 = 2.000$$

Comment For either support vector, $y_i = \langle w_i, w_2 \rangle, \phi(x_i)$.

For the first support vector:

$$-1 = \left(\frac{1}{5}\right)(-6) + \left(\frac{-3}{5}\right)(4) = 14/5$$

For the second support vector:

$$1 = \left(\frac{1}{5}\right)(-6) + \left(\frac{-3}{5}\right)(1) = 14/5$$

Part 4: Update weights

Compute the updated weights for both the hidden layer and the output layer by performing one step of gradient descent. Use a learning rate of 0.4.

$$W_{o1} = [w_{1,1,o1}, w_{2,2,o1}, w_{3,3,o1}] = [2.058, -0.449, -1.942]$$

$$W_{A1} = [w_{x1,A1}, w_{x2,A1}, w_{ph,A1}] = [1.500, 2.001, 2.501]$$

$$W_{A2} = [w_{x1,h2}, w_{x2,h2}, w_{ph,h2}] = [-2.500, -1.003, 2.997]$$

Part 4: Update weights

1. 更新输出层权重 W_{o1}

○ 学习率 $\alpha = 0.4$

$$\circ w_{x1,h1}^{new} = w_{x1,h1} - \alpha \frac{\partial L}{\partial w_{x1,h1}} = -2 - 0.4 \times (-0.144) = -2 + 0.0576 = 2.058$$

$$\circ w_{x2,h1}^{new} = w_{x2,h1} - \alpha \frac{\partial L}{\partial w_{x2,h1}} = -0.5 - 0.4 \times (-0.128) = -0.5 + 0.0512 = 0.4$$

$$\circ w_{ph,h1}^{new} = w_{ph,h1} - \alpha \frac{\partial L}{\partial w_{ph,h1}} = -2 - 0.4 \times (-0.145) = -2 + 0.058 = -1.942$$

○ 所以 $W_{o1} = [2.058, -0.449, -1.942]$

2. 更新隐藏层 h_1 的权重 W_{h1}

$$\circ w_{x1,h1}^{new} = w_{x1,h1} - \alpha \frac{\partial L}{\partial w_{x1,h1}} = -1.5 - 0.4 \times 0.000 = -1.5$$

$$\circ w_{x2,h1}^{new} = w_{x2,h1} - \alpha \frac{\partial L}{\partial w_{x2,h1}} = 2 - 0.4 \times (-0.003) = 2 + 0.0012 = 2.001$$

$$\circ w_{ph,h1}^{new} = w_{ph,h1} - \alpha \frac{\partial L}{\partial w_{ph,h1}} = 2.5 - 0.4 \times (-0.003) = 2.5 + 0.0012 = 2.501$$

○ 所以 $W_{h1} = [1.500, 2.001, 2.501]$

3. 更新隐藏层 h_2 的权重 W_{h2}

$$\circ w_{x1,h2}^{new} = w_{x1,h2} - \alpha \frac{\partial L}{\partial w_{x1,h2}} = -2.5 - 0.4 \times 0.000 = -2.5$$

$$\circ w_{x2,h2}^{new} = w_{x2,h2} - \alpha \frac{\partial L}{\partial w_{x2,h2}} = -1 - 0.4 \times 0.008 = -1 - 0.0032 = -1.003$$

$$\circ w_{ph,h2}^{new} = w_{ph,h2} - \alpha \frac{\partial L}{\partial w_{ph,h2}} = 3 - 0.4 \times 0.008 = 3 - 0.0032 = 2.997$$

○ 所以 $W_{h2} = [-2.500, -1.003, 2.997]$

W10 - Day learning , CNN

Suppose an image with dimension 224 × 224 and 3 color channels is passed to the first layer of a neural network.

How many weights must be learned between the input layer and the first hidden layer if it is...

... a fully connected layer with 128 units?

$$24 \times 224 \times 3 \times 128$$

Weights between input units and hidden units in first layer: 192767384

+ ↗ 100%

Bias terms to hidden units in first layer: 128

+ ↗ 100%

... a convolutional layer with 128 filters, each having filter size 3 × 3, stride 1, and 1 padding?

Weights between input units and hidden units in first layer: 3456

+ ↗ 100%

Bias terms to hidden units in first layer: 128

+ ↗ 100%

H10.5: Fine-tune a pre-trained model

You are developing a machine learning model to classify images of humans in various poses as one of:

[‘sitting’, ‘standing’, ‘lying’, ‘sancing’]

You have loaded your training dataset into the variable x_{trn} , and the associated labels into the variable y_{trn} . Each image has dimension (224, 224, 3). The class labels are given as integers (i.e. as an index for the array of labels shown above).

You decide to use a pre-trained model trained on general image data, and add a new classification head for this specific task. You'll also include a dropout layer between the pre-trained base model and your classification head, for regularization. You'll train this for 20 epochs. After training the classification head, you will also fine-tune the last layers of the pre-trained model for this specific task, for another 10 epochs.

Use the blocks below to construct and train your classifier.

