Lecture 10, Fall 2023

$LCS(X[1 \ldots N],\ Y[1 \ldots M]) \triangleq LCS(N, M)$

$LCS(X[1 \ldots i],\ Y[1 \ldots j]) \triangleq LCS(i, j)$

$i \le N \qquad j \le M$

$$|LCS(N, M)| \ge |LCS(i, j)|$$

$$LCS(N, M) \stackrel{?}{=} \begin{cases} LCS(N-1, M) & \checkmark \\ LCS(N-1, M-1) & \times \\ LCS(N, M-1) & \checkmark \end{cases}$$

$A[i, j]$ as the set of common subsequence

... sequences
between $X[1,...i]$ and $Y[1..j]$

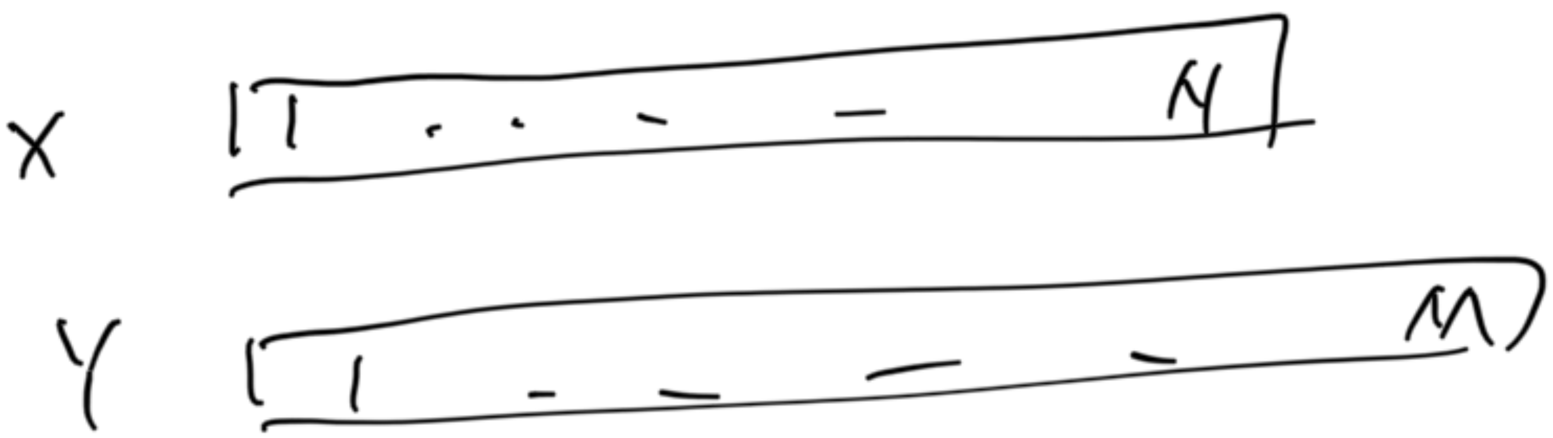
$$A[i_1, j_1] \subseteq A[i, j] \quad \text{if } i_1 \le i,\ j_1 \le j$$

