EL9343 Homework 2

Due: Feb. 6th 5:00 p.m.

1. First use the iteration method to solve the recurrence, draw the recursion tree to analyze.

$$T(n) = T(\frac{n}{2}) + 2T(\frac{n}{8}) + n^2$$

   Then use the substitution method to verify your solution.

2. Use the substitution method to prove that,

$$T(n) = 2T(\frac{n}{2}) + cn \log n$$

   is $O(n(\log n)^2)$, where $c > 0$ is a constant. ($\log \equiv \log_2$, in this and the following questions)

3. Solve the recurrence:

$$T(n) = 2T(\sqrt{n}) + (\log \log n)^2$$

   (Hint: How to make change of variables so that you can apply Master's method?)

4. You want to solve the following three recurrence formulas:

$$\text{A: } T(n) = 5T(\frac{n}{2}) + an$$

$$\text{B: } T(n) = T(\frac{n}{3}) + bn^2$$

$$\text{C: } T(n) = 3T(\frac{n}{3}) + \frac{cn}{\log n}$$

   Can you use Master's method (the version on the lecture slides) for each of these? If yes, write down how you check the conditions and the answer. If not, briefly explain why and solve using other methods. (Hint: when n is very large, $\sum_{k=1}^{n} \frac{1}{k} \approx \log n$)