Correct answer (there may be other correct orders):

1. `INPUT_ME_SHAPES = ([224, 224, 3],)`

2. `base = VGG16(weights='imagenet', include_top=False)`

3. `base.trainable = False`

4. `model = Sequential()`

5. `model.add(base)`

6. `model.add(Dropout(0.5))`

7. `model.add(Flatten())`

8. `model.add(Dense(4, activation='softmax'))`

9. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

10. `model.fit(xtrn, ytrn, epochs=20)`

11. `model.trainable = True`

12. `for epoch in range(10, 20):`

13. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

14. `model.fit(xtrn, ytrn, epochs=1)`

15. `model.trainable = False`

16. `for epoch in range(1, 10):`

17. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

18. `model.fit(xtrn, ytrn, epochs=1)`

19. `model.trainable = False`

20. `for epoch in range(1, 10):`

21. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

22. `model.fit(xtrn, ytrn, epochs=1)`

23. `model.trainable = False`

24. `for epoch in range(1, 10):`

25. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

26. `model.fit(xtrn, ytrn, epochs=1)`

27. `model.trainable = False`

28. `for epoch in range(1, 10):`

29. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

30. `model.fit(xtrn, ytrn, epochs=1)`

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156. `for epoch in range(1, 10):`

157. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

158. `model.fit(xtrn, ytrn, epochs=1)`

159. `model.trainable = False`

160. `for epoch in range(1, 10):`

161. `model.compile(optimizer='Adam', loss='categorical_crossentropy', metrics=['accuracy'])`

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174. `model.fit(xtrn, ytrn, epochs=1)`

175. `model.trainable = False`

Backpropagation o NN

Short answer grading

```

import numpy as np
def sigmoid(x):
    return 1 / (1 + np.exp(-x))
def relu(x):
    return np.maximum(0, x)
def relu_derivative(x):
    return np.where(x > 0, 1, 0)

# 定义权重
w_xh1l = 1.5
w_xh2l = 2
w_xh2h = -2.5
w_xh2h2 = -1
w_h1o1 = 2
w_h2o1 = -0.5
w_xbh1 = 2.5
w_xbh2 = 3
w_bbh1 = -2

# 输入值
x1 = 0
x2 = 1

# sigmoid input
u_h1l = sigmoid(x1 * w_xh1l + x2 * w_xh2l + w_xbh1)
u_h2l = sigmoid(x1 * w_xh2h + x2 * w_xh2h2 + w_xbh2h)

# relu input
# net_h1 = x1 * w_xh1l + x2 * w_xh2l + w_xbh1
# u_h1l = relu(net_h1)
# net_h2 = x1 * w_xh2h + x2 * w_xh2h2 + w_xbh2h
# u_h2l = relu(net_h2)

# linear input
# u_o1l = u_h1l * w_h1o1 + u_h2l * w_h2o1 + w_bbh1
# sigmoid output
net_o1l = u_h1l * w_h1o1 + u_h2l * w_h2o1 + w_bbh1
u_o1l = sigmoid(net_o1l)

# 真实值
y = 1

# 计算 o1 节点的反向传播误差, linear output
# delta_o1l = (u_o1l - y)
# 计算 o1 节点的反向传播误差, sigmoid output
delta_o1l = (u_o1l - y) * u_o1l * (1 - u_o1l)
delta_h1l = delta_o1l * w_h1o1 * u_h1l * (1 - u_h1l)
delta_h2l = delta_o1l * w_h2o1 * u_h2l * (1 - u_h2l)

# relu input
# delta_h1l = delta_o1l * w_h1o1 * relu_derivative(net_h1)
# delta_h2l = delta_o1l * w_h2o1 * relu_derivative(net_h2)

grad_w_xh1l = delta_h1l * x1
grad_w_xh2l = delta_h1l * x2
grad_w_xbh1 = delta_h1l
gradients_h1l = (grad_w_xh1l, grad_w_xh2l, grad_w_xbh1)

grad_w_xh2h = delta_h2l * x1
grad_w_xh2h2 = delta_h2l * x2
grad_w_xbh2h = delta_h2l
gradients_h2l = (grad_w_xh2h, grad_w_xh2h2, grad_w_xbh2h)

grad_w_h1o1 = delta_o1l * u_h1l
grad_w_h2o1 = delta_o1l * u_h2l
grad_w_bbh1 = delta_o1l
gradients_o1l = (grad_w_h1o1, grad_w_h2o1, grad_w_bbh1)

# 学习率
learning_rate = 0.4

w_xh1l = w_xh1l - learning_rate * grad_w_xh1l
w_xh2l = w_xh2l - learning_rate * grad_w_xh2l
w_xbh1 = w_xbh1 - learning_rate * grad_w_xbh1
updated_weights_h1l = [w_xh1l, w_xh2l, w_xbh1]

w_xh2h = w_xh2h - learning_rate * grad_w_xh2h
w_xh2h2 = w_xh2h2 - learning_rate * grad_w_xh2h2
w_xbh2h = w_xbh2h - learning_rate * grad_w_xbh2h
updated_weights_h2l = [w_xh2h, w_xh2h2, w_xbh2h]

w_h1o1 = w_h1o1 - learning_rate * grad_w_h1o1
w_h2o1 = w_h2o1 - learning_rate * grad_w_h2o1
w_bbh1 = w_bbh1 - learning_rate * grad_w_bbh1
updated_weights_o1l = [w_h1o1, w_h2o1, w_bbh1]

print("u_h1l", u_h1l)
print("u_h2l", u_h2l)
print("u_o1l", u_o1l)
print("delta_o1l", delta_o1l)
print("delta_h1l", delta_h1l)
print("delta_h2l", delta_h2l)
print("gradients_o1l", gradients_o1l)
print("gradients_h1l", gradients_h1l)
print("gradients_h2l", gradients_h2l)
print("updated_weights_o1l", updated_weights_o1l)
print("updated_weights_h1l", updated_weights_h1l)
print("updated_weights_h2l", updated_weights_h2l)