$$A[N, M]$$

$$A[N-1, M] \subseteq A[N, M]$$

$$A[N, M-1] \subseteq A[N, M]$$

$$A[N, M] \neq A[N-1, M] \cup A[N, M-1]$$



$$CS \in A[N, M], \quad CS \notin A[N-1, M], \quad CS \notin A[N, M-1]$$

$\underline{CS}$ is a subseq of $Y[1..M]$

not a subseq of $X[1..N-1]$

$X[1,..N]$

$CS(k) = X[N]$

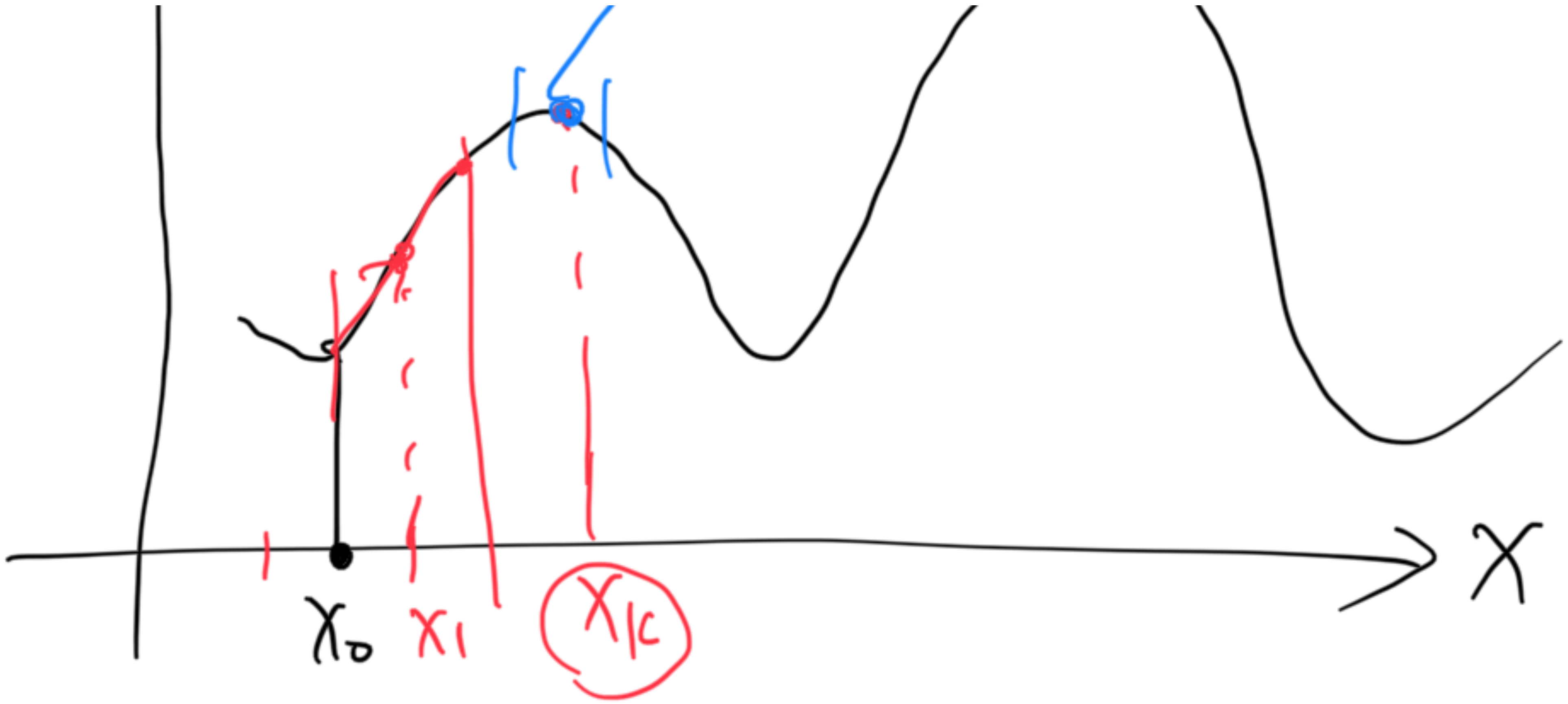
$\|$

$CS(k) = Y[M]$

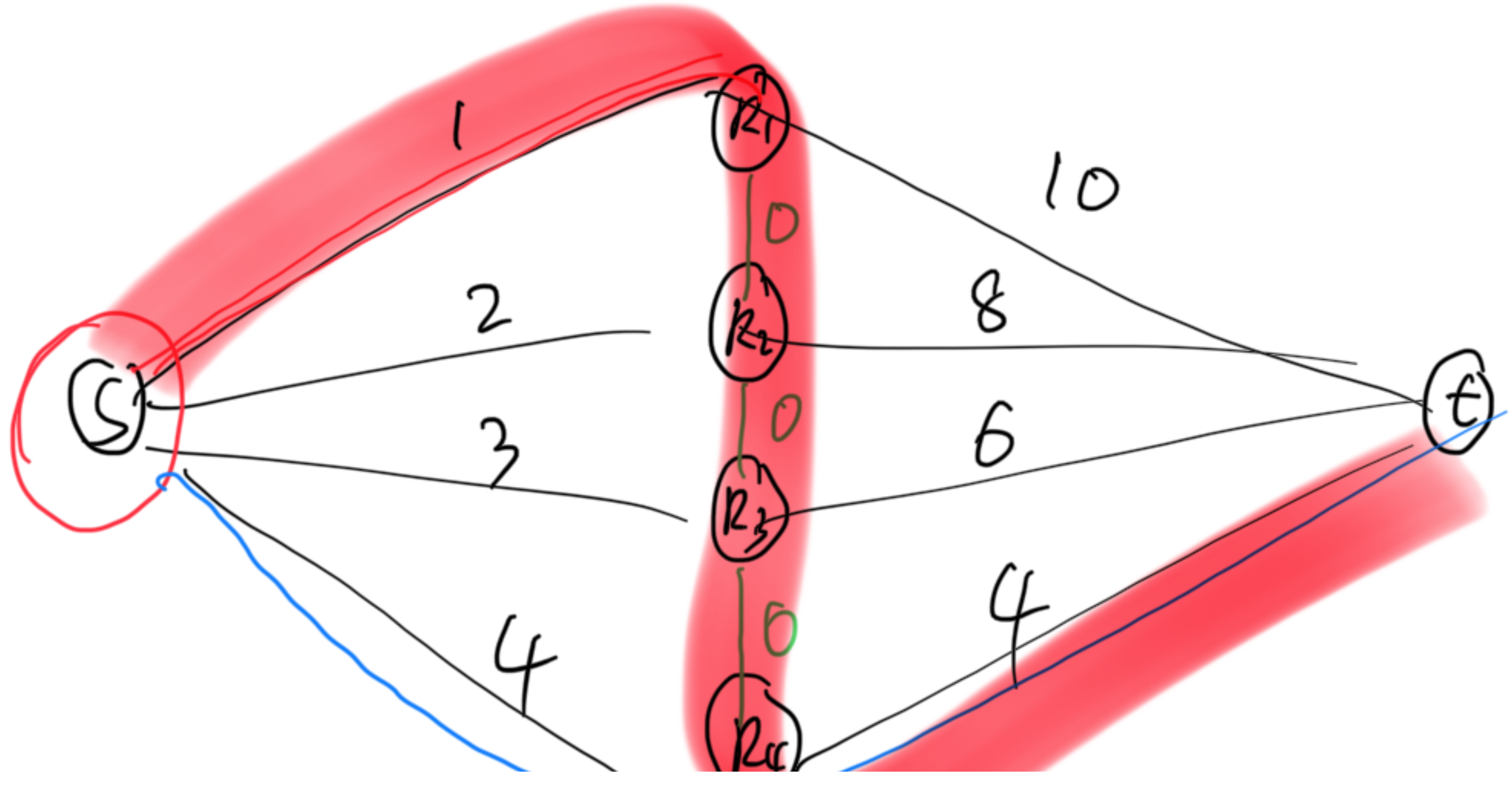
Structure

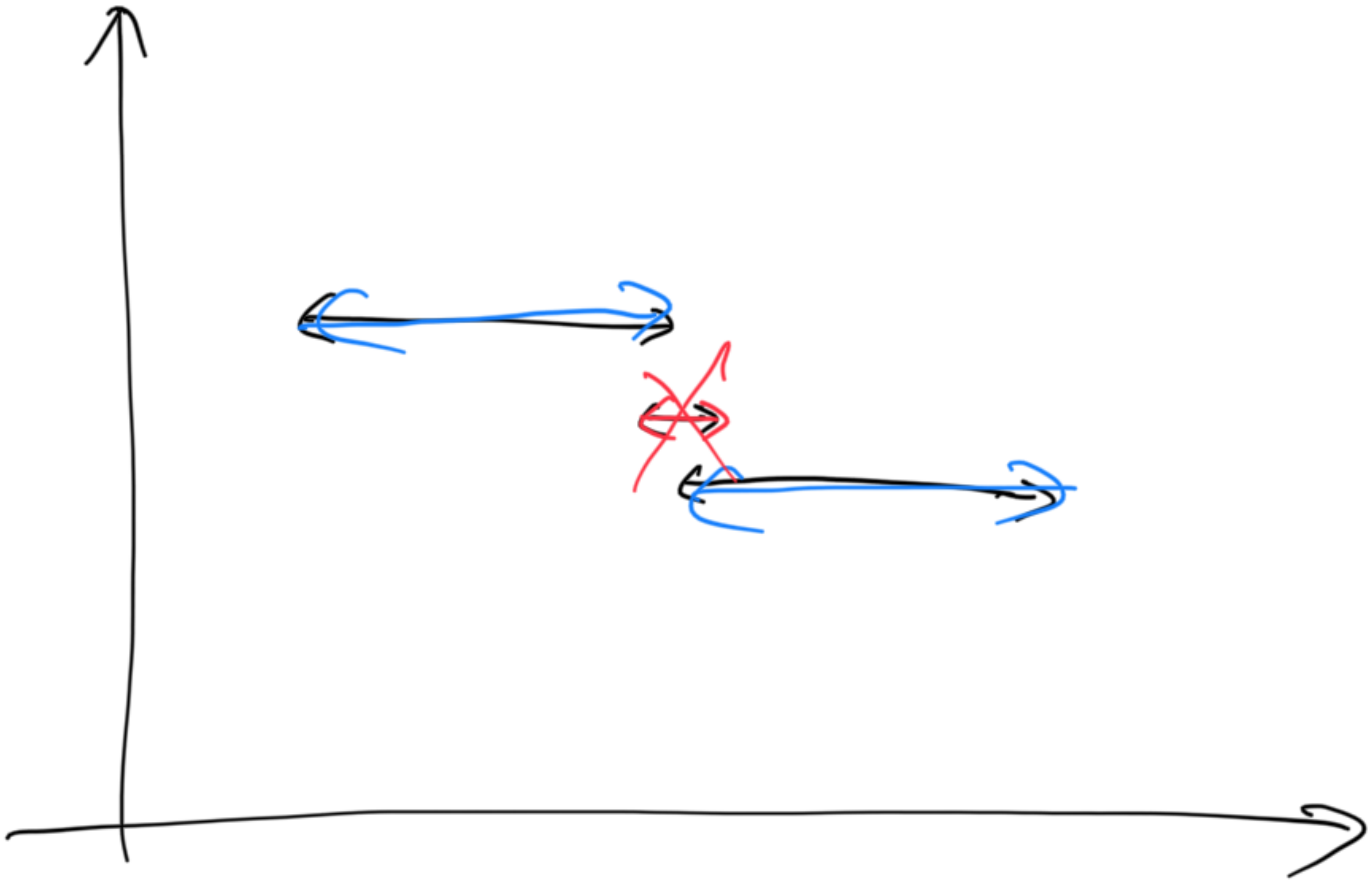
$$A[N,M] = \begin{cases} A[N-1,M] \cup A[N,M-1] & \text{if } X[N] \neq Y[M] \\ A[N-1,M] \cup A[N,M-1] \cup [A[N-1,M-1] + X(N)] & \text{if } X(N) = Y[M] \end{cases}$$

