1.

(a) $n = \omega(\sqrt{n})$

proof: For $\forall c > 0$ $\exists n_0 = c^2+1$ such that $\forall n \geq n_0$, $n > c\sqrt{n}$

Therefore $n = \omega(\sqrt{n})$

(b)

proof: $f(n) = \Omega(g(n)) \Leftrightarrow \exists c_1 > 0, n_1 > 0$ such that $\forall n \geq n_1$ $f(n) \geq c_1 g(n)$

$h(n) = \Theta(g(n)) \Leftrightarrow \exists c_2 > 0, n_2 > 0$ such that $\forall n \geq n_2$ $h(n) \leq c_2 g(n)$

Let $n_3 = \max(n_1, n_2)$ $\quad c_3 = \frac{c_1}{c_2} > 0$

then $\exists c_3 > 0, n_3 > 0$ such that $\forall n \geq n_3$, $f(n) \geq c_1 g(n) \geq \frac{c_1}{c_2} h(n) = c_3 h(n)$

therefore $f(n) = \Omega(h(n))$

(c)

Proof: $f(n) = O(g(n)) \Leftrightarrow \exists c_1 > 0, n_1 > 0$ such that $\forall n \geq n_1$, $f(n) \leq c_1 g(n)$

Let $c_2 = \frac{1}{c_1}$ $n_2 = n_1$ then $\exists c_2 > 0, n_2 > 0$ such that $\forall n \geq n_2$, $g(n) \geq \frac{1}{c_1} f(n) = c_2 f(n) \Leftrightarrow g(n) = \Omega(f(n))$

Thus, $f(n) = O(g(n)) \Rightarrow g(n) = \Omega(f(n))$

$g(n) = \Omega(f(n)) \Leftrightarrow \exists c_3 > 0, n_3 > 0$ such that $\forall n \geq n_3$, $g(n) \geq c_3 f(n)$

Let $c_4 = \frac{1}{c_3}$, $n_4 = n_3$, then $\exists c_4 > 0, n_4 > 0$ such that $\forall n \geq n_4$, $f(n) \leq \frac{1}{c_3} g(n) = c_4 g(n)$

$\Leftrightarrow f(n) = O(g(n))$

Thus, $g(n) = \Omega(f(n)) \Rightarrow f(n) = O(g(n))$

Therefore, $f(n) = O(g(n))$ if and only if $g(n) = \Omega(f(n))$

2.

	A	B	O	o	Ω	ω	Θ
a	$\lg^k n$	n^ϵ	yes	yes	no	no	no
b	n^k	c^n	yes	yes	no	no	no
c	$\sqrt{n}$	$n^{\sin n}$	no	no	no	no	no
d	2^n	$2^{n/2}$	no	no	yes	yes	no
e	$n^{\lg c}$	$c^{\lg n}$	yes	no	yes	no	yes
f	$\lg(n!)$	$\lg(n^n)$	yes	no	yes	no	yes

3.

(a) $4(3^{3\log_5 n}) + 12n + 9527 = \Theta(n^3 + n) = \Theta(n^3)$

(b) $\sqrt[5]{3n!} = \sqrt[5]{3\sqrt{2\pi n}(\frac{n}{e})^n} = \sqrt[5]{3}\,(2\pi n)^{\frac{1}{10}}(\frac{n}{e})^{\frac{n}{5}} = \Omega(n^n)$

(c) $\frac{1}{6}(5^{\log_{10} n})^2 + 4n + 17 = \Theta(n^{\log_{10} 25})$

(d) $3n\log_3 n + (\log_2 n)^3 = \Theta(n\log n)$

(e) $\log_4 \log_2 n + 61 = \Theta(\log\log n)$

(f) $2^{5\log_4 n} = \Theta(n^{2.5})$

(g) $(\log_2 n)^2 + \log_3 \log_3 n = \Theta((\log_2 n)^2)$

	a	b	c	d	e	f	g
A1					✓		✓
A2				✓			
A3	✓	✓				✓	
A4			✓	✓	✓	✓	✓
A5	✓	✓				✓	

4.

(a) $O(n)$

(b) [1, 1, 1, 2, 2, 3]

(c)
```
count = 0
for i = 1, 2, ..., n do
    if v = L[i] then
        count = count + 1
    end if
end for
if count > n / 2 then
    return v
else
    return -1
end if
```