```

relu (Rectified Linear Unit, 修正线性单元)

公式: $f(x) = \max(0, x)$

优点: 当输入小于 0 时, 输出为 0; 当 x 大于等于 0 时, 输出等于 x , 计算简单直观。没有复杂的梯度计算, 能够解决梯度消失问题, 在 $x >$ (梯度消失点为 1) 符合生物神经元特性, 模拟大脑神经元兴奋-抑制机制。

应用场景: 广泛应用于神经网络模型中, 尤其在图像识别的深度学习模型中表现突出。不过存在“神经元死亡”问题, 当某些神经元输出值为负时, 输出值为 0, 不再激活。

sigmoid

公式: $f(x) = \frac{1}{1+e^{-x}}$

优点: 将真实值映射到(0,1)区间, 可表示概率, 具有非线性特性, 能帮助神经网络解决数据的复杂关系。但输出值在很大或很小时, 处于饱和态, 梯度趋近于 0, 会导致梯度消失问题, 影响训练收敛。

应用场景: 可用于二分类问题, 在单神经元网络中应用较多, 比如逻辑回归中由于非线性输出转化为概率。

tanh (双曲正切函数)

公式: $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

优点: 把实数映射到[-1,1]之间, 函数形状与 sigmoid 类似, 相比 sigmoid, 其均值更接近 0, 收敛速度可能更快, 不过梯度较小或为 0 时, 输出平均, 梯度较小, 也存在梯度消失问题。

应用场景: 可用于将数据归一化到单位区间的场景, 但在深层网络中也很难将此函数使用受限。

softmax

公式: $f(x) = \frac{e^{x_i}}{\sum_j e^{x_j}}$, x_i 表示第 i 个类别的输入值。

优点: 用于多分类任务, 大输入的多个实数转换为 0 到 1 之间的概率分布, 所有输出数值之和等于 1, 通过概率映射大输入类别, 并进行归一化。

应用场景: 神经网络输出层处理多分类问题, 如图片分类、文本分类等, 将模型输出的 logits 值转换为不同类别的概率分布。

linear (线性激活函数)

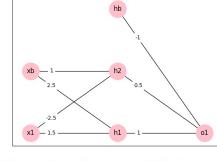
公式: $f(x) = x$

优点: 输出与输入保持完全线性关系, 无论任何非线性变换, 计算简单, 输出范围无限制, 可输出任意实数值, 但不引入非线性, 限制模型对复杂非线性关系的表示能力。

应用场景: 适用于回归问题, 当预测目标值与输入特征在统计学关系时, 如房价预测, 温度等连续值, 或者为线性回归模型。多类感知机 (MLP) 同时预测多类别的分类函数。

Consider the following fully connected neural network, with one input unit, one output unit, and two hidden units. Weights are indicated on the edges. xb and hb represent bias inputs.

There is a sigmoid activation at the hidden units, and a linear activation at the output unit.



What is the output of the network when the input is $x = -2$? (Notice the negative sign.)

$$u_{h_1} = -0.12$$

Comment

First, we compute the input to the activation function at each of the hidden units:

$$z_{h_1} = 1.5 \times -2 + 0.5 = -0.5$$

and

$$z_{h_2} = -2.5 \times -2 + 1 = 6$$

Then, we compute the output of the activation function at each of the hidden units:

$$u_{h_1} = \sigma(-0.5) = 0.377540667981454$$

and

$$u_{h_2} = \sigma(6) = 0.9975273768433653$$

Finally, at the output unit, the input to the activation function will be

$$z_{o1} = 1 \times 0.377540667981454 + 0.5 \times 0.9975273768433653 + -1$$

and since the output unit uses a linear activation function, the output is

$$u_{o1} = -0.1230956427801787$$

Neural network - summary

Things that are “given”

For a particular problem, these are “given”:

- the number of inputs
- the number of outputs
- the activation function to use at the output
- the loss function

Things that we decide

We still need to decide:

- the number of hidden units
- the activation function to use at hidden units

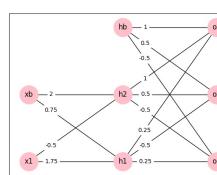
Training the network

- Still need to find the \mathbf{W} that minimizes $L(\mathbf{W})$.
- How?

Consider the following fully connected neural network for a three-class classification problem, with one input unit, three output units, and two hidden units. Weights are indicated on the edges. xb and hb represent bias inputs.

There is a **ReLU** activation at the hidden units, and a **softmax** activation at the output units.

Note: the ReLU activation function is $f(x) = \max(0, x)$.



What is the predicted class when the input is $x = -2$? (Notice the negative sign.)

Comment

First, we compute the input to the activation function at each of the hidden units:

$$z_{h_1} = 1.5 \times -2 + 0.5 = -2.5$$

and

$$z_{h_2} = -2.5 \times -2 + 2 = 3$$

Then, we compute the output of the activation function at each of the hidden units:

$$u_{h_1} = \text{ReLU}(-2.5) = 0$$

and

$$u_{h_2} = \text{ReLU}(3) = 3$$

Finally, at the output layer, the input to the softmax activation function will be

$$z_{o1} = 0.25 \times 0 + -0.5 \times 3 + 0.5 = -2$$

$$z_{o2} = 0.5 \times 0 + 0.5 \times 3 + 0.5 = 2$$

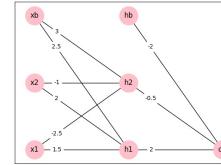
$$z_{o3} = 0.25 \times 0 + 1 \times 3 + 1 = 4$$

We use these values as input to the softmax function

$$g_{\theta}(\mathbf{x}) = \frac{e^{x_i}}{\sum_j e^{x_j}}$$

which estimates the per-class probability as 0.0023, 0.1189, and 0.8769 for class 1, 2, and 3, respectively. The most probable class is therefore 3.

