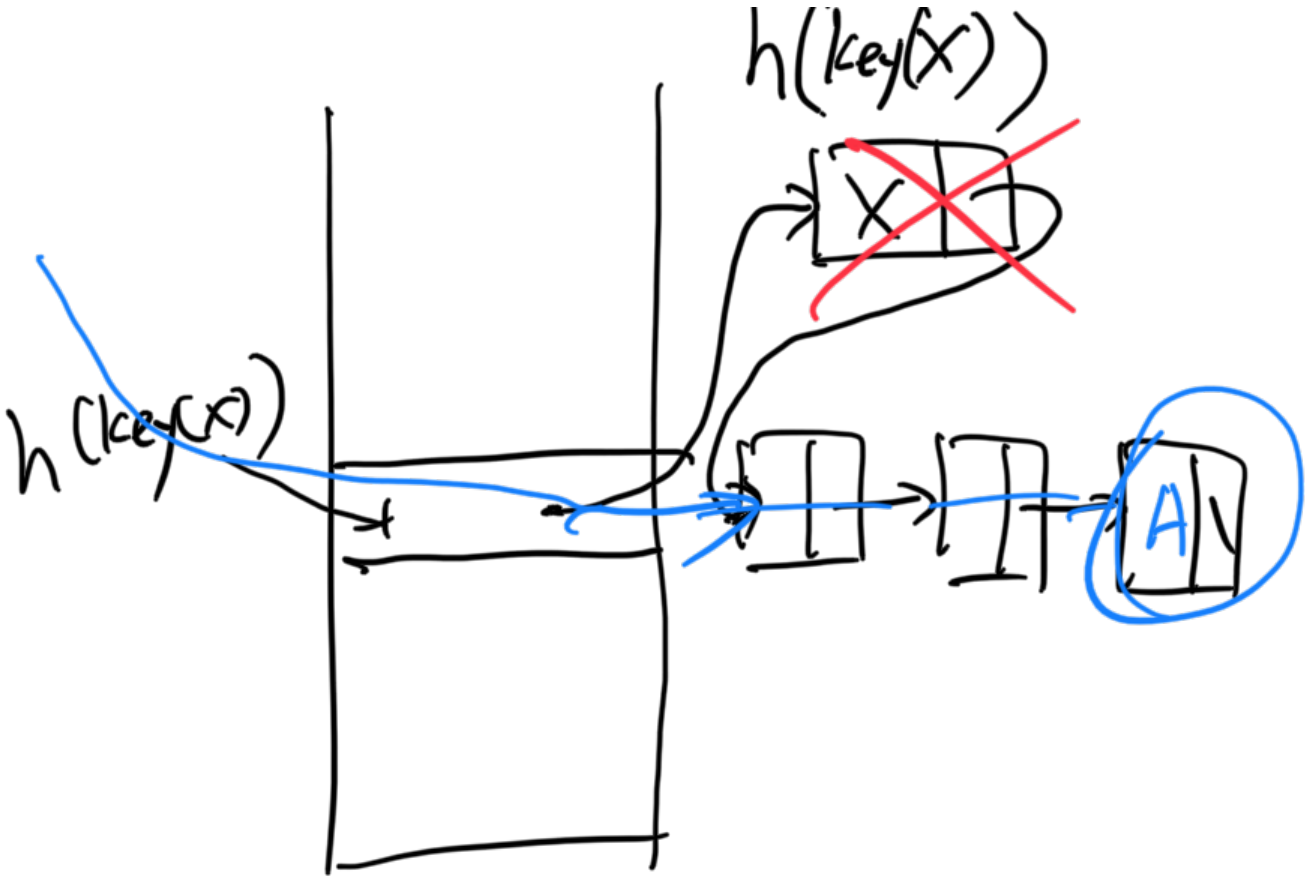
Fall 2022, Lecture 6



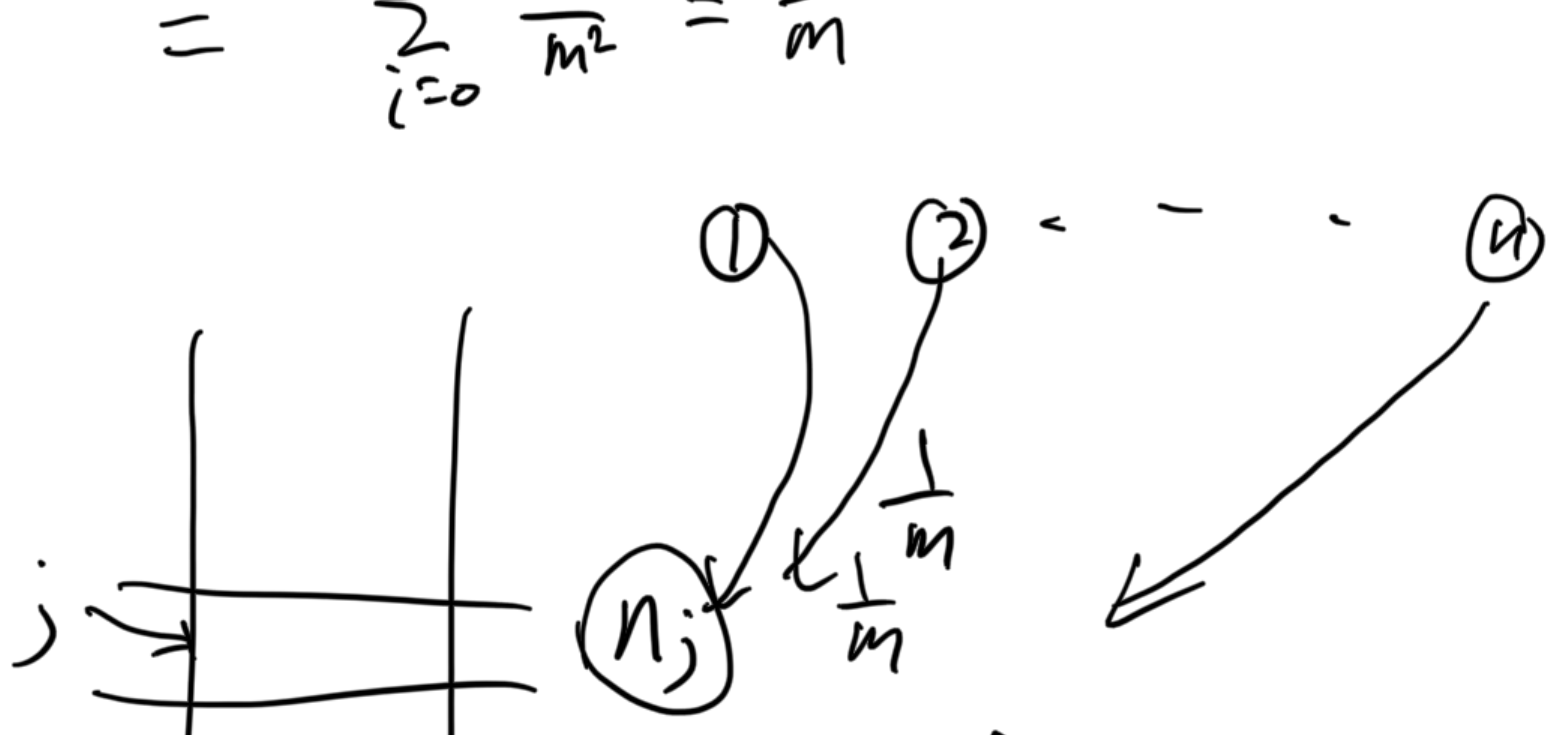
prob $(h(x)=i) = \frac{1}{m}$ prob $(h(y)=i) = \frac{1}{m}$

prob $(h(x)=h(y)=i) = \frac{1}{m} \times \frac{1}{m} = \frac{1}{m^2}$

prob (x and y have hash collision ?)

$= \text{prob}\left(\sum_{i=0}^{m-1} (h(x)=h(y)=i) \right)$

$= \sum_{i=0}^{m-1} \frac{1}{m^2} = \frac{1}{m}$



$p = \frac{1}{m}$

$P(n_j = k) = \binom{n}{k} \left(\frac{1}{m}\right)^k \left(1-\frac{1}{m}\right)^{n-k}$

$P(n_j = n) = \binom{n}{n} \left(\frac{1}{m}\right)^n = \left(\frac{1}{m}\right)^n$

