EL9343 Homework 4

Due: Feb. 20th 5:00 p.m.

1. Demonstrate the operation of HOARE-PARTITION on the array $A =< 14, 12, 14, 19, 5, 3, 4, 14, 7, 22, 16 >$. Show the array after each iteration of the while loop in the lines of 4 to 11 in the code of lecture notes.

2. For the following array: $A =< 3, 9, 5, 8, 15, 7, 4, 10, 6, 12, 16 >$,

   (a) Create a max heap using the algorithm BUILD-MAX-HEAP.

   (b) Remove the largest item from the max heap you created in 2(a), using the HEAP-EXTRACT-MAX function. Show the array after you have removed the largest item.

   (c) Using the algorithm MAX-HEAP-INSERT, insert 11 into the heap that resulted from question 2(b). Show the array after insertion.

3. For an disordered array with n elements, design an algorithm for finding the median of this array. Your algorithm should traverse the array only once.

   Notes: You can imagine the array as a flow which means you can get the data one by one, and you need to do some *cheap* operation at the time you see each element. The size of this array, n, is big and you know n from the start. Please do not sort the array, or you cannot get full mark. A hint to solve this problem is to use heap.

4. In previous homework, we have proven that if $T(n)$ satisfies

$$T(n) \leq 2T(\frac{n}{2}) + Cn \log n$$

   , then $T(n) = O(n \log^2 n)$.

   Now let $A[1, \ldots, n]$ be an array of n integers, and each of them could be positive or negative. Give an $O(n \log^2 n)$ algorithm to find a consecutive subarray $A[i : j], 1 \leq i \leq j \leq n$, for which $\sum_{h=i}^{j} A[h]$ is closest to 0, i.e. minimizing $|\sum_{h=i}^{j} A[h]|$. (**Hint:** Please consider divide-and-conquer algorithms and try to find a recurrence as mentioned above.)