Consider a neural network for classification with two features at the input, two hidden layer nodes in a single hidden layer, and one output node. There is a sigmoid activation function at the hidden layer nodes and at the output layer.



Part 1: Forward pass

For the input $x_1 = 0, x_2 = 1$, do a forward pass on the network and compute the outputs.

$$u_{h_1} = 0.899$$

$$u_{h_2} = 0.881$$

$$u_{o1} = 0.366$$

Part 1: Forward pass

For the input $x_1 = 0, x_2 = 1$, do a forward pass on the network and compute the outputs.

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Part 1: Forward pass

For the input $x_1 = 0, x_2 = 1$, do

Train a RandomForestRegressor

```
import numpy as np
from sklearn.neighbors import KNeighborsRegressor
from sklearn.metrics import r2_score

# Fit the model on the training data. Then, use it to make predictions for the test samples, and save this prediction in y_ts_hat.
```

In this question, we will try to predict the time it will take for an order to be delivered.

First, we'll load the data:

```
# If you run this cell, you will see the warning: "UserWarning: Unknown metric 'r2_score'". You can ignore this warning.
```

```
# X = pd.read_csv("data.csv", names=["id", "cost", "average_cost", "average_time_hr", "average_time_min", "unfilled_orders", "time", "header", "index", "col_id"])
```

We can add some code here to inspect the data, see the names of features, and see the data types - the cell below will not be graded.

```
# df.head()
```

id	cost	average_cost	average_time_hr	average_time_min	unfilled_orders	time
13757.0	55.95529	12.95256	24.00010	28.14029	1.0	2.58182 21.88510
1376.0	58.06812	13.95569	22.95377	105.95563	0.0	4.03039 85.34303
1377.0	58.06812	14.04904	36.00000	54.15081	1.0	4.03039 36.00000
1378.0	58.06812	14.04904	37.00000	54.15081	1.0	5.57283 9.02254
1379.0	6.510663	21.79350	41.01542	56.79793	1.0	5.14290 6.510663

(But, note that your code will be evaluated on different data organized in a data frame with the same columns - so in your solution, you should not hard-code anything specific to this data)

Now we will split into training and test sets, using `train_test_split`:

- Reserve 20% of the data for testing.

- Use the random state specified on the question page.

The following code should work:

```
X = X.drop(['header', 'index', 'col_id'], axis=1)
y = X['time']
X = X.drop(['time'], axis=1)
```

```
X_train, X_ts, y_train, y_ts = train_test_split(X, y, test_size=0.2, random_state=rndm_state)
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

If the model on the training data. Then, use it to make predictions for the test samples, and save this prediction in `y_ts_hat`. Evaluate the R^2 score of the model on the test data, and save this in `r2_ts`.

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

Now we are ready to fit the `KNeighborsClassifier`. Using

- The training data described in the question page

- and setting the number of trees in the forest to 10

- and the default settings otherwise.

```
#model = RandomForestClassifier(n_estimators=10).fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy score of the model on the test data, and save this in acc_ts.
```

If the model on the training data. Then, use it to make predictions for the test samples, and save this prediction in `y_ts_hat`. Evaluate the accuracy score of the model on the test data, and save this in `acc_ts`.

```
#model = RandomForestClassifier(n_estimators=10).fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy score of the model on the test data, and save this in acc_ts.
```

Train a KNN classifier

```
#df.head()
```

First, we'll load the data:

```
# X = pd.read_csv("data.csv", index_col = "Event_ID")
```

You can add some code here to inspect the data, see the names of features, and see the data types - the cell below will not be graded.

```
# df.head()
```

Event_ID	Address	Presence_of_Pets	Percentage_of_Frames_with_Human_Presence	False_Alarm	Time_of_Day	Duration_of_Motion_Events	Events_in_Last_Hour	Outdoor_Temperature
1	2425 White Rd	0	90.07802	0	5	10	1	53.57250
2	905 Main Ct	1	90.05291	0	17	20	1	70.45250
3	4228 Chestnut Ln	0	25.55573	0	4	25	2	45.54510
4	5620 Maple Rd	0	26.37178	0	18	25	0	69.03480
5	9363 Oak Rd	0	99.21168	0	19	7	1	67.95860

(But, note that your code will be evaluated on different data organized in a data frame with the same columns - so in your solution, you should not hard-code anything specific to this data)

Now we will split into training and test sets, using `train_test_split`:

- Reserve 20% of the data for testing.

- Use the random state specified on the question page.

The following code should work:

```
# X = X.drop(['header', 'index', 'col_id'], axis=1)
```

```
#y = X['events_in_last_hour']
```

```
X_train, X_ts, y_train, y_ts = train_test_split(X, y, test_size=0.2, random_state=rndm_state)
```

```
#model = KNeighborsClassifier(n_neighbors=9).fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = KNeighborsClassifier(n_neighbors=9).fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

Train a linear SVM

```
#df.head()
```

First, we'll load the data:

```
# X = pd.read_csv("data.csv")
```

You can add some code here to inspect the data, see the names of features, and see the data types - the cell below will not be graded.

```
# df.head()
```

Address	Walkability_Score	Part_Nearby	Grocery_Stores_Nearby	Schools_Nearby	Public_Transit_Nearby
0	935 Maple Ave, Bloomington, TX	0	2	1	2
1	500 Cedar Bluff, Greenville, NC	0	2	2	1
2	795 Cedar Bluff, Madison, NC	1	1	3	2
3	742 Main St, Riverside, OR	0	2	2	0
4	515 Maple Ave, Brooklyn, NY	0	2	1	3

(But, note that your code will be evaluated on different data organized in a data frame with the same columns - so in your solution, you should not hard-code anything specific to this data)

Now we will split into training and test sets, using `train_test_split`:

- Reserve 20% of the data for testing.