$E(n_j) = \frac{n}{m}$

$E(n_j) = \frac{1}{m}$ ... m

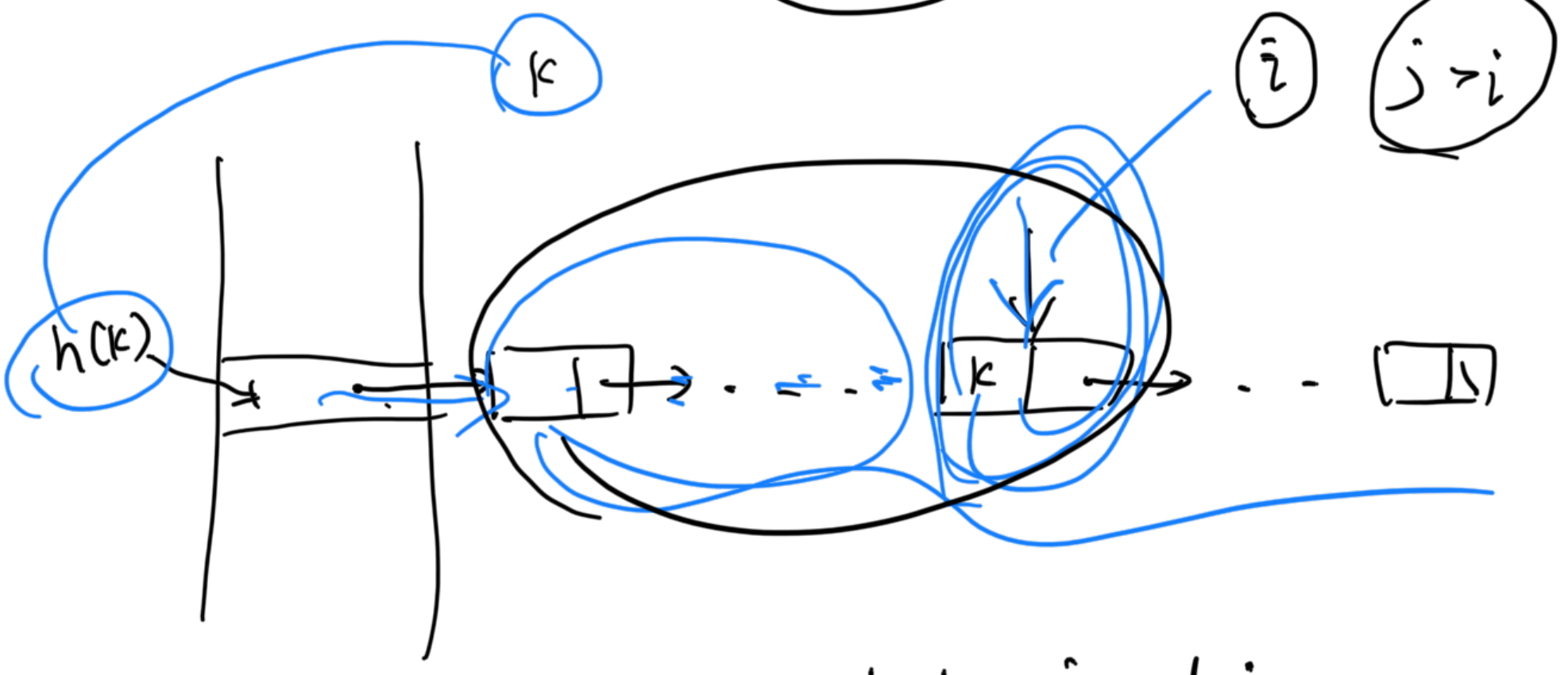
$$E(n_i) = E\left(\sum_{i=1}^{n} \mathbb{I}(\text{element } i \text{ hashes to slot } j)\right)$$

$$= \sum_{i=1}^{n} E\left[\mathbb{I}(-)\right]$$

$$= \sum_{i=1}^{n} P(\text{element } i \text{ hashes to slot } j) \quad \frac{1}{m}$$

