$$a^{\log_b x} \stackrel{?}{=} x^{\log_b a} \leftarrow \log_b(\quad)$$

$$\log_b\left(a^{\log_b x}\right) = \log_b x \log_b a$$

$$\log_b\left(x^{\log_b a}\right) = \log_b a \log_b x$$

$$\log_b x = \frac{\log_a x}{\log_a b} \Leftrightarrow \log_a b \cdot \log_b x = \log_a x$$

$$a^{(\ )}$$

$$a^{(\log_a b \cdot \log_b x)} = \left(a^{\log_a b}\right)^{\log_b x} = b^{\log_b x} = x$$

$$a^{(\log_a x)} = x \qquad b^{\log_a a}$$

$$x(1+x+x^2 \cdots +x^n) = S\cdot x$$

$$x+x^2+x^3 \cdots +x^n+x^{n+1} = S\cdot x$$

$$S-1+x^{n+1} = S\cdot x$$

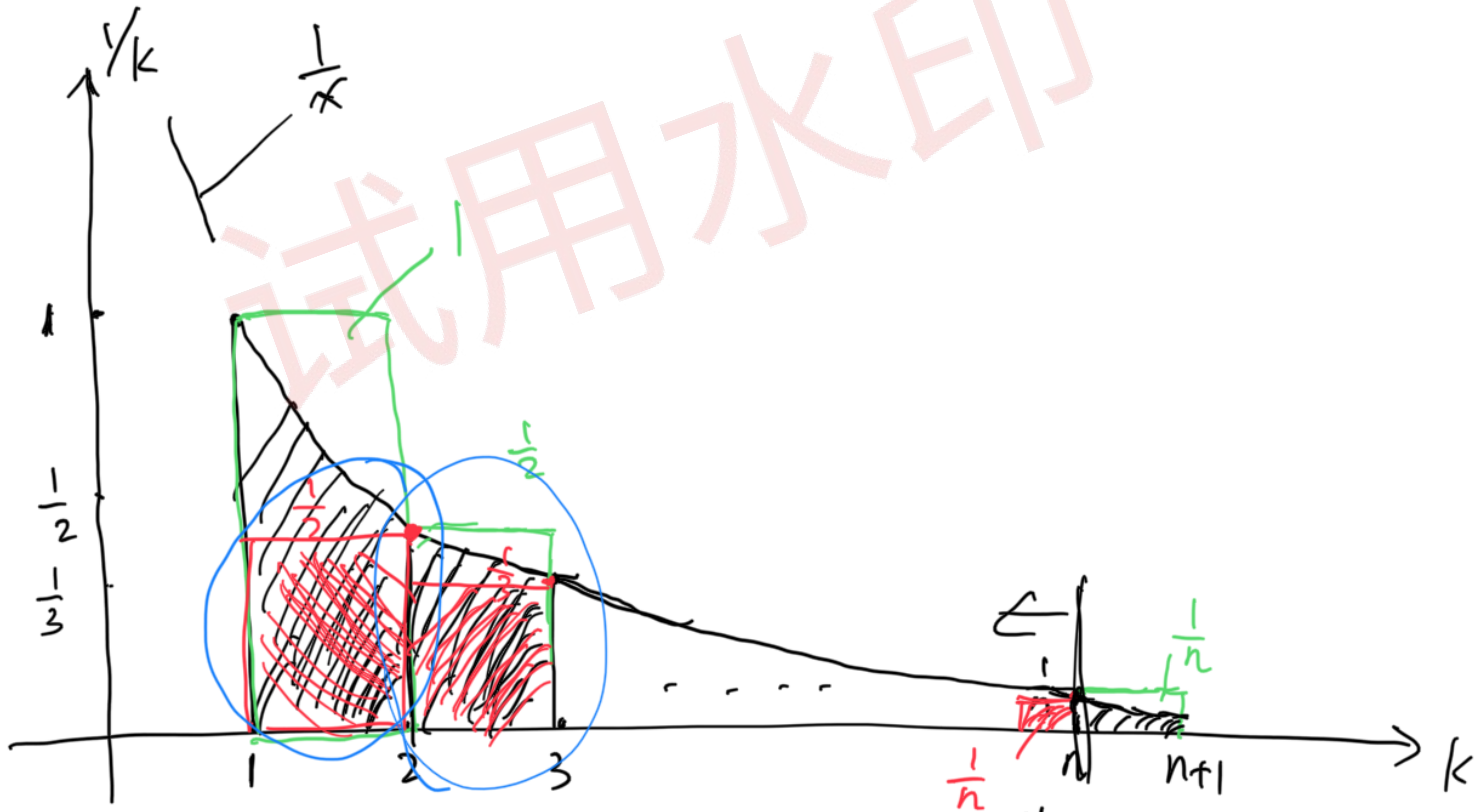
$$x^{n+1}-1 = S(x-1)$$

$$S = \frac{x^{n+1}-1}{x-1}$$

$$|x|<1,\quad n\to\infty \qquad x^{n+1}\to 0$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \text{ if } |x|<1$$

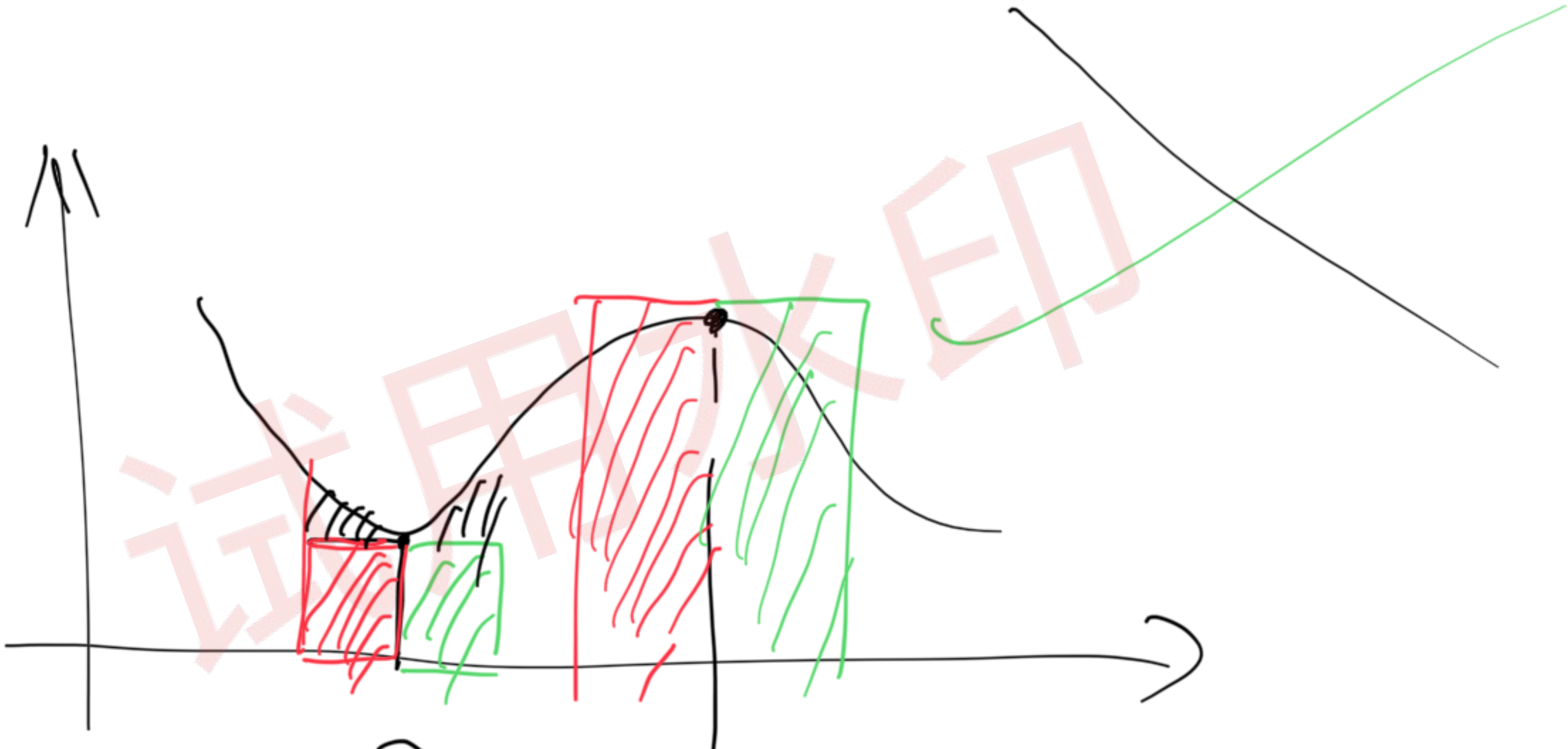
$$\boxed{1+\frac{1}{2}+\frac{1}{3}\cdots+\frac{1}{n}} = \sum_{k=1}^{n}\frac{1}{k}$$