- Use the random state specified on the question page.

The following code should create:

```
# X = X.drop(['header', 'index', 'col_id'], axis=1)
```

```
#y = X['walkability_score']
```

```
X_train, X_ts, y_train, y_ts = train_test_split(X, y, test_size=0.2, random_state=rndm_state)
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

```
#model = SVC().fit(X_train, y_train)
```

```
#y_ts_hat = model.predict(X_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the accuracy of the model on the test data, and save this in acc_ts.
```

Train a KNeighborsRegressor

```
import numpy as np
from sklearn.neighbors import KNeighborsRegressor
from sklearn.metrics import r2_score
```

```
X_ts = np.load("Xt.npy")
y_ts = np.load("yt.npy")
y_hat_ts = np.load("yhat_ts.npy")
r2_ts = r2_score(yts, y_hat_ts)
```

Fit a `KNeighborsRegressor` on the training data (according to the specific R2 score of the model's predictions), and save this value in `r2_ts`.

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

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#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

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#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
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#y_hat_ts = model.predict(X_ts)
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#y_hat_ts = model.predict(X_ts)
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```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
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```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
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```

```
#y_hat_ts = model.predict(X_ts)
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r2_ts = r2_score(yts, y_hat_ts)
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#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
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#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
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#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
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r2_ts = r2_score(yts, y_hat_ts)
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#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
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```

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#y_hat_ts = model.predict(X_ts)
```

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r2_ts = r2_score(yts, y_hat_ts)
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#y_hat_ts = model.predict(X_ts)
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r2_ts = r2_score(yts, y_hat_ts)
```

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#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```

```
#model.score(X_ts, y_ts) # Evaluate the R^2 score of the model on the test data, and save this in r2_ts.
```

```
#model = KNeighborsRegressor(n_neighbors=9).fit(X_train, y_train)
```

```
#y_hat_ts = model.predict(X_ts)
```

```
r2_ts = r2_score(yts, y_hat_ts)
```


2. Train an SVM classifier - stratified split

The second problem is easy to fix. To make sure that the ratio of the class labels is similar across folds, we can use a `StratifiedKFold` instead of the `KFold`.

Copy your code from Part 1, but replace the `KFold` with a `StratifiedKFold`, to ensure that the distribution of y is consistent across folds. Otherwise, keep everything else the same.

You will see the same metrics as before:

- accuracy: `acc_xk_fold`
- balanced_accuracy: `bal_xk_fold`
- sensitivity: `xm_xk_fold`
- specificity: `xp_xk_fold`
- and the data to generate the confusion matrix (`cm_xk_fold`)

```
##(write your code in this cell and DO NOT DELETE THIS LINE)
nfold = 3
acc_xk_fold = np.zeros(nfold)
bal_xk_fold = np.zeros(nfold)
sensitivity_xk_fold = np.zeros(nfold)
specificity_xk_fold = np.zeros(nfold)
cm_xk_fold = np.zeros(shape=(2, 2, nfold))
for fold in range(nfold):
    print("Training fold %d" % fold)
    Xtrain, Xtest, ytrain, ytest = train_test_split(X, y, test_size=1./nfold)
    cm_xk_fold[:, :, fold] = confusion_matrix(ytrain, ytest)
    # fit classifier
    clf = SVC()
    clf.fit(Xtrain, ytrain)
    # predict
    ypred = clf.predict(Xtest)
    # calculate metrics
    acc_xk_fold[fold] = accuracy_score(ytest, ypred)
    bal_xk_fold[fold] = balanced_accuracy_score(ytest, ypred)
    xm_xk_fold[fold] = recall_score(ytest, ypred, pos_label=1)
    xp_xk_fold[fold] = recall_score(ytest, ypred, pos_label=0)
    # print results
    print("Bal. Accuracy: ", bal_xk_fold[fold])
    print("Sensitivity: ", xm_xk_fold[fold])
    print("Specificity: ", xp_xk_fold[fold])
    print("Accuracy: ", acc_xk_fold[fold])
    print("Confusion Matrix: ", cm_xk_fold[:, :, fold])
print("Accuracy: ", acc_xk_fold.mean())
print("Bal. Accuracy: ", bal_xk_fold.mean())
print("Sensitivity: ", xm_xk_fold.mean())
print("Specificity: ", xp_xk_fold.mean())
Accuracy: 0.446595166565595
Bal. Accuracy: 0.446595166565595
Sensitivity: 0.446595166565595
Specificity: 0.446595166565595
for i in range(nfold):
    print("Fold %d" % i)
    print("Confusion Matrix: ", cm_xk_fold[:, :, i])
    disp = ConfusionMatrixDisplay(confusion_matrix = cm_xk_fold[:, :, i], display_labels = [0, 1])
    disp.plot()
    plt.show()
    disp.remove()
    print("Balanced accuracy score (%d): %.2f" % (i, bal_xk_fold[i]))
    print("Recall score (%d): %.2f" % (i, xm_xk_fold[i]))
    print("Precision score (%d): %.2f" % (i, xp_xk_fold[i]))
```

3. Train an SVM classifier - oversampling

The root cause of the first problem – that the classifier does “prediction by mode” – is the class imbalance. To “force” the classifier to focus on both classes, and not only the majority class, we can oversample the minority class to balance the class distribution before fitting our classifier.

