More information can be found here.

The general expression of a PID is :

$$C(p) = G + \frac{1}{Ti*p} + Td*p$$

with :

- G the proportional gain
- Ti the integral time constant
- Td the derivative time constant

The digital version can be derived by changing the complex variable p by another compex variable z^{-1} with $p = \frac{1-z^{-1}}{Ts}$ with Ts the sampling period.

This give : *

$$C(z) = G + \frac{1}{Ti*\frac{1-z^{-1}}{Ts}} + Td*\frac{1-z^{-1}}{Ts}$$

$$C(z) = G + \frac{Ts}{Ti*(1-z^{-1})} + Td*\frac{1-z^{-1}}{Ts}$$

$$C(z) = \frac{G*(1-z^{-1})}{(1-z^{-1})} + \frac{\frac{Ts}{Ti}}{(1-z^{-1})} +* \frac{\frac{Td}{Ts}(1-z^{-1})^2}{(1-z^{-1})}$$

$$C(z) = \frac{G*(1-z^{-1}) + \frac{Ts}{Ti} + \frac{Td}{Ts}(1-z^{-1})^2}{(1-z^{-1})}$$

$$C(z) = \frac{G - Gz^{-1} + \frac{Ts}{Ti} + \frac{Td}{Ts} - 2\frac{Td}{Ts}z^{-1} + \frac{Td}{Ts}z^2}{(1-z^{-1})}$$

$$C(z) = \frac{Y}{X} = \frac{G + \frac{Ts}{Ti} + \frac{Td}{Ts} - (G + 2\frac{Td}{Ts})z^{-1} + \frac{Td}{Ts}z^2}{(1-z^{-1})}$$

That finally gives :

$$Y(1-z^{-1}) = X(G + \frac{Ts}{Ti} + \frac{Td}{Ts} - (G + 2\frac{Td}{Ts})z^{-1} + \frac{Td}{Ts}z^2)$$

$$Y[0] = Y[1] + X[0]*A0 + X[1]*A1 + X[2]*A2$$

Where :

- Y[i] means $Y\,z^{-i}$
- $A0 = G + \frac{Ts}{Ti} + \frac{Td}{Ts}$
- $A1 = -(G + 2\frac{Td}{Ts})$
- $A2 = \frac{Td}{Ts}$