$$1+\frac{1}{2}\cdots+\frac{1}{n} > \int^{n+1}\frac{1}{x}\,dx = \ln(\ldots)$$

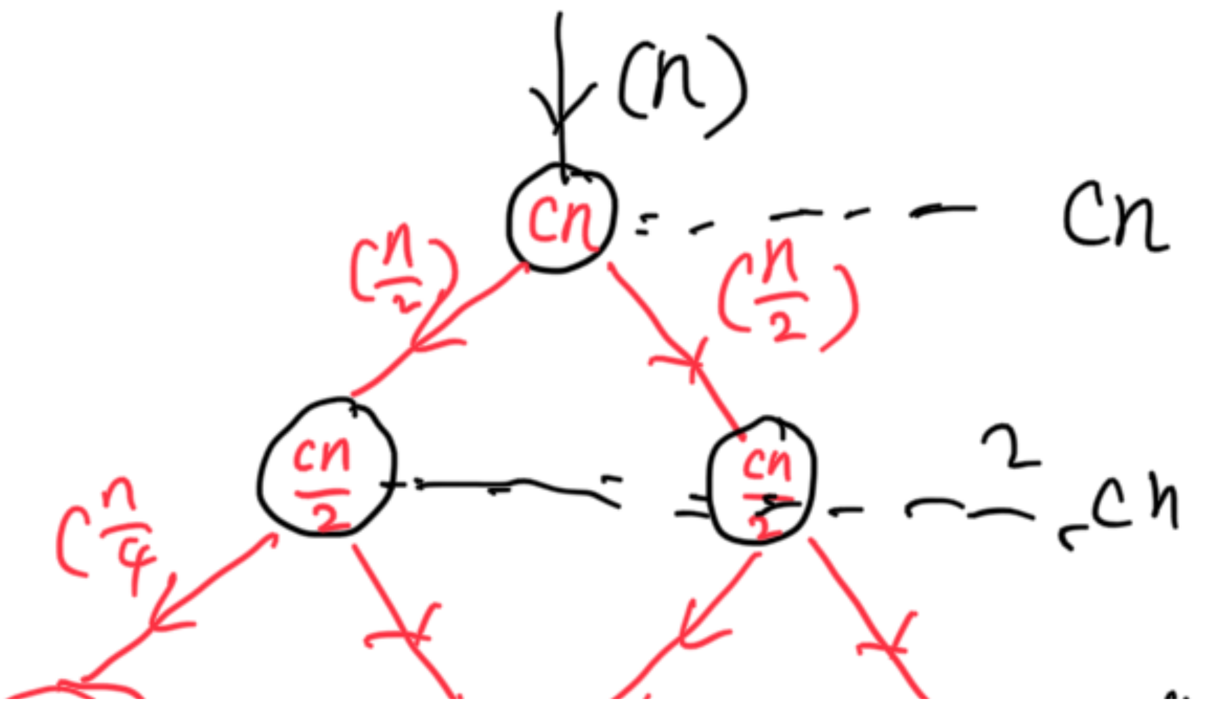
$1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n} \geq \int_1^{n+1} \frac{1}{x}\,dx = \ln(n+1)$

$$1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n} \leq 1+\int_1^n \frac{1}{x}\,dx = 1+\ln(n)$$



$$T(n) = 2T\left(\frac{n}{2}\right) + \cancel{\theta(n)}\ cn$$

$$T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{4}\right) + \frac{cn}{2}$$



$$T\left(\frac{n}{4}\right) = 2T\left(\frac{n}{8}\right) + \frac{cn}{4}$$

$\vdots$

$$T(1) = C_0$$

$$T(n) = \Theta(n \log n) \ ?$$

$\Updownarrow$

S(n)

$$\exists\, c_1, c_2, n_0,\quad c_2 n \log n \le T(n) \le c_1 n \log n, \ \forall n \ge n_0$$