• **Oversampling:** Generate more samples from the minority class, so that it matches the count of the majority class.

• **Undersampling:** Remove some samples from the majority class, so that it matches the count of the minority class.

In this example, since we have a very small data set, undersampling is not practical. We will try oversampling the minority class.

We will use `ADASYN` oversampling, as we will see!

Adaptive Synthetic Sampling (ADASYN) is an oversampling technique in which we generate synthetic samples of the minority class, and focus especially on generating synthetic samples for the minority classes which are harder to learn.

ADASYN also helps to learn the difficulty of a minority class sample by calculating the proportion of majority class samples among the sample's K nearest neighbors. More majority neighbors imply a

higher learning difficulty for the minority sample.

Here is the ADASYN algorithm in detail:

- Calculate the degree of class imbalance

$$\delta = \Delta_1 / K_1$$

where Δ_1 is the number of minority class examples and K_1 is the number of majority class examples.

- If $\delta < \delta_{th}$ (where δ_{th} is a preset threshold for the maximum tolerated degree of class imbalance) do:
 - Calculate the number of synthetic data examples that need to be generated for the minority class:

$$G = (\Delta_1 - \delta \cdot K_1) \times \beta$$

where β is a parameter used to specify the desired balance level after generation of the synthetic data. $\beta = 1$ means a fully balanced data set is created after generalization process.

- For each example x_i c minority class, find K nearest neighbors based on the Euclidean distance in n-dimensional space, and calculate the ratio r_i defined as:

$$r_i = \Delta_1 / K_i$$

where Δ_1 is the number of examples in the K nearest neighbors of x_i that belong to the majority class, therefore $r_i \in [0, 1]$.

- Normalize r_i according to

$$r_j = r_i - \sum_{j \neq i} r_j$$

- Calculate the number of synthetic data examples that need to be generated for each minority example x_i :

$$g_i = r_i \cdot G$$

- For each minority class data example x_i generate g_i synthetic data examples according to the following steps:
 - o Loop from 1 to g_i :
 - o Randomly choose one minority example from the K nearest neighbors for data x_i
 - o Generate the synthetic data example:

$$x_i + \eta_i = x_i + (\alpha - x_i) \cdot r_i$$

where η_i is a random number, $\alpha \in [0, 1]$

You may refer to the documentation for the ADASYN oversampling implementation.

It is similar to `sklearn` “transforms” that we have worked with before, like `StandardScaler` or `CouvertVectorizer`; where those transformers had a `.fit_transform` method, the ADASYN implementation has `.fit`, `sample`.

In the following cell, we will use ADASYN oversampling to oversample the minority class.

- Create an ADASYN oversampler. Specify the random state from the `random_state` variable defined above, and `n_neighbors = 5`.

• Use it to generate `X_oversamp` and `y_oversamp`, the oversampled feature data and label data respectively.

```
##(write your code in this cell and DO NOT DELETE THIS LINE)
from imblearn.over_sampling import ADASYN
X_oversamp, y_oversamp = ADASYN(n_neighbors=5).fit_resample(X, y)
```

Then, copy your code from Part 2, But

- split on the oversampled data, not the original data

• when defining `Xtr_kfold`, `Xts_kfold`, `ytr_kfold`, `yts_kfold`, select rows from the oversampled data, not the original data.

Otherwise, keep everything else the same.

You will see the same metrics as before, in:

- accuracy: `acc_xk_fold`

• balanced_accuracy: `bal_xk_fold`

• sensitivity: `xm_xk_fold`

• specificity: `xp_xk_fold`

• and the data to generate the confusion matrix (`cm_xk_fold`)

```
##(write your code in this cell and DO NOT DELETE THIS LINE)
```

```
##(write your code in this cell and DO NOT DELETE THIS LINE)
```

```
##(write your code in this cell and DO NOT DELETE THIS LINE)
```

```
##(write your code in this cell and DO NOT DELETE THIS LINE)
```

```
##(write your code in this cell and DO NOT DELETE THIS LINE)
```

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Autodiff

```
import numpy as np
from sklearn import display, Image, display, SVC
from tensorflow.keras import Sequential, Model
from tensorflow.keras.layers import Dense, Activation, Input
from tensorflow.keras.optimizers import Adam
import tensorflow as tf
model.add(Dense(10, input_dim=dim, activation='relu', name='hidden'))
model.add(Dense(1, activation='linear', name='output'))
```

We saw how the backpropagation algorithm can be used to find gradients for any neural network, as long as each step in its computational graph is differentiable. Working from the end of the computational graph (output) towards the inputs, we can compute the gradients for each node in the graph. This is what Autodiff does - it takes your code, finds all the operations in the graph, and then computes the gradients for each node in the graph. It does this by first creating a computational graph, then implementing rules to avoid this derivative and then implementing them in code. Modern neural networks are enabled by frameworks for automatic differentiation (like TensorFlow and PyTorch), which build the computational graph and apply the backpropagation algorithm to it without us having to compute derivatives by hand.