hill climbing

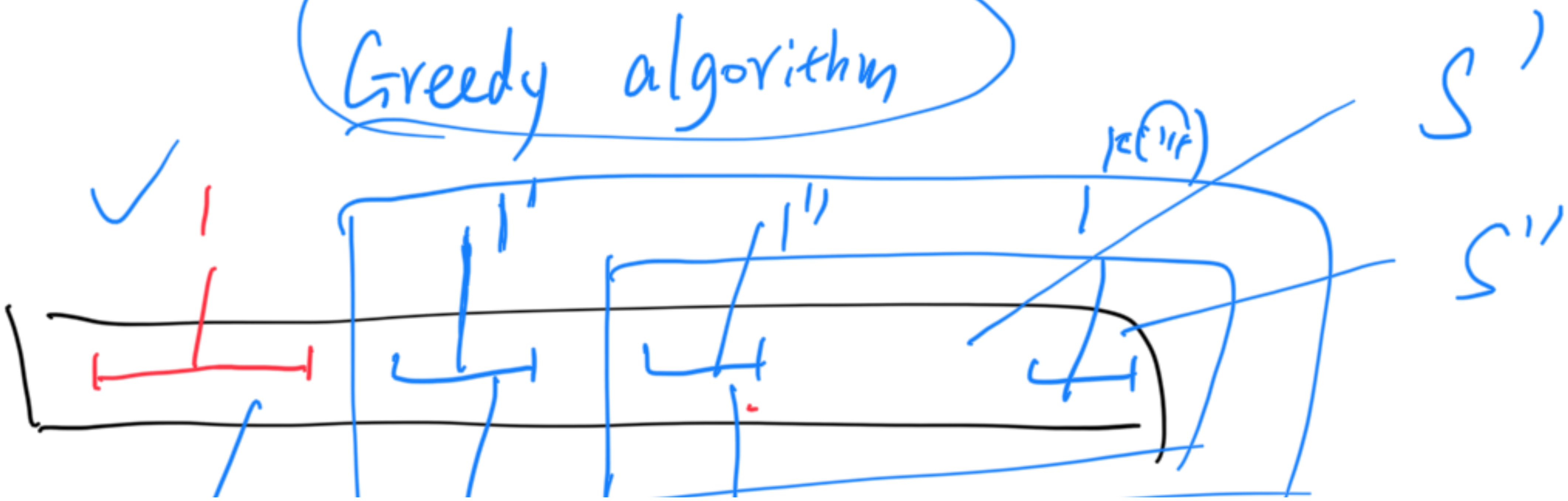


Local optimum vs. global optimum





Greedy algorithm



A $\quad A'$ $\quad A''$

$W, \quad 1 \ldots n$ —— S

$W - w_j, \quad 1 \ldots j-1, j+1, \ldots n$ —— $S' = S - \{j\}$

× S' ~~is~~ not optimal

S'' is optimal

$S'' + \{j\}$ will be a better packing than S

$\text{weight}(S'') \leq W - w_j$

$$\text{weight}(S'' + \{j\}) \le W$$

$$\text{value}(S'' + \{j\}) = \underline{\text{value}(S'')} + \text{value}(j)$$

$$> \text{value}(S') + v_j$$

$$= \cancel{\text{value}(S)}$$