$$T(n) = 2T\left(\frac{n}{2}\right) + cn$$

hypo. if $T(k) \le c_1 k \log k \quad \forall k < n$

induction: we need to show $\underline{T(n) \le c_1 n \log n}$

$$T(n) = 2T\left(\frac{n}{2}\right) + cn \qquad \frac{n}{2} < n$$

$$\le \cancel{2}\, c_1 \frac{n}{\cancel{2}} \log\frac{n}{2} + cn$$

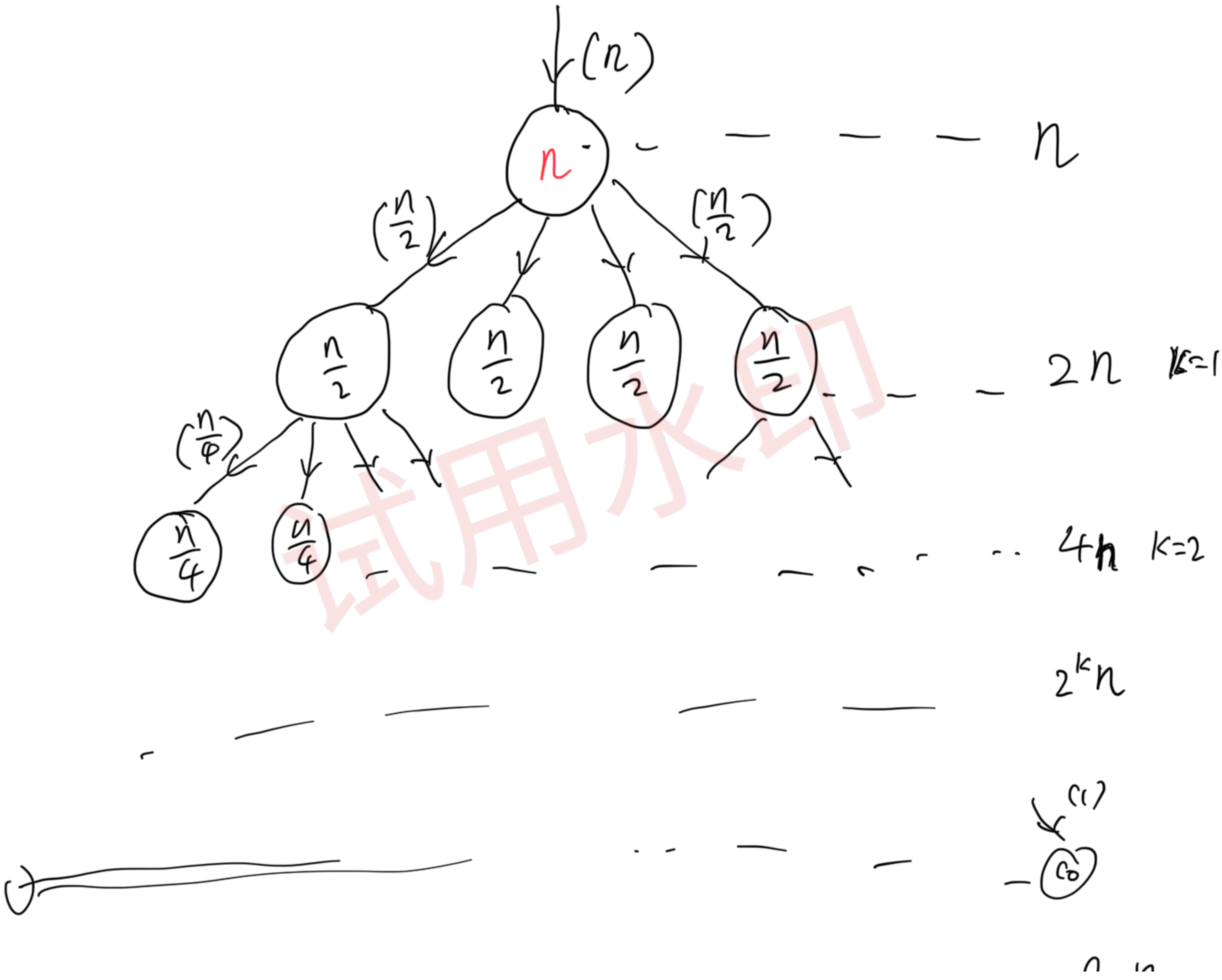
$$= c_1 n(\log n - \log 2) + cn$$

$$= \cancel{c_1 n \log n} + n(c - c_1 \log 2) \le \cancel{c_1 n \log n}$$

$$n(c - c_1 \log 2) \le 0$$

$$c_1 \ge \frac{c}{\log 2}$$

$$T(n) = 4T\left(\frac{n}{2}\right) + n$$


(n)
n
(n/2)
(n/2)
n/2
n/2
n/2
n/2
(n/4)
n/4
n/4
n
2n k=1
4n k=2
2^k n
(1)