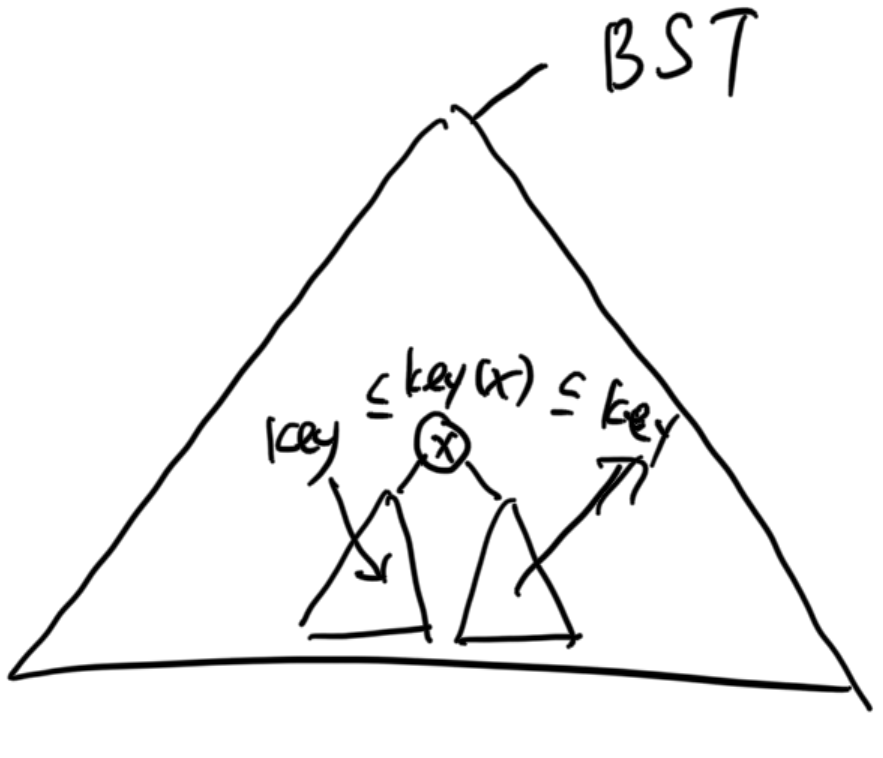
$$= \frac{n}{m}$$

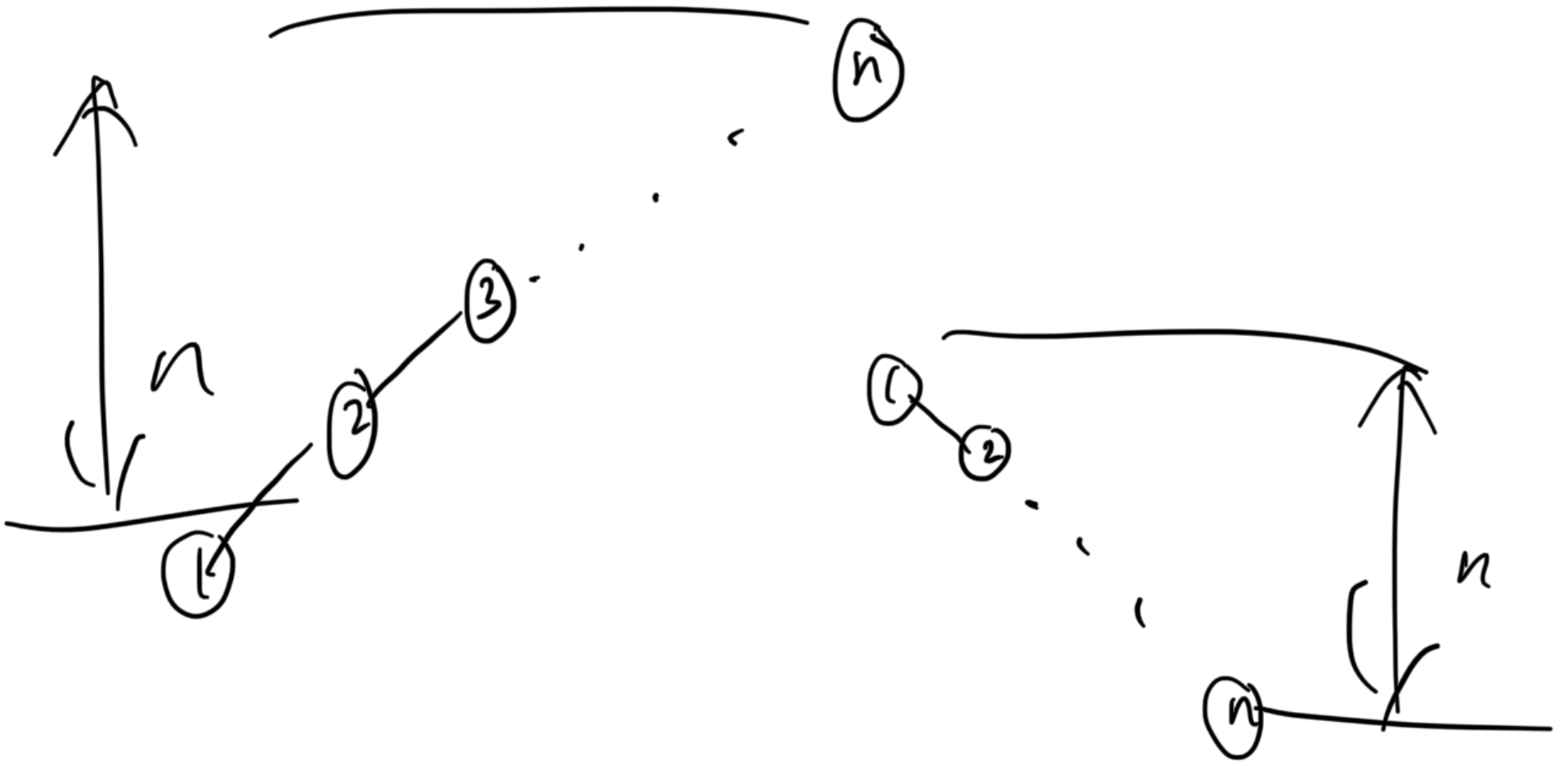
1 .. n

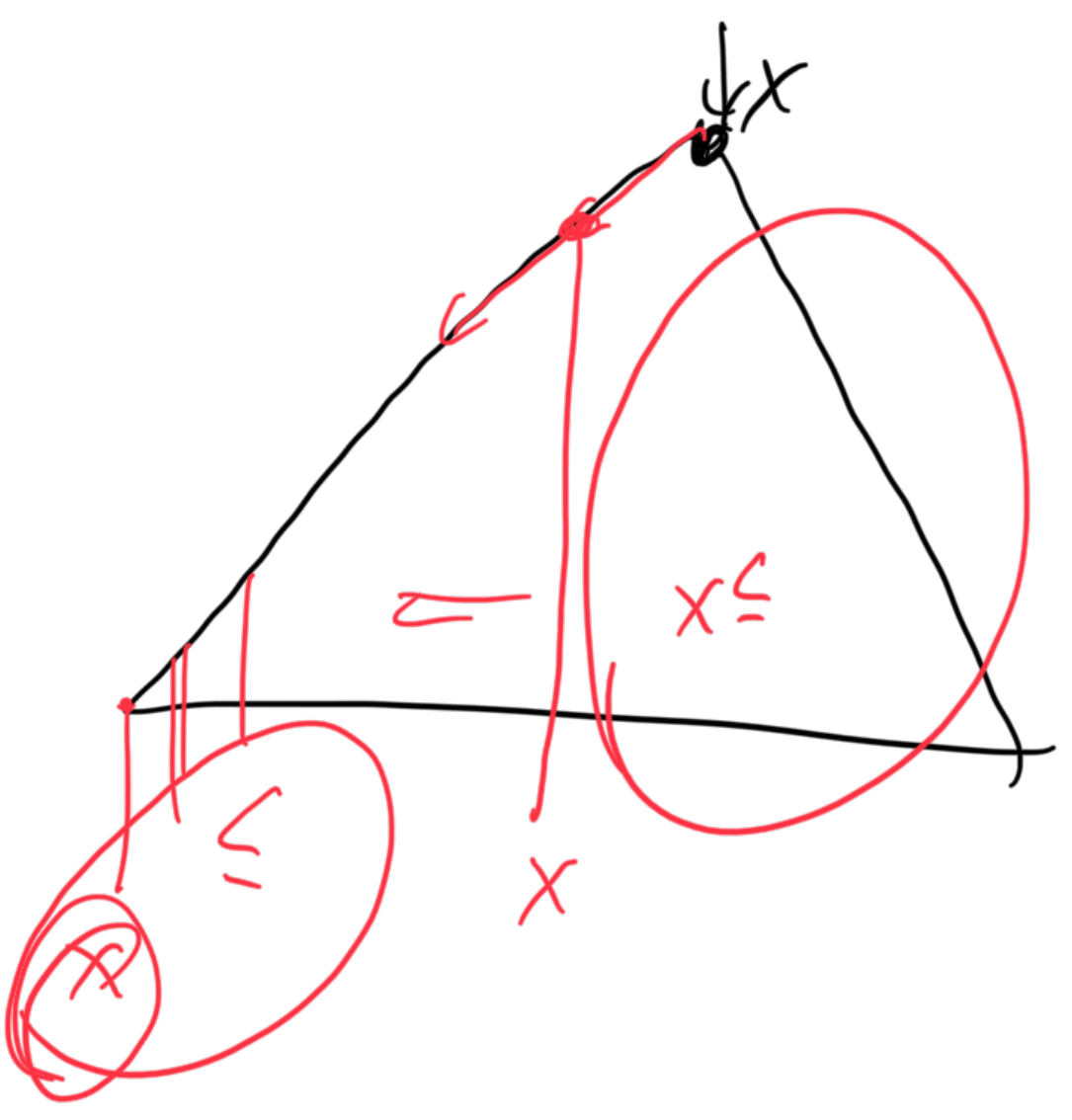


X_{ij} = whether i and j are inserted in the same slot?

$$E\left[\frac{1}{n}\left(\sum_{i=1}^{n}\sum_{j=i+1}^{n} X_{ij}\right)\right]$$




n
3
2
1
n
1
2
n
n


X
C
C
X
X
X