In this exercise, we will build our own very basic framework for automatic differentiation, to better understand how it works.

Build a Variable class and some operations

In our framework, a `Variable` will represent a node in the computational graph and holds these essential pieces of information:

- `value` - the value of the variable
- `grad_fn` - the function that computes the derivative, i.e. the derivative of the operation that this node performs with respect to its arguments.
- `accumulated_gradient` - the accumulated gradient will compute the backwards pass along the computational graph

(We also add a `name` attribute so that we can visualize our computational graph.)

Here is our `Variable` class:

```
class Variable:
    def __init__(self, value, local_gradients=None, name=None):
        self.value = value
        self.local_gradients = local_gradients
        self.name = name
        self.grad_fn = None
```

Of course, we want to support some operations on our variables. Let's implement addition (add) and multiplication (mul).

For each operation, we should return a new instance of a `Variable` with:

```
def add(x, y):
    """Create the variable that results from adding two variables."""
    value = x.value + y.value
    local_gradients = {
        'x': x.local_gradients,
        'y': y.local_gradients
    }
    name = f'{x.name} + {y.name}'
    return Variable(value, local_gradients, name=f'({x.name},{y.name})')

def mul(x, y):
    """Create the variable that results from multiplying two variables."""
    value = x.value * y.value
    local_gradients = {
        'x': x.local_gradients * y.value,
        'y': y.local_gradients * x.value
    }
    name = f'{x.name} * {y.name}'
    return Variable(value, local_gradients, name=f'({x.name},{y.name})')
```

Now we can use this basic framework to do a forward pass on a graph. We'll define two variables, `x` and `y`, and give them values:

```
a = Variable(name='a', value=2)
b = Variable(name='b')
```

Note that both of these have "empty" `local_gradients` and zero `accumulated_gradient`, to start:

```
a.local_gradients, b.local_gradients
(D, D)
a.accumulated_gradient, b.accumulated_gradient
(0, 0)
```

If we add the variables:

```
x = a + b
print(x.value)
```

we get a new `Variable` whose `value` is the output of the "forward pass":

```
x.value
Value: 4.0
and also has local_gradients with respect to the variables x and y:
```

```
for grad in x.local_gradients:
    print(f'Gradient with respect to {grad.name}: {grad.value}')
Gradient with respect to x: 1.0
Gradient with respect to y: 1.0
```

```
x.local_gradients
graphviz.graphtools.DotGraph at 0x7f8000000000
Note: each node in this graph is its own Variable.
```

```
def get_gradients(variable, path_value=1):
    """Get the gradients of variable with respect to child variables.
    path_value is the path length from the variable to the output node.
    """
    def compute_gradients(variable, path_value):
        for child, value in variable.local_gradients.items():
            path_value *= child.path_value
            local_gradient = value * path_value
            child.accumulated_gradient += path_value * local_gradient
            compute_gradients(child, path_value)
    compute_gradients(variable, path_value)

    # Compute the gradients of the output node.
    for child in variable.local_gradients:
        child.accumulated_gradient *= path_value
```

```
get_gradients(x)
Value: 4.0
x.local_gradients
graphviz.graphtools.DotGraph at 0x7f8000000000
graphviz.graphtools.DotGraph at 0x7f8000000000
```

Let's try it now. We will use the example from Figure 20 in the lecture handout.

```
a = Variable(name='a', value=2)
b = Variable(name='b', value=3)
c = a * b
d = c + a
e = d * a
f = e + a
g = f + a
h = g + a
i = h + a
j = i + a
k = j + a
l = k + a
m = l + a
n = m + a
o = n + a
p = o + a
q = p + a
r = q + a
s = r + a
t = s + a
u = t + a
v = u + a
w = v + a
x = w + a
y = x + a
z = y + a
y.local_gradients
graphviz.graphtools.DotGraph at 0x7f8000000000
get_gradients(z)
Value: 16.0
y.accumulated_gradient, z.accumulated_gradient
(1, 1)
y.local_gradients
graphviz.graphtools.DotGraph at 0x7f8000000000
(you can confirm that this matches Figure 20 in the lecture handout)
```

We have correctly computed the value in the forward pass and the backward pass.

```
from tensorflow.keras.models import Model, Sequential
from tensorflow.keras.layers import Dense, Activation, Input
from tensorflow.keras.optimizers import optimizers
import tensorflow.keras.backend as K
import tensorflow as tf
model.add(Dense(10, input_dim=dim, activation='relu', name='hidden'))
model.add(Dense(1, activation='linear', name='output'))
```

Create an optimizer and compile the model. Select the appropriate loss function for this type of problem, and use an appropriate metric.

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
opt = optimizers.Adam()
model.compile(optimizer=opt, loss='mean_squared_error', metrics=['mse'])
```

Extend the framework

Using the framework we have developed so far, implement these additional operations, similar to the way `add` and `mul` are implemented above:

- `Sub` (`subtract`)
- `Square` (`square`)
- `Sigmoid`

Each function should return a new `Variable` with a correct `value` and tuple of `local_gradients`, as well as a `name`.

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
```

```
def sub(x, y):
    """Create the variable that results from subtracting x-y."""
    value = x.value - y.value
    local_gradients = {
        'x': 1,
        'y': -1
    }
    name = f'{x.name} - {y.name}'
    return Variable(value, local_gradients, name=f'({x.name},{y.name})')
```

Note: for a function that takes a single input, `local_gradients` will have one element but should still be defined as a tuple. When you define `local_gradients`, you will use e.g.

```
local_gradients = (
    ('x', 2),
)
```

with the comma being strictly required in order for it to be a tuple and not just a value.