$\log n$

$2^{(\log_2 n)} \cdot n = c_0 n^2$

$n + 2n + 4n + \cdots n^2$

$n(1+2+4 \cdots n)$

$\frac{2n-1}{2-1} = 2n-1$

$\theta(n^2)$

$c_2 n^2 \le T(n) \le c_1 n^2, \quad \forall n \ge n_0$ $S(n)$

$T(n) = 4T(\frac{n}{2}) + n$

$T(k) \le c_1 k^2, \quad \forall k < n$ $\frac{n}{2} < n$

$T(n) \le 4 \cdot c_1 (\frac{n}{2})^2 + n$

$$= \quad c_1 n^2 + n \quad \le c_1 n$$

$$T(n) = \Theta(n^2) = \underline{\Theta(n^2 - n)}$$

$$T(k) \le c_1 (k^2 - k)$$

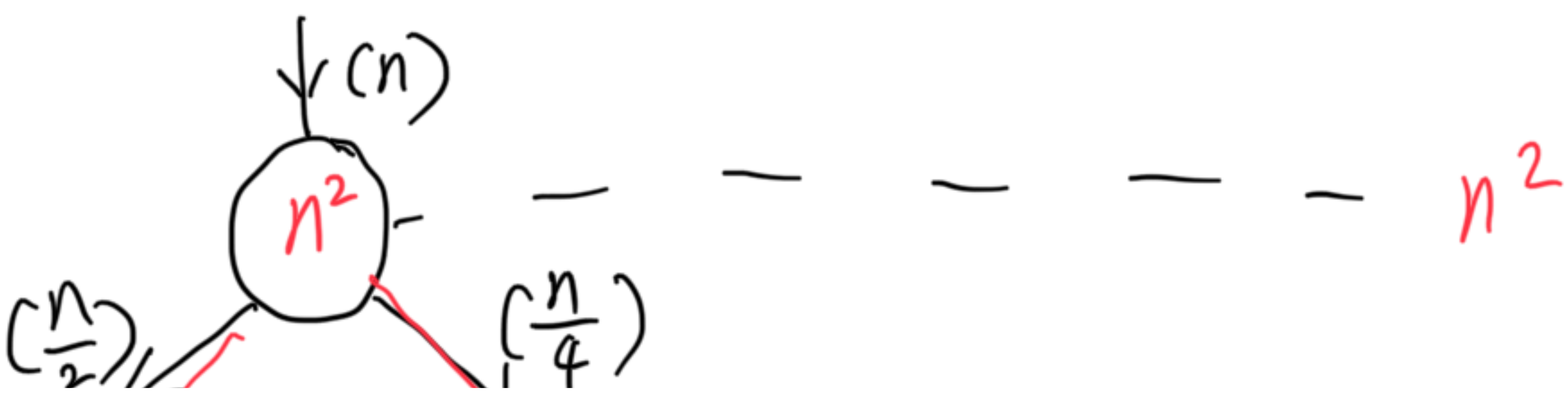
$$T(n) \le 4 c_1 \left(\frac{n^2}{4} - \frac{n}{2}\right) + n$$