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
```

```
def square(x):
    """Create the variable that results from squaring the variable x, i.e. x^2."""
    value = x.value ** 2
    local_gradients = {
        'x': 2 * x.value
    }
    name = f'{x.name}^2'
    return Variable(value, local_gradients, name=f'({x.name}^2)')
```

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
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```
def sigmoid(x):
    """Create the variable that results from applying a sigmoid function to x."""
    value = 1 / (1 + np.exp(-x.value))
    local_gradients = {
        'x': value * (1 - value)
    }
    name = f'sigmoid({x.name})'
    return Variable(value, local_gradients, name=f'sigmoid({x.name})')
```

Build a neural network

With these new operations, we can construct the computational graph shown in Figure 15 in the lecture handout, where:

data sample -

let the value of `x` be 4

let the value of `y` be 1

activation and loss functions -

let the activation function on the hidden unit be sigmoid

let the activation function at the output unit be linear

use the mean squared error loss function: $y - \hat{y}$

```
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```
# Input Variables
x = Variable(4, name='x')
y = Variable(1, name='y')
```

```
# Variables created by operations on those Input Variables
```

```
h = sigmoid(x)
l = linear(h, y)
u = z
L = square(u, y)
```

Visualize the forward pass:

```
vis_forward_pass()
graphviz.graphtools.DotGraph at 0x7f8000000000
```

Then, do a backward pass to get the gradient of the loss function with respect to each of the weights.

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
```

```
get_gradients()
vis_backward_pass()
graphviz.graphtools.DotGraph at 0x7f8000000000
```

PCA for dimensionality reduction before classification

```
import numpy as np
from sklearn.datasets import make_moons
from sklearn.decomposition import PCA
from tensorflow.keras import Sequential
from tensorflow.keras.layers import Dense, Activation, Input
from tensorflow.keras.optimizers import Adam
from tensorflow.keras import metrics
from tensorflow.keras import Model
from tensorflow.keras import optimizers
import tensorflow as tf
model.add(Dense(10, input_dim=dim, activation='relu', name='hidden'))
model.add(Dense(1, activation='linear', name='output'))
```

Let's try it now. We will use the example from Figure 20 in the lecture handout.

We will need to change the `get_gradients` function. It will operate as follows, using recursion:

- First, it will set the final output node of the computational graph. We will call `compute_gradients` on that output. Variables will pass a `path_value` of 1 to `compute_gradients` - this is the path from the output node to the variable.
- Inside the `compute_gradients` function, we get the `local_gradients` of the node and iterate over these - these are the child nodes of this node when traversing the graph for the backward pass.

We multiply the values along the edges: `path_value` up until this node times the `local_gradient` of this node with respect to this child.

And we add this to the `accumulated_gradient` of the child node - i.e. we'll sum the path values for all paths to this child.

Finally, we set the `local_gradients` of the child node to be the `path_value` times the `local_gradients` of the parent node, prepared to continue recursing on the graph.

When we reach the input nodes, they do not have any `local_gradients` so the recursion will end.

```
def get_gradients(variable, path_value=1):
    """Get the gradients of variable with respect to child variables.
    path_value is the path length from the variable to the output node.
    """
    def compute_gradients(variable, path_value):
        for child, value in variable.local_gradients.items():
            path_value *= child.path_value
            local_gradient = value * path_value
            child.accumulated_gradient += path_value * local_gradient
            compute_gradients(child, path_value)
    compute_gradients(variable, path_value)

    # Compute the gradients of the output node.
    for child in variable.local_gradients:
        child.accumulated_gradient *= path_value
```

Let's try it now. We will use the example from Figure 20 in the lecture handout.

```
a = Variable(name='a', value=2)
b = Variable(name='b', value=3)
c = a * b
d = c + a
e = d * a
f = e + a
g = f + a
h = g + a
i = h + a
j = i + a
k = j + a
l = k + a
m = l + a
n = m + a
o = n + a
p = o + a
q = p + a
r = q + a
s = r + a
t = s + a
u = t + a
v = u + a
w = v + a
x = w + a
y = x + a
z = y + a
y.local_gradients
graphviz.graphtools.DotGraph at 0x7f8000000000
get_gradients(z)
Value: 16.0
y.accumulated_gradient, z.accumulated_gradient
(1, 1)
y.local_gradients
graphviz.graphtools.DotGraph at 0x7f8000000000
(you can confirm that this matches Figure 20 in the lecture handout)
```

We have correctly computed the value in the forward pass and the backward pass.

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
opt = optimizers.Adam()
model.compile(optimizer=opt, loss='mean_squared_error', metrics=['mse'])
```

Create an optimizer and compile the model. Select the appropriate loss function for this type of problem, and use an appropriate metric.

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
model.compile(optimizer=opt, loss='mean_squared_error', metrics=['mse'])
```

Finally, compute the optimal `u_pc1` according to the one-SE rule, and save this in `u_pc1_opt`.

```
#grade (write your code in this cell and DO NOT DELETE THIS LINE)
best_pc1 = np.argmax(np.abs(u_pc1))
u_pc1_opt = u_pc1[:, best_pc1]
```

threshold = best_pc1 - best_pc1_x

u_pc1_opt = u_pc1[:, best_pc1_x]