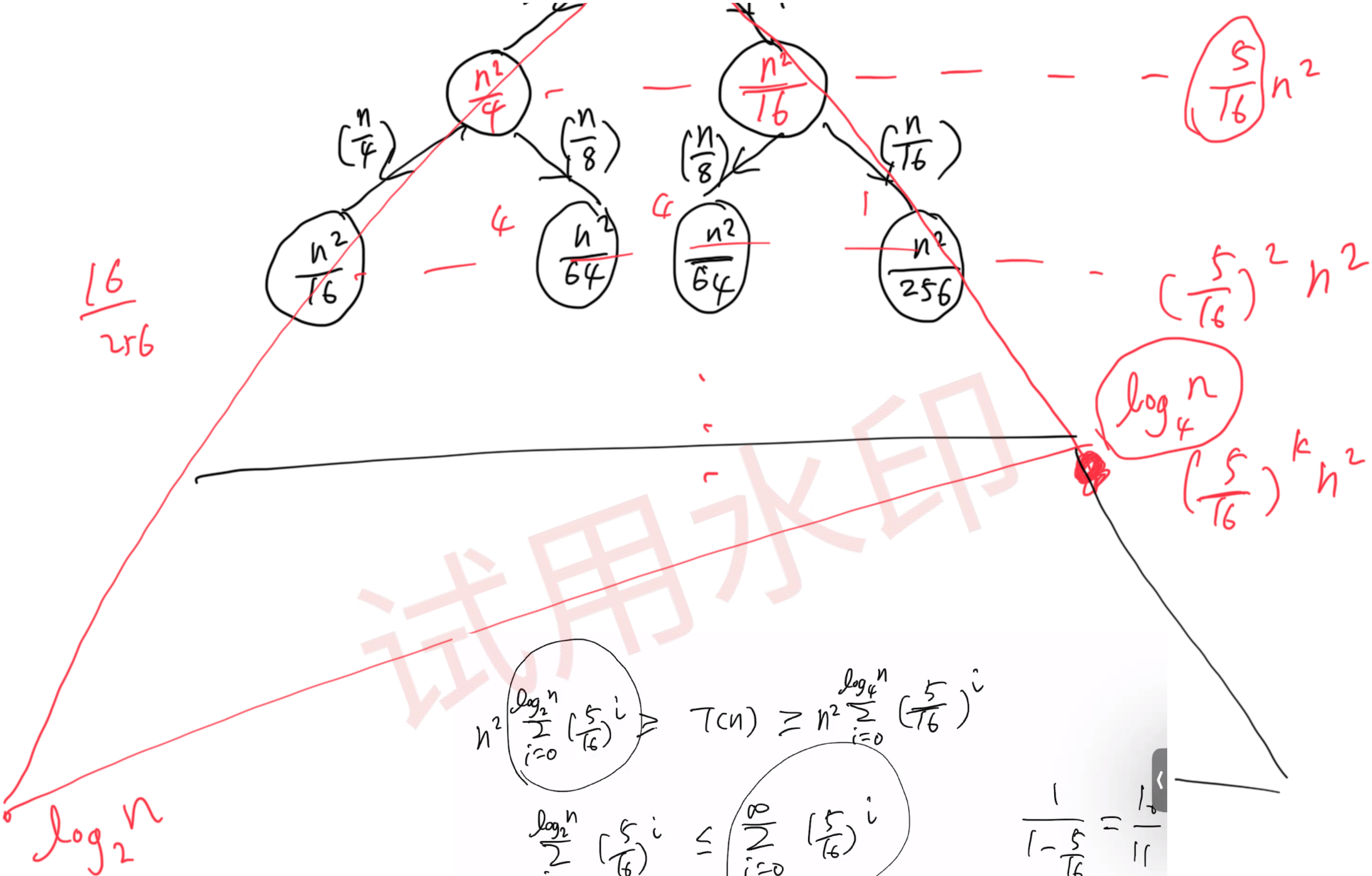
$$= \cancel{c_1 n^2} - 2c_1 n + n \quad \le \cancel{c_1 (n^2} - n)$$

$$-2c_1 n + n \le -c_1 n$$

$$c_1 \ge 1$$

$$T(n) = T\left(\frac{n}{2}\right) + T\left(\frac{n}{4}\right) + n^2$$





$$n^2 \sum_{i=0}^{\log_2 n} \left(\frac{5}{16}\right)^i \geq T(n) \geq n^2 \sum_{i=0}^{\log_4 n} \left(\frac{5}{16}\right)^i$$

$$\sum_{i=0}^{\log_2 n} \left(\frac{5}{16}\right)^i \leq \sum_{i=0}^{\infty} \left(\frac{5}{16}\right)^i$$

$$\frac{1}{1-\frac{5}{16}} = \frac{16}{11}$$

$$c\, n^2 \geq T(n) \geq n^2$$

$$T(n) = \Theta(n^2)$$